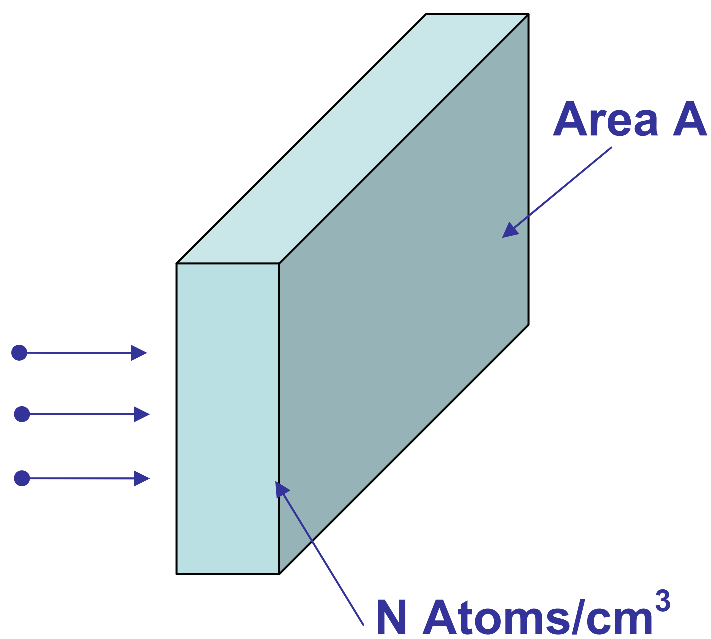
22.05 Reactor Physics - Part Nine

Neutron Flux, Reaction Rates, Group Constants, and Neutron Current

1. We wish to be able to compute reaction rates within materials. To do this, we first define a neutron flux.

Consider a beam of monoenergetic neutrons passing through a thin slab of material whose microscopic cross-section for the reaction of interest is σ and whose number density is N.



The neutrons are monoenergetic so they are all traveling with the same speed v. It is important to note that we are using the word "speed" and not velocity. V is a scalar for this application. The volume swept out by these neutrons in one second is vA. Denote the number density of the neutrons as n cm^{-3}.

Thus, the number of neutrons that strike a unit area, A, that is perpendicular to the beam per second, is nvA. The number striking the unit area per cm^2 per second is therefore nv.

The quantity nv is the neutron flux. It has units of neutrons/cm^2 x s. It is denoted by the symbol ϕ. Note that flux is a scalar quantity.

2. **Reaction Rate:** From the earlier discussion of the good geometry experiment, we defined the probability that a neutron will interact as it passes through a target as:

$$\sigma N \Delta x$$

where σ is the microscopic cross-section, N is the number density of the medium, and Δx is the target thickness. The total number of interactions in the target per second will then be:

$$(nvA)(\sigma N \Delta x)$$

Where (nvA) is the number of neutrons that strike the target per second and ($\sigma N \Delta x$) is the fraction that interact. The total number of interactions per second is then:

$$(nv)(\Sigma)(A\Delta x)$$

Where $\Sigma = \sigma N$ and upon division by $A\Delta x$ which is the target volume:

$$\text{\# interactions/cm}^3 \cdot \text{s} = nv\Sigma$$

$$= \Sigma\phi$$

Thus, a reaction rate is the product of a macroscopic cross-section and the neutron flux.

3. **Example:** What is the power output of a reactor given both the flux and the macroscopic fission cross-section? The power level is the fission reaction rate which at a given point in the reactor core is given by $\Sigma_f\varphi$. To get the total power, we have to integrate over the core volume. Thus,

$$\text{Power} = \left[\int_v \Sigma_f \phi dV\right]\left[\frac{200\,\text{MeV}}{\text{Fission}}\right]\left[\frac{10^6\,\text{eV}}{1\text{MeV}}\right]\left[\frac{1.6\text{x}10^{-19}\,\text{J}}{\text{eV}}\right]$$

Where Σ_f is for the fission process. Evaluation of the integral is non-trivial because:

(i) Σ_f will be a function of neutron energy as well as the fuel type and fuel burnup (if any).

