22.05 Reactor Physics – Part Four

Cross-Sections for Neutron Reactions

1. Interactions:

Cross-sections deal with the measurement of interactions between moving particles and the material through which they pass. Electrons, protons, neutrons, photons, etc., all have cross-sections. A definition of a cross-section is developed here by explaining how cross-sections are measured.

2. Good Geometry Experiment:

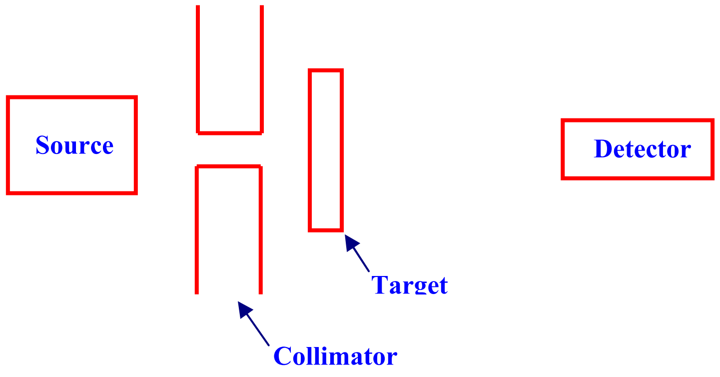
This experiment is applicable to any type of particle – not merely neutrons.

Assumptions:

(1) Monoenergetic (single energy), collimated beam of particles with an initial intensity of I_o particles/cm^2 s;

(2) Target has a number density N and is thin. That is, its thickness, Δx, is small enough so that no atom (or nucleus) obstructs another and so that no multiple interactions occur;

(3) Detector is located far from the target and "sees" only the uncollided intensity.



The premise of assumption (2) that no multiple interactions occur means that the target has only a few layers of atoms.

Measurement: We measure the beam intensity that is transmitted through the target. These are the uncollided particles, the ones that did not interact. Denote this as I.

3. Cross-section:

We define a cross-section for the type of particle in question at the energy of the beam and for the target material as:

$$\sigma \equiv \left[\frac{I_o - I}{I_o}\right]\left[\frac{1}{N\Delta x}\right]$$

$$= \left[\begin{array}{l}\text{Fraction of Beam} \\ \text{that Interacted}\end{array}\right]\left[\frac{\text{cm}^2 \perp \text{Beam}}{\text{Atom}}\right]$$

The units of cross-section are “barns” with 1 barn equal to 10^{-24} cm^2.

So, σ, is the measure of the fraction of particles that have interacted as the beam passes a distance Δx through the target. It is a function of particle type, particle energy, and target material.

σ is known as a “microscopic” cross-section. Its units are those of area and it can be likened to an effective target area. But, it is <u>not</u> the actual size of the target atom or nucleus.

It is a measure of the probability that a given reaction will take place between an incident particle and a single target atom or nucleus.

4. Reaction Rate:

I(x) is the intensity of the beam that has not undergone a collision after penetrating a distance x into the target. If the beam traverses an additional distance Δx then its intensity will decrease by $\Delta I(x)$. Thus

$$-\Delta I(x) = \sigma N I(x) \Delta x$$

Or, as Δx becomes small,

$$-dI(x) = \sigma N I(x) dx$$

This equation can be integrated to give:

$$\begin{aligned} I(x) &= I_o e^{-\sigma N x} \\ &= I_o e^{-\Sigma x} \end{aligned}$$

Where $\Sigma \equiv \sigma N$. Remember that I(x) is the intensity of the uncollided flux that emerges from the target.

5. **Macroscopic Cross-Section:**

The quantity Σ, which is the product of the microscopic cross-section and the number density of the target, is defined as the macroscopic cross-section. It has units of inverse distance.

