18.712 Introduction to Representation Theory
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Introduction to representation theory

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INTRODUCTION

Very roughly speaking, representation theory studies symmetry in linear spaces. It is a beautiful mathematical subject which has many applications, ranging from number theory and combinatorics to geometry, probability theory, quantum mechanics and quantum field theory.

Representation theory was born in 1896 in the work of the German mathematician F. G. Frobenius. This work was triggered by a letter to Frobenius by R. Dedekind. In this letter Dedekind made the following observation: take the multiplication table of a finite group G and turn it into a matrix X_G by replacing every entry g of this table by a variable x_g. Then the determinant of X_G factors into a product of irreducible polynomials in x_g, each of which occurs with multiplicity equal to its degree. Dedekind checked this surprising fact in a few special cases, but could not prove it in general. So he gave this problem to Frobenius. In order to find a solution of this problem (which we will explain below), Frobenius created representation theory of finite groups. [1]

The present lecture notes arose from a representation theory course given by the first author to the remaining six authors in March 2004 within the framework of the Clay Mathematics Institute Research Academy for high school students, and its extended version given by the first author to MIT undergraduate math students in the Fall of 2008. The lectures are supplemented by many problems and exercises, which contain a lot of additional material; the more difficult exercises are provided with hints.

The notes cover a number of standard topics in representation theory of groups, Lie algebras, and quivers. We mostly follow [FH], with the exception of the sections discussing quivers, which follow [BGP]. We also recommend the comprehensive textbook [CR]. The notes should be accessible to students with a strong background in linear algebra and a basic knowledge of abstract algebra.

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[1] For more on the history of representation theory, see [Cu].