MIT 2.852
Manufacturing Systems Analysis
Lecture 7

Transfer Lines – models and bounds

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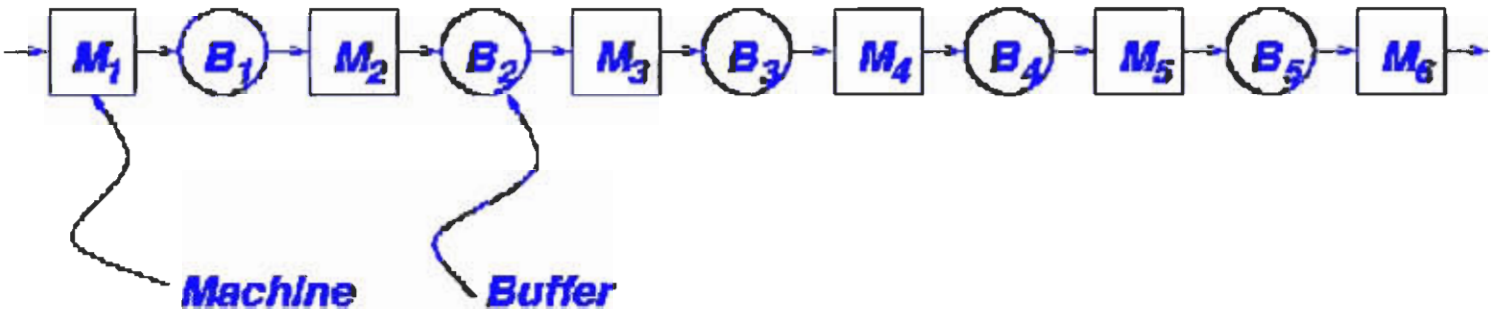
Flow Line

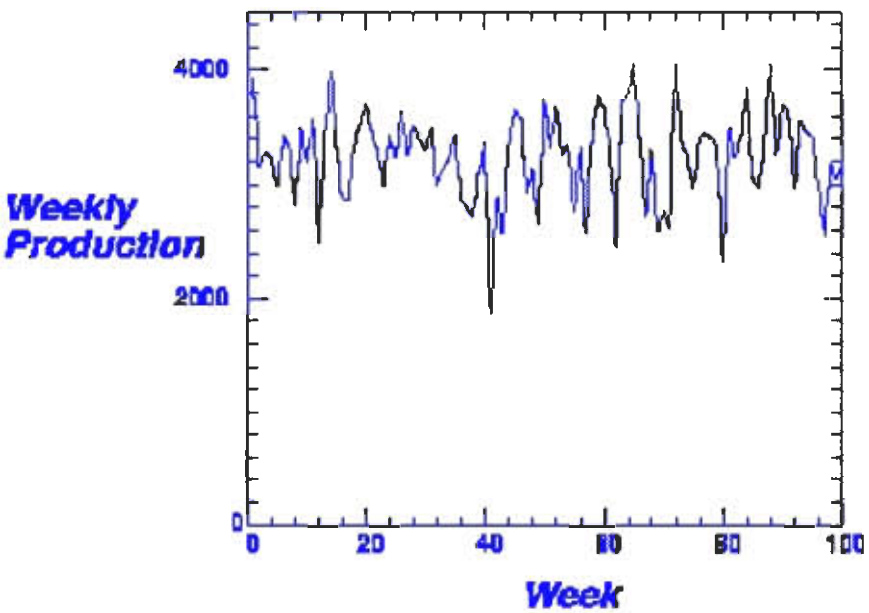
... also known as a Production or Transfer Line.



- Machines are unreliable.
- Buffers are finite.

Flow Line

Output Variability



Production output from a simulation of a transfer line.

Single Reliable Machine

- If the machine is perfectly reliable, and its average operation time is τ, then its maximum production rate is $1/\tau$.
- *Note:*
  - ⋆ Sometimes *cycle time* is used instead of *operation time* , but *BEWARE:* cycle time has two meanings!
  - ⋆ The other meaning is the time a part spends in a system. If the system is a single, reliable machine, the two meanings are the same.

