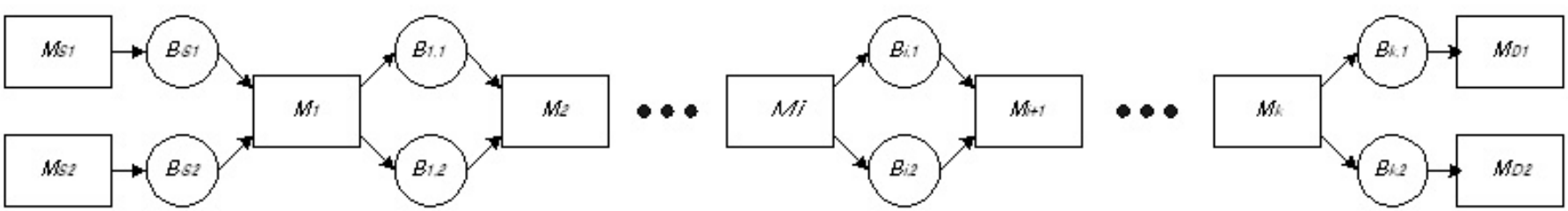
Mathematical Modeling of the Two-Part Type Machine

Young Jae Jang
Massachusetts Institute of Technology
May 12, 2004

Contents

Introduction

- **Multiple-Part-Type Processing Line**
- **Example of Two-Part-Type Processing Line**
  - **Supply Machines: M_{s1}, M_{s2}**
  - **Demand Machines: M_{d1}, M_{d2}**
  - **Processing Machine: M_i**
  - **Homogeneous Buffer: $B_{i,j}$**
    - ***Size: N_{ij},***
    - ***Number of Parts at Bij at time t = $n_{ij}(t)$***
- **Priority Rule: Type one has a priority over type two**
- **Why supply and demand machines are needed?**

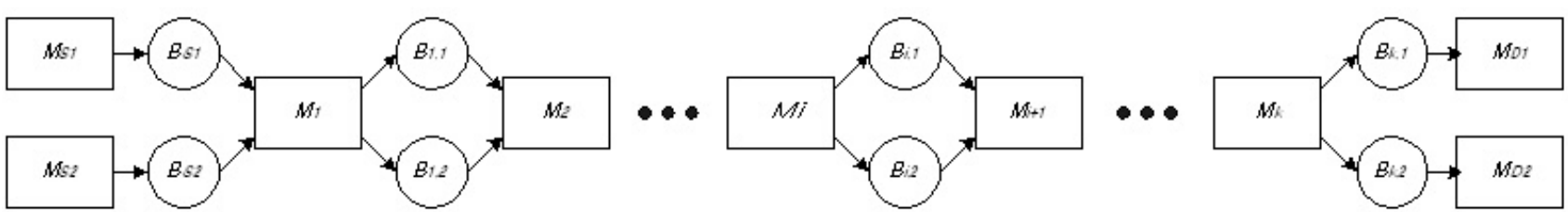


Introduction - Parameters

- Discrete time Model
- Identical processing time
- One time unit = processing time for one part

$$
\begin{aligned}
r_i &= Pr\left[\alpha_i(t+1)=1 | \alpha_i(t)=0\right] \\
p_i &= Pr\left[\alpha_{i,1}(t+1)=0 | \{\alpha_{i,1}(t)=1 \cap n_{i-1,1}(t)>0 \cap n_{i,1}(t)<N_{i,1}\} \cup \right. \\
&\qquad \{\alpha_{i,1}(t)=1 \cap (n_{i-1,1}(t)=0 \cup n_{i,1}(t)=N_{i,1}) \\
&\qquad \left. \cap\, n_{i-1,2}(t)>0 \cap n_{i,2}(t)<N_{i,2}\}\right].
\end{aligned}
$$

for $i = 1, \ldots, k$



Introduction – Importance

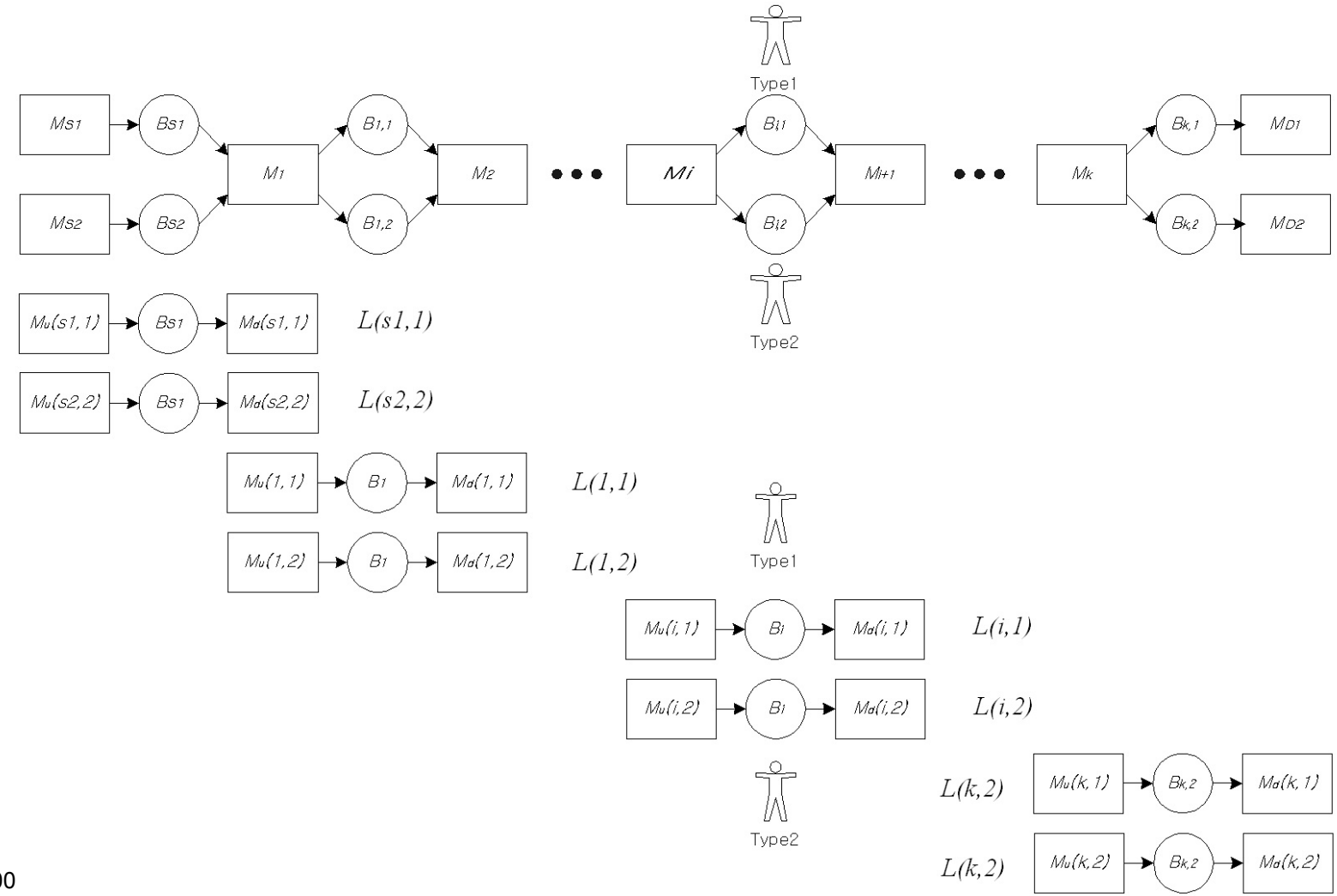
- Most of processing machines these days are able to process several different part types
- Lines are usually processing more than one type of products
- In LCD or Semiconductor FAB multi-loop processing is common
- Better scheduling for multiple part type line
- GM Needs
- It will be a part of analysis of CPP of multiple part type line

