UNDER THE ROOF: AN INVESTIGATION OF THE INTERACTION OF RATIONAL BUILDING STRUCTURE WITH ENCLOSED SPACE

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ABSTRACT

As an architect works, his or her design grows and shifts, contracts and metamorphoses through many different shapes and configurations. Each shape and length of span imposes an order on the structure of a building; The Order of Gravity.

This thesis proposes a way of understanding structure in a way that allows it to swing through limits of shape that can then be related by a designer to the space that a design suggests, and interact with it, proposing new forms. A description of malleable structure will be invaluable to an architect, for it will allow the structural elements, and their relation to each other, to contribute to the form of a building.
A SENSE OF PLACE
"...to rise in undress maisonry upstanded (joygrantit!) a waalworth of a 
skyscraper of most eyeful hoyth entowrly, erigenating from next to nothing 
and celescalating the himals and all, 
hierarchitectitiptitoploftical, with a 
burning bush abob off its bauble top and 
with larrons o'toolers cluttering up 
and tombles a'buckets cluttering down."

J. Joyce Finnegan's Wake
STRUCTURE, (from the Latin "structura,"

1. Manner of building, constructing, or organizing
2. Something built or constructed, as a building or dam
3. The arrangement or interrelation of all the parts of a whole; manner of organization or construction as... the structure of the atom, the structure of society

The use of the term "structure" in this thesis is with multiple definitions, hopefully clarified within their contexts. While it can mean an assemblage or an organization, it will most frequently be used in reference to the load bearing elements of a building. The application of the term "structure" is no accident though, for I want to tie it to the basic organization and character of space. It can, as in a grid, be a strong organizational element, or as in a strong masonry core, be a landmark which references location, most importantly, though, it should be inherently of the spaces themselves.

What do I want to do? I want to believe in what stands up in the same way Lou Kahn believed in materials. If brick says, "I like an arch," arch says "I like a place." Kahn wrote, "In consultation with nature you will discover the order of water, the order of wind, the order of light, and the order of certain materials," one more should be included; that is the order of gravity, meaning that as a building stands it is always interacting with the space around it in a manner that is either working against or working with the pull of forces within it. Buildings that try to hide this are less for the effort. As the brick makes an arch, it makes a place, surrounds it, encloses it, focuses on it. Later Kahn refers to this order in his notebooks, "The spaces defined by the members of a structure are as important as the mem-
bers...design habits leading to this concealment of structure have no place in this implied order." The structural elements of a building will take very special forms in their attempt to bear weight and natural forces. These forms can, and should, influence and interact with the nature of usable space.

As an architect I feel that how something is built is inseparable from its development of form and design, and as a person who moves and thinks and uses buildings on a day to day basis I feel that it is important that the inherent character of any built thing be understandable. A place that I understand, whether I am directly conscious of it or not, becomes more secure, and more familiar with that security, and hence more comfortable. This understanding of a space may be called "Sense of Place," and can be organized into three categories.

If I were to blindfold you, walk you ten paces forward, ten to the right, and up a flight of stairs you could "know" within that described space where you were, and could construct a location as with Cartesian coordinates, and return without tracing the path. That is, within a limited scope, a person can actually locate himself or herself within a Euclidian space relative to former position. Call this First Person.

At some point this method becomes too complicated and we must rely on a second way to orient ourselves. If you were to give directions to a friend, you would probably describe a series of landmarks, and events, and if your friend was totally unfamiliar with the area he or she would have to return using their experience to guide them. On the first trip the directions would probably consist mostly of identifiable objects; buildings, stop lights, stairways, colored signs, etc., but on the
return trip, the "landmarks" would be qualities of space as well; experiences remembered from the first trip. The qualities could include things like size (majestic entrances or tight tunnels), emotional stress (busy intersections or enclosed dark rooms), light, sounds, or any of a million other things. If you think back, you'll probably remember sometime when you've said, "I don't know where we are, but this feels like the right direction." At this point "spatial orientation" begins to transcend objective space and acquire characteristics of an overall understanding. This "path-finding" method of orientation may be thought of as the Second Person.

The third, the Omniscient Observer can see without looking because of an understanding of how a process or system develops. It often seems that all space is a stream of movement and thought, but the difference between William Faulkner's Benjy as he acts, reacting to light, sounds, colors, and memories touched into recurrance, and James Joyce's Leopold Bloom as he navigates the streets of Dublin, is Bloom's overall understanding and acceptance of the matrix of dependent events in which he lives. The matrix includes most apparently social structure, time, and physical force. Of these the latter, gravity, "heirarchitecturally" involves us with the built environment.

As we become more familiar with a place we begin to understand the nature of it, or rather as we begin to understand the nature, organization, and character of a place we feel more comfortable in it. This includes not only navigation, but social expectations, protection and security. There are many, many factors which contribute to these feelings, and perhaps the most important is the nature of the built environment. Gaston Bachelard, discussing the security felt when safely protected from bad weather quotes
Henri Bosco's simple analysis, "When the shelter is strong, the storm is good." In the same manner, when we can understand how our abode is resisting the natural forces (essentially weather and gravity) we understand its nature, and feel more comfortable. It follows then, that the nature of a space, its shape, detail, and material, will be positively effected if it is influenced by how it is made and stands.

A visually strong, and apparent structural system can enhance the "sense of place" in a building; by making it a recognizable place, by establishing a sense of strength and security, and by creating areas of habitable space unique and identifiable from their surroundings.

Since roofs are not able to directly effect our circulation or use of a space, in the same way that the ground does, they must influence it by strong implication. Japanese teahouses are given a ceiling height based on the number of tatami in a room. Palladio, in The Four Books of Architecture suggests that the height of a ceiling be based on a mediating proportion between the length and width of a room. In The Dynamics of Architectural Form Rudolph Arnheim discusses the ratio of distance to perceived mass that is required by form to effect a place, he comes to no conclusions, but it is made apparent that (depending on shape) to have its greatest effect a form needs to be related to human size, and mediated by our distance from it. If we are able to inhabit, touch and be enclosed by a structure, and see and feel its material and joints we will find ourselves highly influenced, and in some cases restricted by it. If, though, it is positioned out of our reach we will be influenced only by how we project ourselves within it, and organize our lives by its suggested form.
A blind person living in a room in the eaves of a roof would arrange his or her furniture and daily routine compliant to the shape and slope of the eave, but a blind person living in a double height room could only be aware of the change of ceiling height and configuration through subtle clues of air movement or acoustics.

In the same way that our perception is affected by form, structure and the forms it takes should be affected by and adjust to all of our senses of perception, that is be designed to influence the use of space, and to communicate its purpose to a building's inhabitants as well as perform its own physical role.
THE DIAGRAM
Structures, when built, are certainly not malleable, but in design, when structures are not yet bound by the constraints of reality, we might ask them to take any form, and with our imaginations mold them, and shape them in any way we wish, to describe space. Space is the quantity and structure a means of definition. This is the process of design.

The Diagram that is the basis of this thesis, attempts to define some parameters for these imaginative manipulations that will synthesize a structure's physical action with its spatial form.