Single Reliable Machine

ODFs

- Operation-Dependent Failures
  - ⋆ A machine can only fail while it is working.
  - ⋆ *IMPORTANT!* MTTF *must* be measured in working time!
  - ⋆ This is the usual assumption.
- *Note:* MTBF = MTTF + MTTR

Single Reliable Machine

Production rate

- If the machine is unreliable, and
  - ⋆ its average operation time is τ,
  - ⋆ its mean time to fail is MTTF,
  - ⋆ its mean time to repair is MTTR,

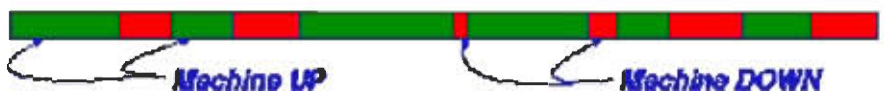
then its maximum production rate is

$$\frac{1}{\tau}\left(\frac{\text{MTTF}}{\text{MTTF}+\text{MTTR}}\right)$$

Single Reliable Machine

Production rate

Proof



- Average production rate, while machine is up, is $1/\tau$.
- Average duration of an up period is MTTF.
- Average production during an up period is MTTF/τ.
- Average duration of up-down period: MTTF + MTTR.
- Average production during up-down period: MTTF/τ.
- Therefore, average production rate is (MTTF/τ)/(MTTF + MTTR).

Single Reliable Machine

Geometric Up- and Down-Times

- *Assumptions:* Operation time is constant (τ). Failure and repair times are *geometrically* distributed.
- Let p be the probability that a machine fails during any given operation. Then $p = \tau/\text{MTTF}$.

Single Reliable Machine

Geometric Up- and Down-Times

- Let r be the probability that M gets repaired in during any operation time when it is down. Then $r = \tau/\text{MTTR}$.
- Then the *average production rate* of M is

$$\frac{1}{\tau}\left(\frac{r}{r+p}\right).$$

- *(Sometimes we forget to say "average.")*

Single Reliable Machine

Production Rates

- So far, the machine really has *three* production rates:
  - ⋆ $1/\tau$ when it is up *(short-term capacity)* ,
  - ⋆ 0 when it is down *(short-term capacity)* ,
  - ⋆ $(1/\tau)(r/(r+p))$ on the average *(long-term capacity)* .

Infinite-Buffer Line

Assumptions:

- A machine is not idle if it is not starved.
- The first machine is never starved.

Infinite-Buffer Line

- The production rate of the line is the production rate of the *slowest* machine in the line — called the *bottleneck* .
- *Slowest* means least average production rate, where average production rate is calculated from one of the previous formulas.

Infinite-Buffer Line

- Production rate is therefore

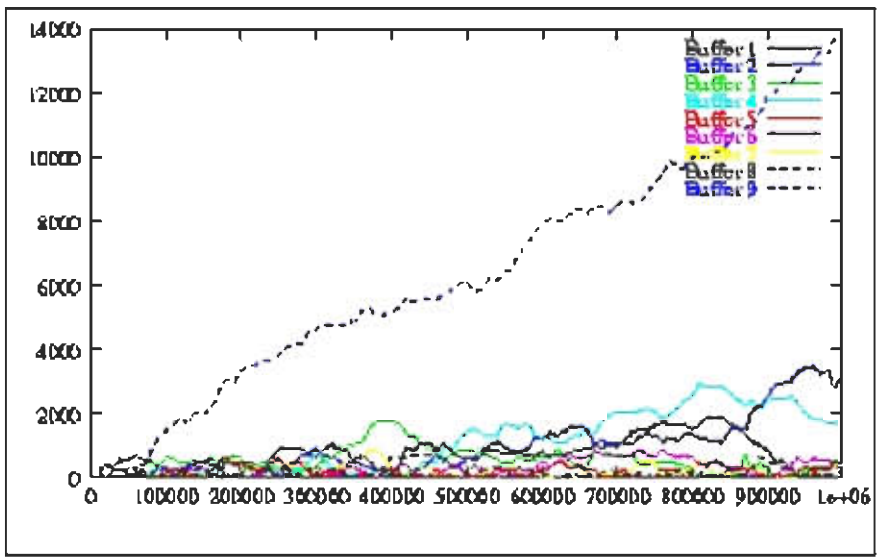
$$P = \min_i \frac{1}{\tau_i}\left(\frac{\mathrm{MTTF}_i}{\mathrm{MTTF}_i + \mathrm{MTTR}_i}\right)$$

- and M_i is the bottleneck.