Its physical meaning follows from:

$$-dI(x) = \sigma N I(x) dx$$

or

$$\frac{-dI(x)}{I(x)} = \Sigma dx$$

The quantity on the left is the probability that a particle that survives up to x will interact in the next dx. So, Σ is the probability per unit path length of an interaction.

Nuclear engineers use the symbol Σ**.** Physicists describe the exact same process and use the symbol μ which is termed a "linear attenuation coefficient." Thus, in describing radiation effects, one sees equations of the type:

$$\mathrm{I_o} = \mathrm{I_o e^{-\mu x}}$$

While in reactor physics applications, one notes

$$I(x) = I_o e^{-\Sigma x}$$

6. **Buildup Factors**:

Cross-section measurements are made under conditions of good geometry. However, most realistic scenarios include "bad" geometry in which the target is thick and a person or an instrument is located close to the far side of the target. Under such conditions, the relation

$$I(x) = I_o e^{-\Sigma x}$$

greatly underpredicts beam intensity and hence associated radiation levels. The reason is that many particles undergo multiple collisions and some of those that are initially removed from the beam are scattered back into it. This is addressed by use of "build-up factors" which are functions of Σ and x. Thus, we have:

$$\mathrm{I(x) = B(\Sigma, x) I_o e^{-\Sigma x}}$$

This is a transcendental equation that is generally solved by iteration. Buildup factors are mostly empirical and tabulations exist for various geometries.

7. **Types of Nuclear Reactions**:

Most reactions involving neutrons occur in two steps. First a compound nucleus forms and second the compound nucleus decays via one of several possible methods including elastic scattering, inelastic scattering, radiative capture, charged particle emission, or fission.

The compound nucleus results from the incident neutron being swept into the potential well of the existing nucleus. As a result, the compound nucleus gains energy equal to the sum of the incident neutron's kinetic energy and the binding energy associated with adding the incident neutron to the nucleus. (The latter is sometimes referred to as the separation energy because it equals the energy that would have to be put into a nucleus to remove or separate a neutron).

Compound nuclei are most likely to form if the energy associated with the incident neutron (kinetic plus binding) equals that needed to form an excited nuclear state. Under this condition, a resonance is said to exist and the cross-section takes on a maximum.

A nuclear reaction involves a collision between a neutron and a target nucleus. In all cases, energy and momentum are conserved. In some cases, an additional quantity, kinetic energy, is conserved. Such collisions are called elastic. As an illustration, if two objects collide and the collision results in the emission of sound or heat, total energy has been conserved but not kinetic energy. Some of the incident particle's kinetic energy has been transformed to heat or sound. But if there is no sound or heat (or any other form of energy produced) then kinetic energy is conserved. The colliding particles may have different speeds before and after the collision but the sum of the mv^2 terms for each particle before the collisions will equal that after the collisions. Such collisions are called "elastic."

a) **Elastic Scatter:** Elastic scattering implies that kinetic energy (mv^2) is conserved. This means that the compound nucleus is not left in an excited state. Cross-sections for elastic scattering can be divided into three regions:

(i) Low Energy (Potential Scattering): There is no formation of a compound nucleus. The scattering is the result of forces exerted on the neutron as it passes near the nucleus. The cross-section is given by

$$\sigma_s = 4\pi R^2$$

Where R is the radius of the nucleus.

(ii) Intermediate Energy (Resonance Region): Compound nuclei are formed.

(iii) High Energy (Smooth Region): The resonances can be longer be resolved so the cross-section appears smooth.

The resonance region starts at lower energies as the mass of the target nuclei increases. This is because heavier nuclei have more possible excited states.

Example: Cross-section of C-14.

b) **Inelastic Scatter:** The cross-section is zero until the incident neutron can provide sufficient energy to raise the target nucleus to its first excited state. This occurs earlier for heavy nuclei.

c) **Radiative Capture (n,γ)**: This cross-section may also be divided into three regions.

