22.05 Reactor Physics – Part Five

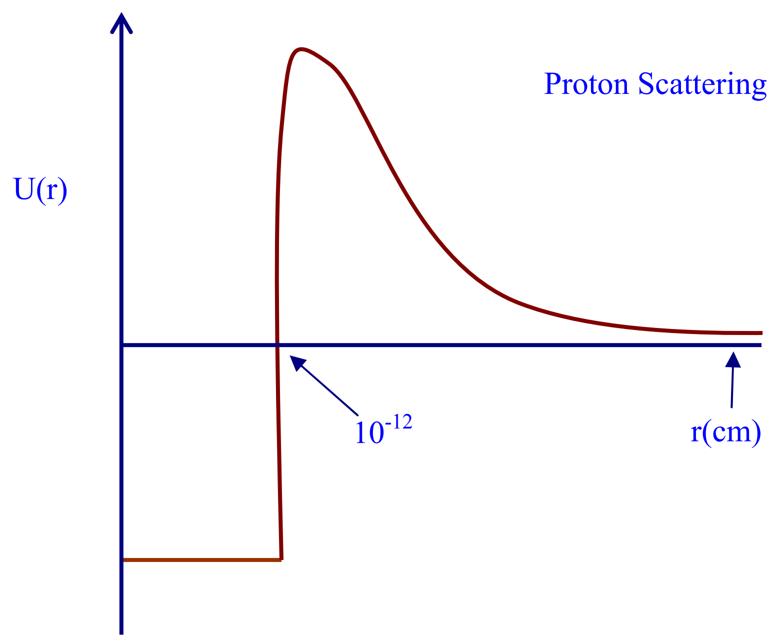
The Fission Process

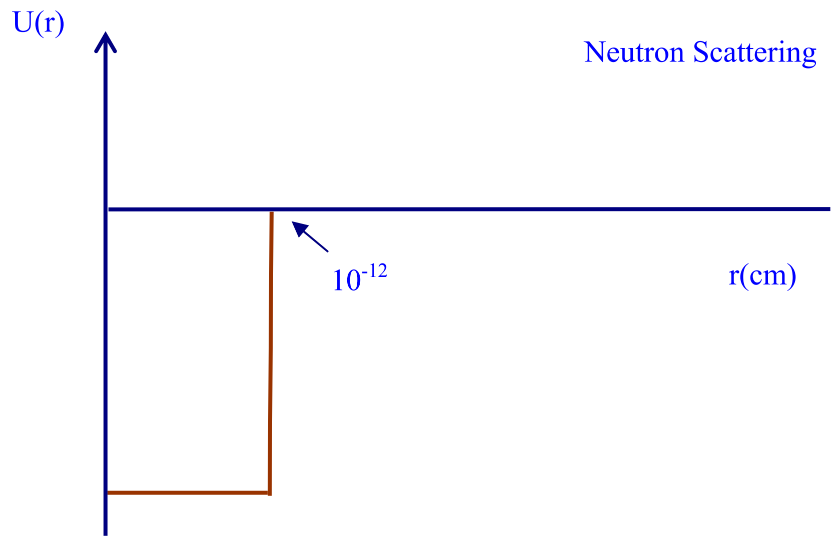
1. **Saturation**:

We noted earlier that the strong (nuclear) force (one of four fundamental forces – the others being electromagnetic, weak, and gravity) has several characteristics. These are:

- Very strong
- Short-range
- Charge independent
- Saturated

Evidence for the first three properties is obtained by scattering protons (or alphas) and neutrons off of nuclei. The result is:

