A set $S$ of rays is called a direction if it satisfies the following laws:

1. Any two rays in $S$ have the same direction.
2. Every ray that has the same direction as some member of $S$ is in $S$.

A vector $\vec{A}$ consists of a non-negative real number, called the magnitude of the vector, and a direction. We denote the magnitude of $\vec{A}$ by $|\vec{A}|$. A vector $\vec{A}$ such that $|\vec{A}| = 1$ is called a unit vector.

We will now describe four basic algebraic operations with vectors in $\mathbb{E}^3$:

1. Multiplication by a scalar

   Let $\vec{A}$ be a vector, and let $c$ by a real number. Multiplying $\vec{A}$ by the scalar $c$, we obtain a vector denoted by $c\vec{A}$. The magnitude of the result is given by $|c\vec{A}| = |c||\vec{A}|$. The direction of $c\vec{A}$ is the same as the direction of $\vec{A}$ if $c \geq 0$, and opposite to the direction of $\vec{A}$ if $c < 0$.

2. Addition of vectors

   The sum of two vectors $\vec{A}$ and $\vec{B}$ is denoted simply by $\vec{A} + \vec{B}$. We can define this sum geometrically. Translate $\vec{B}$ such that its start point is the end-point of $\vec{A}$. Then $\vec{A} + \vec{B}$ will be the vector having the same start-point as $\vec{A}$ and the same end-point as $\vec{B}$.

3. Scalar product (Dot product)

   The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity denoted by $\vec{A} \cdot \vec{B}$. Its value is $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta$, where $\theta$ is the angle made by $\vec{A}$ and $\vec{B}$ if we translate $\vec{B}$ such that it has the same start-point as $\vec{A}$. We observe that $|\vec{B}|\cos\theta$ is the length of the projection of $\vec{B}$ on $\vec{A}$. 


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The magnitude of $\vec{A}$ is equal to the square root of the dot product of $A$ with itself:

$$|\vec{A}| = \sqrt{\vec{A} \cdot \vec{B}}$$

Hence $\frac{\vec{A}}{|\vec{A}|}$ is a unit vector with the same direction as $A$.

4 Vector product (Cross product)

The cross product of two vectors $\vec{A}$ and $\vec{B}$ is a vector denoted by $\vec{A} \times \vec{B}$. The magnitude of the cross product is given by:

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin\theta.$$ 

Let $\vec{A}$ and $\vec{B}$ have the same start-point $P$ and end-points $Q_1$ and $Q_2$, respectively. Let $M$ be the plane of $\vec{A}$ and $\vec{B}$. The direction of $\vec{A} \times \vec{B}$ is normal to $M$ in the manner established by the right-hand rule: if a right hand is placed at $P$ and the fingers are curling from $PQ_1$ to $TQ_2$ through the angle smaller than $\pi$, then the thumb indicates the direction of $\vec{A} \times \vec{B}$.

The following are some properties of these basic vector operations.

1. $(a + b)\vec{C} = a\vec{C} + b\vec{C}$
2. $(ab)\vec{C} = a(b\vec{C})$
3. $a(\vec{B} + \vec{C}) = a\vec{B} + a\vec{C}$

Figure 1: Vector addition
4. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
5. $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
6. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$