It is apparent that certain structures have certain definite actions, and certain definite spatial forms, but what is less obvious, and what this thesis attempts to describe, is their relationship to each other:

1) that elements with similar structural actions share related spatial characteristics.
2) that structural elements can be understood as all sharing a path of development of shape from their structural action.
3) that elemental structural action can be described on a continuous path of development between limits, that is, between Arch Action and Catenary Action.
4) that elemental spatial form can be described on a continuous path of development between limits, that is between concave (enclosing and collective) form, and convex (exclusive and dispersive) form.
5) that these paths are related, and correspondant.
6) that these structures may be further defined in two categories that are also related to, and define the sections of the paths; "Mechanisms" are structures whose forms are defined by resolution of structural action into simple action. "Assemblages"
are structures whose form is defined by a multiple use and integration of different structural actions.
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THE HYBRIDIZATION OF STRUCTURE
I GROUND RULES
HANGERS AND COLUMNS:
SIMPLE COMPRESSION AND SIMPLE TENSION

A vertical load is carried easily by either a hanger or a column, both common elements in most buildings, and the most elemental structural pieces because they bear force directly along their principle axis. In the case of the column this pushes and compresses it, and in the hanger pulls and tensions it. These situations are referred to as "Simple Compression," and "Simple Tension."
HANGERS AND COLUMNS: USABLE SPACE

Hangers and columns effect some dimension in a space by their mere presence because they define a location that is referential to any person sharing that space. At close proximity, a column is usually at least human size, probably taller, and frequently broader, one can lean on it, things can be attached to it, or bear upon it, be hung from it, or be pushed against it, locations can be defined both mentally and verbally as distances from it, but no space is enclosed by it.
Two columns define an area between them that is discrete from the surroundings, but only as thick as the columns. As a boundary, columns in a plane can imply a wall, and define a gate, but no more. Imagine two columns alone, or imagine columns in a plane associated to each other one by one, discrete as an element in a field. There is only a division of a continuous space, and no definition of that space; two infinites now, instead of one, no protection, and no security. Humans seem to need to define whole spaces, always within some larger continuum. It is both model, and reason, for which we mentally organize our world; almost as a series of concentric spheres.

Three elements, three columns are needed to define a cartesian three dimensional space. Height, width, and now depth become comprehensible quantities. Walls, or walls and columns may act similarly, in each case containment is only implied, but now there is an opportunity to make a stable structure, and a chance to support a roof.
SPANNING STRUCTURE:
PARTIAL DEFINITION

This way of describing the initial developments of spatial definition ignores the effect of the necessary, and always present, ground plane until there is some element spanning it, working in a direction parallel to it, and reciprocating its movements. The interaction between the ground and the roof is the most appropriate primal definition of enclosed space because, realistically, it is the simplest enclosure that provides shelter.

People have long associated themselves with trees as examples for buildings, and more abstractly for many systems of growth and life. There is a solid strength in a tree, battered from the elements, but still spreading its branches, sheltering below it the ground where humans can reside.

Although there is only one 'column,' security and containment are provided by this special protection from above. Carl Sagan suggests that a genetic memory from a time when our ancestors feared weather, and predators from the sky as well as the ground, makes us wish for this cover, and in fact to climb into it, arboreal ancestors apparent, to live like Calvino's Cosimo.
SPANNING STRUCTURE:
INCLINED ELEMENTS AND SUPPORT REACTIONS

First consider both a compression structure and a tension structure in the plane of this page, assuming them to be stable in any direction intersecting the page. Two elements inclined against each other, and in compression from a centered, vertical point load will transmit that load to the two supports through simple compression of the elements. A cable hung between two points, with similar loading will be in simple tension, and transfer the load to the supports through a capable connection. The ground reactions can be considered in terms of vertical and horizontal components. Increasing the rise or sag of the structure decreases the horizontal component by increasing the angle of inclination, and decreasing the rise or sag increases the horizontal reaction by decreasing the angle of incli-
nation.

In the compression structure, the horizontal reaction is termed 'thrust' and in a sense is the continuation of compression through the supports. A continuous support condition such as this, where the structure is spanning a horizontal distance, and all of the elements are acting in internal compression, is termed 'Arch Action.'

When a structure is spanning a horizontal distance, and carrying its loads wholly by internal tension, this action is termed 'Catenary Action.'

When $\theta = 45^\circ$

$$\text{Thrust} = \frac{P}{2}$$

When $\text{Rise}_2 = \text{Rise}_1/2$

$$\text{Thrust}_2 = \text{Thrust} \times 2$$

Thrust is inversely proportional to Rise
SPANNING STRUCTURE:
INCLINED ELEMENTS AS SPATIAL ENCLOSURE

Three columns provide a stable support for a roof, and suggest a completed enclosure, but the simplest complete spatial enclosure that is also self-supporting is composed of three elements inclined against each other, and acting as both support and roof. A section of this would be equivalent to the two inclined elements, the arch, that begins the structural discussion.
Throughout history great importance has always been put on the image of roofs. From the exterior they complete the image of a building, complete the facade, and take the final step in the transition from the ground to the sky. The domestic image of a temple, and the pompous image of a corporate tower are each linked to specific building types, and each associated with certain roof forms.

Strong roof forms are connected in our minds to habitable places, for most strong roofs are of a type that collect or carve through great chunks of space.

The characteristics of roofs exemplify the essence of shelter. The most primitive buildings were essentially all roof; this 'essence of shelter' is a protection and containment, and even if we watch a building from outside, we expect a roof that encloses space to be habitable, and interpret it by imaginatively placing ourselves within.
DESIGN CONNOTATIONS I:
THE HABITABLE ROOF

One should be aware of these innate desires for protection, and make use of them. Beautiful structures are often, in many senses, wasted because one is kept separated from them.

At some places in a building, the habitable areas should get high enough to become involved with the roof structure, and maybe even penetrate it...
THE STRUCTURAL DIAGRAM:
THE FUNICULAR CATENARY

After the hanger, the catenary is the simplest possible tension form of structure, and in many ways may be considered simpler than the arch. Most materials are much stronger in tension than compression so that, for a stress imposed by equivalent loading conditions, a tensile element need use a much smaller cross-section of material than a compression element to maintain its form. This natural slenderness allows a flexibility in the cable such that it may conform its shape to any loading condition and maintain an equilibrium of forces by assuring that its single element remains in simple tension.
The 'funicular' shape of a cable structure under a given loading condition may be inverted to form an equivalent arch carrying the same load, and maintaining simple compression. The word 'funicular' derives from the Latin for 'rope', but in common usage has come to refer to any structure which derives its form from its loading. The arch described, then, may be referred to as a 'funicular arch.' The architect Antonio Gaudi used catenaries as a means of determining the shape of arches and vaults in his buildings. After determining the forces created by the loads in his schematic designs, he would hang cables of appropriate
length, and from them hang weights to model the loading conditions. The resulting catenaries were then photographed, flipped bottom to top, and sketched over to develop the funicular arches for which his buildings are so famous.

What is worth understanding is that under similar loading conditions the shapes taken by structures in simple compression are directly analogous to shapes taken by structures in simple tension.
The "simplest spatial enclosure" is a roof come to ground, but a simple triangulated space leaves little comfortable room. Where the inclined planes meet the ground at an acute angle few things fit, it is hard to sit here, and a standing person is kept away from the edge of the building. The obvious tendency is to make walls to a building that stand vertical, or close to it, in the same way people stand. One will find this type of definition made independent of the roof, in the ground, or integral with the roof in a structure that stands above the ground. In both cases a more complicated space is taking form, and in the latter, a more complicated structural system.
DESIGN CONNOTATIONS II:
THE ROOF COME TO GROUND

...And in some places the roof structure should come to ground, like a tree dipping its branches, so that there may be a meeting of the high and the low, and so that it may be seen and touched from out as well as in.

An interesting transition can be made between the roof and the ground by developing a column which is not wholly column, nor wholly roof structure. Both Riccardo Morandi in his "Pavillion d' Automobile" in Turin and Antonio Gaudi in the Coloni Guell chapel have used inclined columns which very visually transmit the thrust from the span of the slightly vaulted roofs to the ground. These inclined columns are special in another way as well. Besides expressing a strength and action in the structure, they differentiate between one side and another. It is quite different to stand within the span, and enclosed by the vault rather than behind it. Each has a feeling of protection, but behind the column, in line with the thrust, is an active space, and the feeling is one of an aggressive defiance of force.