Previous Work

- Joe Nemec: MIT PhD Thesis 1998
  - Single Failure Mode
  - Too complicated
  - Lines are not very well converged
- Diego Syrowicz: MIT MS Thesis 1999
  - Multiple failure mode
  - Did not complete decomposition equations
- Tulio Tolio, 2003
  - Two machine two buffer building block, discrete time, multi-failure model
  - Too complicated, some equations are not clear

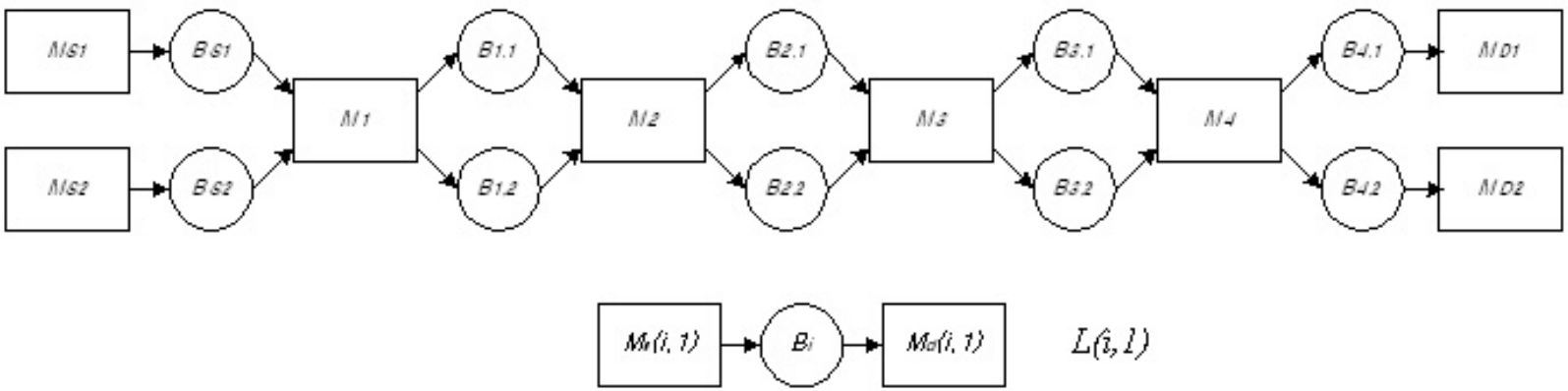
Approach

- Observers
  - Think that machine is working only single part type
  - Type2 guy thinks that machine is down, when the machine is working on part type one



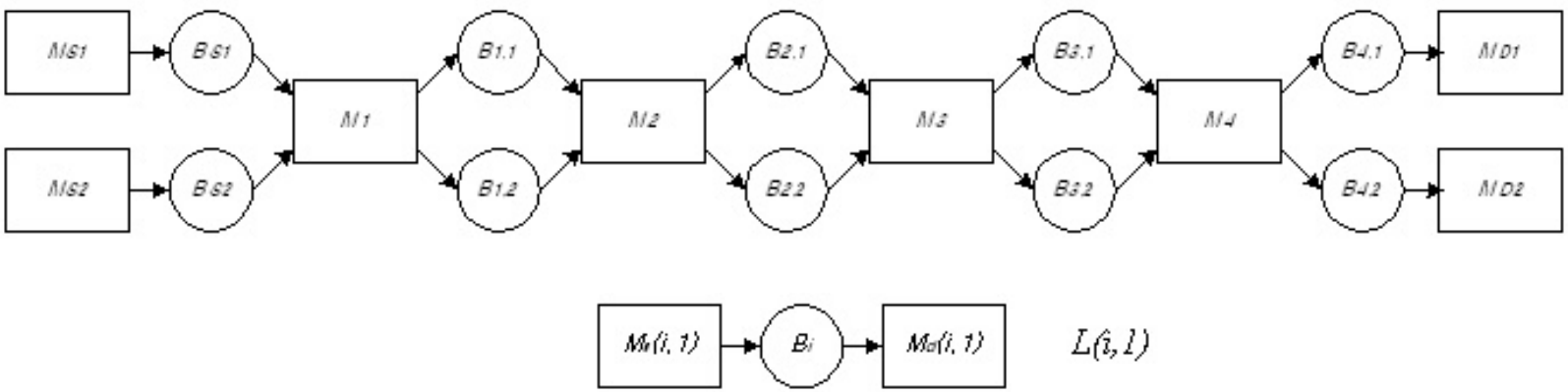
Issues on Multiple-Part-Type Line – Idleness Failure

- Idleness failure: Failure while machine is starved or blocked
- There is no idleness failure in a single part type case
- Example
  1. M_3 down
  2. $n_{2,1} = N_{2,1}$, $n_{2,2} <= N_{2,2}$, $n_{s,1} > 0$, $n_{s,2} > 0$
  3. M_2 blocked for type 1, starts working type two
  4. M_2 goes to down, but $n_{2,1} = N_{2,1}$, $n_{s,1} > 0$
- Observer for part type one sees that M_1 goes to down when it is blocked for part type one!



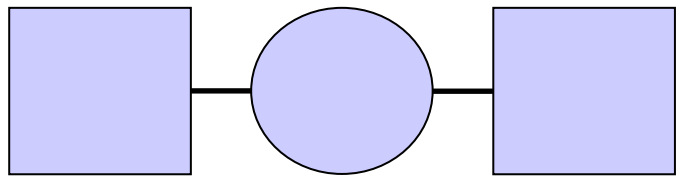
Issues on Multiple-Part-Type Line – Failure Mode Change

- Failure mode shift might be detected by an observer
- There is no failure mode change in a single part type line
- Example
  - M_4 down
  - $n_{3,1} = N_{3,1}$, $n_{3,2} < N_{3,2}$
  - M_3 is blocked for type one and start working type twp part
  - M_3 is down while working on type two
  - M_4 is up and $n_{3,1} < N_{3,1}$
  - M_3 is still down



Two Machine Line Parameters

- Two machine line
  - Single-up, multiple-down, failure mode changing Markov chain
  - $\alpha(t)$: machine states at time t
  - Υ: Machine is up, Δ_i: Machine is down at mode i



$$r_j^u = Pr[\{\alpha^u(t+1) = \Upsilon^u\} \mid \{\alpha^u(t) = \Delta_j^u\}]$$

$$r_k^d = Pr[\{\alpha^d(t+1) = \Upsilon^d\} \mid \{\alpha^d(t) = \Delta_k^d\}]$$

$$p_j^u = Pr[\{\alpha^u(t+1) = \Delta_j^u\} \mid \{\alpha^u(t) = \Upsilon^u\} \bigcap \{n(t) < N\}]$$

$$p_k^d = Pr[\{\alpha^d(t+1) = \Delta_k^d\} \mid \{\alpha^d(t) = \Upsilon^d\} \bigcap \{n(t) > 0\}]$$

