22.05 Reactor Physics Part Twenty-Eight

Dynamic Period and Inhour Equations

1. Review

In the previous section, the point kinetics equations were solved analytically subject to the assumption that there was only one delayed neutron group. In reality, there are six. If one allows for all six, then the solutions for T(t) and C(t) will be a sum of seven exponentials -- six for the six precursor groups and one for the prompt neutrons.

2. Inhour Equation

The traditional approach to analyzing a reactor's response with six groups of delayed neutrons is a relation called the inhour equation. It is:

$$\rho = \omega\Lambda + \sum_{i=1}^{N} \frac{\beta_i \omega}{\omega + \lambda_i}$$

Where the name comes from the fact that, in the early days of reactor technology values of ω were quoted in "inverse hours." The value of ρ such that $\omega = 1\ hr^{-1}$ is one inhour. The λ_i are the decay constants for the N precursor groups. This equation is widely noted in the nuclear literature. It is therefore important to understand its applicability. A plot of the inhour formula is given in the next figure. The scale is logarithmic because $-\lambda_6 = -3.0\ s^{-1}$ and β/Λ might be $-125\ s^{-1}$. The quantity $f(\omega)$ in the plot can be taken as the reactivity. Analysis of the figure, from Henry (p. 312) is:

"We see that the roots are always real and that five of them always lie in the range $-\lambda_6 < \omega < \lambda_1$. At most, one of the roots is positive, and then only when $\rho > 0$. The Inhour equation shows that, when $|\omega| \gg \lambda_i$, $0 \approx \omega\Lambda + \beta$ or

$$\omega = \frac{\rho - \beta}{\Lambda}$$

Thus, since $\beta/\Lambda \gg \lambda_i$, there is always one large negative root for $\rho \le 0$. The corresponding most positive (least negative) root for this case is seen from the figure to lie between $-\lambda_1$ and zero. For positive values of reactivity, the condition $\omega \gg \lambda_i$ will occur for $\rho \approx 1.5\beta$, and the most negative root will move over toward the negative side of $\omega = -\lambda_6$ for the super-prompt-critical case. Note that the estimate of the large roots for this case of six precursor groups is the same as that for one. We thus expect that the qualitative behavior derived for the one-precursor-group case ought to be similar to what we would find if all precursor groups were considered." (Henry, pp. 314-315)

The above means that the response of a reactor to a step insertion of reactivity is qualitatively the same for six groups of delayed neutrons as it is for one group. Prompt jumps (or drops) are observed, the rate of change of power becomes asymptotic once the less dominant exponentials die out, and the reactor shows a destructive response for $\rho \geq \beta$.

3. Graphical Solution of Inhour Equation

The plot of f(ω) can be used to solve the inhour equation. Take f(ω), which is on the vertical axis, to be reactivity. The horizontal axis is then ω, the inverse of the reactor period. A horizontal line drawn through the selected value of the reactivity will intersect the plot of the inhour equation. Vertical lines drawn from each intersection gives the values of the ω_i. The following two figures are examples.

