22.05  Reactor Physics – Part One

**Course Introduction**

1. **Instructor:** John A. Bernard

2. **Organization:**
   - Homework (20%)
   - Four Exams (20% each; lowest grade is dropped)
   - Final Exam (3.0 hours) (20%)

3. **Text:**
   The text book for this course is: *Introduction to Nuclear Engineering*, 3rd Edition, by John Lamarsh. This covers basic reactor physics as part of a complete survey of nuclear engineering. Readings may also be assigned from certain of the books listed below:

   - *Nuclear Reactor Analysis* by A. F. Henry
   - *Introduction to Nuclear Power* by G. Hewitt and J. Collier
   - *Fundamentals of Nuclear Science and Engineering* by J. Shultis and R. Faw
   - *Atoms, Radiation, and Radiation Protection* by J. Turner
   - *Nuclear Criticality Safety* by R. Kneif
   - *Radiation Detection and Measurement* by G. Knoll

4. **Course Objective: To quote the late Professor Allan Henry:**

   “The central problem of reactor physics can be stated quite simply. It is to compute, for any time \( t \), the characteristics of the free-neutron population throughout an extended region of space containing an arbitrary, but known, mixture of materials. Specifically we wish to know the number of neutrons in any infinitesimal volume \( dV \) that have kinetic energies between \( E \) and \( E + \Delta E \) and are traveling in directions within an infinitesimal angle of a fixed direction specified by the unit vector \( \Omega \).

   If this number is known, we can use the basic data obtained experimentally and theoretically from low-energy neutron physics to predict the rates at which all possible nuclear reactions, including fission, will take place throughout the region. Thus we can...
predict how much nuclear power will be generated at any given time at any location in the region."

There are several reasons for needing this information:

- Physical understanding of reactor safety so that both design and operation is done intelligently.
- Design of heat removal systems.
- Fuel management.

There are several approaches to the estimation of the neutron population:

- Neutron Life Cycle Analysis: Used for design of the original reactors in the 1950s and early 1960s before computers were available. Very useful for physical understanding.
- One-Velocity Model: A form of diffusion theory in which all neutrons are assumed to have the same speed. Hence, it allows for geometrical and material, but not energy, effects. Useful for designing unreflected (bare) fast reactor cores.
- Diffusion Theory: Design tool for most existing PWRs/BWRs. Also called few-group theory or multi-group theory because the neutrons are treated as in distinct energy ranges or groups.
- Transport Theory: Methods for solving the Boltzmann transport equation (not covered in this course).
- Monte Carlo Methods: Design technique that is currently in use. Reactor is precisely modeled as to its material and spatial properties. Individual neutron case histories are projected using probability theory. Case histories are run until the statistics are sufficient to assure that an accurate picture of the overall neutron behavior has been generated. The technique is computer intensive.

The choice of an analysis technique is largely determined by the available computing power.

5. **Course Organization:**

The course is organized into four sections: 1) information on the fission process; 2) definitions of neutron flux and current in conjunction with development of the one-velocity model for analysis of both non-multiplying and multiplying mediums; 3) neutron diffusion theory and its use for the determination of the neutron flux’s shape and energy; 4) reactor kinetics and the achievement of safe reactor designs. We begin with certain useful background information.
6. **Nuclear Reactor Layout:**

Neutrons that are both cooled and moderated by light water are currently used for both the generation of electricity and the propulsion of naval vessels. Such reactors are designated as LWRs (light water reactors). There are two types: pressurized water (PWR) and boiling water (BWR). The figure on the next page is a 1D schematic of a PWR. The following factors should be noted:

- There are two cooling loops. The first is the “primary.” It consists of the core, the pressurizer, steam generator, and a pump. The fission reaction occurs in the reactor core. The energy (200 MeV/fission) released by this process is conducted through the fuel assemblies (fuel pellets contained in a clad material) to the coolant. The heated coolant exits the core and flows past the pressurizer to the steam generator. The section of pipe that forms this flow path is called the “hot leg.” The steam generator consists of an inlet plenum, a U-shaped tube bundle, and an outlet plenum. The two plenums are separated by a divider plate. Hence, the coolant flows through the tube bundle which consists of thousands of small diameter tubes. Upon exiting the steam generator, the coolant enters the “cold leg” section of the piping which contains the coolant pump. The coolant then reenters the core.