Two Machine Line – Markov Model

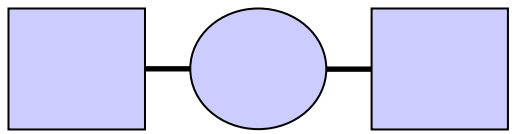
- Two machine line
  - Failure mode change and Idleness failure
  - z^*_{ij} : Failure mode change parameters
  - $q^*_{l,j}$: Idleness failure parameters
    (= u or d)*
  - *These parameters are zero*

$$q_j^u = Pr[\{\alpha^u(t+1) = \Delta_j^u\} \mid \{\alpha^u(t) = \Upsilon^u\} \bigcap \{n(t) = N\}]$$

$$q_k^d = Pr[\{\alpha^d(t+1) = \Delta_k^d\} \mid \{\alpha^d(t) = \Upsilon^d\} \bigcap \{n(t) = 0\}]$$

$$z_{j,j'}^u = Pr[\{\alpha^u(t+1) = \Delta_{j'}^u\} \mid \{\alpha^u(t) = \Delta_j^u\}]$$

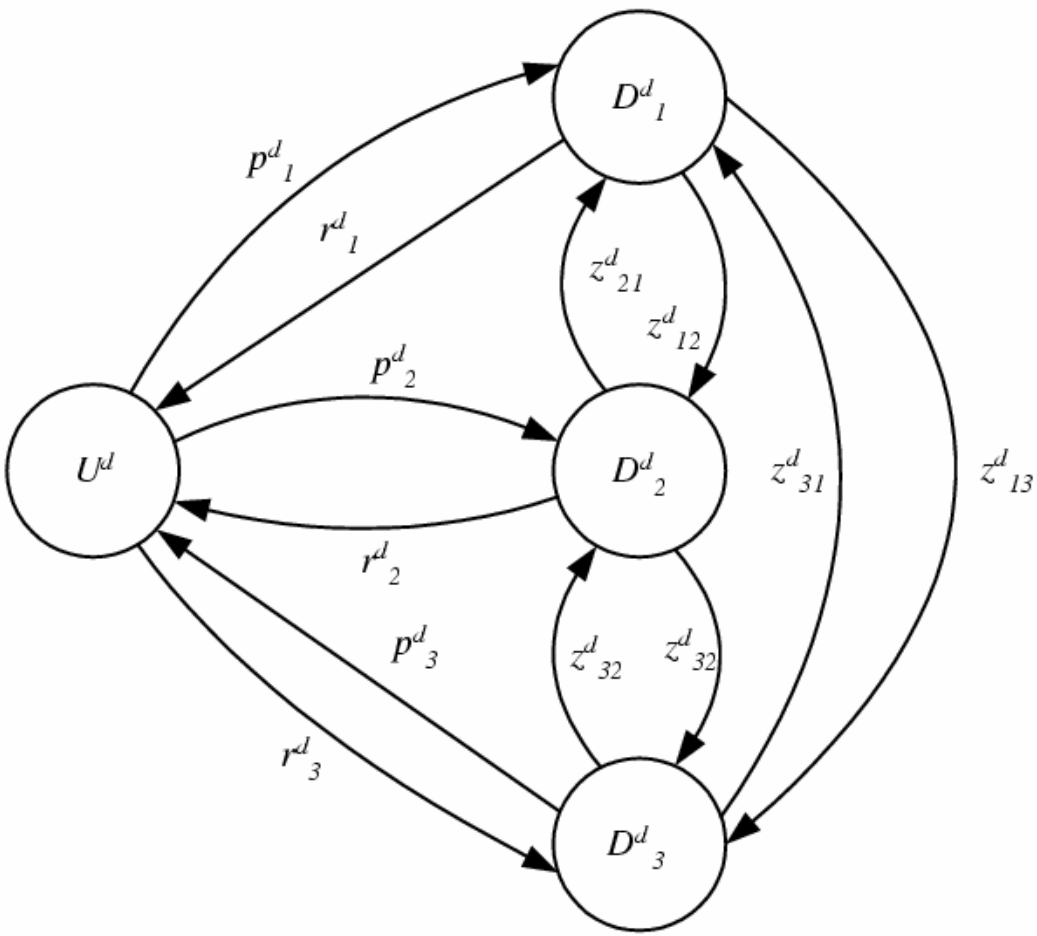
$$z_{k,k'}^d = Pr[\{\alpha^d(t+1) = \Delta_{k'}^d\} \mid \{\alpha^d(t) = \Delta_k^d\}]$$



$$Q^u = \sum_{j=1}^{J} q_j^u, \qquad J = \text{Total number of failure mode of } M^u$$

$$Q^d = \sum_{l=1}^{L} q_l^u, \qquad L = \text{Total number of failure mode of } M^u$$

One Machine Markov Model



- Single-up, Multiple-down, Failure Mode Changing Markov Chain
- Example of 3 down modes Markov Chain

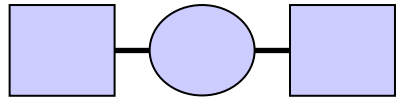
Two Machine Line – Efficiency of the line

Single part type case (without idleness failure), efficiency is

$$E^u = Pr[\{\alpha^u(t) = \Upsilon^u\} \bigcap \{n(t) < N\}]$$

$$E^d = Pr[\{\alpha^d(t) = \Upsilon^d\} \bigcap \{n(t) > 0\}]$$

$$E^u = E^d$$



However, with idlness failure,

$$Pr[\{\alpha^u(t) = \Upsilon^u\} \bigcap \{n(t) < N\}] \neq Pr[\{\alpha^d(t) = \Upsilon^d\} \bigcap \{n(t) > 0\}]$$

Efficiency needs to be derived from the definition,

$$E^u = Pr[\{\alpha^u(t+1) = \Upsilon^u\} \bigcap \{n(t) < N\}]$$

with the fact $r_j^u(Pr[\alpha^u = \Delta_j^u \cap n < N] + Pr[\alpha^u = \Delta_j^u \cap n = N])$

$$= p_j^u Pr[\alpha^u = \Upsilon_j^u \cap n < N] + q_j^u Pr[\alpha^u = \Upsilon_j^u \cap n = N]$$

