22.05 Reactor Physics - Part Twenty-Six

Reactor Operation Without Feedback Effects

1. Reference Material:

See pp. 363-368 of the article, "Light Water Reactor Control Systems," in Wiley Encyclopedia of Electrical and Electronics Engineering.

2 Critical – Point of Adding Heat

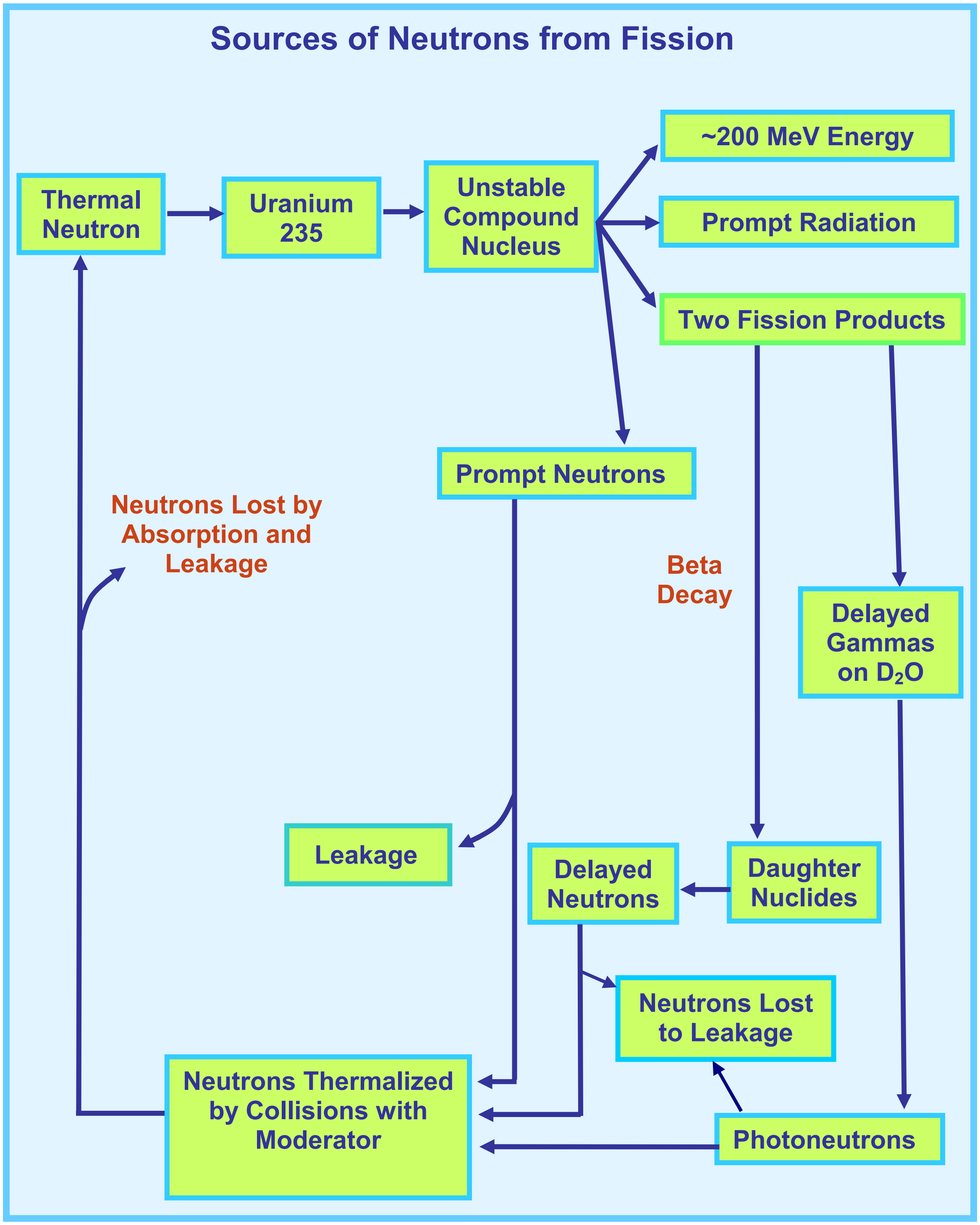
a) Crucial concepts are:

