22.05 Reactor Physics - Part Twenty-Five

Subcritical Multiplication and Reactor Startup

1. Reference Material

See pp. 357-363 of the article, "Light Water Reactor Control Systems," in Wiley Encyclopedia of Electrical and Electronics Engineering.

2. Reactor Physics and Operation

Nuclear fission reactors operate by maintaining a precise neutron balance. The reactor is in a critical condition if the number of neutrons created by the fission process equals the number that are either lost by leakage or captured within the reactor's structural materials. Neutrons produced from fission are called "fast" because they are traveling at very high speeds. Uranium-235, which is the principal fuel in most reactors absorbs very few high-energy or fast neutrons. In contrast, it will absorb, in large quantity, those neutrons that are moving slowly. Such neutrons are referred to as being "thermal." Hence, in order for the fission process to be maintained, it is necessary that the fast neutrons produced from fission be slowed down or thermalized. The need for the efficient thermalization of fast neutrons drives both the design and the operation of nuclear reactors

3. Reactor Operating Regimes

Reactor Operation is traditionally divided into three regimes. These are:

a) Subcritical – Critical

- Subcritical Multiplication

b) Critical – Point of Adding Heat

- Inhour Equation
- Dynamic Period Equation

c) Point of Adding Heat – Hot Operating

- Temperature Feedback
- Doppler Effect
- Xenon Feedback

The same physical relations describe all three regimes. However, because different terms in those relations dominate during each regime, the equations often look different. We will cover certain aspects of all three regimes with emphasis on subcritical multiplication, the dynamic period equation, and feedback mechanisms.

4. Subcritical – Critical

Crucial concepts are:

- Need for neutron sources
- Source-detector geometry
- Neutron thermalization
- Subcritical multiplication
- '1/M' plots

5. Need for Source Neutrons

a) It should be possible to monitor the neutronic condition of a reactor at all times, including when shutdown.

b) Nuclear instruments (power level and rate of change of power level) must be on scale prior to initiating a startup. Otherwise, the operator has no means of determining if his actions are having the intended effect.

c) If a reactor core is brand new or if a reactor has been shutdown for many months, then the neutron population will be so low as to be undetectable. Small movements of the control devices can therefore result in large rates of change – rates that are so rapid that power can rise to the level where fuel damage occurs before the nuclear safety system is capable of responding.

d) The installation of a neutron source ensures safety by keeping all instruments on scale.

6. Neutron Sources

a) Photo-Neutron

$$ {}^{0}_{0}\gamma + {}^{2}_{1}\mathrm{D} \rightarrow {}^{1}_{0}\mathrm{n} + {}^{1}_{1}\mathrm{H} $$

Note: Fission products provide the gamma rays. So, the reactor must have a power history for this source to be effective. The needed fission products decay within 2-3 months.

b) Plutonium-Beryllium

$$^{239}_{94}\mathrm{Pu} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{235}_{92}\mathrm{U}$$

$$^{4}_{2}\mathrm{He} + {}^{9}_{4}\mathrm{Be} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n}$$

Thought Question #1: PuBe sources could be made by mixing Pu and Be powders together or by surrounding a solid Pu rod with Be. Which is used?

Thought Question #2: The alpha particle produced by the Pu decay is monoenergetic. The second reaction $^{9}\mathrm{Be}(\alpha, n)^{12}\mathrm{C}$ is a two-body collision. From conservation of energy and momentum, one can easily show that for a two-body collision, the energy spectrum is discrete. That is, the C-12 nucleus and the neutron should each appear with a fixed energy. Those energies are:

$$E_n = \frac{12}{15} Q_{alpha}; \; E_c = \frac{1}{12} Q_{alpha}$$

Why then is the measured spectrum of the neutrons produced from a Pu-Be source observed to be continuous up to some maximum?

CAUTION: PuBe sources are doubly encapsulated in steel. Hence, heat transfer is poor and PuBe source must not be left in a reactor if power exceeds a few hundred Watts.

