# Basic Physics of Underwater Acoustics

*Reference used in this lecture: Lurton, X. 2002. An introduction to underwater acoustics. New York: Springer.*

# Definitions

p: pressure, measured *relative to hydrostatic*, Pa

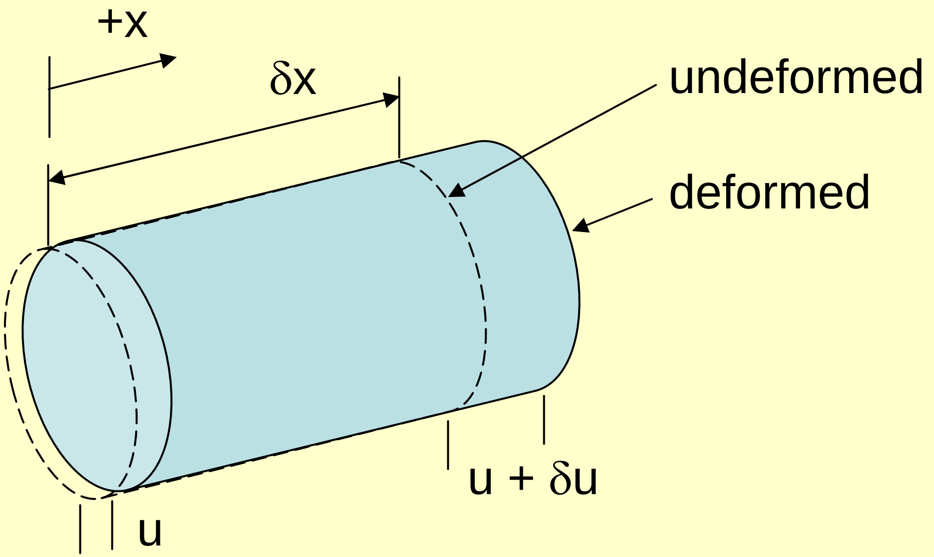
$\rho$: density, measured *relative to hydrostatic*, kg/m$^3$

E: bulk modulus of the fluid, Pa, $\delta p = E [ \delta\rho / \rho ]$

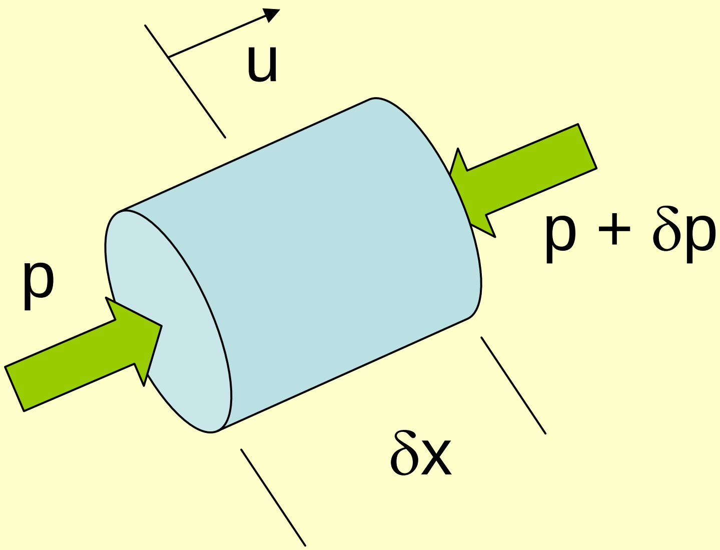
[u,v,w]: deflections in [x,y,z]-directions, relative to the hydrostatic condition, m

Then in one dimension *(pipe)*

$p = E [ -\delta u / \delta x ]$



# One-dimensional Case *cont.*




Newton's Law:

$\delta p = - \rho \, u_{tt} \, \delta x$   OR

$p_x = - \rho \, u_{tt}$

diff wrt x

Constitutive Law:

