22.05 Reactor Physics – Part Three

Useful Tools

1. Maxwell – Boltzmann Distribution:

The energy distribution of the prompt neutrons at the time of fission has been described. It is also necessary to describe neutron energy distributions at other times such as when the neutrons attain thermal energies.

The number density of neutrons in a reactor core is very low, even at high power levels, when compared to that for normal matter. Hence, it is possible to describe the neutron population as a very dilute gas. This is turn makes the Maxwell-Boltzmann distribution of use. In a gas, the energies of the atoms or molecules are distributed according to the Maxwellian distribution function. Thus, if N(E) dE is the number of particles per unit volume having energies between E and E + dE, then N(E) is given by the relation:

$$N(E) = \frac{2\pi N}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT}$$

Where N is the total number of particles per unit volume and k is Boltzmann's constant, which has units of energy per degree Kelvin:

$$\begin{aligned} k &= 1.3806 \times 10^{-23} \ joule/°K \\ &= 8.6170 \times 10^{-5} \, eV/°K \end{aligned}$$

T is the absolute temperature of the gas in degrees Kelvin.

The *most probable energy* is defined as that corresponding to the maximum of the curve. This is obtained by taking the derivative of N(E) with respect to temperature and then setting the result equal to zero. Upon so doing, one obtains:

$$E_p = \frac{1}{2} kT$$

The *average energy*, $\overline{E}$ is defined by the integral

$$\overline{E} = \frac{1}{N} \int_0^{\infty} N(E) E \, dE$$

or

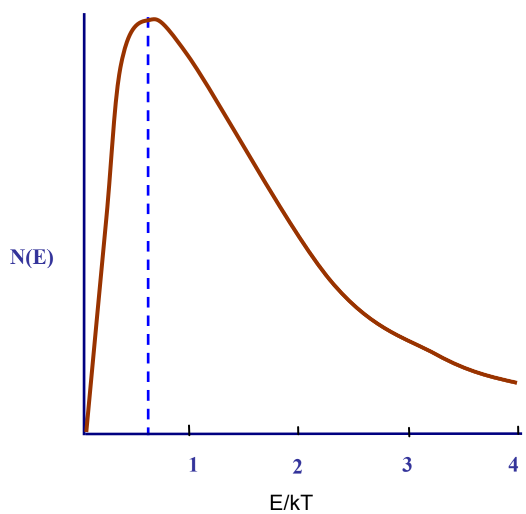
$$\overline{E} = \frac{3}{2}kT$$

Note that the average and most probable energies are not the same. This happens because the distribution is not symmetric. The most probable is at the peak; the average is higher because of the tail that extends to high energies.

The quantity kT often appears in nuclear applications. Its value is:

$$kT_0 = 0.0253 eV \cong 1/40 eV \text{ for } T_0 = 293^\circ K \text{ (room temperature)}$$

The energy (0.025 eV) is defined as that of a thermal neutron.



Maxwellian Distribution Function

2. <u>Number Density</u>:

Many nuclear engineering calculations require knowledge of the number of atoms per unit volume. Examples are fuel depletion and reaction rate calculations. This capability is needed for elements, molecules, and mixtures.

a) <u>Elements</u>: If ρ is an element's physical density in g/cm^3 and M is its gram atomic weight, there are ρ/M gram moles of the element in 1 cm$^{3.}$ Because each gram mole contains N_A atoms, where N_A is Avogadro's number, the atom density N, in atoms per cm^3, is

$$N = \frac{\rho N_A}{M}$$

For example, the density of carbon is 1.6 g/cc. Its atomic weight is 12.0. Its number density is:

$$\mathrm{N} = \frac{1.6 \times 6 \times 10^{23}}{12.0} = 1.33 \times 10^{22}$$

b) Molecules: Same as above except use the molecular density and the gram molecular weight. For example, the density of water is 1.00 g/cc. It contains 2 hydrogen and 1 oxygen atoms, so its molecular weight is 18. Its number density is:

$$\mathrm{N} = \frac{1.0 \times 6.02 \times 10^{23}}{18.0} = 3.34 \times 10^{22}$$

Sometimes the number density of a particular atom in the molecule is desired. This is obtained by multiplying the molecule's number density (as obtained above) by the number of the atoms in question per molecule. For example, there are two hydrogen atoms per water molecule. Hence, the number density of hydrogen in this example is $2(3.34 \times 10^{22})$ or 6.68×10^{22} atoms/cc.

c) Isotope: Isotopic abundances are given in atom percent. Hence, the number density of a particular isotope is the atomic density of the element multiplied by the atomic percent of the isotope. Thus,

$$\mathrm{N_i} = \frac{\gamma_i \rho \mathrm{N_A}}{\mathrm{M}}$$

Where γ_i is the isotopic abundance and M is the molecular weight of the element. For example, the density of uranium is 19.05 g/cc. The molecular weight of natural uranium (0.7% U-235 and the rest U-238) is

$$.007(235) + (1 - .007)(238) = 237.98$$

The number density of the U-235 is:

$$\frac{.007 \text{ x } 19.05 \text{ x } 6.02 \text{ x} 10^{23}}{237.98} = 3.45 \text{ x } 10^{20}$$

d) Alloys: Each constituent is usually specified by its percent weight. Hence, if ρ is the physical density of the mixture, the average density of the i^{th} component is:

$$\rho_i = \frac{w_i \rho}{100}$$

Where w_i is the weight percent of the i^{th} component. The number density of the i^{th} component is:

$$N_i = \frac{w_i \rho N_A}{100 M_i}$$

Where M_i is the gram atomic weight of the i^{th} component.

Suggested reading: Section 2.13 of Lamarsh, p. 37, and in particular, example 2.15.

3. Definition of the Electron-Volt:

The electron-volt is a unit of energy. It is defined as the kinetic energy acquired by an electron when that electron is accelerated through a potential difference of 1 Volt.

$$1 \text{ eV} = 1.6 \text{ x } 10^{-19} \text{ J}$$

4. Mathematical Functions:

The student should be familiar with certain mathematical functions including:

- Exponential
- Trigonometric (sine, cosine)
- Hyperbolic (sinh, cosh)
- Bessel Function

In particular, the student should be able to draw the shape of each of these.

5. **Differential Equations:**

The student should be able to solve ordinary differential equations by a variety of methods and to solve partial differential equations via separation of variables.