(i) Low Energy (1/v region): The cross-section varies inversely with the square root of the neutron's kinetic energy. Given that

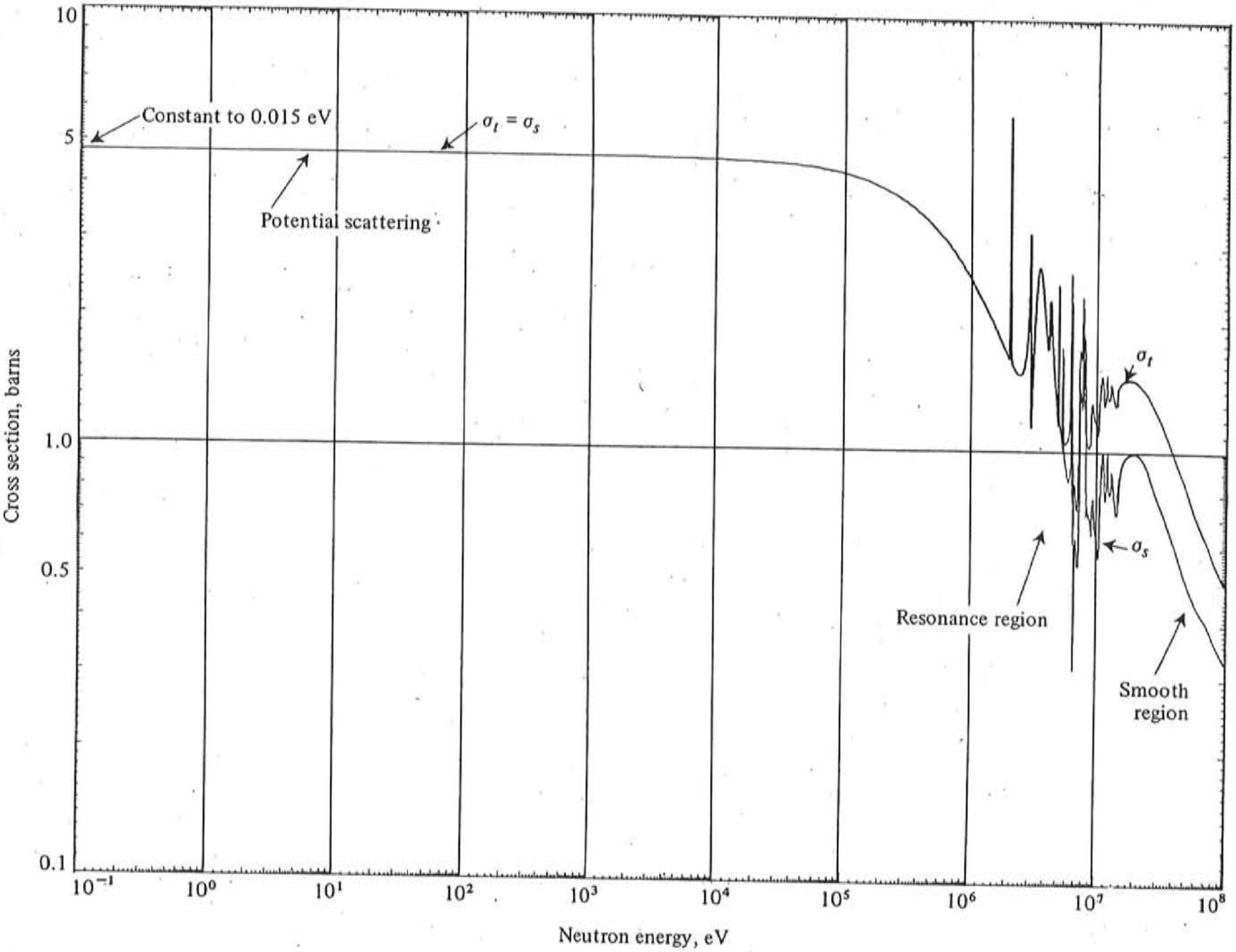


Fig. 3.3 The elastic scattering and total cross section of ^{12}C. (Plotted by machine from data on tape at the National Neutron Cross Section Center, Brookhaven National Laboratory.)