Answer to Thought Questions

Thought Question #1

Only the former will work. The reason is that the alpha particles are of such short range that the Pu and Be must be in close contact for the source reaction to occur. If the rod configuration were used, most of the alphas would be absorbed within the rod. Only those at surface would escape and these would interact only with the Be that was adjacent to the rod surface.

Thought Question #2

The alpha particles are monoenergetic when born. But, each slows down via collisions with atomic electrons before colliding with Be nuclei So, the alpha energies that initiate the $^{9}Be(\alpha, n)^{12}C$ reactions are all different. Hence, the spectrum of the neutrons that are produced is continuous.

c) Antimony-Beryllium

$$ {}^{1}_{0}n + {}^{123}_{51}Sb \rightarrow {}^{124}_{51}Sb $$

$$ {}^{124}_{51}Sb \rightarrow {}^{0}_{0}\gamma $$

$$ {}^{0}_{0}\gamma + {}^{9}_{4}Be \rightarrow {}^{1}_{0}n + 2\left({}^{4}_{2}He\right) $$

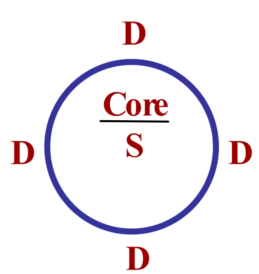
$$ {}^{4}_{2}He + {}^{9}_{4}Be \rightarrow {}^{1}_{0}n + {}^{12}_{6}C $$

Note: Must have radioactive antimony as initial condition.

7. Source-Detector Geometry

a) Photoneutrons are homogeneous because neutrons are produced throughout the entire volume of the core. The other types of sources are discrete entities and care must be taken to ensure proper source-detector geometry.

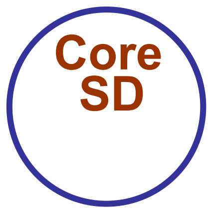
b) A discrete source should be placed in the center of the reactor core with fuel surrounding it. The detectors should be located beyond the fuel. Thus:



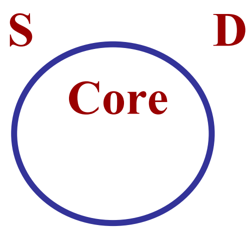
c) Power reactors are required to have four detectors, one in each quadrant.

8. Examples of Incorrect Source-Detector Geometry

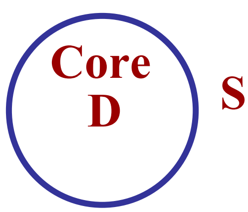
Configuration | Problem



Both source and detector are at core center. Hence, detector registers only source neutrons.