Two-Machine Line – Efficiency

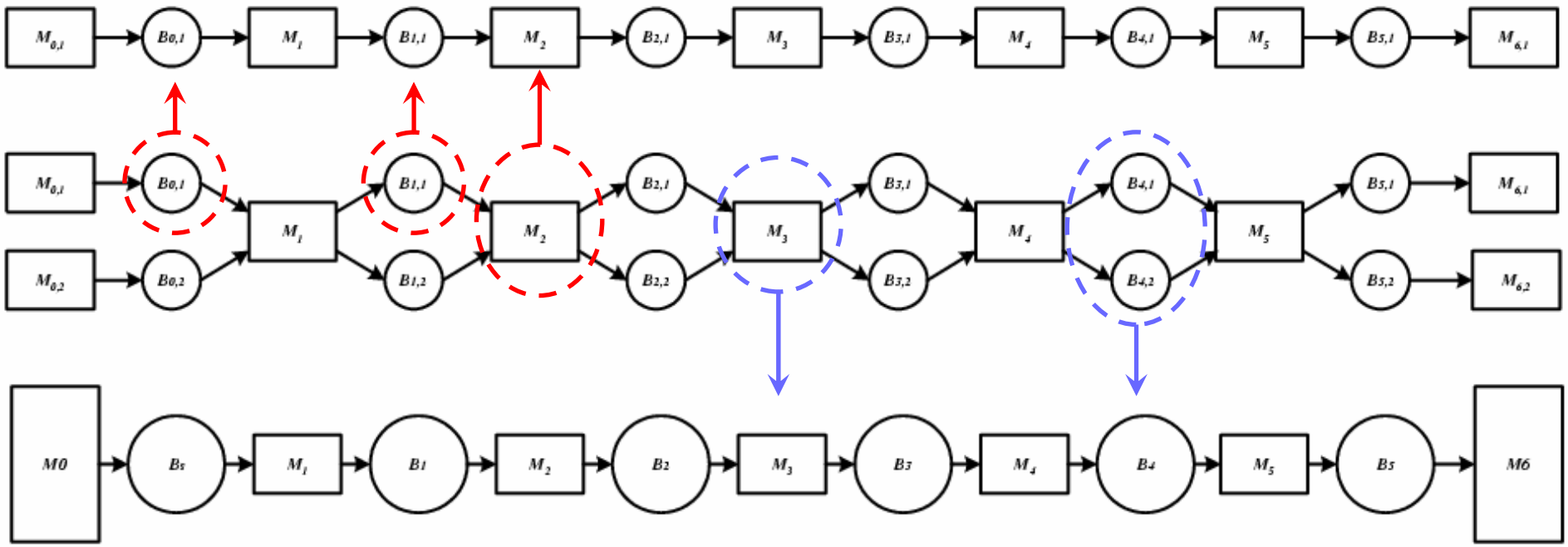
- $E^u = E^d$

$$
\begin{aligned}
E^u &= \sum_{n=0}^{N-1} Pr(n, \Upsilon^u, \Upsilon^d) + \sum_{n=0}^{N-1} \sum_{l=1}^{L} Pr(n, \Upsilon^u, \Delta_j^d) \\
&+ Q^u \sum_{l=1}^{L} Pr(N, \Upsilon^u, \Delta_l^d) - \sum_{j=1}^{J} r_j^u \sum_{l=1}^{L} Pr(N, \Delta_j^u, \Delta_l^d)
\end{aligned}
$$

$$
\begin{aligned}
E^d &= \sum_{n=1}^{N} Pr(n, \Upsilon^u, \Upsilon^d) + \sum_{n=1}^{N} \sum_{j=1}^{J} Pr(n, \Delta_j^u, \Upsilon^d) \\
&+ Q^d \sum_{j=1}^{J} Pr(0, \Delta_j^u, \Upsilon^d) - \sum_{l=1}^{L} r_l^d \sum_{j=1}^{J} Pr(0, \Delta_j^u, \Delta_l^d)
\end{aligned}
$$

Heuristics

- Two pseudo-machines: Lp1 and Lp2
- Analyze two lines separately using single part type machine line decomposition

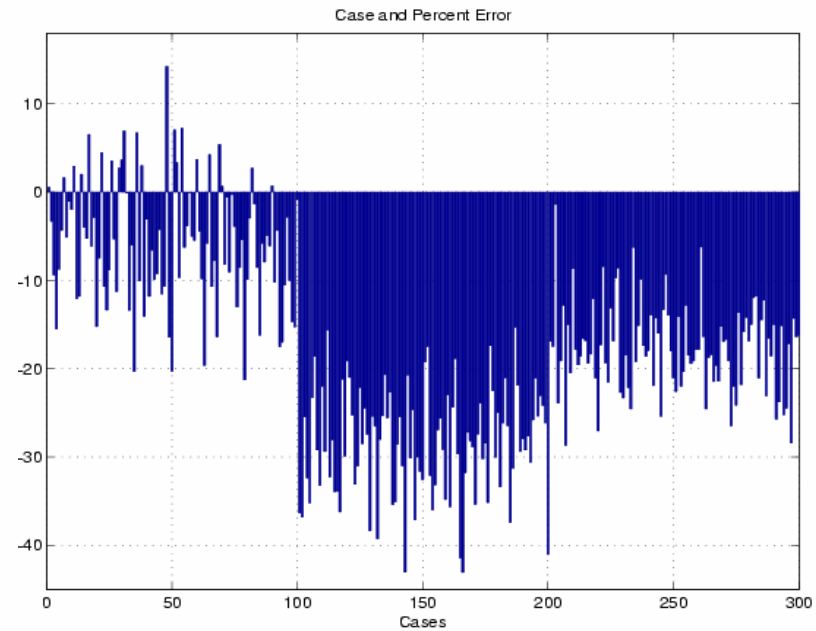


Heuristic Results

- Type one production rate % error: 2%
- Type two production rate % error:

Case	Parameter	Mean	Expected Demand Rate
1-100	r	0.1	
	p_1	0.08	$\overline{E}_1 = 0.55$
	p_2	0.08	$\overline{E}_2 = 0.55$
101-200	r	0.1	
	p_1	0.08	$\overline{E}_1 = 0.55$
	p_2	0.15	$\overline{E}_2 = 0.40$
201-300	r	0.1	
	p_1	0.15	$\overline{E}_1 = 0.40$
	p_2	0.08	$\overline{E}_2 = 0.55$