From Lamarsh.

$mv^2/2=E$, the cross-section therefore varies inversely with v, the neutron velocity, so this region is called "1/v."

(ii) Intermediate Energy (Resonance Region): This is the region of resonances.

(iii) High Energy (Smooth Region): The cross-section becomes smooth and is usually quite small.

Example: Cross-section of Au-197.

d) **Charged Particle**: These reactions are endothermic because of the energy needed to expel the charged particle. Hence, the cross-section is zero below some threshold and usually small above it. (Note: There are some very important exceptions to the rule that charged particle reactions are endothermic. The three reactions that were noted in Section Two of these notes for neutron detection are examples of exothermic charged-particle reactions that occur in light nuclei. The three reactions are:

$$^{10}B(n,\alpha)^{7}Li$$

$$^{6}Li(n,\alpha)^{3}H$$

$$^{3}He(n,\rho)^{3}H$$

e) **Fission**:

- The fission cross-section for U-235 is as described for the radiative capture process. Three regions are apparent: 1/v, resonance, and a smooth low value at high energy.
- For U-238, the fission cross-section is zero below a threshold.

Example: U-235 and U-238 cross-sections.

8. Total Cross-Section:

At low energies, cross-sections can be described by a formula of the type:

$$\sigma_T = 4\pi R^2 + C/\sqrt{E}$$

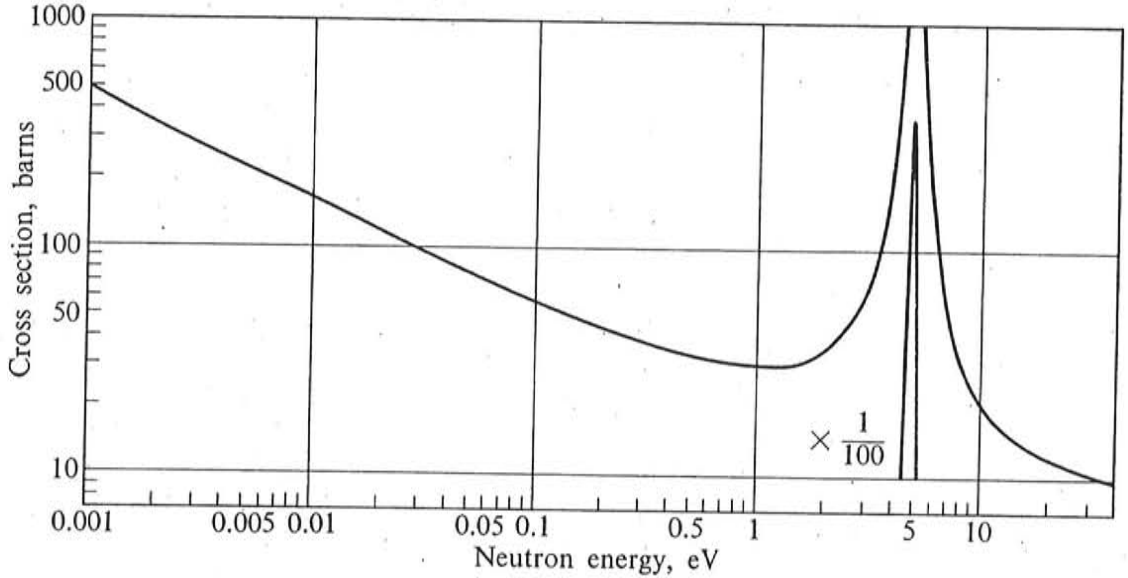


Fig. 3.4 The radiative capture cross section of ^{197}Au at low energy. (From *Neutron Cross Sections,* Brookhaven National Laboratory report, BNL-325, 2nd ed., 1958.)