- Prompt and delayed neutrons
- Importance of delayed neutrons
- Power-period relation
- Reactivity
- Dynamic period equation
- Step and ramp reactivity transients.

b) The 'point of adding heat' is the power level above which a change in temperature is observed following a change of power. Reactors may be critical at a few Watts or even less. At such low powers, no change in temperature results because of the core's heat capacity. In general, there are no reactivity feedback effects below the point-of-adding heat.

3. Prompt and Delayed Neutrons

The reactor multiplication factor, K, has been defined as the ratio of neutrons in one generation to those in the immediately preceding generation. Combining this definition with a physical understanding of the neutron life cycle allowed us to write an equation that predicted the equilibrium neutron count rate in a subcritical reactor. Unfortunately, that relationship is not valid for a critical or supercritical reactor. We need to develop a means of describing both the neutron level and its rate of change in a supercritical reactor. The first thing that we should do is to examine the fission process and determine what types of neutrons are produced. The fission of a U-235 nucleus normally yields two fission fragments, an average of 2.5 neutrons, and an assortment of beta particles, gamma rays and neutrinos. The neutrons that are produced directly from the fission event are referred to as prompt because they appear almost instantly. Most of the neutrons produced in a


Sources of Neutrons from Fission
Thermal Neutron
Uranium 235
Unstable Compound Nucleus
~200 MeV Energy
Prompt Radiation
Two Fission Products
Prompt Neutrons
Neutrons Lost by Absorption and Leakage
Beta Decay
Delayed Gammas on D_2O
Leakage
Delayed Neutrons
Daughter Nuclides
Neutrons Lost to Leakage
Neutrons Thermalized by Collisions with Moderator
Photoneutrons

reactor are prompt. However, certain fission fragments, which are called precursors, undergo a beta decay to a daughter nuclide that then emits a neutron. Neutrons produced in this manner are referred to as delayed. The delay is the time that must elapse for the precursor to undergo its beta decay. Delayed neutrons constitute an extremely small fraction of a reactor's total neutron population. Nevertheless, they are crucial to the safe operation of a reactor during power transients. (**Thought Question:** Why do fission products undergo beta decay and why do some excited daughters produce delayed neutrons? See pp. 10-11 of Part One of these notes.)

4. Importance of Delayed Neutrons

a) The time required for a prompt neutron to be born, thermalize, and cause a fission is on the order of $1 \cdot 10^{-4}$ s. This is too rapid for human or machine control.

b) Delayed neutrons have an average lifetime of 12.2 s.

c) The effective fraction of neutrons that are delayed at thermal energies in a typical light-water reactor is 0.0065. This quantity is denoted by the Greek letter $\overline{\beta}$ and is called Beta.

d) Assume that a reactor has 100,000 neutrons present. The lifetime of an 'average' neutron is therefore:

$$\frac{(\#\text{ prompt n's})(\text{prompt lifetime}) + (\#\text{ delayed n's})(\text{delayed lifetime})}{\text{total \# neutrons}}$$

or $$\frac{(99350)(0.0001) + (650)(12.2)}{100{,}000}$$

or 0.079 s

Thus delayed neutrons lengthen the average neutron lifetime and result in a controllable reactor.

5. Effect of Slowing Down Process on Delayed Neutron Fraction:

In the previous section, the term “effective” was used to describe the delayed neutron fraction. This is because the delayed neutron fraction changes as the neutrons slow down. Denote the fraction that is delayed at birth by the symbol β. Both delayed neutrons and prompt neutrons are born at fast energies. However, they exhibit very different spectrums and, on average, the delayed neutrons are born at lower energies then the prompt ones. This means that delayed ones are subject to fewer losses during the slowing down process. In particular, losses to leakage are less. Hence, as the neutrons slow down, the fraction that originally were delayed will increase. Thus, a different symbol, $\overline{\beta}$, is used to denote the delayed neutron fraction at thermal energies and it is called "beta effective." A numerical example (artificial numbers) may help to illustrate the concept.