(ii) ϕ will be a function of neutron energy and position within the core.

4. **Physical Interpretation of Flux**: The flux can be given a physical interpretation as the total track length swept out by all the neutrons per unit volume per second. To see this, again consider the monoenergetic beam. $v\Delta t$ is the track length swept out by a single neutron with speed v in time Δt. If n is the number density of the neutrons, then $nv\Delta t$ is the total track length of all neutrons with speed v in a unit volume. Division by Δt gives the flux.

This physical interpretation of flux also meshes with the definition of reaction rate:

$$R = \Sigma\phi$$

$$= \left[\frac{\text{Probability of an interaction}}{\text{neutron} - \text{cm}}\right]\left[\frac{\text{Total track length of all neutrons}}{\text{cm}^2 - \text{s}}\right]$$

$$= \text{Interactions} / \text{cm}^3 - \text{s}$$

5. **Polyenergetic Systems:** Let n(v) be the speed distribution of the neutrons in a unit volume. Thus,

$$n(v)dv = \frac{\text{Number neutrons with speeds between v and (v + dv)}}{\text{cm}^3}$$

Hence, the contribution to the flux by all neutrons with speeds between v and (v + dv) is:

$$d\phi = vn(v)dv$$

and

$$\phi = \int_0^\infty vn(v)dv$$

Alternatively, the flux in a polyenergetic system could be computed in terms of the average neutron speed, $\overline{v}$. We start with the same equation, but solve it differently:

$$\phi = \int_0^\infty vn(v)dv$$

Multiply top and bottom by $\int_0^\infty n(v)dv$ to get:

$$\begin{aligned}\phi &= \int_0^\infty vn(v)dv \times \frac{\int_0^\infty n(v)dv}{\int_0^\infty n(v)dv} \\ &= \frac{\int_0^\infty vn(v)dv}{\int_0^\infty n(v)dv} \times \int_0^\infty n(v)dv \\ &= (\overline{v})(n)\end{aligned}$$

Where n is the total number of neutrons per cm^3.

What about the reaction rate? For a monoenergetic system $R = \Sigma\phi$. But in general, the cross-sections are functions of energy and we need to integrate over energy. Thus,

$$dR = \Sigma(v)d\phi$$

Where we write Σ as a function of neutron speed. (We could also write Σ(E) given that E and v are related.) $d\phi = n(v)vdv$, so

$$dR = \Sigma(v)n(v)vdv$$

Hence,

$$R = \int_0^\infty \Sigma(v)vn(v)dv$$

Multiply top and bottom by $\int_0^\infty vn(v)dv$:

$$\begin{aligned}R &= \frac{\int_0^\infty \Sigma(v)vn(v)dv}{\int_0^\infty vn(v)dv} \bullet \int_0^\infty vn(v)dv \\ &= \overline{\Sigma}^\phi \phi\end{aligned}$$

Where $\overline{\Sigma}^{\phi}$ is the macroscopic cross-section for the process in question (fission, scatter, absorption) averaged over the flux. Such cross-sections are called "group" constants. These will play an important role later in analysis of reactor systems. For now, recognize that group constants are averaged over an energy range (or velocity range). Here, we chose 0 to ∞. But, we could integrate over the thermal, epithermal, or fast regions. Or we could subdivide the energy range into several thousand small groups.

One final and very important point. Evaluation of the group cross-sections requires knowledge of the flux and we would not know the flux at the time that we wished to evaluate the integrals. Thus, this integration is a major challenge in reactor physics.