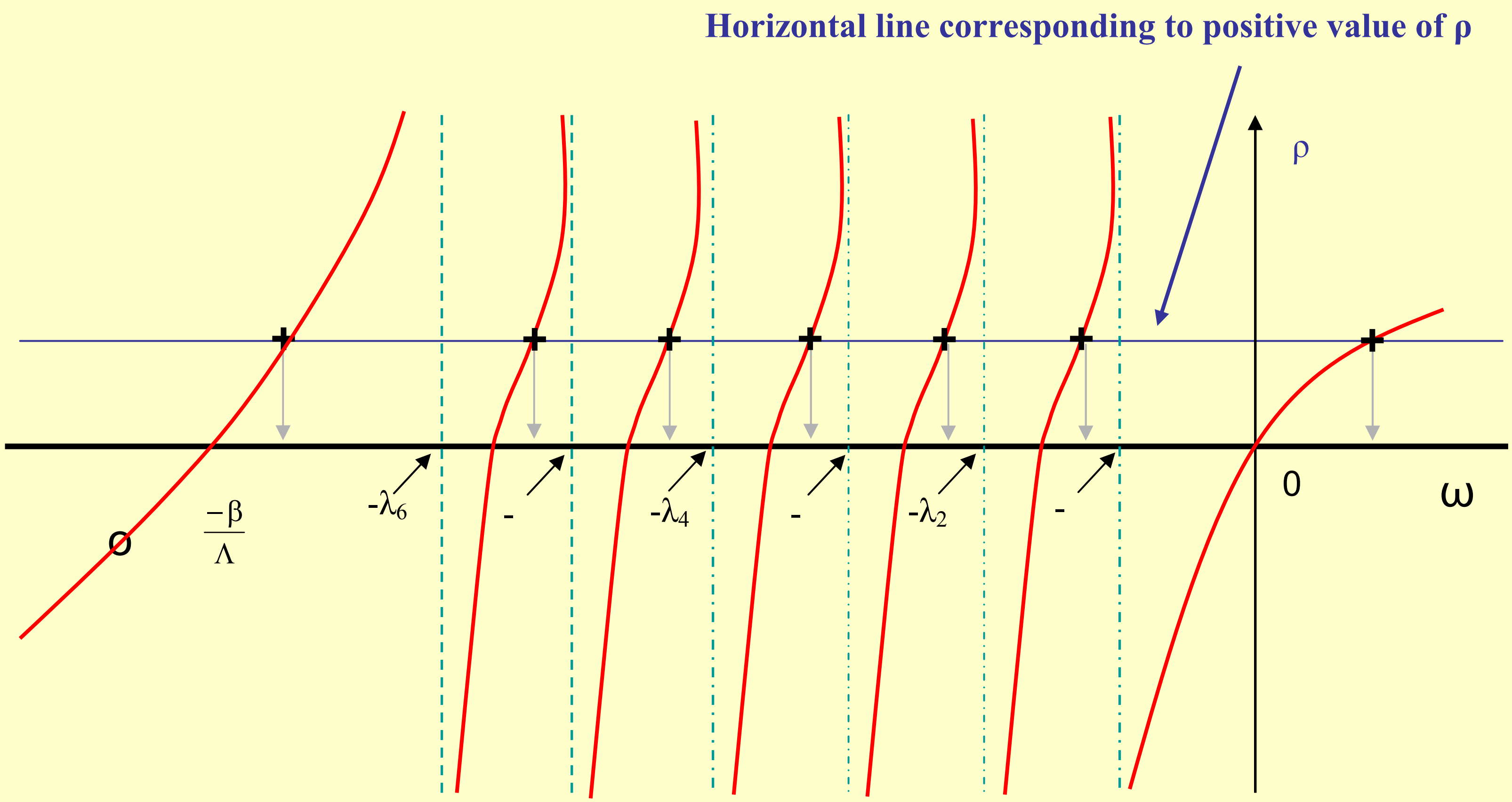
a) In the first figure, a positive value of reactivity is selected. There are seven values of the ω_i, but only one is positive. The reactor power will rise on a period (τ=1/ω) corresponding to this value.

b) In the second figure, a negative value of reactivity is chosen. Again there are seven values of the ω_i, but all seven are negative. The power decreases on each and finally settles on the one that is the least negative. This corresponds to the decay of the longest-lived precursor group.