From Lamarsh.

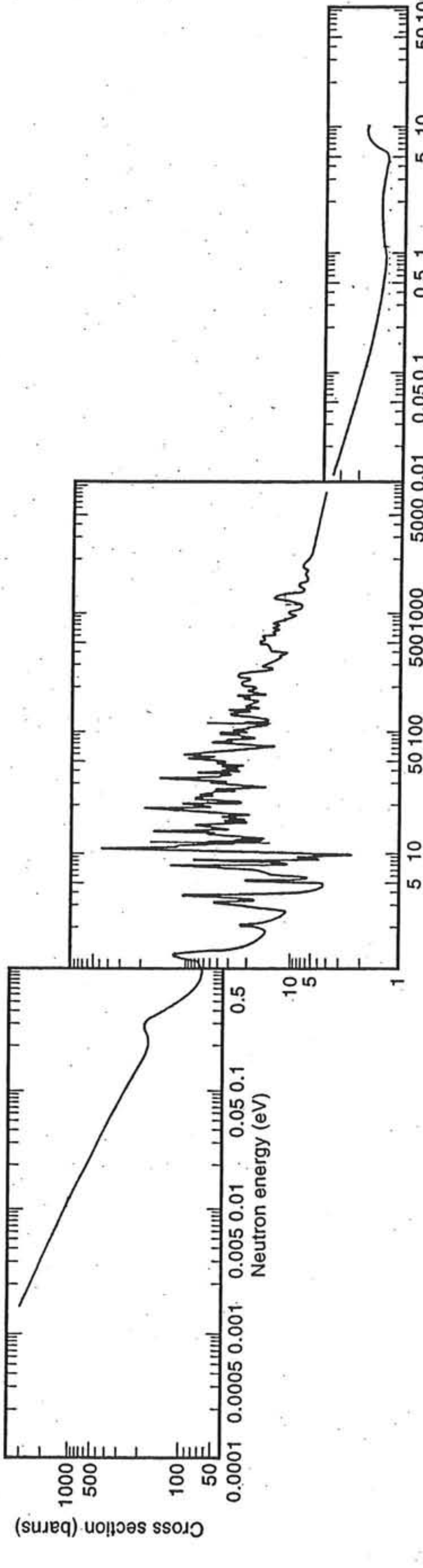


Figure 1. Microscopic cross section of ^{235}U. Source: BNL-325.

From Henry, A. F. *Nuclear Reactor Analysis*. Cambridge, MA: MIT Press, 1975.
Courtesy of MIT Press. Used with permission.