6. **Evaluation of Group Constants**: Examination of the U-235 fission cross-section shows three distinct regions: 1) thermal where the cross-section varies inversely with the neutron speed and is termed 1/v behavior; 2) resonance region; and 3) a low smooth variation at fast-energies. We can use this information to illustrate a possible approach to the evaluation of group cross-sections. For the thermal region, we can write the cross-section as:

$$\sigma_{th} = \frac{k}{v}$$

Where k is a constant. From the previous section, we can define a group cross-section over the thermal range (energy from 0 to E_T) as:

$$\begin{aligned}
\overline{\Sigma} &= \frac{\int_0^{E_T} \Sigma(v)vn(v)dv}{\int_0^{E_T} vn(v)dv} \\
&= \frac{\int_0^{E_T} N\sigma(v)vn(v)dv}{\int_0^{E_T} vn(v)dv} \\
&= \frac{\int_0^{E_T} N(k/v)vn(v)dv}{\int_0^{E_T} vn(v)dv} \\
&= \frac{Nk}{\frac{\int_0^{E_T} vn(v)dv}{\int_0^{E_T} n(v)dv}} \\
&= \frac{Nk}{\overline{v}}
\end{aligned}$$

Where $\overline{v}$ is the average neutron speed which is given by:

$$\overline{v} = \frac{\int_0^{E_T} vn(v)dv}{\int_0^{E_T} n(v)dv}$$

The point is that knowledge of the energy dependence of the microscopic cross-section will allow us to evaluate the group constants. So, one of our goals is to investigate the variation of flux with energy. We will address this later.

7. **<u>Neutron Current</u>:** Another concept that is very useful in reactor physics is neutron current. As with group cross-sections, the concept will be discussed in greater detail later. Here, a physical basis is given and then a formal definition.

Neutron flux was defined as the product of neutron density and speed. It is a scalar and has units of neutrons/cm^2 x s. Neutron current is defined in terms of neutron density and velocity. It is a vector and also has units of neutrons/cm^2 x s. For the case of monoenergetic neutrons, we define the neutron current density, $\mathbf{J}$, as a vector that points in the direction of net neutron flow and has a magnitude equal to the net number of neutrons that cross a unit area perpendicular to the net rated flow. Thus,

$$\mathbf{J} = \sum_i n_i \mathbf{v}_i$$

Where n_i is the number of neutrons per unit volume with velocity $\mathbf{v}_i$.

We now give a formal mathematical definition of flux and current for purposes of comparison.

Flux: $\Phi(\mathbf{r},E) \equiv \int_\Omega d\boldsymbol{\Omega} v(E) N(\mathbf{r},\boldsymbol{\Omega},E)$

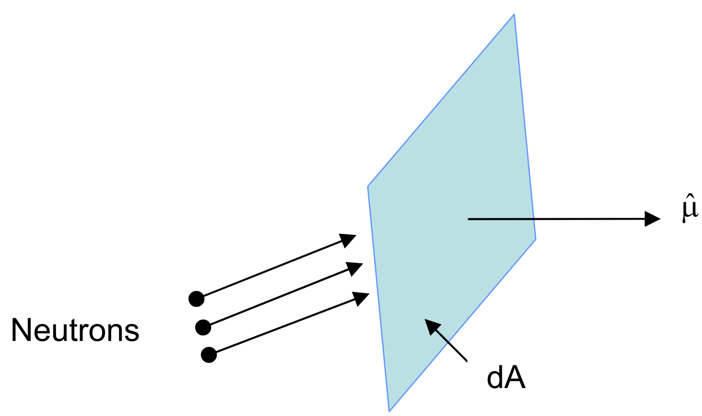
Current: $\mathbf{J}(\mathbf{r},E) = \int_\Omega d\boldsymbol{\Omega}\, \boldsymbol{\Omega} v(E) N(\mathbf{r},\boldsymbol{\Omega},E)$

Where $\mathbf{J}(\mathbf{r},E)$ is the neutron current,
$\mathbf{r}$ is position,
E is energy,
$\boldsymbol{\Omega}$ is direction of travel
v is the neutron speed, and
N is the neutron number density