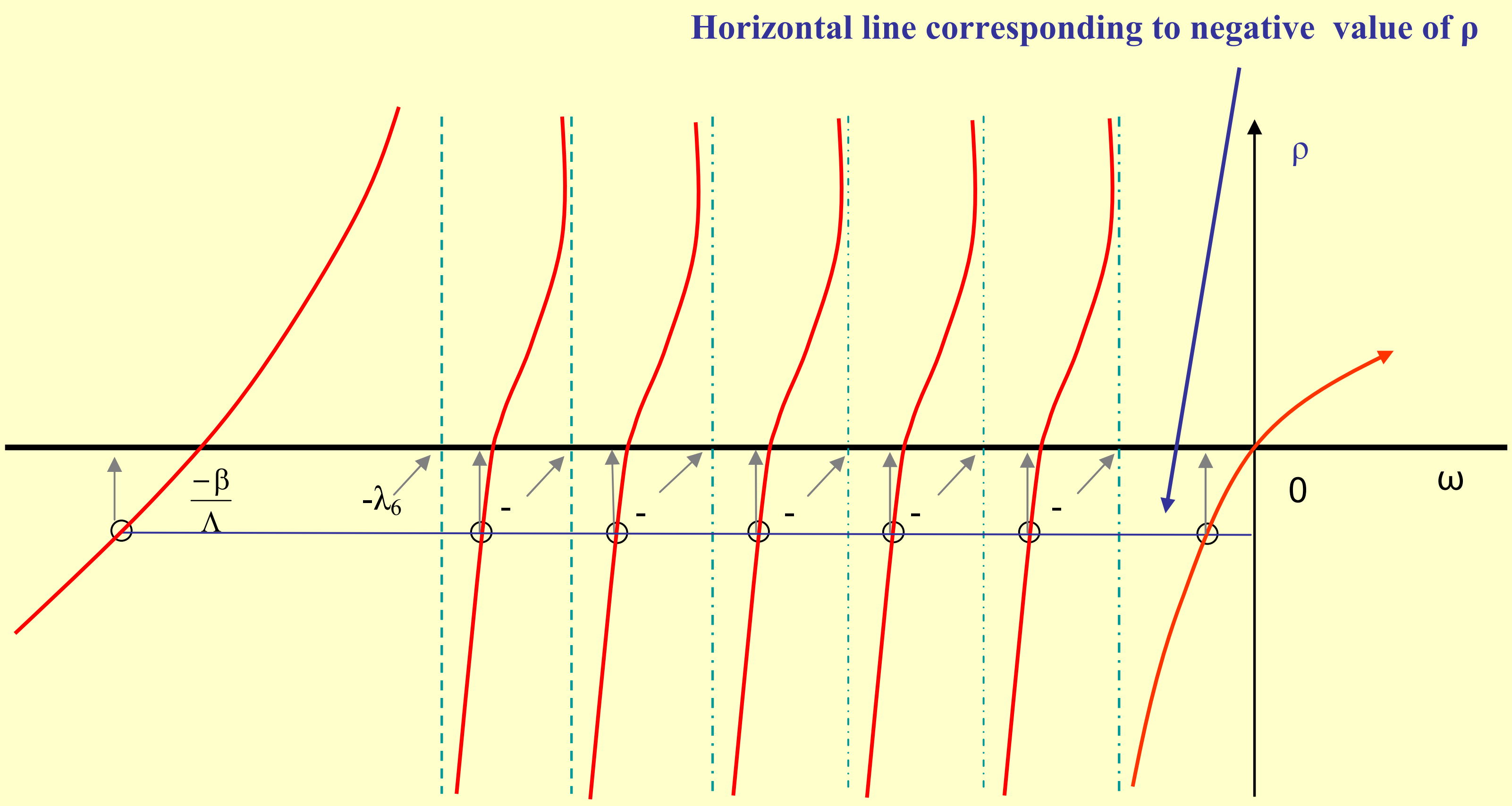
4. Limitations to the Inhour Equation:

Both the inhour equation and our earlier one-group solution of the point kinetics equations give the reactivity in terms of the exponentials that describe T(t) and C(t). That is, in order to obtain a solution, it was assumed that T(t) and C(t) could be described as exp(ωt). We also required that the reactivity be a constant. The following restrictions were imposed:

- Reactivity is changed as a step function and is held constant throughout the transient.
- The solution (asymptotic behavior described by exp(ωt)) is valid a long time after the step change has occurred.



Graphical Solution of Inhour Equation: Positive Reactivity



Graphical Solution of Inhour Equation: Negative Reactivity

These are significant restrictions. They are avoided entirely by the dynamic period equation which is a more general solution of the point kinetics equations. We give its derivation and then show its relation to the inhour equation.

5. **Derivation of the Standard Dynamic Equation**

For this the reader is referred to Part II of the paper, "Dynamic Period Equation Derivation, Relation to Inhour Equations, and Precursor Estimation," by J. Bernard and L. Hu, *IEEE-NS*, 46(3), June 1999. The limitations on it are the same as those in point kinetics – constancy of the flux shape. This is, of course, also a limitation in the inhour equation because it too is a solution of point kinetics.

6. **Relation of the Inhour and Dynamic Period Equation**

The inhour equation is obtained by assuming (1) a constant reactivity (i.e., a step change) and (2) that sufficient time has elapsed since the step change for asymptotic conditions to be attained. As a result, the inhour equation cannot be used to explain such observed phenomenon as the dependence of period on the rate of change of reactivity. In contrast, the dynamic period equation requires no such limitations. It treats the variable reactivity case. It can be shown that the inhour relation is in fact a special case of the dynamic period equation. For this, the reactor is referred to Part III of the paper, "Dynamic Period Equation Derivation, Relation to Inhour Equations, and Precursor Estimation," by J. Bernard and L. Hu, *IEEE-NS*, 46(3), June 1999.