Let $w = f(x, y)$ be a differentiable function of $x$ and $y$. The linear approximation of $f$ is given by

$$\Delta f_{\text{app}} = f_x(x, y)\Delta x + f_y(x, y)\Delta y.$$ 

We introduce a new notation, the differential notation for the increments $\Delta f, \Delta x, \Delta y$, namely we write $df, dx, dy$ instead: $df = f_x dx + f_y dy$. This expression is called the differential of $f$. For example, the differential of

$$w = x^2 + y^2 - 1 \quad \text{is} \quad dw = 2x dx + 2y dy.$$ 

For any function $w = f(x, y)$, the equality

$$dw = \left( \frac{\partial w}{\partial x} \right)_y dx + \left( \frac{\partial w}{\partial y} \right)_x dy$$

holds. By the elimination method, we can find $\frac{\partial w}{\partial x}_y$ and $\frac{\partial w}{\partial y}_x$ if $w$ is not given directly as a function of $x$ and $y$ but can be reduced to such a function. We illustrate this in the following example. Consider the these two equalities:

$$w = f(x, y, z) = xyz, \quad z = g(x, y) = e^{xy}.$$ 

Then the differentials of $w$ and $z$ are

$$dw = yzdx + xzdy + xydz \quad \text{and} \quad dz = ye^{xy}dx + xe^{xy}dy.$$ 

Substituting $dz$ in the first equality, we get

$$dw = (yz + xy^2e^{xy})dx + (xz + yx^2e^{xy})dy.$$ 

Then the derivative of $w$ with respect to $x$ when $w$ is seen as a function of $x$ and $y$ is precisely the term of $dx$ in the equality above:

$$\left( \frac{\partial w}{\partial x} \right)_y = yz + xy^2e^{xy} \quad \text{and} \quad \left( \frac{\partial w}{\partial y} \right)_x = xz + yx^2e^{xy}.$$