Event	Prompt	Delayed	Total	Delayed Fraction
Birth	1000	100	1100	0.0909
Loss to Leakage	50%	40%	-	-
Thermal	500	60	560	0.1071

Thus, because a smaller percentage of delayed neutrons are lost to leakage during the slowing down process, their effective fraction increases.

Thought Question: How does the effective delayed neutron fraction change over core life of a small research reactor such as the MITR? To answer this, consider the effective size of the reactor. At the beginning of core life (fresh fuel), the control blades are low in the core. The core volume is small and the surface to volume ratio is large. There is a lot of leakage. As the core ages, the blades are withdrawn. Core volume increases, the surface to volume ratio decreases, and core leakage decreases. So, the effective delayed neutron fraction also decreases. The effect is slight though.

6. Reactivity

a) Reactivity is a measure of the departure of a reactor from criticality. It's mathematical definition is:

$$\rho \equiv \frac{K-1}{K}$$

where K is the reactor's multiplication factor.

b) If the reactivity is negative, the reactor is subcritical. Conversely, if it is positive, the reactor is supercritical. If the reactivity is zero, the reactor is exactly critical.

c) Reactivity may be thought of as the 'fractional change in the neutron population per neutron generation.'

d) Reactivity is a global property of a reactor. Nevertheless, it is common practice to speak of the reactivity worth of a control rod or of the soluble poison. Withdrawing a rod or diluting the poison is said to 'add reactivity.'

7. Allowed Magnitude of Reactivity

a) We noted earlier that there are two kinds of neutrons: prompt and delayed. The latter are produced on a time scale that is controllable by humans and instruments. Thus, it is essential that reactor transients always be conducted in a manner such that the delayed neutrons are the rate determining factor.

b) The effective fraction of delayed neutrons is 0.0065. Therefore, the amount of positive reactivity present in a reactor should never be allowed to exceed some small percent of the effective delayed neutron fraction.

c) Reactivity, being dimensionless, has no units. But, it is common to measure it relative to the effective delayed neutron fraction. We say that:

$$1 \text{ Beta} = 0.0065\ \Delta K/K$$

Reactivity is sometime also measured in dollars and cents with 1 Beta = \$1 = 100 cents.

8. Rationale for Limiting Reactivity

a) The following table illustrates the importance of limiting reactivity additions. Shown are three cases, all with the same initial condition: reactor critical with a population of 10,000 neutrons.

b) For the first case, no change is made. One generation later there are 9935 prompt neutrons and 65 delayed ones. Criticality can NOT be maintained without the delayed neutrons. Hence, they are the rate-determining step.

c) For the second case, we add 0.500 Beta of reactivity. This corresponds to (0.0065 ΔK/K) (0.50) or 0.00325 ΔK/K or 33 neutrons in the first generation. Thus, after one generation there are 9968 prompt neutrons and 65 delayed ones for a total of 10,033. The delayed neutrons are still controlling because it takes 100,000 neutrons to stay critical and there are only 9968 prompt ones.

d) For the third case, we add 1.5 Beta of reactivity. This corresponds to (0.0065)ΔK/K (1.5) or 0.009 ΔK/K or 98 neutrons in the first generation. Thus, after one generation there are 10,032 prompt neutrons and 66 delayed ones. There are more than enough prompt neutrons to maintain criticality. The prompt neutrons are controlling with their 10^{-4} s life cycle.