Where roof structure is inhabited is a special place in a building, and anything that happens there will take on, and should be expected to assume a function that is related to the strength of the structure. Henri Bosco's statement, "When the shelter is strong the storm is good," at these places, seems applicable, for at these places the strength of the shelter will be felt.
CONSERVATION OF FORM:
STRAIN & GEOMETRIC DEFORMATION

A structure in a condition of static equilibrium will deform in two ways under variations in loading conditions; by deformation of elements of the structure under stress from the imposed forces, called Strain, or by a change in configuration to conform to more efficient lines of force, called Geometric Deformation.
CONSERVATION OF FORM:
ELASTIC AND GEOMETRIC INSTABILITY

Loads can vary in two ways to cause deformation of a structure.

1.) The magnitude of the load can be increased or decreased without changing any other characteristics of the loading condition. This variation of force always results in a change to the existing deformation of a structure, but will not produce failure and collapse of the structure until either the yield point of the material is reached, or a condition of elastic instability arises. Elastic Instability is the tendency of an element in apparent simple compression to buckle due to bending moments that are a result of inconsistencies within the material of the element. A tensile force tends to straighten an element, but in compression, non-uniformity of material can cause one part to compress more than another, and throw a load off center so that a moment creates large bending (tension and compression) stresses in the extreme fibers.
2.) The configuration of a load can be varied without varying the total load. This variation in configuration will usually result in small changes in the shape of deformation of a structure, but if the structure must change its overall general shape to re-establish an equilibrium of forces it may be considered in a condition of Geometric Instability.

The catenary, because of its flexibility always describes the most efficient lines of force to its supports. Where each element of the catenary meets its "ground," the force is transmitted through a capable connection, and there will be a tensile reaction. Lateral loading will increase the reaction on the same side as the loading until it fails in tension or the opposite reaction decreases to zero.
Consider again the arch and catenary previously discussed. In contrast to the flexibility allowed by a material's strength in tension, the rigidity necessitated by its elastic instability in compression demands a large enough cross-section to counteract any bending moment caused by the element's tendency to buckle. This dimension will always contain within it the possible lines of forces that describe funicular arches of some varying loading conditions.

Where each element of the arch meets the ground there will be a compressive reaction. Lateral loading increases the opposite reaction, but the structure will remain intact to the point of compressive failure in the far element, or to the point of tensile failure in the near element, which would topple the structure by rotating it around the far support.
DIRECTIONAL PAUSE II: FATIGUE AND RESONANT FREQUENCY

It is possible to "fatigue" a structure by repetitive application of a force which pushes it near, but not over its yield point. It is also possible to bring a structure to failure by applying even a small load in a rhythm that coincides with the resonant frequency of the structure so that each small load is essentially added to the previous one, the result being a large amount of movement and deformation of the structure. The forces are added in somewhat the same way as small pushes of a playground swing can make it swing as high as one big push.

CONSERVATION OF FORM: EQUILIBRIUM OF FORCES

Structures that are acting in simple tension or simple compression depend on their shape to equilibrate the forces generated from their loads. If flexible enough, they will change their shape to equilibrate a change in loading configuration, or if "thick" enough, will change their lines of force internally to equilibrate a change in loading configuration. If neither is possible, the structure will collapse.

Structures that are rigid and act in bending, will equilibrate the forces...
generated from their loads internally, and hence their stability is independent of their overall shape. Simply supported beams may take on a wide variety of shapes without changing the diagram of bending moments within them.
II ASSEMBLAGES
D'Arcy Thompson describes the assemblage of posts and walls with beams and frames as the most elementary way of making spatial enclosures because the vertical elements are both linear and analogous to trees and human stance, and because the horizontal elements are a natural addition of linear elements and are referential to the ground plane. Historically, inclined elements were used to build first, because of their structural simplicity, and because of the need in most climates to shed water from rain or snow. The next step, as has been noted, was most likely the elevation of these inclined elements onto some kind of vertical walls, but in any case we may consider the rectilinear system of wall and flat roof as the middle of the spatial diagram for it is spatially and formally simpler, and adds no new direction to our predominantly vertical and horizontal world.
As in all things, there are many reasons why roofs take certain shapes and configurations, and climatic influences are always a great factor. While gabled roofs and other types of sloped roofs are certainly conducive to strong structural design, it was probably the need to shed water from thatch, or other types of gravity dependent roofing materials that originally accounted for the wide use of the form.
It has been assumed so far that a point load be considered as a general condition of structural loading. In reality, there is a continuous uniform load imposed by at least the weight of the structure, and usually by various live load conditions.

Structural elements working in pure compression, or pure tension, are more efficient than elements with a combination of forces (bending), because the stress is distributed uniformly through the material of the element. An element, such as a beam, in bending contains within it both catenary and arch action. The extreme lower fibers of the beam carry the largest tensile stress, and its extreme upper fibers carry the largest compressive stress. It can be seen in a cross-section that there is a transition through the beam of these stresses, from compression to tension, and that there must then be a "neutral axis"
where the fibers are under no stress, and a relatively large zone surrounding this where the stresses are small. If a structure can be designed so that all of the stresses are being distributed evenly as elements acting in simple compression, or simple tension, the material is being used most efficiently. A truss is such a structure, but a beam may be designed to be more efficient by using as much material as possible in its extreme fibers. Some material is necessary to transfer horizontal shear, but this can be a relatively small amount. "I" beams, box beams and variable cross-section beams have all been designed with this intention.
The bending moment in a simply supported beam is maximum at midspan, hence the most material, at the greatest depth is needed at this point to resist the moment. This moment decreases towards the supports as the lever arm of the load over the length of the beam decreases. To resist this most efficiently, a beam would need to follow a similar progression in the shape and depth of its cross-section, though, as we've seen, the overall shape is unimportant.
DEVELOPMENT OF A SIMPLY SUPPORTED BEAM:
DERIVATION OF THE CANTILEVER

A uniformly loaded, simply supported beam of continuous cross-section is inefficient because the bending stresses only reach their maximum allowable value for the material in the middle of the span. (It is usually unfeasible to manufacture variable cross-section beams except for long-span conditions where material costs become a substantial savings). An obvious way of improving upon this inefficiency is to design a beam which carries the load in such a way as to "balance" and offset the bending moments at mid-span.

A beam of a given length, under uniform, symmetrical loading, will develop its maximum bending stresses at mid-span. Consider first one half of the beam. As it is, this half will be unstable without some force at the cut to support it (provided, of course, by the other half of the beam), or some moment around the original support to balance it. It is obvious that when the beam is extended an equal length to the outside, be-
Beyond the support, the loads are balanced and the forces are in a state of equilibrium. In this case, the length of each piece is equal to the original span, and each is carrying the same load as the original beam, so the moment and resultant fiber stresses around each support will be equal to those created in the original beam. This may then be seen as a complete system where over each support there are large moments, but no moment, and no stresses at mid-span. This situation, where the cantilevers are equal to half the original span, will be considered as the opposite limiting condition to a beam with no cantilever, for any further extension of the beam will create a further negative moment at mid-span.

A proportionate length of cantilever to span may be found somewhere between these limits that will use the material of the beam most efficiently, by creating offsetting moments. It works out that cantilevers of approximately $\frac{1}{\sqrt{8}}$ (about $1/3$) times the span create the most even distribution of moments.
DESIGN CONNOTATIONS III:
THE PHENOMENA OF CLOSURE (I)

In the early 1900's the Gestalt school of psychology began to define some of the rules of interpretation of perception that have helped explain various architectural phenomena. Among these were the "phenomena of closure" which said that an individual would tend to interpret a complicated field of events by reducing it to a more recognizable pattern, thus for example, a series of dots may be read as a line.

It is very difficult to explain logically (such as in a computer language) that a series of parallel lines may be read as a rectangle except by defining the lines in terms of their endings, and the relative positions of these endings.