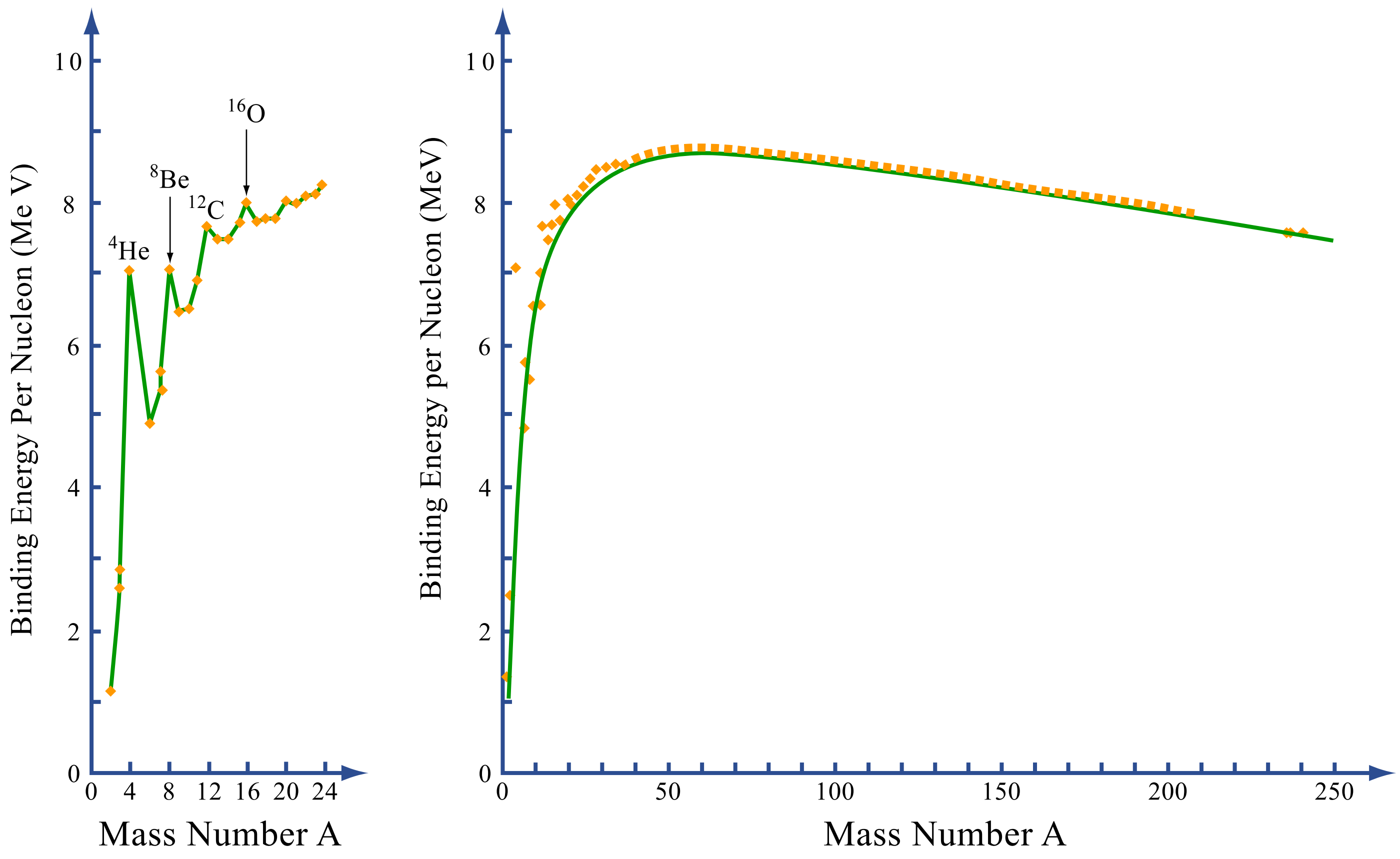




For charged particles, U(r) rises because work is being done against Coulomb repulsion of the protons that are in the nucleus. At distances below 10E-12 cm, U(r) drops sharply as the nuclear force dominates. This indicates both high strength and short range. Comparison of U(r) for charged and neutral particles shows charge independence. Both are attracted for R<10E-12 cm. Also, if one compares the binding energies of low Z nuclides such as ${}^{2}_{1}H$, ${}^{3}_{1}H$, ${}^{3}_{2}He$, it is empirically observed that the nuclear force between a neutron-neutron, proton-proton, or proton-neutron pair is the essentially the same.

But what about the fourth property, saturation?