Table 1: Randomly generated demand machine parameters

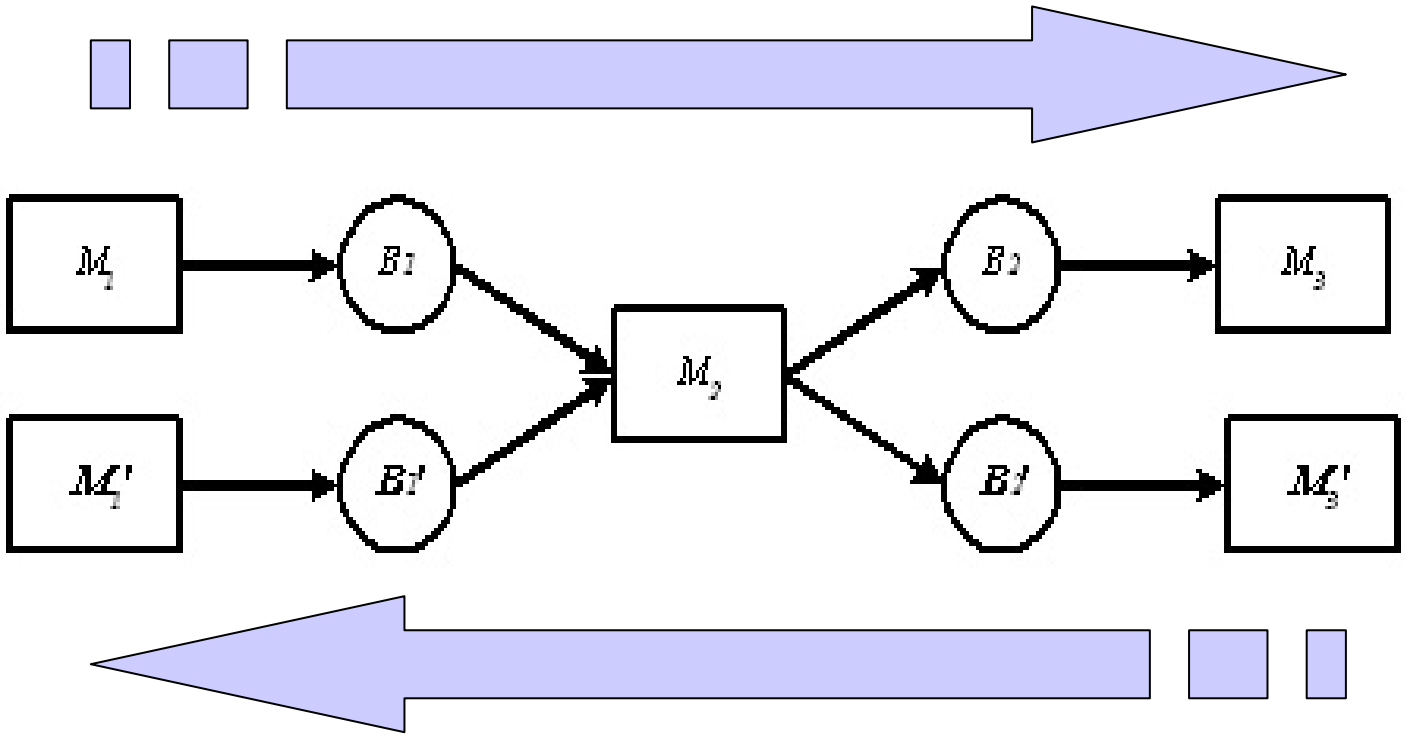


Decomposition Equations:

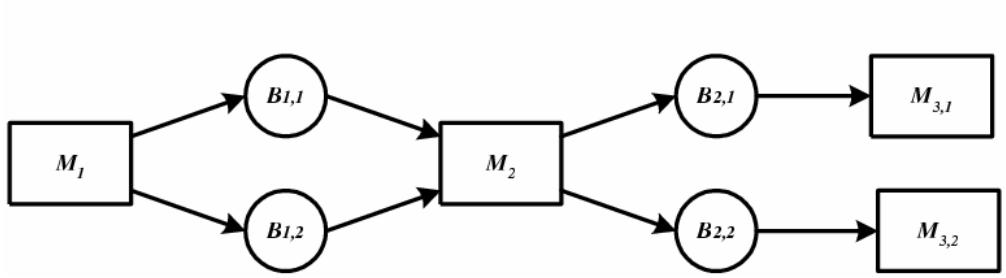
- I don’t want you to get bored…

Algorithm

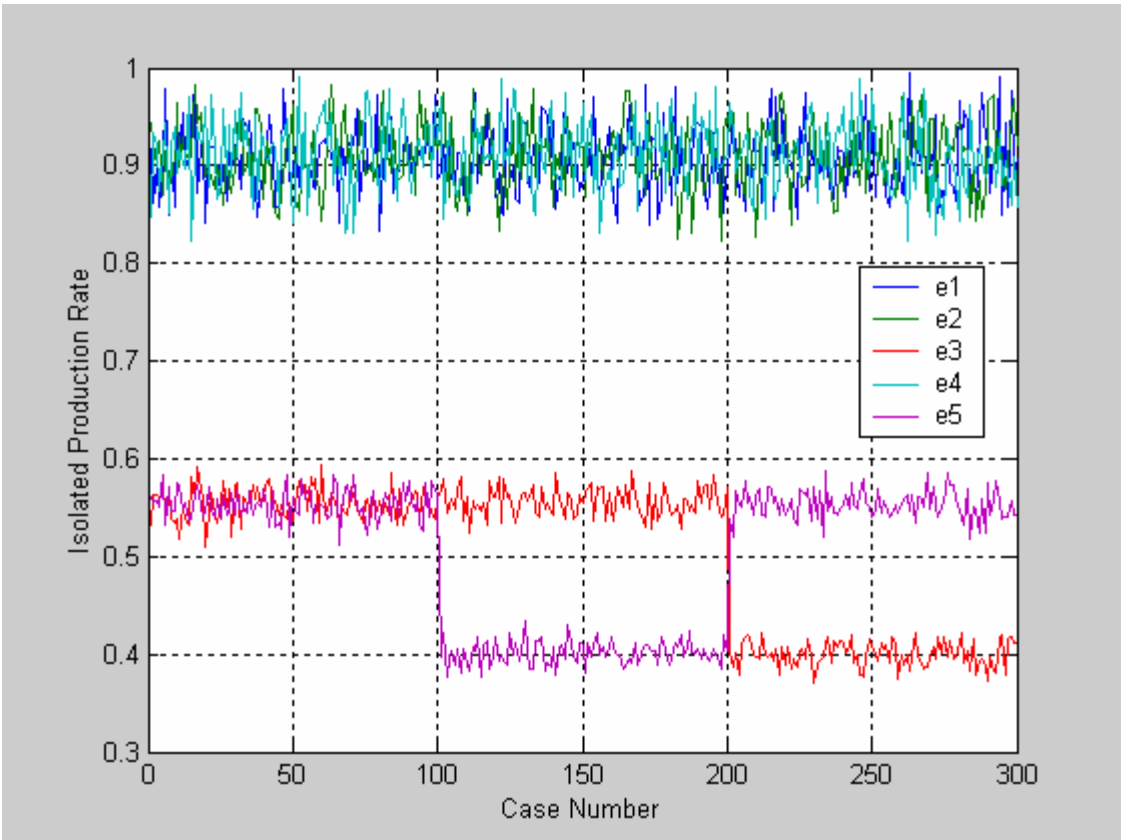
- Jang DDX algorithm
- 6 unknowns, 13 equations => Over constrained equations
- Very robust -> always converges



Simulation Case



Random Number Generation
- Triangular distribution
- Two processing machines two supply machine and two demand line case

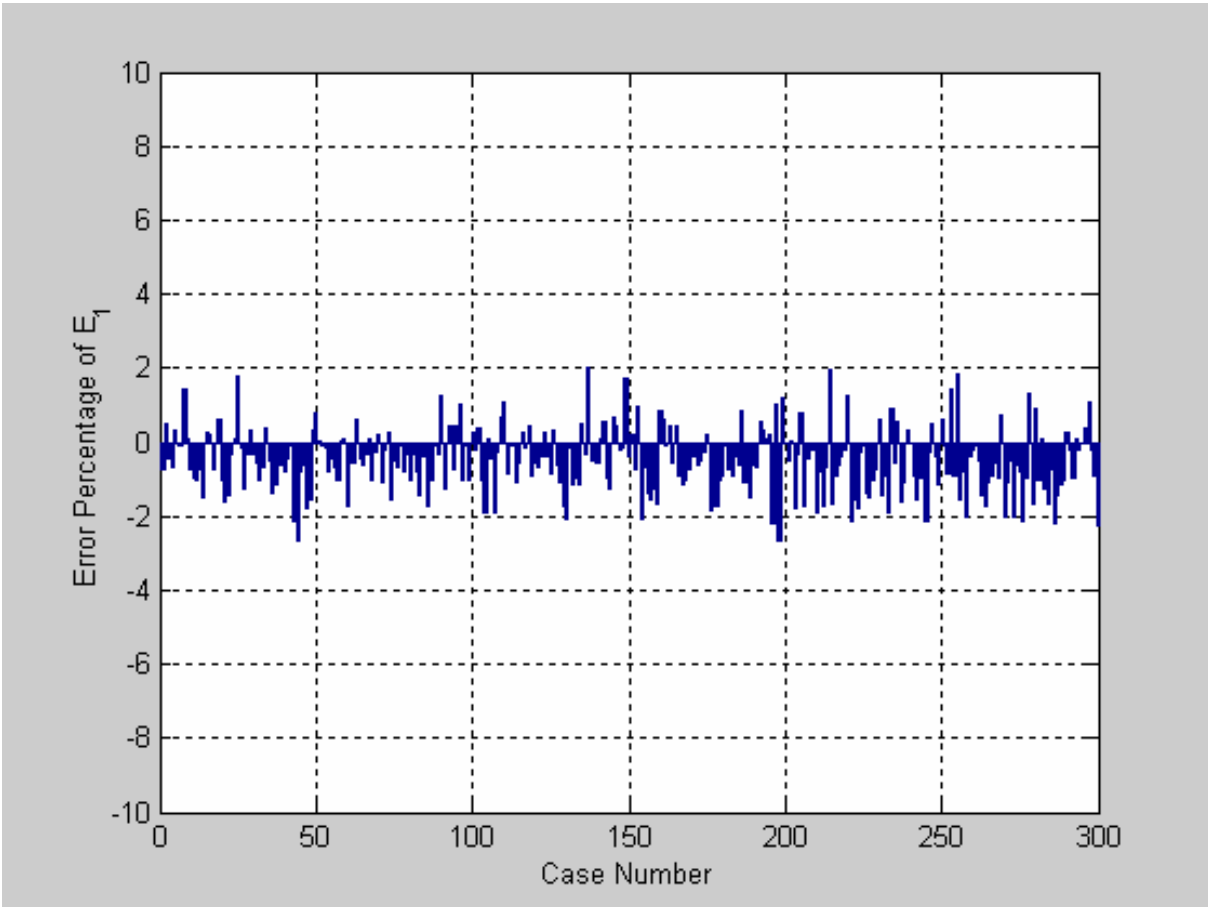


e4= efficiency of M'_{d1}
e5= efficiency of M'_{d2}

E_1 Error

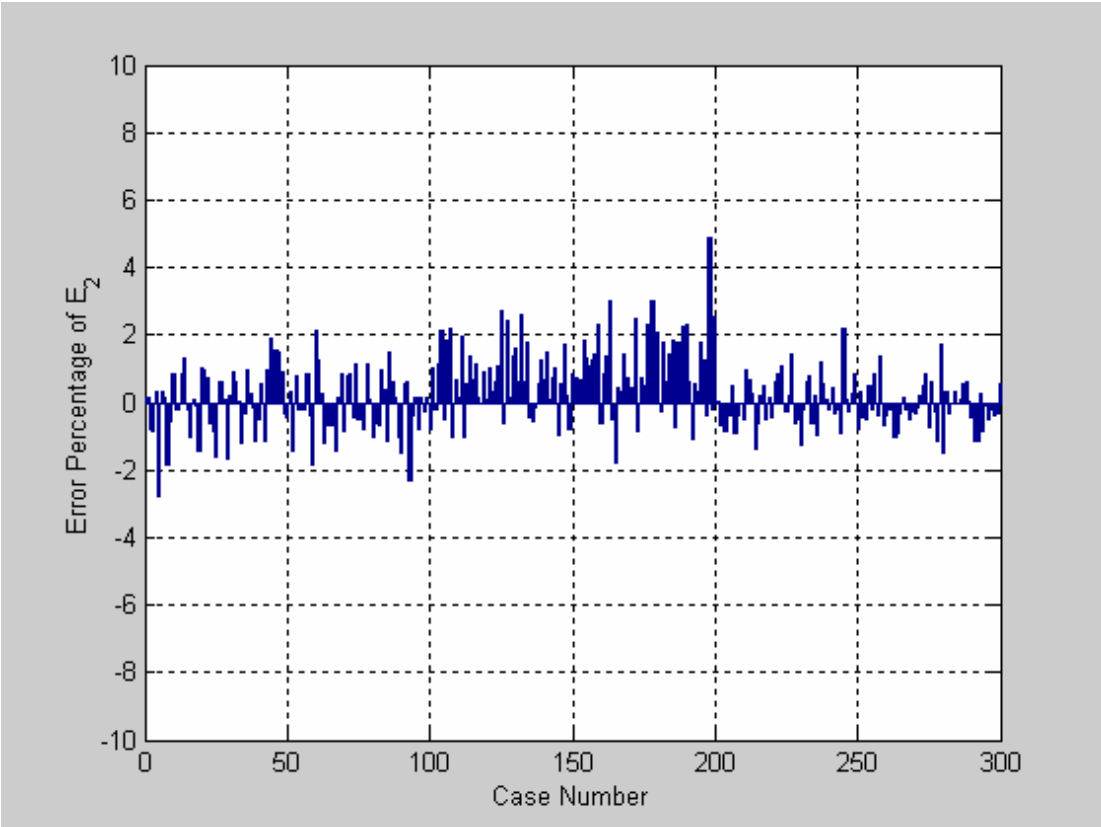
- Abs(E1 error) = 0.80712 %
- Std(E1 error) = 0.8627

$$\%Error = 100 \times \frac{E_{decomp} - E_{sim}}{E_{sim}}$$



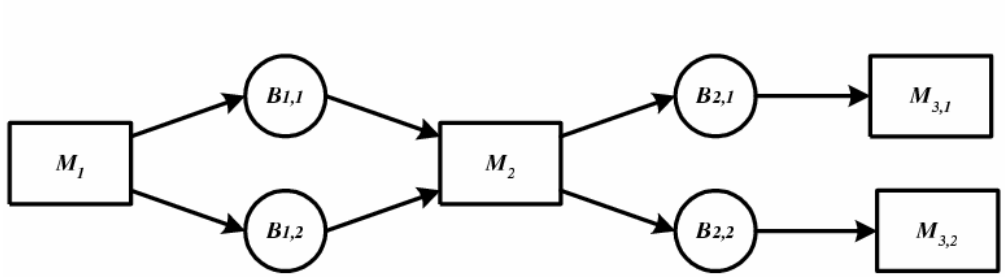
E_2 Error

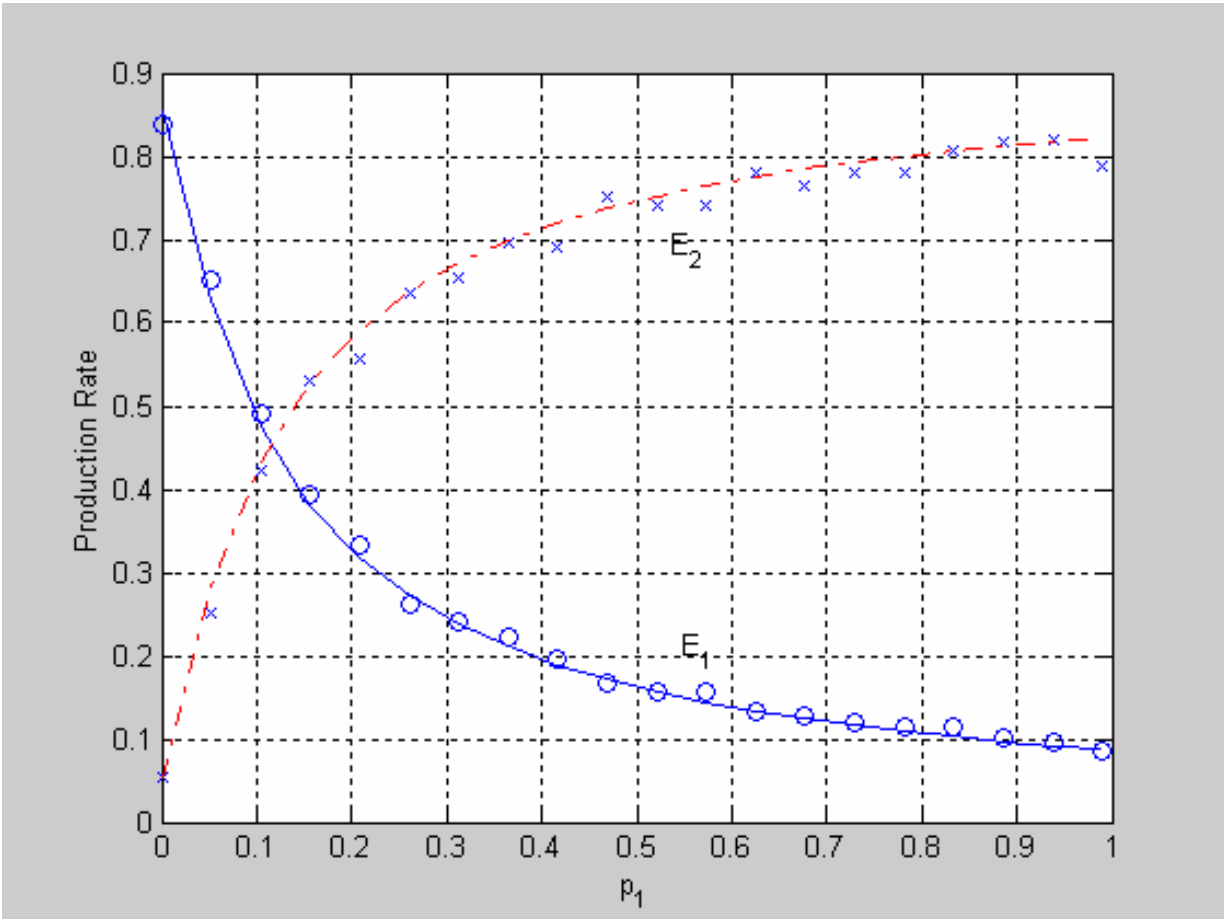
- Abs(E2 error) = 1.24 %
- Std = 0.8934



Case 1

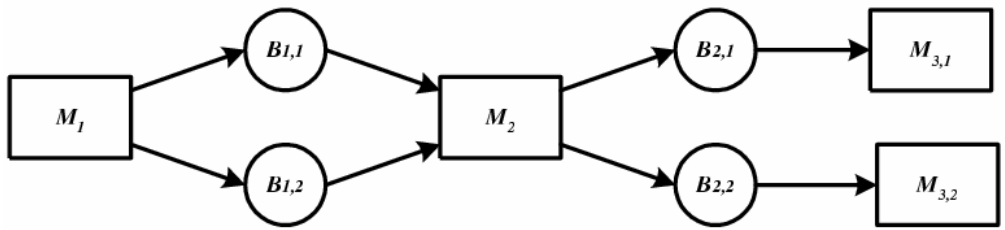
- Type one supply varies
- Reliable type two machines

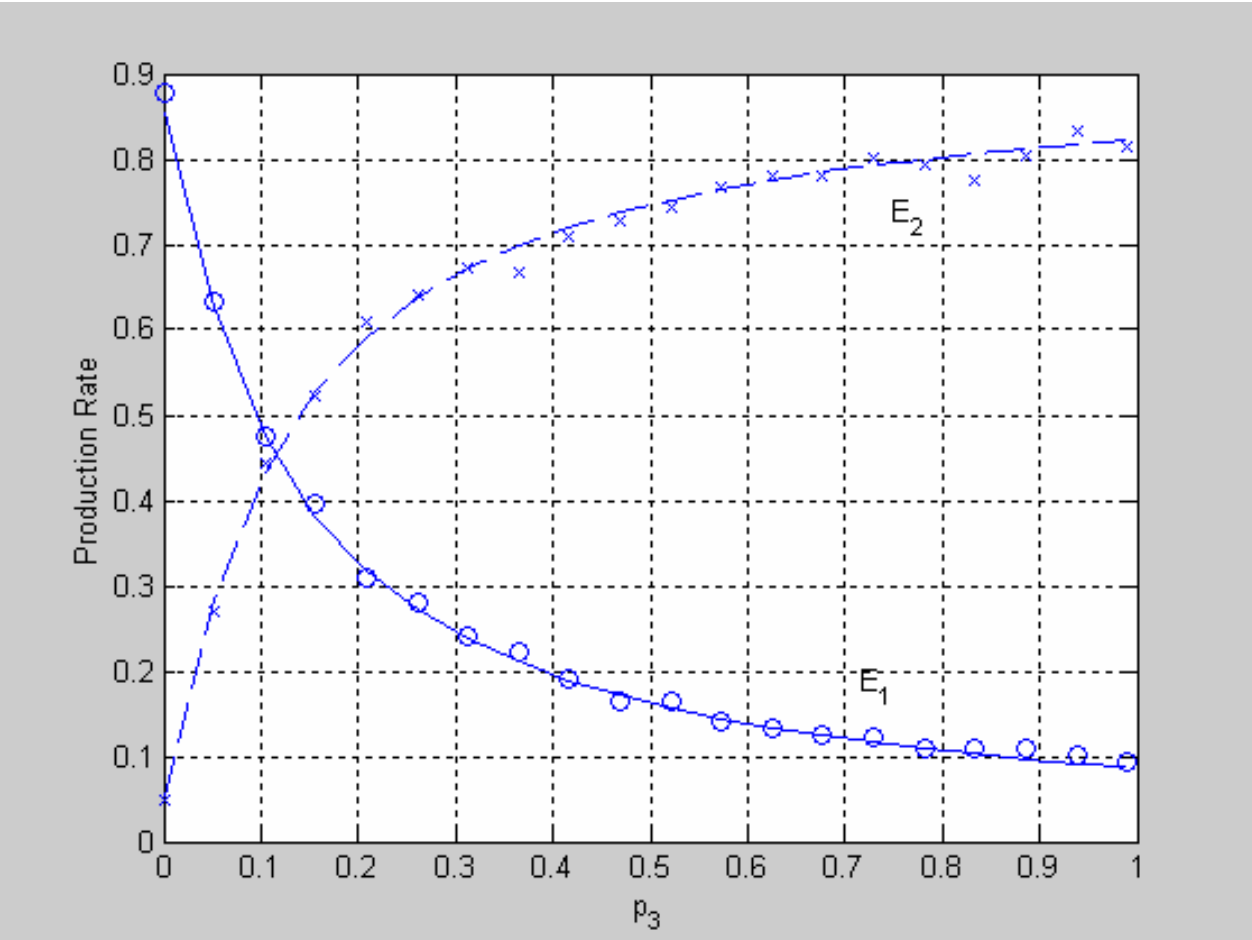




Case 2

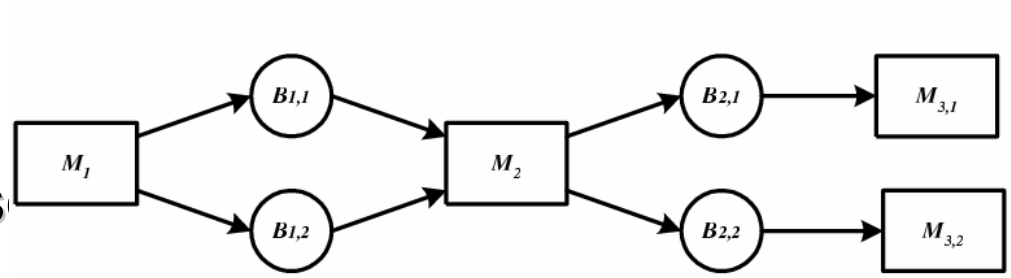
- Type two demand varies
- Reliable type two machines