It is interesting to note that while four dots may be perceived as defining a square, two parallel lines (with four endings) are difficult to see as such. The directionality of the line is stronger when it is seen as an independent element. It is improbable that one can
draw any directly applicable conclusions from the Gestalt, for everything is relative to and dependent on its context (though several theorists of the modern movement have tried to do so), but it is possible to understand given situations in terms of Gestalt interpretations. Christopher Alexander, writing in *The Responsive House*, and in *Pattern Language* continually emphasizes that one need not define a whole space with walls, but that column placement and corners may define as much containment as is needed except in the most private areas of a house. Frank Lloyd Wright, the continually proven master of space applied this form of definition in the open plan, and in the sections of his buildings. Continually throughout one finds definition of the living areas from cantilevered roofs or balconies, or changes in ceiling heights. A cantilevered roof can carve a little piece of enclosure from the outdoors without separating it. As has been noted already, there is a feeling of security in being protected from above.
DESIGN CONNOTATIONS IV: CANTILEVERS

At many places, if not most places, the roof structure should cantilever beyond its supports. This is structurally efficient, and if the closure is taken to the outside edge, frees the supports to be used within the building, or if the closure is kept within the supports, creates an area of partial enclosure, similar to an arcade, where one may be within a building without being inside it. The experience of structure soaring above one, without immediately apparent means of support, can be as exciting an experience as it can be fearful or suppressing.

For a uniformly loaded, simply supported beam of continuous cross-section, the most efficient, and longest cantilever that should be used, is no more than 1/3 of the span of the beam on each end. If any assymmetric or non-uniform loading conditions are expected these cantilevers should not exceed 1/4 of the length of the span. It should be noted that this is only a general rule, and any situations that might be questionable should be calculated for their true values.
The bending moment in the cantilever of a simply supported beam is maximum over the support, hence the most material, at the greatest depth, is needed here to resist bending. This moment decreases towards the end of the cantilever, and a beam, to act most efficiently, would follow a similar progression in the shape and depth of its cross-section. Shear is greatest over the support, as well, and decreases towards the ends where it is non-existant. No end-beam is needed to resist shear, but several cantilevered beams together will need some form of lateral bracing.
A rigidly supported beam acts similarly to a simply supported beam with double cantilevers, in that the bending moment and subsequent stresses reach their maximum values over the supports and minimum values at mid-span, as the strength of the connection increases. Very long span, rigidly connected, beams may increase their efficiency by increasing the shape and depth of their cross-section towards the supports, and hence increasing the depth and resistance in the connections, and by decreasing the amount of material as it approaches the center of the beam, consistent with the reduction of the bending moment. This reduction of material reduces the
dead load's moment at the supports. At its limit this produces a beam of continuously varying cross-section, curvilinear in elevation, with minimum depth at mid-span, and maximum depth over the supports. Unless its supports are absolutely static, the rigidly supported beam becomes part of a complete system, such as a rigid frame, transfers its forces through its connections to the other elements, and should be considered in terms of this system.
DESIGN CONNOTATIONS: UNDERSTANDING THE INDIVIDUAL ELEMENTS

By its mere presence a single spanning element, such as a beam or truss, can be definitive in the same way as a column or hanger. As these elements become more structurally "refined," their shape becomes more visually complex, and often more identifiable as an element in space. An intense "structural expressionism" of the individual element is an architecturally honest type of ornamentation, but one that is perhaps not so easily understood in its more subtle forms. How do we understand structure? Moore and Bloomer suggest that much of it is in relation to our bodies. They suggest that elements bulging slightly may possibly recall "to the human observer the feeling in his own muscles that he knows from holding a burden aloft." Continuously through our lives we lift and grasp, push and pull, and feel weight and lightness, so one can expect people to understand similar phenomena in a building's structure. Are we anthropomorphizing the elements of a building, or understanding our actions through them? Probably both, but a designer wanting to orient the inhabitant through the structure should be aware of the experience, and describe the structure, especially the joinery, in those terms. For obvious reasons, spanning elements are rarely independent of what they are supporting, and so one is infrequently aware of them, but when, at whatever times they can be made independent, great opportunities exist. As landmarks, as light gatherers, as sound baffles, or as implications of partial enclosures, or all of this and more, the parts can define a much greater whole and give rhythm and harmony to a space. As one example, consider Alvar Aalto's structure in the church at Riola. So gracefully, it does much more than support the roof. One becomes aware of the nature of the supports, of the light,
and of the form of the space, and its integration.

Structural elements should at some place in a building, be allowed to occupy some space of their own, and be able to act as three-dimensional elements that can then be integral, but not necessarily coincident with the other building elements.
One-way systems such as beams may be arranged side by side to cover an area, but such arrangements are impractical because any single load is transferred only along the length of the beam on which it is acting. Generally, the more widely and evenly stress from a load can be dispersed through a system, the better, because the stresses on each element are then proportionately reduced.

BEAM ACTION IN SPATIAL STRUCTURE:
DERIVATION OF THE MONOLITHIC PLATE AS A FLAT ROOF

Beams may be analyzed and designed individually if they are acting only in a closed system where the loads are distributed one-dimensionally within the plane of the beam, but any structure to be relevant in a three-dimensional world must be considered in terms of systems that occupy space.
A two-way bearing system is designed most commonly as a primary set of one-way elements with a perpendicular secondary set of elements whose purpose is to accept the initial load and distribute it among the primary elements. This is more efficient than a single series of beams, but the hierarchy of elements does not allow all of the structure to be used in every loading condition, since a point load on one of the secondary elements will cause it and any of the primary elements on which it bears to deflect, but will not effect any of the other secondary elements.

Two beams acting perpendicular to each other and connected at some intersection of their spans will distribute any load to all four supports. Since the two beams must deflect by the same amount at their intersection, if one beam is stiffer it will carry a proportionately
greater share of the load, whether it is stiffer because its span is shorter, it is of larger cross-section, or it is of stronger material.

In the standard case a beam grid is most efficiently designed so that under uniform loading there is an equal transfer of loads in each direction and all of the supports are carrying equal loads (in reality this may not always be possible or desirable, depending on the support conditions). A square beam grid does this naturally without any needed variation in the beams, because all of the spans are equal. Any other shape of grid will create situations with varying spans in the beams, and require extra strength of material, or depth in the longer beams so that they are of equal stiffness to the shorter beams.

A change in orientation of the grid may achieve similar aims, though, for example, in a rectangular area, the grid can
be turned at some angle to the edge so that there is variation in the spans of the beams only at the corners where the beams are shorter (and therefore stiffer). This will allow a greater consistency in construction of the beams, but more importantly, it demonstrates that there is a progression in beam direction that stiffens the entire grid.

Thus the development of the beam grid can be seen to progress two ways; through means of construction of the beams and their connections, and through the orientation of the grid. At its limit the grid is composed of an infinite number of beams infinitesimally close to each other, and acting in the most efficient direction at any point. That is, the grid becomes a monolithic plate.

At any point in a plate with uniform loading and given support conditions, there are perpendicular directions for which the stresses are either maximum or minimum, and shear is zero. These define the principal stress lines, sometimes called the isostatic lines, and are the aforementioned most efficient localized grid directions. The principal stress lines are unique for every combination of loading and support, but once understood, can be calculated, and formed into a concrete floor slab to make the extreme most efficient use of material, and a very beautiful, structurally expressive pattern.
BEAM ACTION IN SPATIAL STRUCTURE:
THE RIGIDLY CONNECTED BEAM GRID

If perpendicular beams that are part of a beam grid are rigidly connected to each other and to their supports rather than pinned, the entire system will be stiffened in the same manner that a rigidly connected beam is stiffer than a simply supported beam. The primary action in each element will be the bending directly imposed by the loading, but a secondary action similar to that in a rigid frame will be introduced through the connection and will transfer the bending of one beam into a twisting of any beam perpendicularly connected to it. This second beam's resistance to the twist will carry part of the load, and act to further distribute stress within the system.
BEAM ACTION IN SPATIAL STRUCTURE:
THE FOLDED PLATE