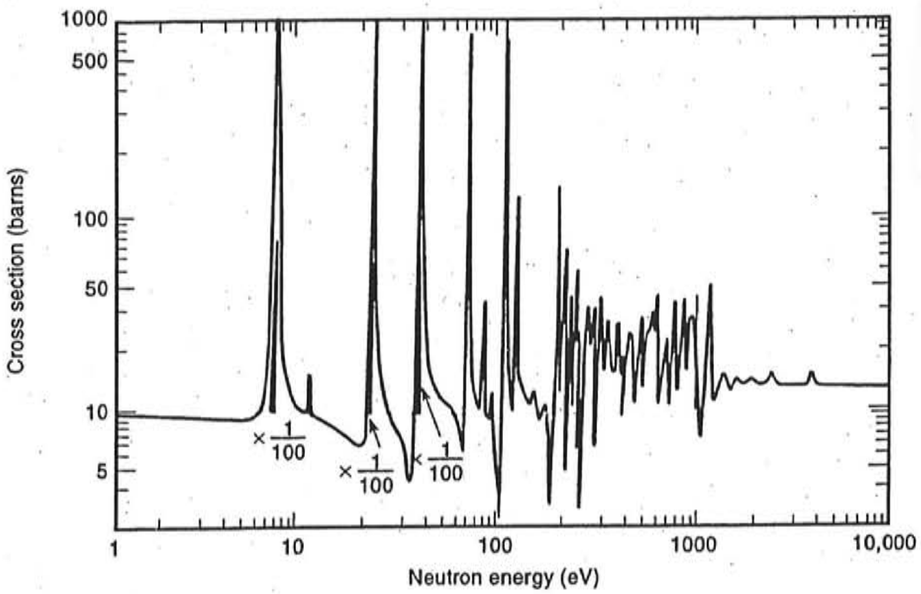


Figure 2. Microscopic cross section of ^{238}U. Source: BNL-325.

From Henry, A. F. Nuclear Reactor Analysis. Cambridge, MA: MIT Press, 1975.
Courtesy of MIT Press. Used with permission.