Initial Condition	Initial Population	Reactivity Addition	One Generation Later	Condition
Critical	10,000 neutrons 9,935 prompt 65 delayed	0% ΔK/K or 0.0 Beta or 0 neutrons	10,000 9,935 65	Steady-State; Not critical on prompt neutrons alone
Critical	10,000 neutrons 9,935 prompt 65 delayed	0.325% ΔK/K or 0.5 Beta or 33 neutrons	10,033 9,968 65	Supercritical; Not critical on prompt neutrons alone
Critical	10,000 neutrons 9,935 prompt 65 delayed	0.98% ΔK/K or 1.5 Beta or 98 neutrons	10,098 10,032 66	Power Runaway; Critical on prompt neutrons alone

9. **Power-Period Relation**

a) Reactivity is not directly measurable and hence most reactor operating procedures do not refer to it. Instead, they specify a limiting rate of power rise, commonly called a 'reactor period.'

b) Reactor period is denoted by the Greek letter, τ, and is defined as:

$$\tau \equiv n(t)/(dn(t)/dt)$$

where n(t) is the reactor power. Thus, a period of infinity corresponds to the critical condition.

c) If the period is constant, then the relation between power and period is:

$$P(t) = P_o e^{t/\tau}$$

where P(t) is the power level, P_O is the initial power, e is the exponential, and t is time.

10. Examples of Power-Period Relation

a) Suppose the period is 100 s and the initial power is 10% of rated. How long before 100% power is attained?

$$P(t) = P_o e^{t/\tau}$$

$$100\% = 10\% \; e^{t/100}$$

$$\ln(100/10) = t/100$$

$$230s = t$$

b) Suppose the reactor period is equal to the prompt neutron lifetime of $1 \cdot 10^4$ s. By what factor would power rise in 1.0 ms?

$$P(t) = P_o e^{t/\tau}$$

$$P/(t)/P_o = e^{1 \cdot 10^{-3}} / 1 \cdot 10^{-4}$$

$$= 22{,}026$$

The reactor is uncontrollable.

c) Repeat problem #2 with a period of 0.079 s. The answer is 1.013. Hence the value of delayed neutrons.

11. Reactor Kinetics

In order to understand the time-dependent behavior of a reactor we need equations that describe the response of the prompt and delayed neutron populations to changes in reactivity. This problem is mathematically complex because the neutron population in a reactor is a function of both space (i.e., position in the core) and time. For many practical situations, we can assume that the spatial and temporal behavior are separable. This allows us to write equations of reactor kinetics as a function of time alone. This approach is acceptable for purposes of personnel training and for routine reactor operation including transients. It is often NOT acceptable for reactor design analysis and for some safety studies.

The space-independent equations of reactor kinetics, which are often called the "point kinetics" equations are:

$$\frac{dn(t)}{dt} = \frac{\left(\rho(t) - \overline{\beta}\right)}{\ell^*} n(t) + \sum_i^N \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\overline{\beta}_i n(t)}{\ell^*} - \lambda_i C_i(t)$$

where $n(t)$ = is the reactor power,

$\rho(t)$ = is the net reactivity,

$\overline{\beta}$ = is the effective delayed neutron fraction,

ℓ^* = is the prompt neutron life time,

λ_i = is the decay constant of the ith precursor group,

C_i = is the concentration of the ith precursor group, and

N = is the number of delayed neutron precursor groups

Quantity	Physical Meaning
$\rho(t)$	Fractional change in the total neutron population per generation.
$\overline{\beta}$	Effective fraction of neutrons that are delayed.
$\left(\rho(t)-\overline{\beta}\right)$	Fractional change in the prompt neutron population per generation.
$1/\ell^*$	Number generations per unit time.
$n(t)$	Total neutron population.
$\dfrac{\left(\rho(t)-\overline{\beta}\right)}{\ell^*}n(t)$	Change in prompt neutron population per unit time.
$\lambda_i C_i$	Rate of decay of delayed neutron precursors. This equals the rate of appearance of the delayed neutrons.
$\dfrac{\overline{\beta}_i n(t)}{\ell^*}$	Rate of production of delayed neutron precursors per unit time.

The first kinetics equation describes the behavior of the neutrons. It states that the rate of change of the total neutron population equals the sum of the rates of change of the prompt neutrons and the delayed ones. The second kinetics equation describes the behavior of the precursors. It says that the rate of change of the precursors is the difference between their production and loss.