We define a force as "unsaturated" if each particle interacts with every other one. Examples are gravity and the electromagnetic force. What about the nuclear force? If every neutron interacted with every other one, then the number of interacting pairs would be: A(A-1)/2 for a system of A particles. This gives us a criterion for determining if the nuclear force is saturated. If the binding energy varies as A^2, it is not saturated. If it varies with A, then it is saturated. Examination of the curve of binding energy versus mass number (see figure) shows that for A>60, the binding energy per neutron (BE/A) is flat or slowly declining. We therefore approximate BE/A as a constant which means that the binding energy is proportional to A for A>60. So, the nuclear force is saturated



The average binding energy BE/A of the nucleons in a nucleus versus the atomic mass number A for the naturally occurring nuclides (and ^{8}Be). The left-hand figure shows the detailed variation of the average binding energy for the lightest nuclei, while the right-hand figure shows the overall variation.

Figure by MIT OCW. From Shultis.

in this range of mass numbers. So, the curve does not exhibit the characteristic of being unsaturated.

What about for A<60? Here the BE/A is rising as a function of A and the binding energy is proportional to A^2. So, the nuclear force has not yet saturated.

We conclude that:

a) For small nuclei, the $BE \propto A^2$ and the force is not saturated.

b) For large nuclei, $BE \propto A$ and the force is saturated.

c) The transition occurs at $A \cong 60$ (iron).

Thus, in a heavy nucleus a proton does not attract a proton on the other side of the nucleus. But, it does repel it because the Coulomb force is not saturated. So, as nuclei get heavier, the neutron to proton ratio must rise so as to provide more attraction than repulsion.

2. <u>Indicators of Nuclear Stability</u>: We note some indicators of stability:

a) <u>Nuclear Pairing</u>: Close examination of the Chart of Nuclides shows that there are far more stable nuclides with an even-even number of neutrons and protons than there are with an odd-odd number. The data is:

Configuration of Stable Nuclei

		Protons	
		Even	**Odd**
Neutrons	**Even**	**156**	**48**
	Odd	**50**	**5**

This suggests that pairing of neutrons and protons is important for stability.

b) Magic Numbers: Nuclides that contain certain numbers of neutrons and protons tend to be more stable than others. The magic numbers are 2, 8, 20, 28, 50, 82, and 116.

c) Radioactive Decay: The longer a half-life, the closer a nuclide is to being stable. Also, when heavy nuclei do decay, they don't emit individual nucleons. Rather, alpha particles are emitted. This suggest that nucleons are grouped as alpha particles within a nucleus.

3. Nuclear Density: The radius of a nucleus is given by:

$$R = 1.1A^{1/3} x 10^{-15}\ cm$$

This indicates that each nucleon has the same volume because the total volume is then that of a sphere $(4/3)\pi R^3$ and hence proportional to A.

4. Liquid Drop Model: The liquid drop model of the nucleus is semi-empirical. Its basis is argued in terms of analogy to the forces in a drop of liquid while its numerical coefficients are determined by fitting the model to observation. For a drop of liquid:

— Density is near constant as is that of a nucleus.

— Each molecule of liquid interacts with its nearest neighbors as do nucleons in a saturated system (A>60).

— The liquid is nearly incompressible as is a nucleus.

These similarities suggest the following approach to modeling a nucleus. The nuclear mass of a nucleus that contains Z protons and (A-Z) neutrons would be to a first approximation:

$$M\left({}^{A}_{Z}x\right) = Zm_p + (A - Z)m_n$$

This overestimates the true mass because energy (the binding energy) is released upon formulation of the nucleus. We estimate that binding energy as a volume term that is proportional to the mass number. Thus,

$$BE_v = a_v A$$

So, our model becomes:

$$M\left({}^{A}_{Z}x\right) = Zm_p + (A - Z)m_n - a_v A$$

This correction is too great. We use the analogy with the liquid drop to develop corrections:

a) Surface Term: Nucleons near the surface are not as tightly bound as are interior ones because they aren't surrounded on all sides by other nucleons. The analogy is that of surface tension so we expect the effect to be a function of the surface area of the nucleus and hence proportional to R^2 or $A^{2/3}$. Thus,

$$BE_s = -a_s A^{2/3}$$

b) Coulomb Term: The protons within a nucleus cause repulsion. This decreases stability. The effect is not saturated and is therefore proportional to

$$\begin{aligned} &Z(Z-1)/R \\ &= Z(Z-1)/A^{1/3} \\ &\cong Z^2/A^{1/3} \end{aligned}$$