- The other PWR loop is termed the “secondary.” It consists of the steam generator, high and low pressure turbines, condenser, condensate pumps, booster pumps, main feed pumps, and a preheater. Heat from the primary coolant is transferred through the steam generator U-tubes thereby causing the secondary coolant to boil. It passes through steam separators that remove entrained liquid and then flows to the turbines and condenser. Condensate is then pumped back to the steam generator.

- Primary flow rate exceeds secondary flow rate by roughly a factor of ten. Yet, the same amount of energy is transported by both the primary and secondary loops. The reason is that there is a phase change in the secondary and the latent heat allows more energy to be absorbed per unit mass of coolant.

- The source of pressure in the primary is the pressurizer. This is a tank that is physically located above the primary loop. Water is allowed to boil in this tank so that there is a steam-liquid interface. The primary coolant itself is subcooled- that is, its temperature is below the boiling point. Pressure from the pressurizer is transmitted through the entire primary loop. To increase pressure, electric heaters are energized to boil more
Pressurized Water Reactor (PWR)
water in the pressurizer. To decrease pressure, a spray nozzle is opened to condense some of the steam in the pressurizer.

- The primary coolant pump’s function is to provide flow sufficient to offset friction losses as the coolant moves through the core and steam generator tube bundle.
- The source of spray to the pressurizer is the cold leg. The driving force for spray flow is the pressure drop across the core.
- Emergency cooling lines connect to the cold leg.
- The pressure drop across the thin walls of the steam generator’s U-tubes is roughly 1000 psi.
- The energy conversion cycle for the secondary is a Rankine cycle which consists of:
  - Heat absorption at constant temperature (steam generator).
  - Expansion at constant entropy (turbine).
  - Heat rejection at constant temperature (condenser).
  - Compression at constant entropy (pump).

Note: Students should be able to draw this diagram, know values for typical pressures and temperatures, the purpose of each major component, and be able to explain the basics of the energy conversion cycle.

In this course, we focus on the reactor core and, in particular, the neutron population.

7. **Reactor Classification:**

Henry (p. 27) defines a nuclear reactor as “an assembly of fissile and other materials that has been arranged in such a geometrical configuration and is of such concentrations that a sustained nuclear chain reaction occurs.” A “sustained nuclear chain reaction” means that the number of neutrons produced from the fission process equals the number lost from both leakage or absorption (including absorption in the fissile material). If this balance is exact, the reactor is said to be critical. If the neutron production exceeds losses, the reactor is supercritical. If neutron production is less than the loss rate, the reactor is subcritical. The power level is determined by the number of fissions. A reactor can be critical with ten neutrons being produced and ten neutrons being lost to leakage/absorption or with 10 billion neutrons being created and 10 billion neutrons being lost. The power level of the later is \(10^9\) times that of the former.
The reactor core is the fueled region. It consists of the fuel itself, a moderator which is used to adjust the energy of the neutrons and thereby promote the fission reaction, and a reflector which is used to minimize neutron leakage. (Note: Some reactors also have a blanket which is located between the moderator and the reflector. Blankets contain fertile material which on absorption of a neutron produces a fissile material. For example, Th-232 to U-233 or U-238 to Pu-239).

Reactors may be classified by:

- The energy range of the neutrons.
- The material constituents, either the phase (solid, liquid, gas) or the purpose.
- The reactor’s purpose.
- The reactor’s generation or time of design.

a) **Energy Range:** Neutrons are classified by reactor physicists as thermal, epithermal, or fast. There are no fixed definitions for these ranges. One possible scheme is:

<table>
<thead>
<tr>
<th>Neutron Range</th>
<th>Energy Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal</td>
<td>Below 1eV</td>
</tr>
<tr>
<td>Epithermal</td>
<td>1 eV &lt; E &lt; 50 keV</td>
</tr>
<tr>
<td>Fast</td>
<td>50 keV &lt; E &lt; 15 MeV</td>
</tr>
</tbody>
</table>