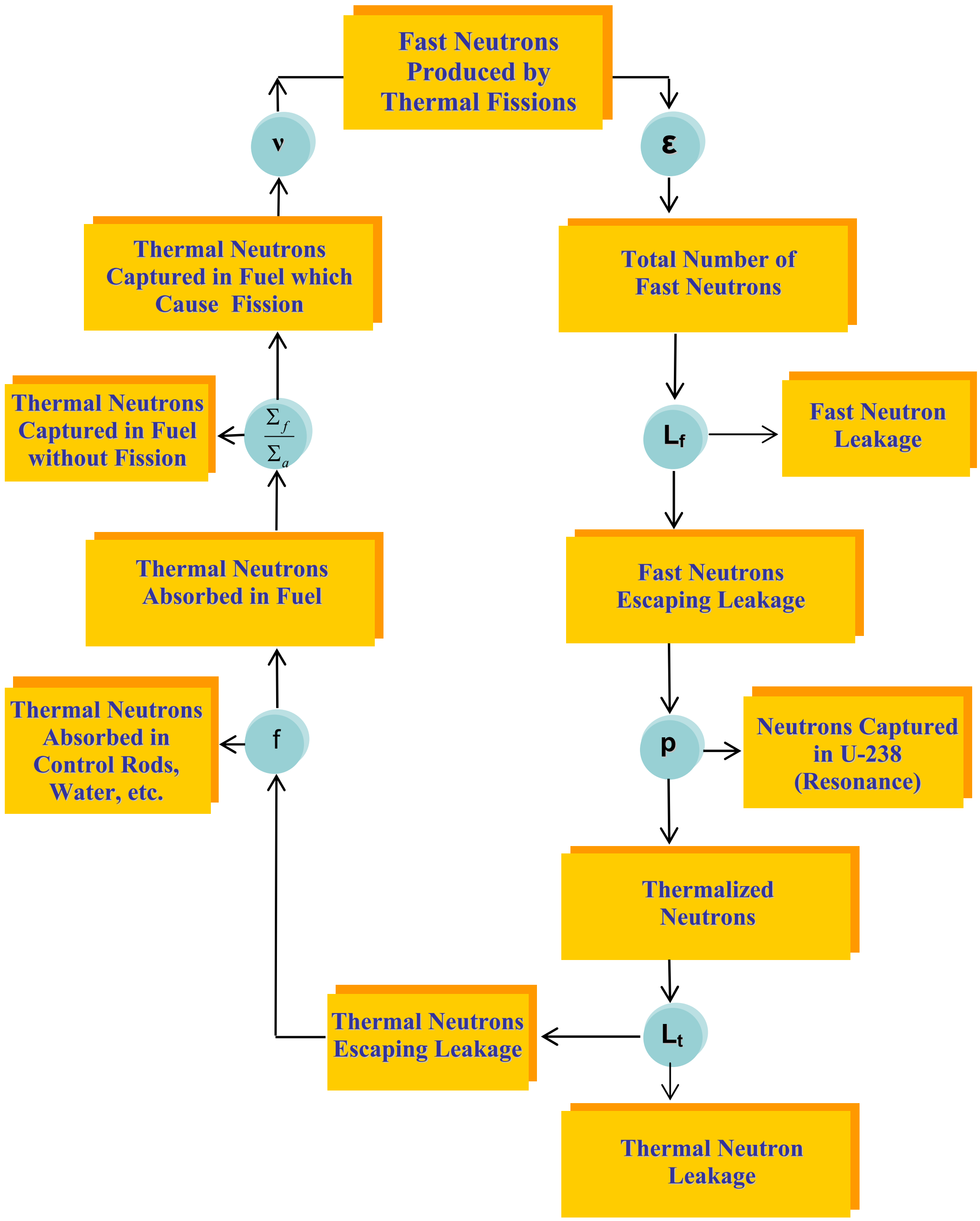


Both source and detector are external to core. Again, the detector only registers source neutrons.



Source is exterior to core. Hence, only a small fraction of the fuel is exposed to the source neutrons.

Neutron Life Cycle



Definitions of Neutron Life Cycle Factors

$$\varepsilon = \frac{\text{Total Number of Fast Neutrons Produced from Fast and Thermal Fission}}{\text{Number of Fast Neutrons Produced from Thermal Fission}}$$

$$L_f = \frac{\text{Total Number of Fast Neutrons Escaping Leakage}}{\text{Total Number of Fast Neutrons Produced from Fast and Thermal Fission}}$$

$$p = \frac{\text{Total Number of Thermalized Neutrons}}{\text{Total Number of Fast Neutrons Escaping Leakage}}$$

$$L_t = \frac{\text{Total Number of Thermal Neutrons Escaping Leakage}}{\text{Total Number of Thermalized Neutrons}}$$

Definitions of Neutron Life Cycle Factors (cont.)

$$f = \frac{\text{Thermal Neutrons Absorbed in Fuel}}{\text{Total Number of Thermal Neutrons Escaping Leakage}}$$

$$\eta = \frac{\text{Thermal Neutrons Captured in Fuel Which Cause Fission}}{\text{Thermal Neutrons Absorbed in Fuel}} \quad \text{or}$$

$$\eta = \frac{\text{Number of Fast Neutrons Produced from Thermal Fission}}{\text{Thermal Neutrons Absorbed in the Fuel}}$$

Of the above factors, the reactor operator can alter 'f' by changing the control rod position or by adjusting the soluble poison content. The leakage terms also vary during routine operation whenever coolant temperature changes. The other terms are fixed by the fuel type.