These equations are not too useful to a reactor operator. One can not measure precursors and it is difficult to visualize the consequences of two simultaneous differential equations. A more useful approach is to combine these equations through a process of differentiation and substitution to obtain the dynamic period equation. This equation relates the reactor period, which is measurable, to the reactivity.

12. Dynamic Period Equation

a) It is useful to relate reactivity to period. Most text books do this by use of the Inhour Equation which is valid only a long time after reactivity changes and then only for step changes. A more general relation, one that is valid under all conditions, is the dynamic period equation. (The full version of this equation was developed at MIT in the mid-1980s and is the basis of MIT's very successful program on digital control of reactors.) A simplified version is:

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)}$$

where

$\tau(t)$	=	is the reactor period,
$\overline{\beta}$	=	is the effective delayed neutron fraction,
$\rho(t)$	=	is the net reactivity,
$\dot{\rho}(t)$	=	is the rate of change of the net reactivity, and
$\lambda_e(t)$	=	is the standard, effective multi-group decay parameter.

Quantity	Physical Meaning
$\dot{\rho}(t)$	Rate of change of reactivity. This is proportional to the prompt neutron population. Changes in the velocity of a control device therefore have an immediate effect on the period.
$\lambda_e(t)\rho(t)$	This term is proportional to the delayed neutron population. Reactivity can not be changed on demand. Rather, a control device's position has to be altered or the burnable poison concentration has to be adjusted. This takes time.

b) It is important to note that the reactor period depends on both the rate of change of reactivity $(\dot{\rho})$ and the total reactivity (ρ). The former corresponds to prompt neutron effects; the latter to delayed ones. Hence:

- The speed at which one changes reactivity alters the period. This is the basis of power cutbacks that involve high speed rod insertions.

- The reactor period is a function of the power history because the decay term reflects the power level that existed when the delayed neutron precursors were created. This is one reason why it is important to approach a final power level slowly.

c) The complete dynamic period equation is:

$$\tau(t) = \frac{\left(\overline{\beta}-\rho(t)\right)+\ell^*\left[\frac{\dot{\omega}(t)}{\omega(t)}+\omega(t)+\lambda_e(t)-\frac{\dot{\lambda}_e(t)}{\lambda_e(t)}\right]}{\dot{\rho}(t)+\lambda_e(t)\rho(t)+\frac{\dot{\lambda}_e(t)}{\lambda_e(t)}\left(\overline{\beta}-\rho(t)\right)}$$

where the standard, effective, multi-group decay parameter is defined as

$$\lambda_e(t) \equiv \Sigma\lambda_i C_i / \Sigma C_i(t) \text{ for } i=1,N$$

and where symbols not previously defined are:

$\dot{\omega}(t)$ is the rate of change of the inverse of the dynamic reactor period,

$\omega(t)$ is the inverse of the dynamic reactor period,

$\dot{\lambda}_e(t)$ is the rate of change of the standard, effective, multi-group decay parameter,

$C_i(t)$ is the concentration of the i<u>th</u> precursor group normalized to the initial power, and

N is the number of groups of delayed neutrons, including photo-neutrons.

d) Students should review the derivation of the equation, which is given in an appendix to Part Twenty-Eight of these notes. Upon doing so, one will see that the $\dot{\rho}$ term originates with the prompt neutrons while the $\lambda_e(t)\rho(t)$ term reflects the delayed neutrons. (The third term in the denominator $\left(\left(\dot{\lambda}_e/\lambda_e\right)\left(\overline{\beta}-\rho(t)\right)\right)$ is the redistribution of the precursors among the defined groups as the power changes. (<u>Note</u>: There are two versions of the dynamic period equation: standard and alternate. They are mathematically equivalent. The alternate one defines $\lambda_e(t)$ differently and, as a result, is easier to program in a computer.)

e) The multi-group decay parameter is time dependent. When power is rising, short-lived precursors dominate. The opposite is true for power decrease – the long-lived ones dominate.