Thus,

$$BE_c = -a_c Z^2 / A^{1/3}$$

c) Asymmetry: For small values of the mass number, A, nuclides have equal numbers of neutrons and protons. For large values of the mass number, the number of neutrons exceeds the number of protons. This occurs because of the Coulumb repulsion. If there were no such repulsion, stability would be enhanced by symmetry – equal numbers of neutrons and protons. We have already corrected for the Coulomb effect with the BE_c term. We create a term to correct for asymmetry (i.e., $N \neq Z$ or $A \neq 2Z$). This term varies inversely with A because the asymmetry effect is greatest for small nuclei. Thus,

$$BE_a = -a_a (A - 2Z)^2 / A$$

d) Pairing: There are many even-even nuclides and very few odd-odd ones. Thus, we add a pairing correction factor of the form:

$$BE_p = -a_p / \sqrt{A}$$

Where a_p is positive for even-even, zero for odd-even or even-odd, and negative for odd-odd.

Upon summing all terms, one obtains

$$m\left({}^{A}_{Z}X\right) = Zm_p + (A - Z)m_n \\ - \frac{1}{c^2}\left\{a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} - \frac{a_p}{\sqrt{A}}\right\}$$

Values for the empirical constants are:

$a_v = 15.835$ MeV, $a_s = 18.33$ MeV, $a_c = 0.714$ MeV, $a_a = 23.20$ MeV

and

$$a_p = \begin{cases} +11.2 \text{ MeV for odd N and odd Z} \\ 0 \quad \text{for odd N, even Z or for even N, odd Z} \\ -11.2 \text{ MeV for even N and even Z} \end{cases}$$

Or, in terms of atomic masses (add Z electrons to both sides of the above equation):

$$M\left({}^{A}_{Z}X\right) = ZM\left({}^{1}_{1}H\right) + (A - Z)m_n \\ - \frac{1}{c^2}\left\{a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} - \frac{a_p}{\sqrt{A}}\right\}$$

5. **Uses of Liquid Drop Model**: The liquid drop model provides no insight about the internal structure of the nucleus. But, it can be used to predict both which nuclides are fissionable and radioactive decay modes. It can also be used to predict the line of stability that we found earlier by plotting the neutron and proton populations of the stable (gray color) nuclides in the Chart of the Nuclides.

For a given value of A, the most stable nuclide is the one with the smallest mass (i.e., most BE lost). Differentiate the model equation with respect to Z and set the result to zero. Thus,

$$\left(\frac{\partial M\left({}^{A}_{Z}X\right)}{\partial Z}\right)_A = m_p - m_n + \frac{1}{c^2}\left\{2a_c \frac{Z}{A^{1/3}} - 4a_a \frac{(A-2Z)}{A}\right\} = 0$$

Solving for Z (at constant A) yields:

$$Z(A) = \left(\frac{A}{2}\right)\frac{1 + \left(m_n - m_p\right)c^2/(4a_a)}{1 + a_c A^{2/3}/4a_a}$$

The result is shown in the attached figure which is from Shultis and Faw (p. 63). (Note: The line denoted as equation 3.18 in the figure is the above equation.)

6. **Shell Model**: This model is useful for explaining the observed magic numbers. The model entails solving the wave equation subject to two major assumptions:

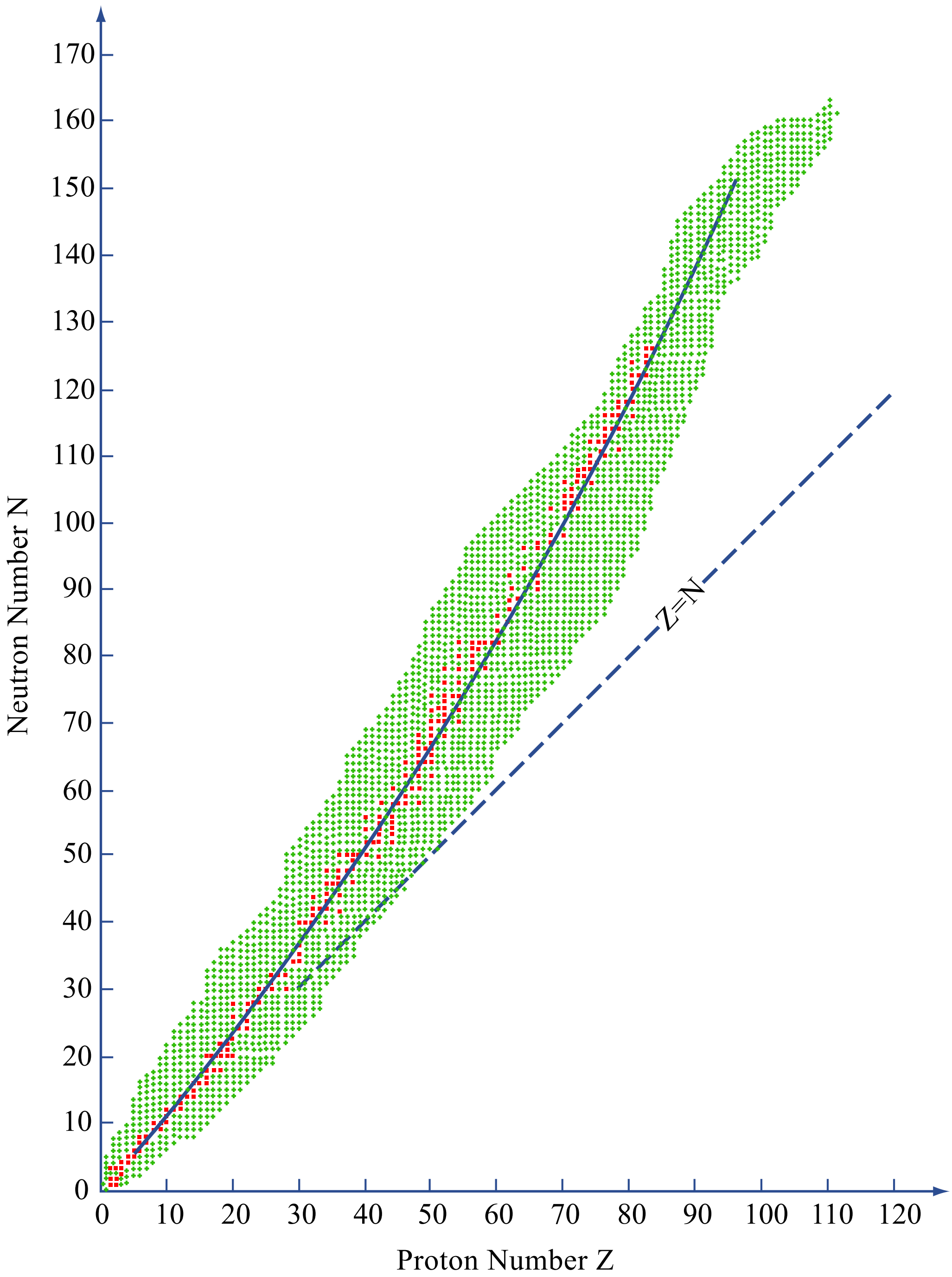
a) Each nucleon moves independently within the nucleus.

b) The nucleus may be represented as a potential well that is constant from the center of the nucleus to its outer radius whereupon it rises rapidly by several tens of MeV.

The model gives results analogous to those obtained by applying the wave equation to electrons in an atom. Specifically, there are sets of energy levels for both protons and neutrons. Large gaps in energy exist between filled shells which correspond to the observed magic numbers.

7. **Nuclear Fission**: The liquid drop model can be used to provide a qualitative explanation of fission. Consider a large nucleus that has at least 200 nucleons. All nucleons are being acted upon to some degree by their nearest neighbors because all are within the potential well. That is, all are within range of the nuclear force. The degree of attraction is not the same between all pairs of nucleons because of the saturation effect. The nucleons are all moving and forces average out so, over time, nucleons in a similar location (interior or surface) experience the same net force. For those in the interior, individual forces cancel with the net force being zero. For those on the surface, there are no nucleons on the "outside" to balance those in the "inside." So, the surface nuclei are subject to a net force that pulls them inward. On average, the shape of the nucleus is therefore spherical. However, that is only the average. The nucleus, like a drop of liquid, oscillates, and forms an ellipsoid. If the oscillations grow, the ellipsoid splits in two. This splitting in two is the fission process.

The probability of fission increases if energy is added from some exterior source. If the added energy exceeds a certain "critical" value, the likelihood of fission is very great. For nuclides below thorium, the critical energy is quite large (i.e., for Pb-208, its 20 MeV). For nuclides above Thorium, it is much lower, 4-6 MeV.