A "folded" plate is stronger in the direction of its fold than a flat plate of similar depth because, like the waffle slab or "T" section slab, it uses its material further from the neutral axis, and hence has better moment resisting capabilities. In this direction the plate may be analyzed as a beam with depth equal to the fold, and transversely braced. In the direction perpendicular to the fold, the structure does not gain efficiency because under loading it will tend to "flatten" itself (though it does not lose it either since a beam can take any "overall" shape). This same effect applies to long cylindrical shells, except that, depending on the support conditions, arch action may improve the strength in the transverse direction.
A series of parallel lines may be read as a rectangle, and a series of coincident surfaces such as the bottom of beams may be read as a plane. The elements share a common characteristic which may be collectively interpreted as a simple geometry, or a simple spatial element. This latter phenomena is part of our everyday experience; pickets in a fence, bollards lining a walk, trellis work, or the mullions in a greenhouse define boundaries and planes which are visually permeable, but just as real. A series or grid of beams may be understood as a flat surface, but remain permeable, and become texturous and thick. Smooth, hard, flat ceilings only minimally control light or sound, but a ceiling of deep beams or trusses can provide the means to do so, and provide a visual depth as well. One may also be able to understand this roof as a structure with depth and strength, even where it may be impossible to see the edge.
Roof structures such as these slabs with the isostatic lines articulated are very introverted structures, for they are not as responsive to human use as to their own internal problems. A structure which can assimilate all the structural problems, and also take a form which encloses and organizes space and light offers much more exciting architectural possibilities. This is not to say that a structural expressionism has any negative connotations, or rather does so only when it is allowed to take precedence over or interfere with the intended spaces and uses of a building. The structures of Pier Luigi Nervi are some of the most sculpturally beautiful pieces of architecture ever built. If only one in a thousand people who sees them is able to understand from whence they come, to that person they are of even more incredible richness and depth.
DIRECTIONAL PAUSE IV:
PERCEPTUAL SPACE

In *Body, Memory, and Architecture* Kent Bloomer and Charles Moore attempt to describe the difference between Cartesian space and Perceptual space by describing the way in which we relate the world to our body to measure, interpret, order, and understand it. Perceptual space is very complex, more complex than can be described in terms of our body relationships because it involves our perceptual memories; interpretation of the experience, and interpretation of the memory. The body relationship is an object oriented interpretation, and our space can not always be described by physical material, much less defined by physical objects. The important distinction that Bloomer and Moore make is that the Cartesian space does not need a "centerplace" to describe it (this is what is so miraculous about it) but that "Architectural" space is "experiential" space, is "perceptual" space, is "personal" space, and hence is centered, and should be described in terms of the individual.
THE SPATIAL DIAGRAM:
IN A RECTILINEAR SYSTEM, THE PLANE AS ROOF

There seem to be only two attitudes that the individual's perception of a planar element (such as a flat roof!) can take.

An attitude of movement and position: In a space where there is a single very directional plane, one can only move in three ways relative to it: (1) towards it, (2) parallel to its direction and within an associational range, that is within its influence, or (3) away from it in some direction that is irrelevant to its direction. Beyond the plane's associational range of influence one feels lost, adrift in an amorphous, limitless space. Columns or walls or any other type of enclosure are not needed to define the space, for boundaries as strong as any of these are evolved in our perception. It is not a boundary that is directly measurable in any way, but as surely as a swimmer lost to the sea feels the absence of land, one knows when he/she has moved away.

If this is a plane that is wide enough to be similar in width and length, then we discover three zones of position: The first is any place that is out of the range of the influence of the plane, this is a non-directional area. The second is in a zone that is within
range of influence of the edge, and is parallel to it. The third is an area within the range of influence of the plane, but away from the edge, this too is non-directional, though it is not limitless, since it is defined by the edge.

An attitude of frontality: Moving towards the plane is limited, and becomes a confrontation. One can either face towards the plane, at an angle incident to its surface, or away from it. At any angle other than perpendicular, visual movement is experienced. The perpendicularity is singular and static, and in this case irrelevant.

We can conclude that there are two inherent characteristics important in a flat roof; the surface and the edge, and two defining attitudes; parallel, a direction of movement, and perpendicular, a static direction.
DESIGN CONNOTITIONS VIII:

FLAT ROOFS

In spite of many obvious problems, such as their difficulty in shedding water, flat roofs continue to be built, even in the worst climates. Their advantages include producing an outdoor surface which can and should be used, if sometimes only for access to mechanical equipment, and allowing use of building systems which do not always take as easily to construction of inclined surfaces. In this country, it seems, flat roofs are used frequently on public buildings to avoid a domestic imagery. For whatever reasons, if one chooses to use flat roofs, full advantage should be taken of their properties.

As a surface, the flat roof creates a non-directional, and uncentered area. This may not be a form which suggests creative use of space, but it is one that is non-interfering, and can be mediating in an environment that is pushing the limits of complexity. In fact, at times the flat roof can even have collective properties by providing a single surface to which varying and intense movements can react. Because the surface is non-definitive, full use must be made of the supports and ground. (Trusses may support for flat roofs as well as columns).

The edge of a flat roof is its only definitive element, and should be visible, and accessible at some places in the building. Frank Lloyd Wright described the way in which a hole in a wall could make it more of a wall by displaying its thickness, and showing what was being walled out. In the same way the edge turns the roof from a ceiling surface into a structural three-dimensional entity, which begins, ends, and occupies space.
DIRECTIONAL PAUSE:
RECTILINEAR ENCLOSURE

To understand the difference between concave roofs and flat roofs with supports, one must consider the distance that the supports are apart from each other relative to the height of the roof. The transition is ill defined and subjective, but one can see that an approximately cubic space as Palladio frequently used would feel more enclosing than a long, low, flat ceiling as might be used in warehouse construction. Also, the nature of the support is important; any support which transmits thrust becomes associated with the roof structure, as well as the ground, but in terms of walls and columns, columns rarely provide much enclosure (though they do command space in important ways) unless they are close enough to each other to be associated with the arms reach distance of humans. Walls effect some closure if straight (and connected to the roof) and more if they turn in some manner, but in this case, more than any other, distance of separation is important.
III MECHANISMS
DEVELOPMENT OF THE CATENARY:
DERIVATION OF CURVILINEAR FORM FROM THE LOADING CONDITION

It is possible to understand the most appropriate shape for a funicular structure under a condition of uniform loading from the following derivation of a catenary.

A cable hung between two supports with a single load in its center will hang as an equilateral triangle. When the length of the cable is $\sqrt{2}$ times the span, the interior angle at the support will be 45° and the thrust will equal the vertical reaction at the supports. Increasing the length of the cable will decrease the thrust, while decreasing the length of the cable will increase the thrust.

Two loads hung at third points along the length of the cable will form a trapezoid with the middle segment parallel to the span. When the loads are uniformly dis-
When the cable is approximately 1 1/4 times the span, the thrust will equal the vertical reaction at the support.

If the cable were to remain the same length as above, the thrust would be slightly less than the vertical reaction, that is to say that for a given length of cable, a relative distribution of loads will increase the load on the supports at a slightly faster rate than the thrust.

Three loads uniformly distributed along the length of the cable will form a polygon.

If a cable is subjected to an asymmetrical loading, the cable changes its shape, and adapts itself to carrying the load by means of straight sides with different inclinations. The supports develop different vertical reactions, but maintain similar thrust.
As the number of loads on the cable increases, the polygon acquires an increasing number of smaller sides, and at its limit approaches a continuous curve. If the loads had been distributed evenly along the length of the span, instead of along the length of the cable, the curve would have been a parabola. As it is, there is little recognizable difference between the parabola and the true catenary.
DEVELOPMENT OF THE CATENARY:
THE SHAPE OF FUNICULAR STRUCTURES UNDER NON-UNIFORM LOADS

The effect of non-uniform loads on the shape of funicular arches or catenaries can easily be understood as a "summation" of simpler uniform loading conditions. For example, an arch under continuous loading with a concentrated point load in its center may be thought of as the addition of a structure under uniform loading to a structure under a point load. The resultant form is similar to the familiar Gothic Arch.
A single cantilevered element may be loaded perpendicular to its principle axis so that it is in bending only, and compressive and tensile stresses are equal throughout a section of the element, or it may be loaded parallel to its principle axis so that it is in either simple tension or simple compression. If the load is directed at some angle incident to the neutral axis, and toward the support, the force may be described in components parallel to the axis, that put the element in simple compression, and perpendicular to the axis, that put it in bending. If the angle of the force applied by the load is close enough to the principle axis to bring its intersection with the joint within the base of the cantilever, the compression stresses will be large enough, relative to the bending stresses to cancel the tension component of the bending. Each element has a "neutral zone" parallel to the principle axis where if the angle of the applied force intersects the base, all material of the element will be in compression.
It is apparent that the incidence of forces with material is dependent both on the angle of the applied force, and on the ratio of height to base width of the element. The "wider" an element is in relation to its height, the more certain it is that an applied force will fall within the material of the element, that is the more certain it will be that the element will be acting in compression.