Core Multiplication Factor

1. It is useful to define a 'core multiplication factor' which is denoted by the symbol 'K' and which is the product of the six factors that define the neutron life cycle. Thus,

$$K = \varepsilon L_f p L_t f \eta$$

2. The above expression, which is called the 'six-factor formula, has physical meaning:

$$K = \frac{\text{Neutrons Produced from Fission}}{\text{Neutrons Absorbed} + \text{Neutron Leakage}} \quad \text{or}$$

$$K = \frac{\text{Number Neutrons in Present Generation}}{\text{Number Neutrons in Preceding Generation}}$$

$$K = \frac{n_1}{n_0} = \frac{n_2}{n_1} = \frac{n_3}{n_2}$$

when n is the number of neutrons in each generation.

3. If K is unity, the reactor is critical.

4. If we know the K-value for a reactor core, we can determine the rate of change of its neutron population. This is most useful in reactor startups.

Buildup of Neutron Population – A Result of Subcritical Multiplication

	Neutrons From		
Generation	Source	Multiplication	Total
0	100	0	100
1	100	60	160
2	100	60 + 36	196
3	100	60 + 36 + 22	218
4	100	60 + 36 + 22 + 13	231
5	100	60 + 36 + 22 + 13 + 8	239
6	100	60 + 36 + 22 + 13 + 8 + 5	244
7	100	60 + 36 + 22 + 13 + 8 + 5 + 3	247
8	100	60 + 36 + 22 + 13 + 8 + 5 + 3 + 2	249
9	100	60 + 36 + 22 + 13 + 8 + 5 + 3 + 2 + 1	250
10	100	60 + 36 + 22 + 13 + 8 + 5 + 3 + 2 + 1 + 0	250

All neutron quantities have been rounded to the nearest unit. Note that the incremental increase to the total population from the first generation is zero in the tenth generation. Hence, the system has reached equilibrium.

Mathematics of Subcritical Multiplication

Generation #	Source Neutrons	Neutrons from Multiplication
0	S_0	0
1	S_0	KS_0
2	S_0	$K S_0 + K^2S_0$
3	S_0	$K S_0 + K^2S_0 + K^3S_0$
4	S_0	$K S_0 + K^2S_0 + K^3S_0 + K^4S_0$
$\vdots$	$\vdots$	
n	S_0	$K S_0 + K^2S_0 + K^3S_0 + K^4S_0 + \ldots + K^nS_0$

Mathematics of Subcritical Multiplication (cont.)

So, after n generations, the neutron population would be:

$$\text{Total Neutrons} = S_0 + KS_0 + K^2S_0 + K^3S_0 + K^4S_0 + \ldots + K^nS_0$$

$$= S_0\,(1 + K + K^2 + K^3 + K^4 + \ldots + K^n)$$

$$\frac{S_0}{1-K}$$

Note: The fact that the series $1 + K + K^2 + K^3 + K^4 + \ldots + K^n$ does equal $1/(1-K)$ can be
= proved mathematically for values of K that are less than 1.

<u>Subcritical Multiplication</u>

– We now have an expression that may be used to calculate the equilibrium neutron level in a subcritical reactor.

– Of extreme importance to reactor startups is that the total neutron population in a subcritical fissile medium exceeds the source level by a factor of 1/(1-K) or 1/M. This process is called *subcritical multiplication.*

– The following should be noted:

a) The subcritical multiplication formula does <u>NOT</u> allow calculation of the time required for criticality.

b) As the multiplication factor, K, approaches 1.0, the number of generations and hence time required for the neutron level to stabilize gets longer and longer. This is one of the reasons why it is important to conduct a reactor startup slowly. If it isn't done slowly, the subcritical multiplication level won't have time to attain equilibrium.

c) The equilibrium neutron level in a subcritical reactor is proportional to the initial neutron source strength. This is why it is important to have neutron count rates above a certain minimum before conducting a startup.

d) The formula is only valid while subcritical.