13. Step Change in Reactivity

Step changes of reactivity can be analyzed analytically (i.e., without the need for a computer). However, such changes do not usually occur on operating reactors. Exceptions might be the sudden injection of a cold slug of water into a steam generator as might occur if the hot well level control valve on a condenser failed open.

A step insertion of reactivity such that $\rho < \overline{B}$ will cause:

a) A rapid increase in the prompt neutron population. This is called the "prompt-jump" and it represents the start of a nuclear runaway. But the runaway can not continue because the reactivity is less than the delayed neutron fraction.

b) A rise on a period corresponding to the growth of the delayed neutrons.

The magnitude of the step change is given by:

$$P_f = P_i\left(\frac{\overline{\beta}}{\overline{\beta}-\rho}\right)$$

where P_f and P_i are the power levels before and after the step insertion. Note that the effect of a step insertion depends on the initial power level. Suppose a reactivity of $0.2\,\overline{\beta}$ is inserted as a step and also assume the initial power to be 10% of allowed. The final power is 12.5% of allowed – a minor change. But what if the initial power had been 90% of allowed. The final power would now be 112.5% of allowed – a serious problem.

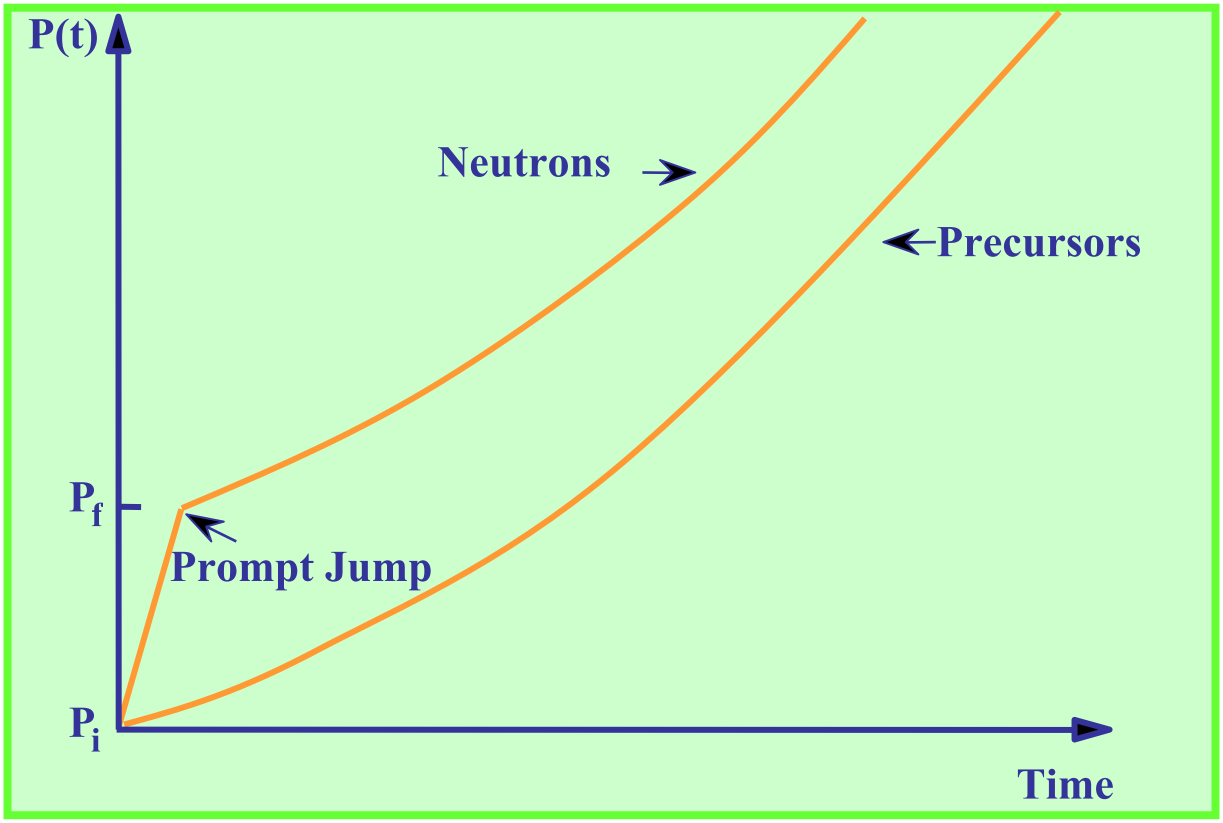
The power behavior following the prompt jump is given by:

$$P(t) = P_f e^{t/\tau}$$

$$\tau(t) = \frac{\overline{\beta}-\rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} \cong \frac{\overline{\beta}-\rho}{\lambda_e\rho}$$

where $\dot{\rho}(t)$ is zero, $\lambda_e(t)$ can be assumed to be 0.1 inverse seconds, and $\rho(t)$ is the step reactivity insertion.

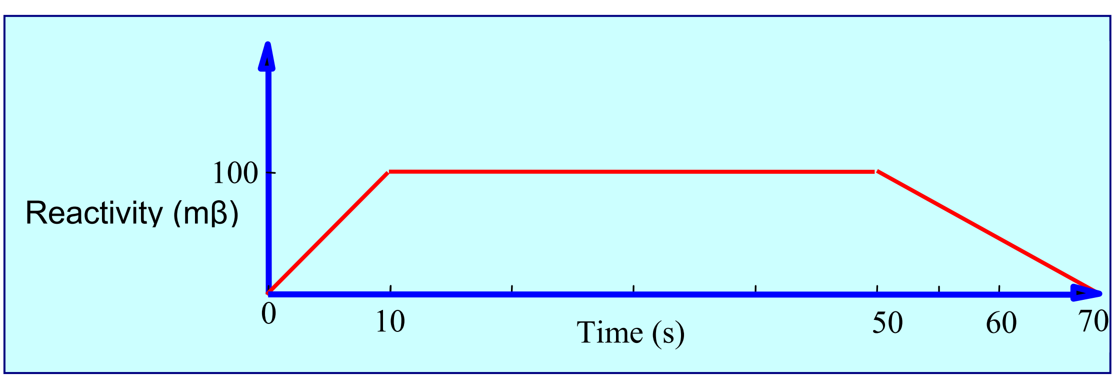
The time-dependent behavior of the reactor power and precursors following a step insertion of reactivity is shown in the following figure. The time scale in this figure is distorted. The prompt jump is over in 100 microseconds and would appear as an instantaneous step rise from P_i to P_f.



15. Ramp Insertion of Reactivity

Ramp reactivity insertions are common in nuclear reactors. These occur when control devices are moved or when the concentration of the soluble boron is adjusted. Even an approximate solution of the power following a ramp insertion requires a computer. The following is a qualitative analysis.

a) **Reactivity** - We assume that the reactivity insertion has the following shape:



Reactivity is inserted at the rate of 10 mbeta/s for 10 seconds, held constant at 100 mbeta for 50 seconds, and then removed at the rate of -5 mbeta/s for 20 seconds.

b) **Period** – The period is defined as the power divided by the rate of change of power. Hence, it is infinite at steady-state and therefore difficult to plot. So, the inverse of the period is plotted.

It is useful to calculate the period immediately before and immediately after each change in the reactivity insertion. Thus, calculations are done at t = 0-, 0+, 10-, 10+, 50-, 50+, 60, 70-, and 70+ where (-) and (+) refer to immediately before and after the indicated time. The relation:

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)}$$

will be used. Also, assume $\lambda_e(t)$ to equal 0.1 s^{-1}

- At t = 0-, both the reactivity and its rate of change are zero. So, the period is:

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.0}{0.0 + (0.1)(0.0)} = \infty$$

- At t = 0+, the reactivity is still zero. But the rate of change of reactivity is now +10 mbeta/s or 0.01 Beta/s. Hence,

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.0}{0.01 + (0.1)(0.0)} = 100\text{s}$$

So, the mere act of initiating a reactivity insertion has immediately placed the reactor on a positive period of 100 s.