The loading on an arch must keep the lines of force within this "neutral zone" to assure that it is acting in simple compression.
DEVELOPMENT OF THE RIGID FRAME:
DERIVATION OF CURVILINEAR FORM FROM A STIFFENING OF THE ELEMENTS

The elements of the rigid frame, acting in both compression and bending, can be analyzed in pieces equivalent to the cantilever also under both compressive and bending stresses, as it moves between its limits; pure bending, and simple compression.

A rigid frame may be designed to enclose a certain area. In its simplest case it is built of three elements; posts and a beam. Considered in elevation, and under uniform loading, it can be seen that if the beam is simply supported it will deflect in continuous bending. If the ends are rigidly connected to the posts, the frame may be considered as a complete system. The
beam will deflect notably less at the center of the span, but will transfer the moments through the rigid connection so that all three members of the frame are subject to both compression and bending. The columns are bent by rotation transferred through the joints and compressed by the loading, and the beam is bent by the loading, and compressed by the thrust from the resistance of the posts. Stiffening the columns reduces their effective length and reduces bending. This essentially stiffens the column and beam joint, and hence stiffens the beam.

In a gabled frame the top members act as two inclined beams, if they are
hinged to each other and to two rigidly supported posts, they act as struts, and their thrust imposes bending on the posts, but if they are rigidly supported, the compression and bending is transferred through each of the members, increasing the number of elements shortens each end and effectively stiffens them. To extend this progression further, one may think of the gable as being rebuilt out of twice as many shorter members, rigidly connected. If the objective is to enclose a given area, the columns need not maintain their height, that is they may be made equal in length to the other elements.

As was shown, this decrease in length will effectively stiffen the elements, and will increase the compression and
decrease the bending in each member. At its limit, a frame with an infinite number of infinitesimally small segments becomes a curvilinear arch acting in simple compression. As a frame becomes braced and strengthened, its joints become larger, and more visually apparent, and the elements lose their individuality. Finally the post, beam, braces, and joints become structurally and visually continuous, and the structure is perceived and treated as a single element.
STABILITY OF FORM:
ARCHES WITHIN ARCHES

The depth of cross-section of an arch allows it to carry a variety of loads without changing its shape because it contains within itself, in effect, many different arches. These configurations are limited, though, and extreme lateral loading will cause first bending, to whatever extent the material will allow it, and then a failure and collapse by what will appear to be a "rotation" around the points where the lines of force touch or surpass the depth of material of the arch.
THE SPATIAL DIAGRAM:
DERIVATION OF CURVILINEAR FORM

"We experience satisfaction in architecture not by aggressively seeking it, but by dwelling in it." (from Body, Memory, and Architecture)

If a single plane must make its definitions by implication, two planes may begin to actually enclose space. The observer, now inhabitant, within the concavity of the roof surfaces experiences the same relationships as to a flat roof, but with the difference that the surfaces, in relation to each other, define a direction that is new to each. Where the perpendicular attitudes were irrelevant with a single plane, here they interact to establish an orientation and organization of the space beneath.

As the interior of a roof is defined by more independent surfaces, its section approximates, and then at its limit becomes, a curve. The parallel attitudes diminish to nothing and the incident attitudes become stronger, at its limit an infinite number, all focussing to a single point. The definition of the form is at its most specific...
CATENARY ACTION IN SPATIAL STRUCTURE: DEVELOPMENT OF SYNCLASTIC FORMS

For a given stress a single cable will produce a greater horizontal reaction in its supports, and a consequently lessened vertical reaction, the less sag it is allowed relative to its span, because with a reduction in sag, the slope of the cable, and hence the line of force, approaches the horizontal. The shape of a cable under uniform loading can be approximately described by a curvature similar to the catenary (and defined by 1/radius that would determine that curvature). The sag to span ratio and the curvature are directly related, and hence, the greater the curvature (the smaller the radius) the greater the vertical reaction will be for a given stress.

Two cables acting perpendicular to each other, and connected at some intersection of their spans, will distribute any load to all four supports. They will have the same sag at their intersection, and will have equal internal stresses. If the cables are spanning unequal distances, the curvature will be greater for the shorter cable, and therefore the vertical reaction at its support will be greater than for the longer cable.
A network of perpendicular cables can be used to cover an area. In the standard case this network is most efficiently designed so that under uniform loading there is an equal transfer of loads in each direction and all of the supports are carrying loads as equal as possible (though in reality this may not be possible or desirable, depending on the support conditions). This will be the case when, for a given situation, an orientation of the perpendicular cable network is found that will allow a minimum of variation of spans in the cables, and hence, a minimum in the variation of their curvatures.

For a given uniform loading, one can easily see that the more cables used, the greater will be the distribution of stress. At its limit the network becomes a membrane and may be considered as composed of an infinite number of cables, infinitessimally close to one another, acting at each point as a set of cables with the most efficient orientation for that point. These define the principal stress lines for the membrane, and are unique for every combination of loading and support.
A cross-section at an angle intersecting the center or principal axis of a simple membrane will show a profile with some degree of curvature. If sections are taken along the principal lines it may be seen that one is of greater curvature, and therefore the line of maximum stress through that point. This direction is transferring most of the load. The perpendicular is the line of minimum stress through that point. At one limit, the curvature of one direction will be maximum, and the other direction will be zero, that is a straight line. This will occur at any point on the surface of a cylindrical membrane where the loads are normal to the surface (such as an air supported membrane), or at a point along the principal axis of an approximately cylindrical membrane weighted by gravity. At the other limit each direction of principal stress is equal, and the curvatures are all equal. This occurs only in a spherical membrane. The limits of form in membranes are singly curved (approximately cylindrical) at one end, and synclastic (ideally a sphere) at the other, of these the synclastic more evenly transfers loads. For a given strength of membrane the greater the curvature, and the less variation in curvature there is, the greater total load that it can carry.
The stresses in a membrane can be considered to be totally tensile, and distributed uniformly across the thickness of the material, and are hence optimal. Membranes are light and economical, but their use has been limited for three reasons.

The first is a technology of joinery and manufacture. It is of considerably greater difficulty to make joints to transmit tension that to transmit compression, especially if they must be flexible, because of the concentration of forces. In Frei Otto's West German Pavilion an intermediary set of heavy cables was used to make the transfer of forces to the supports. Also, because of the induced curvature in a membrane, the covering must be continuous along the surfaces with small slope to close against the weather, this means that most standard roofing systems are inapplicable.

The second is the inherent flexibility of membranes. As we have seen, membranes will change their geometry to equilibrate a change in loads (for example caused by wind), and while this is the root of their strength, it makes them difficult to seal, or incorporate with more static structures. Most membranes tend to be isolated structures. There are many ways that have been tried to stiffen membranes without destroying their integrity, but the most appealing are ones which work with the nature of the membrane rather than against it (such as compressive stiffeners would do.) One way stiffness may be achieved is by means of a double, anticlastic curvature. A primary set of cables is hung to carry the entire load, and then a secondary set is tensioned at right angles against it to steady. Another way that stiffness may be achieved is by pretensioning the cables, this allows the membrane to develop "compressive" stresses up to the values of the pretensioning before they will deform.
The third is spatial preference...