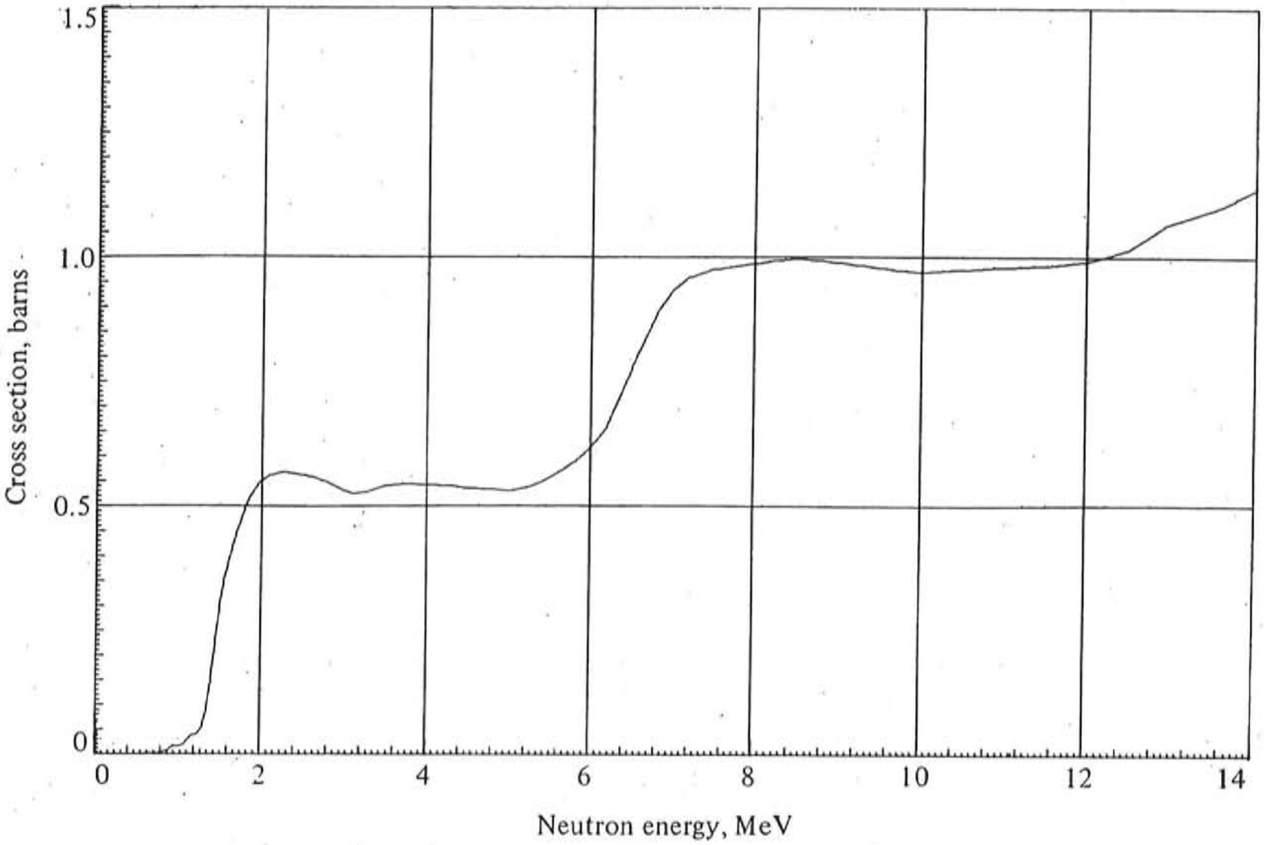


Fig. 3.9 The fission cross section of ^{238}U. (Plotted by machine from data on tape at the National Neutron Cross Section Center, Brookhaven National Laboratory.)

From Lamarsh.

This represents the total cross-section as the sum of a potential scatter term ($4\pi R^2$) and a radiative capture term ($C/\sqrt{E}$). Which term dominates depends on the nuclide.

At intermediate energies, there are resonances and at high energies a smooth low value.

9. Cross-Sections for Mixtures:

Most reactor fuels contain a mix of isotopes. For example, research reactor fuel is uranium-aluminum. Fuel for power producing reactors is uranium dioxide (UO_2). The total type α interaction rate (i.e., total elastic scatter or total radiative capture) is the sum of the individual macroscopic cross-sections. Thus,

$$\Sigma_\alpha = \sum_j \Sigma_\alpha^j$$

Where Σ_α is the total cross-section for process α and the Σ_α^j are individual cross-sections for each isotope j for the α process.

10. Doppler Effect:

The Doppler Effect is an important passive safety feature that exists in reactors that contain a significant amount of U-238 in the fuel. The radiative capture cross-section for U-238 contains six large resonances in the epithermal region. These expand when the fuel heats up. If there is an overpower condition, the fuel temperature will rise and these resonances will expand. As a result, they absorb more neutrons and hence fewer neutrons are available to sustain the fission of U-235. The power level therefore drops and the reactor is protected.

Why do the U-238 resonances expand with rising temperature?

If a neutron's energy equals that of the resonance, it will be absorbed. If it is slightly above or below the resonance energy, it will scatter off the U-238 nuclei without absorption. As the fuel heats up, the nuclei vibrate and this changes the relative speed between the neutron and the nuclei. Hence, the neutron is effectively at a different energy. What are the consequences? Suppose a neutron is initially slightly below the resonance energy. If the nucleus moves toward the neutron, the relative speed between the two goes up. Hence, that neutron will be absorbed. But, for every neutron that is now newly absorbed, one that was previously at the resonance energy is now too high and it only scatters off the nucleus. So, why is there a net increase in absorption? The reason is that the scattered neutron loses only a slight amount of energy (small object bouncing off a large one) and on its next collision, which will likely be with the fuel again, it will be absorbed. Thus, U-238 resonances broaden because (1) the U-238 nuclei vibrate more rapidly on heat up; and (2) the fuel is separate from moderator so that successive interactions occur in the fuel.

It is important to recognize the factors responsible for the Doppler effect when designing fuels for advanced reactors. Proposals that entail increased enrichments (less U-238) for longer fuel life or homogeneous fuels such as molten salt will result in reduced Doppler effects.

11. Temperature Dependence:

It was noted above that cross-section varies with the square root of energy for some processes. This behavior is termed "1/v" where v is the neutron speed. (Note: Kinetic energy is ½ mv^2; therefore v is proportional to $\sqrt{E}$; hence the name.) This behavior is apparent for thermal neutrons for both the radiative capture and the fission reaction. For nuclides that exhibit "1/v" behavior, knowledge of the cross-section at some reference energy, 0.025 eV for example, allows its calculation to another energy. Thus,

$$\frac{\sigma}{\sigma_0} = \frac{v_0}{v} = \left(\frac{E_0}{E}\right)^{1/2}$$

Where σ_0, v_0, and E_0 are the reference cross-section, velocity, and energy, respectively.