$\mathbf{J}$(r,E) is defined as the neutron current. It is “the maximum over all orientations of a unit surface at **r** of the net number of neutrons with energies between E and (E+ dE) crossing that unit surface per second, the direction of the vector being the direction of this maximum net flow.” (Henry, p. 115)

Why do we need the concept of neutron current? Consider a box with a center partition. Gas A is to the left and Gas B is to the right. The motion of all gas atoms is random so the probability of a given atom moving left or right is the same, at 0.50. We remove the partition and the eventual result is a uniform mixture. Why? There was more of gas A on the left. So, even though the probability of left or right movement of a given atom is equal at 0.50 there was a net drift of gas A to the right. The converse was true for Gas B. This process is called diffusion. The analogy also applies to neutrons because, as noted earlier, they can be modeled as a dilute gas.

The general case is illustrated below:



Here the neutrons are traveling at some angle relative to a surface dA for which $\hat{\boldsymbol{\mu}}$ is the surface normal. The net number of neutrons that cross dA is:

$$\mathbf{J} \bullet \boldsymbol{\mu}\, dA$$

If dA is parallel to $\mathbf{J}$, $\mathbf{J} \bullet \boldsymbol{\mu}\, dA = 0$. If dA is perpendicular to $\mathbf{J}$, the direction of net flow, $\mathbf{J} \bullet \boldsymbol{\mu}\, dA$ is a maximum.

$\mathbf{J}$ is the direction of net flow and there may not actually be any neutrons moving in that particular direction. For example, suppose there are two groups of in the plus y direction specified by the unit vector **j**. **J** will be along the 45° line between the x and y axis even though no neutrons actually move in that direction.

8. **<u>Net Neutron Loss Because of Leakage:</u>** The neutron life cycle analysis discussed earlier defined the core multiplication factor as the ratio of neutrons produced from fission to those lost by either leakage or absorption. Current is a very useful concept for quantifying leakage. To see this, consider the reactor core as constituting some arbitrary volume. Let **n** be a unit vector normal to that surface. Then, the neutron leakage rate through the volume's surface is:

$$\text{Leakage Rate} = \int_A \mathbf{J} \bullet \mathbf{n}\, dA$$

This integral is not too useful because it is over a surface and other terms that we use to describe the reactor (such as neutron production from fission; neutron absorption, and neutron scattering) are all reaction rates that are integrated over volumes. We can convert this surface integral to a volume one by use of Gauss's Theorem, namely.

$$\int_A \mathbf{J} \bullet \mathbf{n}\, dA = \int_v \operatorname{div} \mathbf{J}\, dV$$

This change makes the leakage calculation similar (i.e., over a volume) to those of the other neutron terms.

Rather than use Gauss's Theorem, we now derive the divergence integral directly. The following, which is from Nuclear Reactor Analysis by A. H. Henry (p. 119-120) explains the procedure.

"The net current density can be used to obtain an expression for the net loss of neutrons in an energy interval dE due to leakage out of a volume element dV. To do so we take dV = dx dy dz to be a small rectangular parellelopiped as shown the figure. The relation $J(r, E) \bullet \mathbf{n}\, dS)$ states that the net rate at which neutrons in the energy range dE leave dV through the surface of area dy dz at a distance x from the YZ plane is **J**(x, y, z, E) • (**-i**) dy dz dE, where the minus sign on the unit vector **i** is due to the fact that the direction leaving dx dy dz on the back side of the parallelepiped is **–i.**

Similarly the net rate of which neutrons pass through the surface dy dz at x + dx, the front side of the box, is **J**(x + dx, y, z, E) • **i** dy dz dE, where we take y and z to be the same values that appear in **J**(x, y, z, E).