Infinite-Buffer Line

- The system is not in steady state.
- An infinite amount of inventory accumulates in the buffer upstream of the bottleneck.
- A finite amount of inventory appears downstream of the bottleneck.

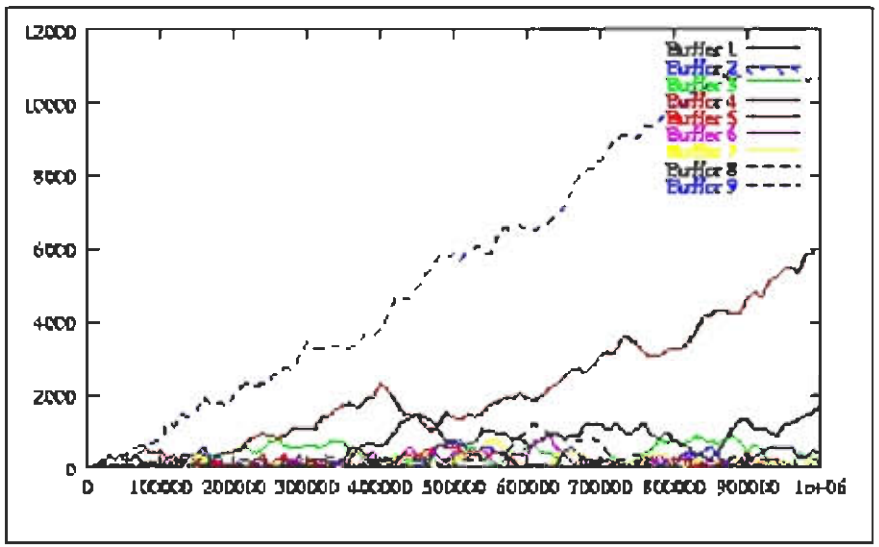
Infinite-Buffer Line



Infinite-Buffer Line

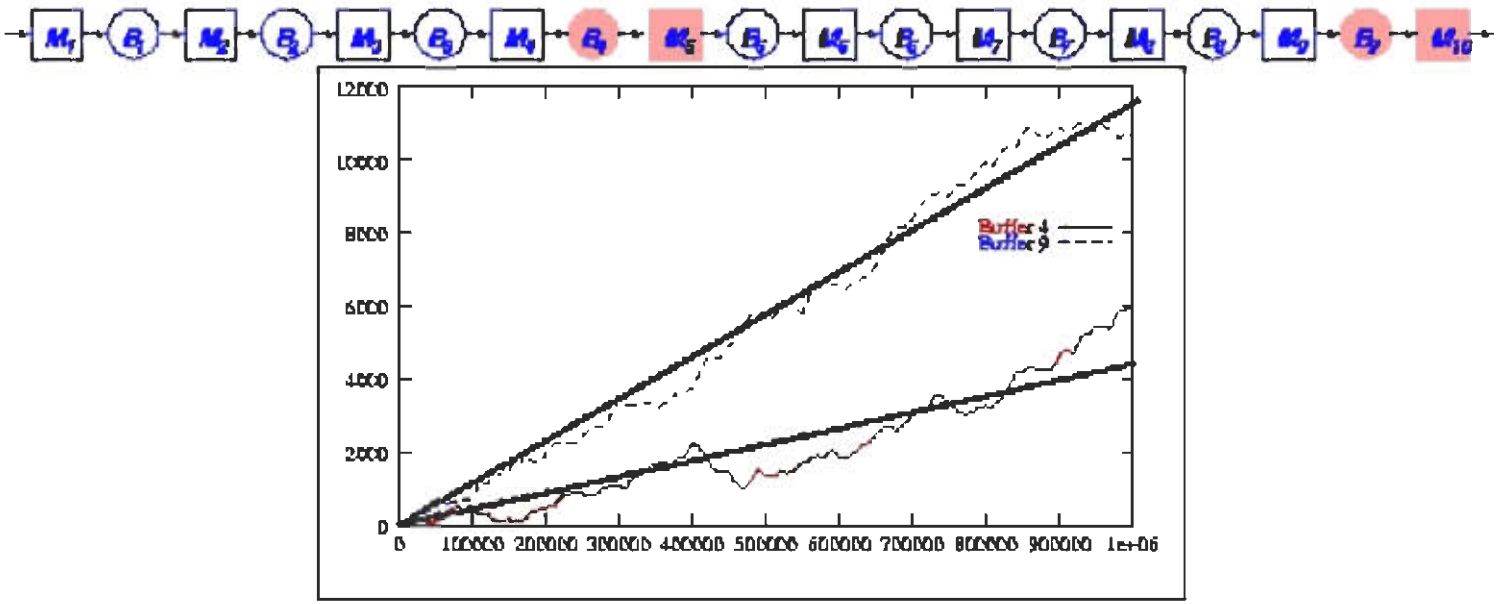
- The *second bottleneck* is the slowest machine upstream of the bottleneck. An infinite amount of inventory accumulates just upstream of it.
- A finite amount of inventory appears between the second bottleneck and the machine upstream of the first bottleneck.
- Et cetera.

Infinite-Buffer Line



A 10-machine line with bottlenecks at Machines 5 and 10.

Infinite-Buffer Line



Question:

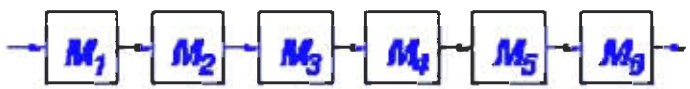
- What are the slopes *(roughly!)* of the two indicated graphs?

Infinite-Buffer Line

Questions:

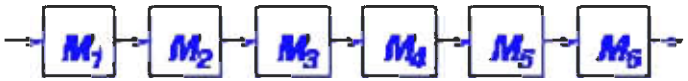
- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

Zero-Buffer Line



- If any one machine fails, or takes a very long time to do an operation, *all* the other machines must wait.
- Therefore the production rate is usually less — possibly much less – than the slowest machine.

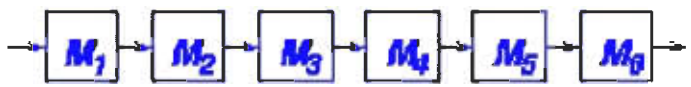
Zero-Buffer Line



- *Example:* Constant, unequal operation times, perfectly reliable machines.
  - ★ The operation time of the line is equal to the operation time of the slowest machine, so the production rate of the line is *equal to* that of the slowest machine.

Zero-Buffer Line

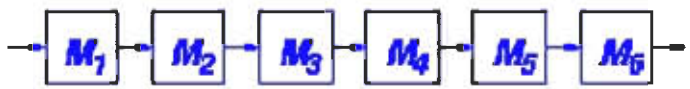
Constant,
equal operation times,
unreliable machines



- *Assumption:* Failure and repair times are *geometrically* distributed.
- Define $p_i = \tau/\text{MTTF}_i$ = probability of failure during an operation.
- Define $r_i = \tau/\text{MTTR}_i$ probability of repair during an interval of length τ when the machine is down.

Zero-Buffer Line

Constant,
equal operation times,
unreliable machines



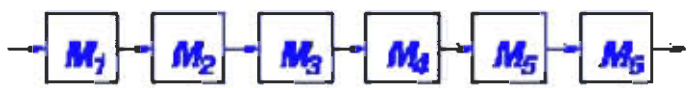
Buzacott's Zero-Buffer Line Formula:

Let k be the number of machines in the line. Then

$$P = \frac{1}{\tau} \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}}$$

Zero-Buffer Line

Constant,
equal operation times,
unreliable machines



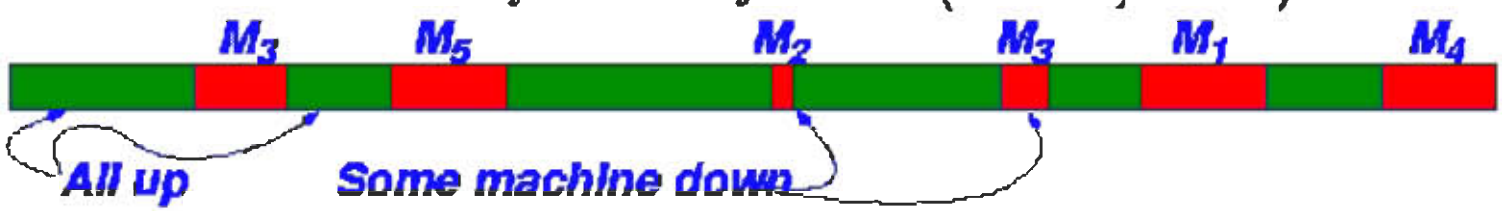
- Same as the earlier formula (page 6, page 9) when $k = 1$. The *isolated production rate* of a single machine M_i is

$$\frac{1}{\tau}\left(\frac{1}{1+\frac{p_i}{r_i}}\right) = \frac{1}{\tau}\left(\frac{r_i}{r_i+p_i}\right).$$

Zero-Buffer Line

Proof of formula

- Let τ (the operation time) be the time unit.
- *Assumption:* At most, <u>one</u> machine can be down.
- Consider a long time interval of length $T\tau$ during which Machine M_i fails m_i times ($i = 1, \ldots k$).



- Without failures, the line would produce T parts.

Zero-Buffer Line

Proof of formula

- The average repair time of M_i is τ/r_i each time it fails, so the total system down time is close to

$$D\tau = \sum_{i=1}^{k} \frac{m_i \tau}{r_i}$$

where D is the number of operation times in which a machine is down.

Zero-Buffer Line

Proof of formula

- The total up time is approximately

$$U\tau = T\tau - \sum_{i=1}^{k} \frac{m_i \tau}{r_i}.$$

- where U is the number of operation times in which all machines are up.

Zero-Buffer Line

Proof of formula

- Since the system produces one part per time unit while it is working, it produces U parts during the interval of length $T\tau$.
- Note that, approximately,

$$m_i = p_i U$$

because M_i can only fail while it is operational.

Zero-Buffer Line

Proof of formula

- Thus,

$$U\tau = T\tau - U\tau \sum_{i=1}^{k} \frac{p_i}{r_i},$$

or,

$$\frac{U}{T} = E_{ODF} = \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}}$$

Zero-Buffer Line

Proof of formula

and

$$P = \frac{1}{\tau} \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}}$$

- Note that P is a function of the *ratio* p_i/r_i and not p_i or r_i separately.
- The same statement is true for the infinite-buffer line.
- However, the same statement is *not* true for a line with finite, non-zero buffers.

Zero-Buffer Line

Proof of formula

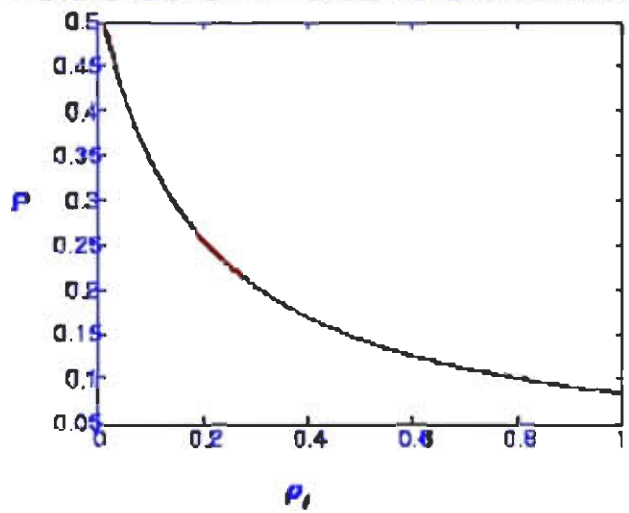
Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

Zero-Buffer Line

P as a function of p_i

All machines are the same except M_i. As p_i increases, the production rate decreases.



Zero-Buffer Line

P as a function of k

All machines are the same. As the line gets longer, the production rate decreases.