$p = - E \, \delta u / \delta x$   OR

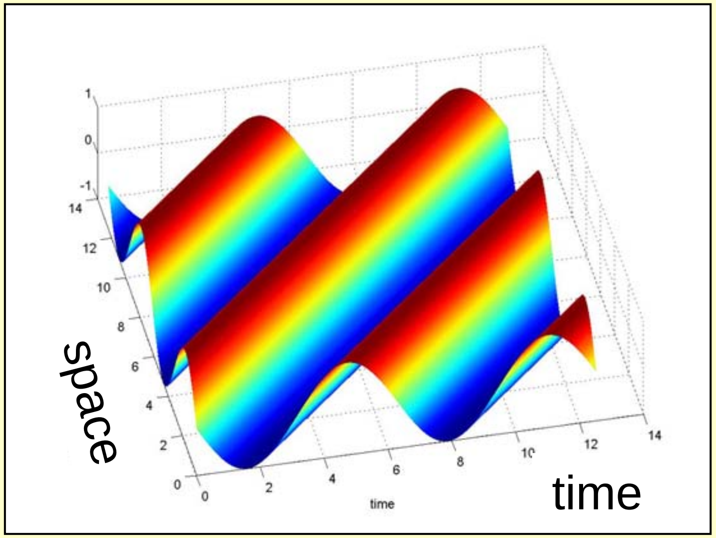
$p = - E \, u_x$

diff wrt tt

$p_{xx} = [\, \rho / E \,] \, p_{tt}$

a wave equation!

Let $p(x,t) = P_o \sin(\omega t - kx)$ ⟶




Insert this in the wave equation:

$$- P_o k^2 \sin(\,) = - [\, \rho / E \,] P_o \omega^2 \sin(\,) \rightarrow$$

$$[\, \omega / k \,]^2 = E / \rho \rightarrow$$

Wave speed $\quad c = \omega / k = [\, E / \rho \,]^{1/2}$

This is *sound speed in water*, independent of pressure, or frequency.

$\rho \sim 1000\ kg/m^3$, $E \sim 2.3e9\ N/m^2$ $\rightarrow$ $c \sim 1500\ m/s$

Wavelength $\quad \lambda = 2\pi/k = 2\pi c/\omega = c/f$; **1kHz : 1.5m**

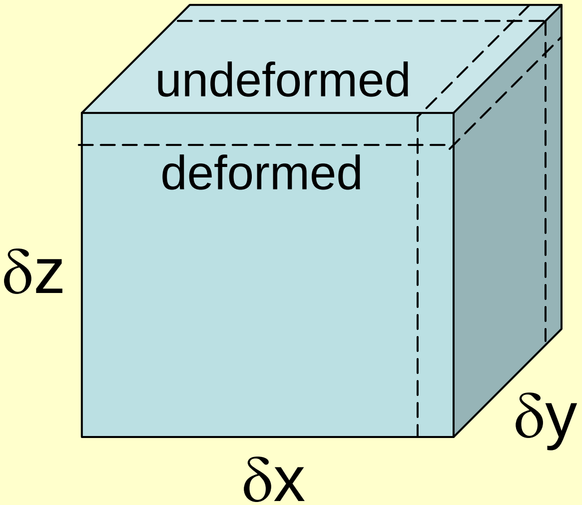
# In Three Dimensions: A CUBE

Newton's Law:

$p_x = -\rho\, u_{tt} \rightarrow p_{xx} = -\rho\, u_{ttx}$

$p_y = -\rho\, v_{tt} \rightarrow p_{yy} = -\rho\, v_{tty}$

$p_z = -\rho\, w_{tt} \rightarrow p_{zz} = -\rho\, w_{ttz}$




Constitutive Law:

$-E\, u_x = p / 3 \rightarrow -E\, u_{ttx} = p_{tt} / 3$

$-E\, v_y = p / 3 \rightarrow -E\, v_{tty} = p_{tt} / 3$

$-E\, w_z = p / 3 \rightarrow -E\, w_{ttz} = p_{tt} / 3$

*All directions deform uniformly*

*Lead to Helmholtz Equation:*

$$p_{xx}+p_{yy}+p_{zz} = p_{tt} / c^2$$
or
$$\Delta p = p_{tt} / c^2$$

*where Δ is the LaPlacian operator*

# Particle Velocity

Consider one dimension again:

$p_x = -\rho\, u_{tt} \rightarrow \quad p_x = -\rho\, (u_t)_t$

If $p(x,t) = P_o \sin(\omega t - kx)$ and $u_t(x,t) = U_{to} \sin(\omega t - kx) \rightarrow$

$-kP_o \cos(\ ) = -\rho\, \omega\, U_{to} \cos(\ ) \rightarrow \quad U_{to} = P_o / \rho\, c$

*Note velocity is in phase with pressure!*

[$\rho$ c]: *characteristic impedance;*

water: $\rho c \sim 1.5e6$ Rayleighs "hard"

air: $\rho c \sim 500$ Rayleighs "soft"

In three dimensions:

$\nabla p = -\rho\, \underline{V}_t$ where

$\nabla p = p_x\, i + p_y\, j + p_z\, k$ and

$\underline{V} = u_t\, i + v_t\, j + w_t\, k$

Note equivalence of the following:

$$\lambda = c / f \quad \text{and} \quad \omega / k = c$$

There is no dispersion relation here; this is the only relationship between $\omega$ and k!