- At t = 10-, there is 100 mbeta of reactivity present and reactivity is still being added at the rate of 10 mbeta/s. Thus,

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.1}{0.01 + (0.1)(0.1)} = 45\text{s}$$

So, the period has gone from 100 s to 45 s. Power is rising faster. What happens when the reactivity insertion stops?

- At t = 10+, the reactivity is 100 mbeta. But the rate of change of reactivity is zero. Hence,

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.1}{0.0 + (0.1)(0.1)} = 90\text{s}$$

  So, the effect of stopping the reactivity insertion is to lengthen the period in a stepwise manner.

- At t = 50-, τ(t) = 90 s because conditions are the same as at t = 10+. This means that the period was a constant for 10 < t < 50 seconds. So, during that segment of the transient, the power rose on a pure exponential.

- At t = 50+, the reactivity is still 100 mbeta, but the rate of change of reactivity is now -5 mbeta/s. Thus,

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.1}{-.005 + (0.1)(0.1)} = 180 \text{ s}$$

  The power is still rising. This is an extremely important observation. The fact that the control devices are being inserted does <u>NOT</u> necessarily mean that the reactor power is decreasing. It may still be rising but at an ever decreasing rate. This behavior is the result of the delayed neutron precursors which have not yet attained equilibrium for the current power level. This is one reason why the operation of a nuclear reactor requires skill and experience. Operators must preplan their actions.

- At t=60 s, the reactivity is 50 mbeta and the rate of change of reactivity is -5 mbeta/s. So,

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - .05}{-.005 + (0.1)(.05)} = \infty$$

  A positive reactivity is exactly balanced by a negative rate of change of reactivity. A still rising delayed neutron population is balanced by a decreasing prompt one and, as a result, the rate of change of power is momentarily zero. This condition is called the "point of power turning."

- At t = 70-, the reactivity is zero, but the rate of change of reactivity is still -5 mbeta/s. So,

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.0}{-.005 + (0.1)(0.0)} = -200 \ s$$

- At $t = 70+$, $\tau(t)$ is again infinite.

c) **Power** – The shape of the power profile can be estimated from the periods using the power-period relation $(P(t) = P_0 \exp(t / \tau))$. However, this equation is only valid for a constant period and that is only true for $10 < t < 50$ seconds. The figure on the next page which gives the result is therefore approximate.

14. Response to Ramp Reactivity Insertion

a) At t=0 initiation of control rod motion (reactivity change) causes a step change in period. This is the effect of the prompt neutrons. Power starts to rise.

b) For 0<t<10, as reactivity is added, the period becomes gradually shorter. Power rises ever more rapidly.

c) At t=10 s, rod motion stops. The period lengthens but remains positive because positive reactivity is present. Power continues to rise, but at a lower and now constant exponential rate.

d) For 10<t<50, the period remains constant (zero rate of reactivity change and positive reactivity). Power rises at a constant exponential rate.

e) At t=50 s, the period lengthens because rod motion is inward. The rate of change of reactivity is now negative.

f) For 50<t<60, for the case shown here the rate of negative reactivity insertion (decreasing prompt neutrons) is insufficient to offset the existing positive reactivity (rising delayed neutrons). So, the period remains positive and slowly lengthens. Power continues to rise but at an ever-decreasing rate.

g) At t=60 s, the negative rate of change of reactivity exactly balances the positive reactivity. The period is infinite. The power stops rising and is momentarily constant.

h) For 60<t<70, for the case shown here, the negative rate of change of reactivity is now greater than the remaining positive reactivity. The period goes negative. Power decreases.

i) At t=70 s, rod motions stops and the reactivity is zero. The period goes to infinity and the power remains constant.

Reactivity-Period-Power Profile