**Note:** 15 MeV is the upper range of the fission spectrum. Other definitions are often given, for example 10 keV – 30 keV for epithermal. Fission reactors have been designed to operate in both the thermal and the fast ranges. Present day light water reactors (PWRs, BWRs) are thermal. They have large cores of slightly enriched U-235 with moderation provided by light water. Spacecraft reactors operate on a fast spectrum which has the advantage of minimizing mass (no moderator). No reactors operate in the epithermal range for reasons to be discussed when neutron cross-sections are considered. However, fission converters, which convert a thermal flux to a fast one and then filter out the thermal and fast neutrons, are in use to produce epithermal beams for medical applications. Such beams are used in neutron capture therapy to treat tumors, particularly ones of the brain. The epithermal neutrons penetrate the intact skull thereby delivering dose to the tumor. Thermal neutrons are not wanted because they create dose to the scalp while not providing any therapy. Similarly, fast ones are not wanted because they would penetrate too deep and generate unnecessary dose to healthy tissue.

b) **Constituents:**

- **Fuel enrichment:** Natural uranium is 0.7% U-235. The Atomic Energy of Canada (AECL) CANDU reactor which stands for
Canadian Deuterium Uranium uses natural uranium. Most electricity-generating reactors use slightly-enriched (2-3% U-235) material. Research and test reactors and spacecraft reactors, which have small cores, use enriched material, sometimes as high as 93%. (Note: For proliferation reasons, an enrichment of 20% or less is preferred.)

- **Moderators:** Moderators should offer high neutron scattering cross-sections, low neutron absorption cross-sections, and high energy losses per collision. A figure of merit for a moderator is:

\[ M.R. = \frac{\zeta \Sigma_s}{\Sigma_a} \]

where M.R. is the moderator ratio, \( \zeta \) is a measure of the energy loss per collision, \( \Sigma_s \) and \( \Sigma_a \) are the scattering and absorption cross-sections. Other desired attributes are high density, chemical stability, and resistance to radiation damage. Typical choices are light water, heavy water, graphite, and beryllium.

- **Coolants:** Coolants should have neutronic properties similar to moderators. They may be either gaseous or liquid. If liquid, they should have low melting points and high boiling points. Light water, helium, carbon dioxide, liquid sodium, and lead-bismuth are common choices. (Note: As will be explained later in this course, the use of the same material for both the coolant and the moderator offers certain safety advantages. The choice of moderator and coolant are therefore interconnected)

- **Other:** Descriptions may also be given in terms of the type of cladding used in the fuel, the type of fertile material (if any), the type of control material (boron, Ag-In-Cd, hafnium, etc.)

  c) **Purpose:** The most widely recognized purpose is to generate electrical power. But other purposes exist including propulsion (military vessel, spacecraft), research/test, and production reactors. Propulsion reactors are used for submarines and other naval vessels. Space reactors have been launched by the former Soviet Union and are under design by the United States. (In 1965 the U.S. Air Force launched a SNAP-10A reactor into orbit which was part of a series of fission reactors and radioisotope power systems that were developed under the Atomic Energy Commission’s SNAP (Systems for Nuclear Auxiliary Power) program. The SNAP-10A reactor was a small zirconium hydride (ZrH) thermal reactor fueled by Uranium-235. This reactor suffered a sensor failure after only 44 days and
was shutdown. To date, it remains the only one ever launched by the U.S. The former Soviet Union launched many TOPAZ reactors. The U.S. has preferred radioisotope generators.) Research reactors are used worldwide for trace element analysis, nuclear medicine, geologic studies, materials studies, etc. Test reactors are used to evaluate new fuels. Production reactors, a vanishing type, are built to convert one material to another (e.g., U-238 to Pu-239).

d) **Generation:** Reactors are classed into four generations which are:

<table>
<thead>
<tr>
<th>Generation #</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Original reactors built as proof of principle in the 1950s.</td>
</tr>
<tr>
<td>II</td>
<td>Present day light water reactors, CANDUs, etc., that were designed in the 1960s.</td>
</tr>
<tr>
<td>III</td>
<td>Passively safe reactors designed after the Three Mile Island accident.</td>
</tr>
</tbody>
</table>
| IV           | Reactors that are capable of producing hydrogen. Six designs are reviewed in the U.S. Information on Generation IV reactors may be found on the U.S. Department of Energy’s website. The six candidates are:

- Very high temperature reactor (VHTR)
- Supercritical water cooled (SCWR)
- Gas-cooled (GFR)
- Lead cooled (LFR)
- Sodium cooled (SFR)
- Molten Salt (MSR)

The VHTR and SCWR are both thermal reactors intended to operate at very high temperatures thereby improving the efficiency of electricity production and enabling hydrogen production. The GFR, LFR, and SFR are all fast reactors intended to improve the recycling of actinides (major component of high level waste). The United States is not active in the MSR effort.