The known nuclides. Stable nuclides are indicated by red dots and radioactive nuclides by green dots. The line of equal neutrons and protons is shown by the heavy dashed line.

Figure by MIT OCW. From Shultis.

Critical Energies for Fission

(Source: LaMarsh, Table 3.3)

Fissioning Nucleus	Critical Energy (MeV)	Separation Energy (MeV)
Th-232	5.9	
Th-233	6.5	5.1
U-233	5.5	
U-234	4.6	6.6
U-235	5.75	
U-236	5.3	6.4
U-238	5.85	
U-239	5.5	4.9
Pu-239	5.5	
Pu-240	4.0	6.4

Note that U-235 is called fissile. It absorbs a neutron and becomes U-236. The U-236 has the low value for the critical energy.

Any process that provides the needed energy can cause fission. Thus, we earlier saw photo fission via gamma ray absorption. Most photons do not have energies of 4-6 MeV and hence photo fission is not a significant factor in nuclear reactors. The most efficient way to cause fission is neutron absorption. When a neutron gets within range of the nuclear force, it falls into the potential well and its original potential energy is converted to kinetic energy. The above table shows the separation energies for several nuclides. Recall that separation energy is the binding energy associated with adding one more neutron. For three nuclides (U-233, U-235, and Pu-239), this energy is greater than the critical energy and fission results upon the capture of any neutron – even one of zero kinetic energy. For other nuclides (Th-232, U-238, actinides such as Np, Am, Cm the capture of a thermal neutron (one with no kinetic energy) is insufficient to cause fission. Such nuclides will only undergo fission if the incident neutron has a kinetic energy of approximately 1 MeV or more. (<u>Note</u>: This explains why "actinide burners," which are reactors that will consume long-lived fission products, such as Np and Am, are designed to have a fast neutron spectrum.)

We repeat the definitions of fissile, fertile, and fissionable here. An isotope is fissile if it undergoes fission upon absorption of a neutron of zero kinetic energy. The principal such isotopes are U-233, U-235, Pu-239, and Pu-241. An isotope is fertile if it becomes fissile upon absorbing a neutron. Examples are Th-232, and U-238. An isotope is fissionable if it undergoes fission upon being struck by a high energy neutron. Examples include Th-232 and U-238 and many actinides.

8. **Consequences of Fission**: We have previously listed many of the consequences including:

— An average 2.5 high-energy prompt neutrons are produced. The symbol for neutron yield per fission is ν.

— The emitted neutrons have a particular spectrum given by the function:

$$\chi(E) = 0.453e^{-1.036E} \sinh\sqrt{2.29E}$$

Where E is in MeV.

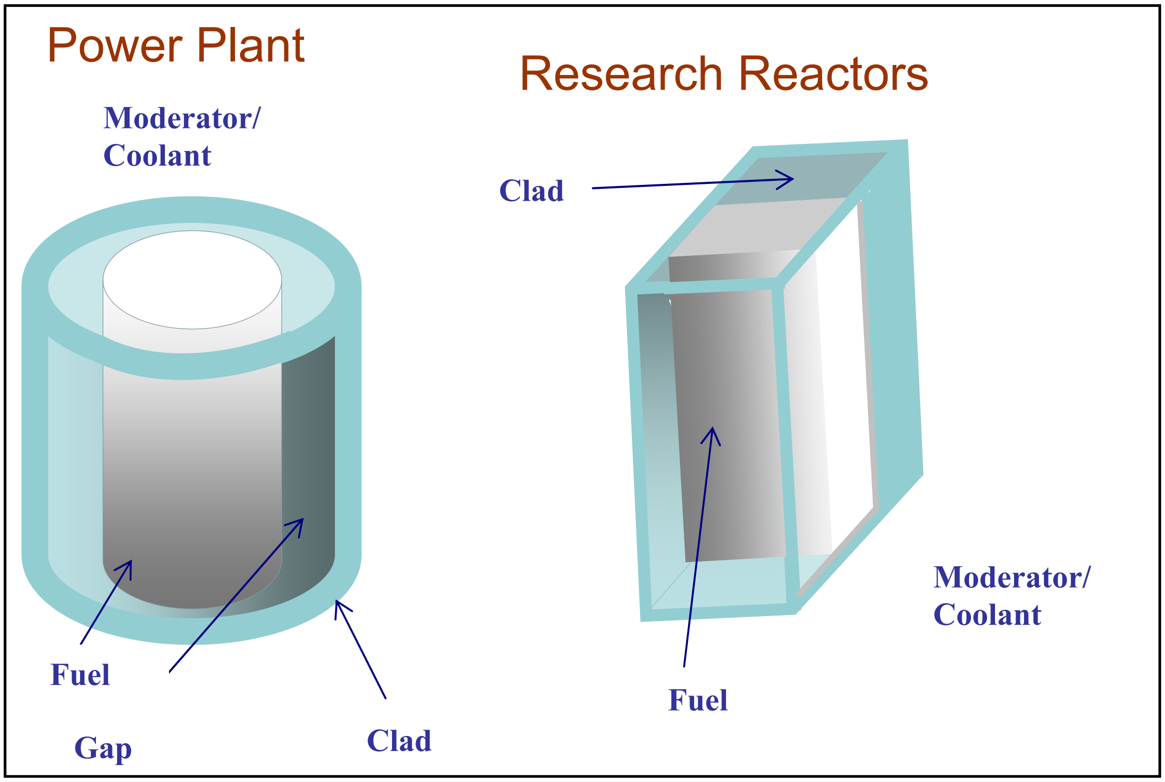
— The peak of the spectrum is about 0.73 MeV and the average is 2 MeV.