If you choose to believe me, good. Now I will tell how Octavia, the spider-web city, is made. There is a precipice between two steep mountains: the city is over the void, bound to the two crests with ropes and chains and catwalks. You walk on the little wooden ties, careful not to set your foot in the open spaces, or you cling to the hempen strands. Below there is nothing for hundreds and hundreds of feet: a few clouds glide past; farther down you can glimpse the chasm's bed.

This is the foundation of the city: a net which serves as passage and as support. All the rest, instead of rising up, is hung below: rope ladders, hammocks, houses made like sacks, clothes hangers, terraces like gondolas, skins of water, gas jets, spits, baskets on strings, dumb-waiters, showers, trapezes and rings for children’s games, cable cars, chandeliers, pots with trailing plants.

Suspended over the abyss, the life of Octavia’s inhabitants is less uncertain than in other cities. They know the net will last only so long.

- ITALO CALVINO
...The understanding of structure is an essential part of daily life, and always has been. People build for themselves, whether a lean-to shelter from the rain, a tent pitched against the wind, or something as simple as a ladder set against the side of a house. We become consciously and unconsciously aware of the shape that things need be made to remain stable, and the shape that they take under the force of wind and gravity. We have become accustomed to these actions and their resultant shapes, and can tell when something is about to break, or how to brace it to keep it from breaking. The shape of a ladder flexing under weight, or a plank across a stream is, from underneath, convex, and when we set a tent against the wind and brace it from within we push out, from the inside the shape is concave. Eero Saarinen wrote about his hesitancy to use a convex cable structure in the Ingalls Skating Rink ("One had worried that a hanging roof might look heavy from underneath,...the Ingalls Rink took it with sweeping lightness") but felt it successful enough to use again at the Dulles Air Terminal.

Structures can take many different shapes, concave and convex, without losing strength, but a designer must always ask himself/herself how that shape will be read by someone inside the building. If one understands that forces push from a concave side, will any convex shape be read as moving with the shape of the force, and if so, can one justifiably put people, without choice, in the way of that force?
ARCH ACTION IN SPATIAL STRUCTURE:
DEVELOPMENT OF THE CYLINDRICAL SHELL

Arches may be analyzed individually only if they are acting in a closed system (such as a wall) where the loads are distributed one-dimensionally within the plane of the arch, though to cover and enclose an area arches may be arranged side by side to make a virtual vault. The stresses from a load will disperse naturally through the material of the vault in simple compression, but no tension is developed, and hence no structural advantage is gained over an arch, and the vault must be continuously supported along its sides to absorb loads and thrust.

The simplest two-way bearing system is one of primary arches with purlins or sheathing acting as a secondary system to distribute the loads. While a vault requires continuous support, a two-way system allows spans perpendicular to the
arches to the capacity of the secondary system. This is usually a fairly minimal amount, but demonstrates that a developed secondary structure increases the flexibility of the whole system.

If the arches are "bound" together with a material that can work in tension, the structure will act both similar to a beam and a membrane. This cylindrical shell is a continuous structure that acts in tension or compression through its thickness, but not bending, and when used in a stable form has great structural advantages over the vault because it has the capacity to span a horizontal distance.

This cylindrical shell is simple, and similar to the folded plate because each improves upon the natural spanning ability of its material by taking a configuration that uses the material further from the neutral axis, and hence with better moment resisting capabilities. Like a beam then, the shell will develop compressive stresses along its upper surface, and tensile stresses towards the bottom. Shear is maximum over the supports, and usually requires extra reinforcing (if the shell is reinforced concrete). If the shell is cantilevered no end beam is needed, but if it is supported in its end plane it should be done so continuously by a rigid frame.
ARCH ACTION IN SPATIAL STRUCTURES: DEVELOPMENT OF THE DOME

Up to this point, for simplicity of description, support reactions have been considered in a plane similar with the structural element under discussion. One can see that this does not have to be true, and that a three dimensional stability can be achieved. An inclined element acting in simple compression needs to be carried vertically by a support, but its thrust can be split between two diagonal elements,
which can also carry any additional transverse loading. A direct application of this is seen in the primary members of a hip roof.

A hip roof is an inherently stable structure because the forces at each "corner" balance the opposite corners. The hip roof is symmetrical around two axes, but this may be progressed to an infinite number of axes. At one limit the structure may have no fewer than three corners to remain stable, but at the other limit there may be an infinite number. By mentally rotating the described element around an interior center one can visualize how cones and shallow domes are inherently stable.

Arches are stable for a limited set of loading conditions, but a dome may be considered as stable for any set of sym-
metrical loads because a dome is laterally supported by these diagonal reactions, referred to as parallels.

A shallow dome under uniform loading does not develop measurable tension in the parallels, though, and may be considered as a series of meridional arches. In the shallow dome the mechanical advantage of a vertical load (imagine a wedge being driven into a log) is enough
to cause a deflection of the meridians inward. This slight motion causes a shortening of the parallels that, in effect, stresses them in compression against the natural thrust of the dome. Below a parallel of approximately 52° in a spherical dome the verticle load is directed within the depth of the material, and the mechanical advantage is lost. The natural thrust then must be resisted by tension in the parallels.
DESIGN CONNOTATIONS X:
SHELLS

Thin shells are probably the most structurally advantageous means of building, for they maintain most of the benefits of membranes, and manage to avoid their disadvantages; mobility and the inability to resolve compressive stresses. Shells are incredibly versatile in their ability to conform to almost any shape or configuration, their only requirement being the use of a geometrically stable form. Many structures have been built of flowing sculptural beauty, with smooth clean surfaces, but for the designer there is danger in this beauty. Far too often the sinuous forms are allowed to exclude participation by other structures, and even the inhabitants of the building. Because of their mobility membranes are difficult to connect to structurally. Shells shouldn't be, and yet time after time one finds beautiful forms dropped onto a space like a big hat in no way in-
tegrated with the forms and materials used for closure (very large shells sometimes need to be allowed room for thermal expansion). As has been noted before, the structure of a building is too important to not use in as many ways as possible; as light gatherers, and spatial definers, if it isn't one can only consider a building a partial success.
THE SPATIAL DIAGRAM:
DERIVATION OF SYNCLASTIC FORM

In space these enclosing forms are sections of gables, hips, vaults, and domes, depending on how the section is used. If another section is taken incident to the first, one can see that similar parallel and perpendicular attitudes describe the character of the form, acting in conjunction with the definitions of the other sections. A plane, cut in perpendicular cross-sections, shows dominant parallel attitudes along both sections, but as one section becomes facetted, and then curved, the definition changes. Movement, the parallel attitude across the curved section, becomes subordinate to the intersecting perpendiculars, but in the long section remains dominant. These forms, gables and cylindrical vaults, have strong directional properties. As the second section becomes facetted, and then curved, the perpendicular attitude dominates in both, and the forms become those of domes and hips, very centroidal, focusing, and collective; indications of centers, pauses, intersections, or endings, but rarely a continuous path.

The perpendicular attitude to a convex surface shows an opposing limit to the completely concave curvilinear form. The perpendiculars are neither intersecting and focussing, nor parallel and irrelevant to each other as in the plane. They disperse, and will tend to separate and polarize activities be-
low them, on either or both axis. Aalto has used this to an advantage in the Vogelvieidplatz Sport and Concert Center, where seating, the habitable area, is separated and pushed to the sides. Underneath the hung, convex portion is an area of activity, the stage.
A common analysis of the relation of spatial objects is the projection of their profiles onto a graphically dissimilar field; that is, a figure-ground analysis. It is an attempt to clarify the interactions and affinities of some set of similar objects by simplifying their description and context. The nature of the study describes a planar element, and can be very necessary where the original context does involve strong planar organization, such as a site plan, but less so where an organization is as three-dimensional as high-rise development in a dense urban context.