There is one other advanced concept which is not part of the Gen-IV effort but which merits mention. It is termed “multi-modular.” The idea
originated in the United States and is now being further explored in Argentina. The idea is for 4 to 6 small nuclear PWR units to supply steam to a common secondary. This concept offers several economic advantages: 1) small PWRs have an excellent operating record; 2) small PWR components can be factory-built and shipped to the site for rapid assembly; 3) the common secondary offers economies of scale; and 4) outages for maintenance and refueling could be staggered. There are open issues, though, including identifying an optimal refueling schedule and operation with unbalanced loads. The latter means that one unit might be run at 100%, another at 80%, another at 60% and so on so as to stagger refuelings. But then each has to be at a different temperature in order to supply steam at a common pressure.

8. **Chart of Nuclides:**

Most students are familiar with the periodic table from chemistry courses. Elements in a given column all have the same configuration of outer shell electrons and hence the same chemical behavior. The Chart of Nuclides differs in that it cannot be used to infer trends or properties. It is merely a very convenient compilation of the known isotopes. Note the following:

- The vertical axis is the number of protons, the horizontal axis is the number of neutrons. Each nuclide has its own box with its properties listed therein.
- The color gray denotes stable nuclides.
- A black horizontal bar at the top of a box indicates a naturally occurring radioactive nuclide. For example, tritium (T-3), carbon-14 (C-14), potassium-40 (K-40).
- A black triangle in the lower right corner of a box indicates a fission product.

The number of protons in the nucleus is called the “atomic number” and is given the symbol Z. The number of protons plus the number of neutrons is called the “mass number” and is given the symbol A. Thus, a given nuclide is uniquely described by the symbol:

\[ _z^A X \]

Where X is the chemical identity (e.g., iron, lead, uranium). With the exception of hydrogen (\(_1^1 H\)) and a rare form of helium \(\left( \frac{3}{2} \text{He} \right)\), all nuclides have \( A \geq 2Z \). Reasons for this are discussed below.
The Chart of the Nuclides facilitates the writing out of both radioactive decay processes and nuclear transformation reactions. For a complete explanation of these processes, the reader is referred to Atoms, Radiation, and Radiation Protection, by J. B. Turner, Wiley. We note here some material of direct use to the study of reactor physics.

- **Neutron Absorption:** \( ^{1}n + ^{A}ZX \rightarrow ^{A+1}ZX + ^{0}_{0}\gamma \)

Here a neutron (zero charge, mass of one) reacts with the nuclide X to produce an “isotope” of that nuclide and a gamma ray (no charge, no mass). An isotope is defined as a nuclide having the same number of protons but a different number of neutrons. A practical example would be:

\[
^{1}n + ^{135}_{54}Xe \rightarrow ^{136}_{54}Xe + ^{0}_{0}\gamma
\]

On the Chart of Nuclides, neutron absorption moves the original nuclide one box to the right.

- **Beta Decay:** \( ^{A}_{Z}X \rightarrow ^{0}_{-1}e + ^{A}_{Z+1}Y + ^{0}_{0}\gamma + Q \)

Beta decay is the process whereby a neutron is converted to a proton and an electron with the electron being ejected from the nucleus. In the process, an anti-neutrino is also produced. The symbol used to describe the nuclide is changed from X to Y because the number of protons (and hence the number of electrons in the neutral atom) has changed. Recall that the number of electrons determines chemical identity. The symbol \( Q \) denotes the energy released in the process. This energy is divided between the electron, the new nucleus, and the neutrino. A practical example would be:

\[
^{135}_{53}I \rightarrow ^{0}_{-1}e + ^{135}_{54}Xe + ^{0}_{0}\gamma + Q
\]

Fission products, which are neutron rich, often undergo beta decay in order to decrease their neutron-to-proton ratio.

On the Chart of Nuclides, beta decay moves the original nucleus diagonally up to the left by one box.