It follows that the total rate at which neutrons are lost to dV by net leakage across surfaces perpendicular to the X axis is the difference of these two terms, which for convenience we write as

$$\left[\frac{\mathbf{J}(x+dx,y,z,E)-\mathbf{J}(x,y,z,E)}{dx}\right]\bullet \mathbf{i}\ dx\ dy\,dz\,dE$$

Thus, in the limit as $dx \rightarrow 0$, the net rate of leakage out of dV across surfaces perpendicular to the X axis is:

$$\left[\left(\mathbf{i}\frac{\partial}{\partial x}\right)\bullet \mathbf{J}(x,\ y,\ z,\ E\ )\right]dx\ dy\ dz\ dE$$

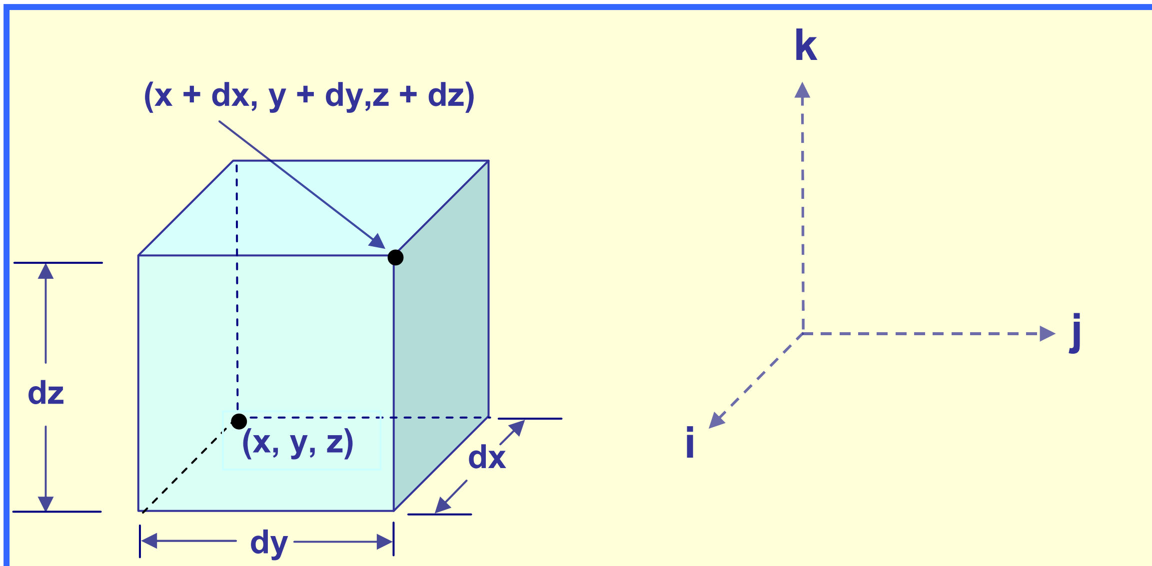
where we have formally associated the derivative operation with the unit vector **i.** Repeating this analysis for the Y and Z directions, we get:

Total rate at which neutrons with energies in the range E to E + dE leak out of dV = dx dy dz

$$=\left[\left(\mathbf{i}\frac{\partial}{\partial x}\right)\bullet \mathbf{J}(x,\ y,\ z,\ E\ )+\left(\mathbf{j}\frac{\partial}{\partial y}\right)\bullet \mathbf{J}(x,\ y,\ z,\ E\ )+\left(\mathbf{k}\frac{\partial}{\partial y}\right)\bullet \mathbf{J}(x,\ y,\ z,\ E\ )\right]dx\ dy\ dz\ dE$$

$$=\nabla \bullet \mathbf{J}(\mathbf{r},E)\,dV\,dE.$$

Thus the leakage rate per unit volume per unit energy is $\nabla \bullet \mathbf{J}$, the divergence of the net current density." (Henry, pp. 119-120)



A Volume Element for the Computation of the Net Neutron Leakage