Application of the Subcritical Multiplication Formula

- How can we apply the subcritical multiplication formula? We can measure neutron counts and source strength. The latter is merely the neutron counts with the reactor shutdown. Hence, we can calculate the multiplication factor, K, and thereby estimate how close a reactor is to criticality.

- We derived the relation:

$$\text{Counts} = S_0/(1\text{-}K)$$

- Rearrange the above to obtain:

$$(1\text{-}K) = S_0/\text{Counts}$$

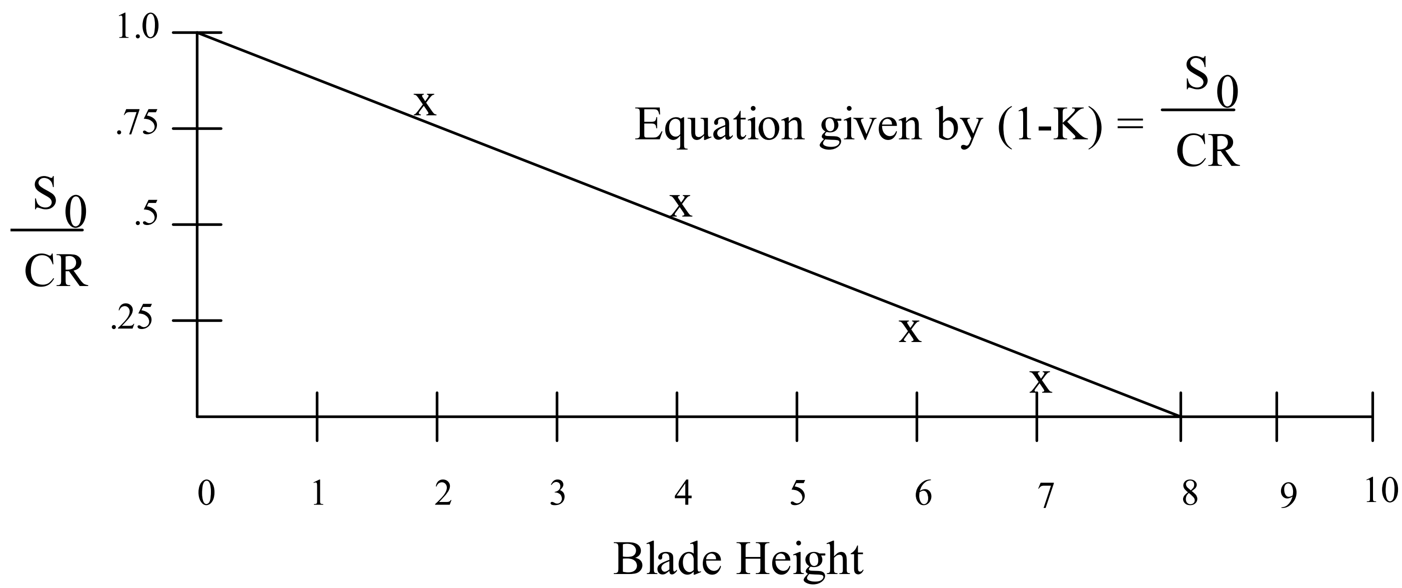
- To make use of this relation, plot inverse counts (S_0/Counts) on the vertical axis and control rod position or poison concentration or fuel loading on the horizontal axis. The result is called a '1/M' plot where M stands for multiplication. The point where the plot is extrapolated to cross the horizontal axis is where K equals 1 and the reactor is critical.

'1/M' Plot

Blade Height	Source Strength	Count Rate	Source/Count Rate
0"	S_0	$_0$	1.0
2"	S_0	1.30 S_0	0.77
4"	S_0	1.98 S_0	0.51
6"	S_0 S	4.15 S_0	0.24
7"	S_0	8.00 S_0	0.125

<u>'1/M' Plot</u> (cont.)

If the data from this example is plotted such that the vertical axis is (Source/Count Rate) or S_0/CR, we get:



– Criticality is expected at 8.0". Actual plots are usually not linear. Instrument noise and improper source-detector geometry cause a non-linear response.