- **Alpha Decay:** \( ^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}He + ^{A-4}_{Z-2}Y + Q \)

A practical example would be:
\[ ^{239}_{94}\text{Pu} \rightarrow ^{4}_{2}\text{He} + ^{235}_{92}\text{U} + Q \]

On the Chart of the Nuclides, an alpha decay moves the original nuclide diagonally down to the left by 2 boxes. The Q value for alpha decay is of special importance in reactor engineering because alpha particles have very short ranges. Hence, the released energy is locally deposited and heat removal is often necessary.

- **Positron Emission:** \[ ^{A}_{Z}\text{X} \rightarrow ^{0}_{+1}\text{e} + ^{A}_{Z-1}\text{Y} + ^{0}_{0}\gamma + Q \]

Positron emission, also called beta plus decay, is the process whereby a proton is converted to a neutron and a positron with the positron being ejected from the nucleus. In the process, a neutron is produced. A practical example would be:

\[ ^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Na} + ^{0}_{+1}\text{e} + ^{0}_{0}\gamma + Q \]

Isotopes that are produced in accelerators often are positron emitters because they are proton rich.

On the Chart of Nuclides, beta plus decay moves the original neutrons diagonally down to the right by one box.

- **Orbital Electron Capture:** \[ ^{A}_{Z}\text{X} + ^{0}_{-1}\text{e} ightarrow ^{A}_{Z-1}\text{Y} + ^{0}_{0}\gamma + Q \]

In this reaction, the neutron captures an electron from the innermost shell (K-shell). The process is equivalent to positron decay and is favored when positron decay is not possible for reasons of energy conservation (see Turner pp. 72-74). An example is:

\[ ^{103}_{46}\text{Pd} + ^{0}_{-1}\text{e} \rightarrow ^{103m}_{45}\text{Rh} + ^{0}_{0}\gamma + Q \]

The Chart of the Nuclides can also be used to infer much about the fission process. To see this, plot the number of protons (vertical axis) versus the number of neutrons (horizontal axis) for some of the stable (gray color) nuclides. For low Z, the nuclides will fall on the 45 degree line because Z, which is the number of protons, equals (A-Z) which is the number of neutrons. For intermediate and high Z, the stability line lies below the 45 degree line. This indicates that stable nuclides have an excess of neutrons. Why? The four known forces of nature are gravity, electromagnetic, weak (responsible for radioactive decay), and strong. The electromagnetic and strong forces are the determining ones in a given nuclide. Their characteristics are:
Saturation implies attraction by nearest neighbors only. Hence, even if two nucleons are sufficiently close to one another that the strong force is active, they may not be attracted to each other because of saturation.

Suppose a proton is to be added to an existing nuclide with atomic number Z. It will be repelled by every one of those existing Z protons because the electromagnetic force is long-range, and because that force does not saturate. The added proton will be attracted via the strong force, which is charge independent. However, that attraction will only be by those nuclides (neutrons/protons) that are nearest to it. So, repulsion exceeds attraction. Now suppose a neutron is to be added. There is no repulsion by the electromagnetic force because it is electrically neutral. There is attraction by the strong force with the nearest neighbors. So, it is possible to add the neutron.

Now consider what happens when a U-235 nucleus fissions. The two fission products have neutron to proton ratios similar to that of U-235 but their atomic numbers are much less than 92. So, these fission product nuclides lie below the line of stability. In order to move toward the stability line (i.e., the line formed by the gray-colored nuclides on the plot of proton versus neutron number), the nuclide needs to decrease its neutron population and increase its proton population. What process does this? Beta decay. So, we expect and observation confirms that fission products often undergo beta decay. Moreover, if that beta decay leaves the daughter nucleus (the one denoted as \( ^{A}_{Z+1}X \) above) in an excited state, that daughter may then emit a neutron. Such neutrons are given the special name “delayed” with the delay being the time for the beta decay to occur. In contrast, neutrons produced directly from fission are called “prompt.” The fission products that give rise to the delayed neutrons are called “precursors.” The formal definition is that a precursor is a fission product that undergoes a beta decay to a daughter nuclide that then emits a neutron. Delayed neutrons constitute a very small fraction, typically 0.65 %, of the total neutron population in a reactor. But they are essential to safety, and will be the subject of extensive study later in the course.