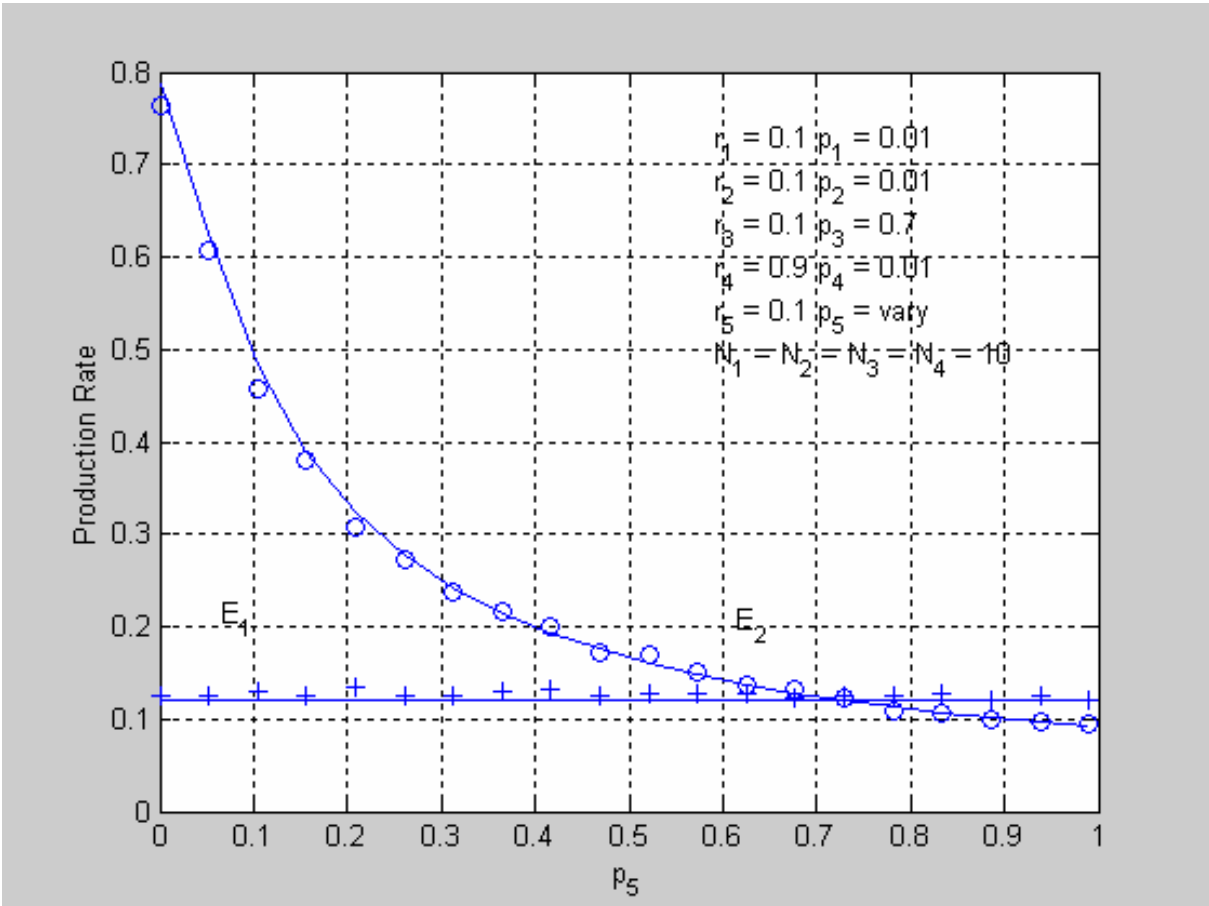




Case 3

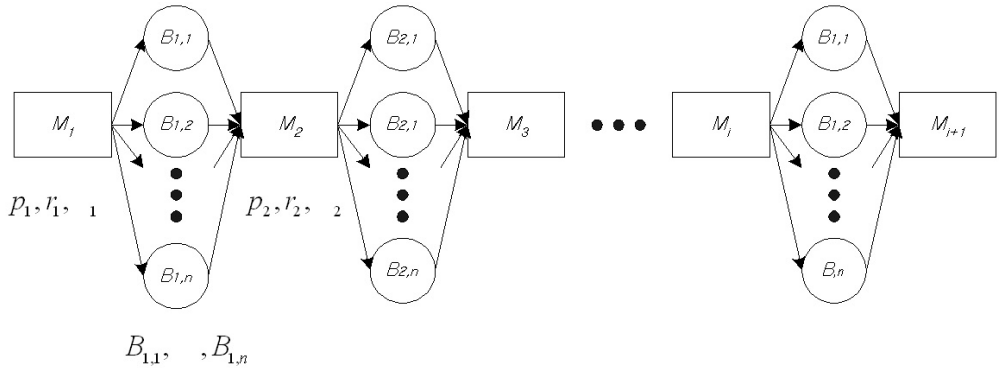
- Type 2 demand decreas

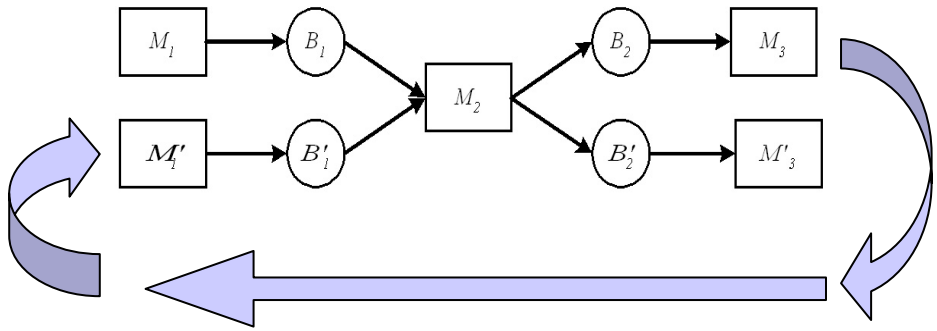




Future Work

- Expand Line to long processing line with five part type cases
- Applying re-entrance flow
- CPP with multiple-part type
- Continuous time line





Reference

- Stanley B. Gershwin, *Manufacturing Systems Engineering*, PTR Prentice Hall, 1994
- Stanley B. Gershwin, An efficient decomposition methods for the approximate evaluation of tandem queues with finite storage space and blocking. *Operations Research,* March 1987
- T. Tolio, S. B. Gershwin, A. Matta, Analysis of two-machine line with multiple failure modes, Technical report, Politecnico di Milano, 1998
- T. Tolio, A. Matta, A method for performace evaluation of automated flow lines, Technical report, Politecnico di Milano, 1998
- Diego A. Syrowicz, Decomposition Analysis of a Deterministic, Multiple-Part-Type, Multiple-Failure-Mode Production Line, Massachusetts Institute of Technology, SM Thesis 1998