— Fission is asymmetric with the fission product yield curve being maximum at mass numbers equal to 100 and 135 and a minimum for 148 (symmetric fission).

— Some fission products which are given the special name precursor, undergo a beta decay to a daughter nuclide that then emits a neutron. These neutrons are termed "delayed" with the delay being the time for the beta decay.

— Approximately 200 MeV is released per fission. Of this 168 MeV is associated with the fission products, 12 with neutrinos, and the rest with prompt neutrons, prompt photons, betas, and decay gammas.

Three items that have not been addressed are the effect of the fission products (particularly the gaseous ones), the need to prevent fission product release, and the distribution of the 200 MeV of energy that is released per fission. To do this, we need some understanding of the core design. The fuel is typically UO_2 for a power plant and UAl_x for research reactors. It is encased in a clad that is intended to contain the fission products.



For power plants, the fuel is in cylindrical pellets that sit inside a cylinder. The outer surface is the clad, typically Zircalloy, which offers high strength and low neutron absorption. There is a gap between the UO_2 pellets and the Zircalloy. Fission product gas accumulates in this gap.

For research reactor fuel, the fuel is a mixture of uranium aluminum alloys (hence UAl_x instead of UAl_2 or UAl_3, etc.) that is clad with aluminum. There is no gas gap. Rather, the UAlx is below theoretical density so the gas can accumulate in the voids.

So, one further consequence of fission is the need to accumulate fission product gas. Recall that xenon, which is gaseous, is at the peak of the fission product yield curve. If allowance is not made for gas buildup, the fuel swells and becomes distorted. In severe cases, it could block the flow of coolant and lead to melting.

A second consequence is that some fission products are both radioactive and readily taken up in the human body. Iodine, also at the peak of the yield curve, is readily taken up in the food chain and accumulates in the thyroid. Other fission products that are a hazard include:

Strontium	–	Bone seeker
Cesium	–	GI Tract
Plutonium	–	Liver-Skeleton

Prevention of the release of such nuclides is done by containing these within the clad, which is in turn contained within the reactor vessel which is in turn housed within a containment building. So, there are three barriers to release in a PWR or BWR. But, the clad is the first and foremost.

A third consequence is energy deposition. The following table gives the distribution of the fission energy:

Approximate Distribution of Fission Energy
(Source: Glasstone Table 1.4)

	MeV
Kinetic energy of fission fragments	165
Instantaneous gamma-ray energy	7
Kinetic energy of fission products	5
Beta particles from fission products	7
Gamma rays from fission products	6
Neutrinos	10
Total fission energy	**~200**

168 of the 200 MeV produced from fission is associated with the fission products. These are large and highly-charged. They slow down rapidly via atomic collisions with the fuel nuclei. So, the 168 MeV shows up in the fuel and is moved by conduction through the fuel and clad to the moderator/coolant. The beta rays stop in the fuel, the clad, and the moderator/coolant. So, even though there is no fuel in the clad, there is heat production in the clad. This complicates the heat transfer analysis. The gamma rays deposit in the fuel, clad, moderator/coolant, and in the shielding that surrounds the reactor. So, the shielding requires cooling.