In describing the effect of three-dimensional form, a sectional projection is useful as a diagram, but still inadequately describes the depth dimension, for the implication of the diagram is also that of an extruded form. The true effect can only be imagined in what essentially would be a spatial figure-ground, and might be termed a "solid-void" analysis, or as Rudolph Arnheim refers to it, "solids and hollows."

In this solid-void study the solid, or built part of the diagram, by its presence, becomes the definitive, but not necessarily dominant element. The domination of a form depends on its shape, and its ability to be interpreted as a simple geometric form, in concordance with the Gestalt "phenomena of closure."

If, when viewed from the "void," a solid is concave, the void is taking a dominant form. It seems to penetrate the solid, and to be contained by it. An observer is able to position himself/herself such that the form dominates his/her field of vision. If the solid...
is convex, it takes both the definitive and the dominant role. It excludes an observer in space, is dispersive, and subtends a relatively smaller percent of his/her field of vision.

Considering the capacity of concavities to be occupied, and the previously discussed interpretation of convexities as force compliant form, one wonders again of the appropriateness of the latter to habitable space.
Spatially enclosing roofs are intense formal complexities, open to interpretations of imagery, construction, and spatial definition, the substance of architectural design. Whereas a flat roof provides a uniform field that form may be played against, the forms of domes, vaults, gables, hips, and hybrids of these become the definition: the figure against the ground. Onto a plan, the roof can act as a primary definition, a shared respondant, a reinforcement, or a contrast. The permutations and combinations are as endless as the interaction of symphonic melodies.

One must always understand the implications of use of form, even in conscious contradiction. Too often what's overhead is forgotten by the architect who works primarily in plan, and results are left to chance or a noncommittal hung ceiling.
IV ASSEMBLAGES
THE STRUCTURAL DIAGRAM:  
DEFINITION OF THE TRUSS

A "Frame" is the most general term for an assembly of interconnected linear and planar structural elements. Frames are usually composed of post and beam like elements with rigid or braced joints rather than full triangulation. This implies that at least some parts of some elements are subjected to bending.

The term "Truss" describes a usually triangulated assembly of structural elements designed to span a horizontal distance, and supported in any of a number of ways, but not dependent on its supports for stability. A truss may be an element as part of a frame, but in its true sense is analyzed as working only in tension and compression, and remains stable with only pin joints. The reality of non-uniform loading will often create bending in the primary members of a truss, but at no time should there be bending in any of the secondary members.

By this definition what is commonly called a Vierendeel is not a truss, but a rigid frame, since its stability depends not on triangulation, but on rigid joints, and bending is found within the primary and secondary members.
DEVELOPMENT OF THE TRUSS:
THE ELEMENTAL TRUSS

The simplest structure which spans a horizontal distance and encloses space is that arch which was first discussed, assuming that there is ground beneath it which is both habitable, and able to carry the thrust of the members. If a tension member, called a tie-rod, is connected between the two compression members, the stability of the structure becomes independent of its supports. In the same manner a compression member may be placed between the tension members of the catenary. In both of these cases, the supports are acting only to carry vertical loads. These are the two simplest examples of a truss. Trusses capable of spanning long distances by means of tension and compression elements only, may be built by combining a series of such triangulated forms.

The development of the truss can be
seen to progress along two lines; a structural efficiency, and a formal adaptability. A true truss is the center of the developmental structural diagram. Structurally it can be considered as a beam with its forces externalized, or as a beam with all inefficient material removed, and as an assemblage of arches and catenaries, just as a beam contains within it both arch action and catenary action.
DEVELOPMENT OF THE TRUSS:
DERIVATION OF CURVILINEAR ELEMENTS AS PRIMARY MEMBERS OF A TRUSS

Again it has been assumed that structure may be analyzed in terms of point loads. Under continuous loading that part of the truss directly carrying the loading would most efficiently do so by adapting its form to the load configuration. The way these primary elements work can be compared to the development of curvilinear form from the loading condition in the catenary and arch previously discussed. A funicular truss is one in which the primary members conform to a loading configuration. The "primary action" of the truss will then transfer the forces generated by the loading to secondary elements. This secondary action will always act along the primary axis of an element, and hence these elements need only be straight compression members or cables.

Bending may occur in the primary members of a truss if they are not funicular, but will never occur in the secondary members until they begin to fail.
The stress in the connections between members of a truss, and the span of its members are usually the determining factors of the maximum spans for a particular configuration. For a given maximum rise, the simplest gable truss increases the stress in its joints as the span increases because the angles between the application of force and its resistant reactions becomes shallower. Before this truss actually reaches a span where the materials are incapable of bearing the stress, the sag of the tie-rod, resulting from its slenderness, usually becomes unacceptable. For this reason, a King Post truss includes a member in its center to pull the tie-rod back up. This member is referred to as a "post" because under extreme deformation of the truss, it acts in compression. Most trusses include mem-
bers such as this along the span which must be capable of working in both tension and compression, depending on the loading condition.

The tension equivalent of the King Post is a materially more efficient truss because it uses lighter tension elements along the longer side of the truss.

Under non-uniform or asymmetrical loading, one inclined member of the King Post truss will act in pure bending while the other acts in simple compression like any support. The basic configuration of the truss can be strengthened by treating each member like a tension equivalent of the King Post. Structurally, then, the truss develops efficiency by stabilizing itself for variations in loading conditions, and by mi-
nimizing its joint stresses. At its li-
mnit, a lenticular truss is the most effi-
cient shape for a uniformly loaded truss
because it makes effective use of its
depth, is funicular in its primary mem-
bbers, and minimizes the joint stresses.
Spatially, though, it is an awkward
shape, and creates a space beneath it
that is similar to that created by a
continuously loaded catenary, and has
similar problems.
DEVELOPMENT OF THE TRUSS:
ADAPTATION TO FORM

Trusses are able to take almost any desirable form, while maintaining their structural integrity, they are essentially the link to a hybridization of structure, because they can be descriptive of both the most efficient form for a given structural task, or the most efficient structure for a given form. The ability of a truss to adapt itself to a form can be demonstrated as follows;
a given truss with loads hung from five points along its span.

The center load is carried by the two diagonal tension members. The compressive upper chord absorbs the horizontal reaction (this part is directly analogous to the most elemental tension truss), and the next two vertical struts transfer the vertical reaction back to the bottom chord where, along with the next load, it is carried by the next diagonal tension member. The bottom chord, acting in tension, equilibrates the horizontal reaction.

Trusses may be considered as analogous to beams with all inefficient material removed, and are used spatially in a manner similar to the beams, either arranged side-by-side, or as a grid, referred to generally as a "space frame,"
which works and deflects so similarly to
the beam brid that the differences need
not be discussed here. Whereas the
beam grid tends to maintain a flat ceil-
ing (usually for ease of construction
in the types of materials commonly used,
more than any specific structural reason)
a space frame can conform to any shape
that can be built up from trusses, and
can mediate between most shapes. A con-
formation of the bottom member of a
truss to a form more spatially intrigu-
ing, or compliant with some other sug-
gested form (such as the shape of the
roof, or a form which reinforces the
use of a place) can be made with only
some increase in joint stresses
THE HYBRIDIZATION OF STRUCTURE

The structures that have been described in this thesis are, in a sense, elemental, for rarely does one find any building, or even any structural system acting in a single way. All structures, though, can be understood as hybrids of different actions and systems. "The Structural Diagram: The Shape of Simply Supported Beams," "Beam Action in Spatial Structure: The Folded Plate," "Development of the Catenary: The Shape of Funicular Structures Under Non-Uniform Loads," and each of the "Development of the Truss" sections have alluded to this within their context.

I have found that in writing this thesis I have come to see buildings with an increased insight and appreciation, and I hope that anyone who reads it will do the same, but more importantly, I hope that a designer beginning the first sketches of a future building will see it with an added light.
BIBLIOGRAPHY


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