Flow Line

... also known as a Production or Transfer Line.

- Machines are unreliable.
- Buffers are finite.
Output Variability

Flow Line

Production output from a simulation of a transfer line.
Single Reliable Machine

• If the machine is perfectly reliable, and its average operation time is $\tau$, then its maximum production rate is $1/\tau$.

• Note:
  ★ Sometimes cycle time is used instead of operation time, but BEWARE: cycle time has two meanings!
  ★ The other meaning is the time a part spends in a system. If the system is a single, reliable machine, the two meanings are the same.
• Operation-Dependent Failures
  ★ A machine can only fail while it is working.
  ★ **IMPORTANT!** MTTF must be measured in working time!
  ★ This is the usual assumption.
• Note: MTBF = MTTF + MTTR
Single Reliable Machine

Production rate

- If the machine is unreliable, and
  - its average operation time is $\tau$,
  - its mean time to fail is MTTF,
  - its mean time to repair is MTTR,

then its maximum production rate is

$$\frac{1}{\tau} \left( \frac{MTTF}{MTTF + MTTR} \right)$$
- Average production rate, while machine is up, is \( \frac{1}{\tau} \).
- Average duration of an up period is MTTF.
- Average production during an up period is \( \frac{MTTF}{\tau} \).
- Average duration of up-down period: MTTF + MTTR.
- Average production during up-down period: \( \frac{MTTF}{\tau} \).
- Therefore, average production rate is \( \frac{(MTTF/\tau)}{(MTTF + MTTR)} \).
Single Reliable Machine

Geometric Up- and Down-Times

- **Assumptions:** Operation time is constant ($\tau$). Failure and repair times are geometrically distributed.

- Let $p$ be the probability that a machine fails during any given operation. Then $p = \tau/\text{MTTF}$. 
Single Reliable Machine

Geometric Up- and Down-Times

- Let $\tau$ be the probability that $M$ gets repaired during any operation time when it is down. Then $r = \tau / \text{MTTR}$.
- Then the average production rate of $M$ is
  \[
  \frac{1}{\tau} \left( \frac{r}{r + p} \right).
  \]
- (Sometimes we forget to say “average.”)
So far, the machine really has three production rates:

- $1/\tau$ when it is up (short-term capacity),
- 0 when it is down (short-term capacity),
- $(1/\tau)(r/(r+p))$ on the average (long-term capacity).
Infinite-Buffer Line

Assumptions:

- A machine is not idle if it is not starved.
- The first machine is never starved.
Infinite-Buffer Line

- The production rate of the line is the production rate of the *slowest* machine in the line — called the *bottleneck*.

- *Slowest* means least average production rate, where average production rate is calculated from one of the previous formulas.
Infinite-Buffer Line

- Production rate is therefore

\[ P = \min_i \frac{1}{\tau_i} \left( \frac{MTTF_i}{MTTF_i + MTTR_i} \right) \]

- and \( M_i \) is the bottleneck.
Infinite-Buffer Line

- The system is not in steady state.
- An infinite amount of inventory accumulates in the buffer upstream of the bottleneck.
- A finite amount of inventory appears downstream of the bottleneck.
Infinite-Buffer Line

- The second bottleneck is the slowest machine upstream of the bottleneck. An infinite amount of inventory accumulates just upstream of it.
- A finite amount of inventory appears between the second bottleneck and the machine upstream of the first bottleneck.
- Et cetera.
Infinite-Buffer Line

A 10-machine line with bottlenecks at Machines 5 and 10.
Question:
- What are the slopes (roughly!) of the two indicated graphs?
Infinite-Buffer Line

Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?
Zero-Buffer Line

- If any one machine fails, or takes a very long time to do an operation, all the other machines must wait.
- Therefore the production rate is usually less — possibly much less — than the slowest machine.
Zero-Buffer Line

- $M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow M_4 \rightarrow M_5 \rightarrow M_6$

**Example:** Constant, unequal operation times, perfectly reliable machines.

- The operation time of the line is equal to the operation time of the slowest machine, so the production rate of the line is equal to that of the slowest machine.
**Assumption:** Failure and repair times are **geometrically** distributed.

- Define $p_i = \frac{\tau}{MTTF_i}$ = probability of failure during an operation.
- Define $r_i = \frac{\tau}{MTTR_i}$ probability of repair during an interval of length $\tau$ when the machine is down.
Zero-Buffer Line

Constant, equal operation times, unreliable machines

Buzacott's Zero-Buffer Line Formula:

Let $k$ be the number of machines in the line. Then

$$P = \frac{1}{\tau} \left( \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}} \right)$$
Zero-Buffer Line

Constant, equal operation times, unreliable machines

- Same as the earlier formula (page 6, page 9) when $k = 1$. The isolated production rate of a single machine $M_i$ is

$$\frac{1}{\tau} \left( \frac{1}{1 + \frac{p_i}{r_i}} \right) = \frac{1}{\tau} \left( \frac{r_i}{r_i + p_i} \right).$$
Zero-Buffer Line

Proof of formula

- Let $\tau$ (the operation time) be the time unit.
- **Assumption**: At most, one machine can be down.
- Consider a long time interval of length $T\tau$ during which Machine $M_i$ fails $m_i$ times ($i = 1, \ldots, k$).

- Without failures, the line would produce $T$ parts.
The average repair time of $M_i$ is $\frac{\tau}{r_i}$ each time it fails, so the total system down time is close to

$$D_{\tau} = \sum_{i=1}^{k} \frac{m_i \tau}{r_i}$$

where $D$ is the number of operation times in which a machine is down.
The total up time is approximately

\[ U_T = T_T - \sum_{i=1}^{k} \frac{m_i T}{r_i} \]

where \( U \) is the number of operation times in which all machines are up.
Since the system produces one part per time unit while it is working, it produces $U$ parts during the interval of length $T \tau$.

Note that, approximately,

$$m_i = p_i U$$

because $M_i$ can only fail while it is operational.
Thus,

\[ U_T = T_T - U_T \sum_{i=1}^{k} \frac{p_i}{r_i} \]

or,

\[ \frac{U}{T} = E_{ODF} = \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}} \]
Zero-Buffer Line

Proof of formula

\[ P = \frac{1}{\tau} \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}} \]

- Note that \( P \) is a function of the ratio \( p_i/r_i \) and not \( p_i \) or \( r_i \) separately.
- The same statement is true for the infinite-buffer line.
- However, the same statement is \textit{not} true for a line with finite, non-zero buffers.
Zero-Buffer Line

Proof of formula

Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?
All machines are the same except $M_i$. As $p_i$ increases, the production rate decreases.
Zero-Buffer Line

$P$ as a function of $k$

All machines are the same. As the line gets longer, the production rate decreases.