9. **<u>Fission Rate and Reactor Power</u>**: Suppose a reactor operates at P MW. What is the fission rate?

$$\begin{aligned}\text{Fission Rate} &= (\text{P MW})(10^{6}\ \text{joules}/\text{MW}-\text{s})\\ &\quad (1\ \text{fission}/200\,\text{MeV})(1\,\text{MeV}/1.6\times10^{-13}\ \text{joules})\\ &\quad (86{,}400\,\text{s}/\text{day})\\ &= 2.70\times10^{21}\,\text{P fissions}/\text{day}\end{aligned}$$

The rate of fuel depletion, which is termed the burnup rate, is found by:

$$\begin{aligned}\text{Burnup Rate} &= \frac{(\text{fission rate})(\text{GAW U}-235)}{\text{Avagadro's \#}}\\ &= \frac{(2.70\times10^{21}\,\text{P fissions}/\text{day})(235\text{g}/\text{mole})}{6.02\times10^{23}\ \text{fissions}/\text{mole}}\\ &= 1.05\,\text{P g}/\text{day}\end{aligned}$$

This relation states that for P equal 1MW, 1.05 grams of U-235 must be fissioned to produce 1 MW. The amount of U-235 consumed is actually greater because, upon nuclear absorption, U-235 may either fission or form U-236. Thus,

$$\begin{aligned}\text{Consumption Rate} &= (1.05\ \text{P})(\sigma_a/\ \sigma_f)\\ &= 1.23\ \text{g/day per MW}\end{aligned}$$

10. **<u>Heat Generation Rate after Shutdown</u>**: When nuclear energy was first proposed, its advocates are claimed to have said that it "would be too cheap to meter." And perhaps it would have been except for the production of heat after shutdown. Reactors are not like fossil-fueled engines which cease to produce energy after shutoff. In a reactor, energy is produced from the decay of the fission products and this process continues even though the fission process itself has been terminated. The following summary of the issue is taken from the book, <u>Nuclear Reactor Analysis,</u> by A. F. Henry.

"Because radioactive decay can be a slow process, the energy of β and γ rays coming from fission fragments and from capture γ rays is transformed into heat relatively slowly. On the average, the fragments from a single fission decay in time approximately as $t^{-1.2}$ $(1\ \text{sec} \le t \le 10^{6}\ \text{sec})$. To be specific, the average decay power (energy released per second) following a *single* fission is

$$P_d(t) = 2.66 t_0^{-1.2} \ MeV/\text{sec}, \ t > 1\text{sec}$$

Where t is the time (in seconds) since the fission took place. If a reactor has been at a constant power corresponding to F fissions per second for a very long time and then at some time t_0 (prior to t) has been turned off, we must add the average decay power from all past fissions to get the overall average decay power. To do this, we note that the contribution to the total decay power at time t due to fissions which occurred in time dt′ equals the number of fissions F dt′ that occurred between t′ and t′ + dt′ times $P_d(t - t')$. As a result the total decay power at t is

$$\int_{-\infty}^{t_0} (F dt') 2.66 (t - t')^{-1.2} = \frac{2.66}{0.2} F (t - t_0)^{-0.2}, \ (t - t_0) > 1 \text{ sec}.$$

Thus the total decay power following sustained, constant power operation of a reactor falls off only as the 1/5th power of the time after shutdown. Only a few percent of the total reactor power is involved in this decay. However, for a reactor which operates at, say, 2400 megawatts, this is a substantial, long-lived decay power. Thus a power reactor must be both cooled and shielded after shutdown."

The energy produced after reactor shutdown is called decay heat. If P_0 is power before shutdown and P is the decay heat (both in same units). Then,

$$\frac{P}{P_0} = 6.1 \text{ x } 10^{-3} \left[(\tau - t_0)^{-0.2} - \tau^{-0.2} \right]$$

Where $(\tau - t_0)$ is the time since shutdown in days (i.e., the cooling period) and τ is the time since startup as shown below: