18.712 Introduction to Representation Theory
Fall 2008

1. 18.712 TAKEHOME ASSIGNMENT

1. Let Q be a quiver, i.e. a finite oriented graph. Let $A(Q)$ be the path algebra of Q over a field k, i.e. the algebra whose basis is formed by paths in Q (compatible with orientations, and including paths of length 0 from a vertex to itself), and multiplication is concatenation of paths (if the paths cannot be concatenated, the product is zero).

(i) Represent the algebra of upper triangular matrices as $A(Q)$.

(ii) Show that $A(Q)$ is finite dimensional iff Q is acyclic, i.e. has no oriented cycles.

(iii) For any acyclic Q, decompose $A(Q)$ (as a left module) in a direct sum of indecomposable modules.

(iv) Find a condition on Q under which $A(Q)$ is isomorphic to $A(Q)^{op}$, the algebra $A(Q)$ with opposite multiplication. Use this to give an example of an algebra A that is not isomorphic to A^{op}.

2. Classify irreducible representations of the group $GL_2(\mathbb{F}_q) \ltimes \mathbb{F}_q^2$ of affine transformations of the 2-dimensional space over a finite field, and find their characters.

3. Compute the decomposition into irreducible representations of all the induced representations from the cyclic subgroups of the preimage Γ of $A_5 \subset SO(3)$ (the group corresponding to the affine Dynkin diagram $\tilde{E}_8$).

4. Find the multiplicities of the irreducible representations of $sl(2)$ in $V^{\otimes n}$, where V is the 2-dimensional vector represenation.