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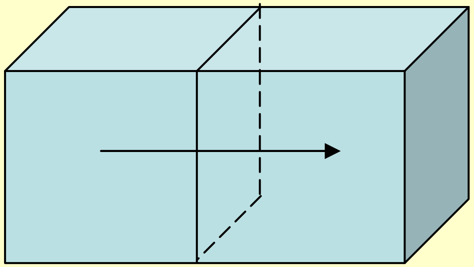
Consider Average Power through a 1D surface:

$$\mathbf{P}(x) = [\, 1 / T \,] \int^{T} p(\tau,x)\, u_t(\tau,x)\, d\tau$$
$$= [\, 1 / T \,] \int^{T} P_o\, U_{to} \sin^2(\omega\tau - kx)\, d\tau$$
$$= P_o\, U_{to} / 2$$
$$= P_o^2 / 2 \rho c = U_{to}^2 \rho c / 2$$

*<u>Acoustic Intensity</u> in* W/m$^2$



*Power per unit area is pressure times velocity*

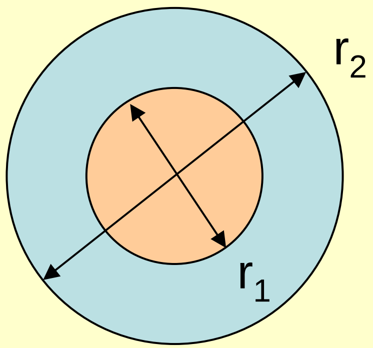
If impedance $\rho c$ is high, then it takes little power to create a given pressure level; but it takes a lot of power to create a given velocity level

# Spreading in Three-Space

At time $t_1$, perturbation is at radius $r_1$; at time $t_2$, radius $r_2$ →

$\mathbf{P}(r_1) = P_o^2(r_1) / 2 \rho c$

$\mathbf{P}(r_2) = P_o^2(r_2) / 2 \rho c$




Assuming no losses in water; then

$\mathbf{P}(r_2) = \mathbf{P}(r_1)\, r_1^2 / r_2^2 = P_o^2\,(r_1)\, r_1^2 / 2 \rho c\, r_2^2$

and

$P_o(r_2) = P_o(r_1)\, r_1 / r_2$

Let $r_1$ = 1 meter (standard!) →

$\mathbf{P}(r) = P_o^2(1m) / 2 \rho c\, r^2$

$P_o(r) = P_o(1m) / r$

$U_{to}(r) = P_o(1m) / \rho c\, r$

← *Pressure level and particle velocity decrease linearly with range*

# Decibels (dB)

$10 * \log_{10}$ (ratio of two positive scalars):

Example:

$x_1 = 31.6$ ; $x_2 = 1 \rightarrow$ 1.5 orders of magnitude difference

$10*\log_{10}(x_1/x_2) =$ **15dB**

$10*\log_{10}(x_2/x_1) =$ **-15dB**

RECALL $\log(x_1^2/x_2^2) = \log(x_1/x_2) + \log(x_1/x_2) = 2 \log(x_1/x_2)$

In acoustics, *acoustic intensity (power) is referenced to* ***1 W/m²*** *;*
*pressure is referenced to* ***1 μPa***

$$10*\log_{10}[\, \mathbf{P}(r) / 1\ W/m^2\,] = 10*\log_{10} \mathbf{[}\, [\, P_o^2(r) / 2\,\rho\, c] / 1\ W/m^2\, \mathbf{]}$$

$$= 20*\log_{10}[\, P_o(r)\,] - 10*\log_{10}(2\rho c)$$

$$= 20*\log_{10}[\, P_o(r) / 1\mu Pa\,] - 120 - 65$$

# Spreading Losses with Range

Pressure level in dB is

$20 \log_{10} [\, P_o(r) / 1\mu Pa \,] - 185 \quad =$

$20 \log_{10} [\, P_o(1m) / r / 1\mu Pa \,] - 185 \quad =$

$20 \log_{10} [\, P_o(1m) / 1\mu Pa \,] - \underline{\mathbf{20 \log10 [r]}} - 185$

**Example:** At 100m range, we have lost

40dB or *four orders of magnitude* in sound intensity

40dB or *two orders of magnitude* in pressure

(and particle velocity)

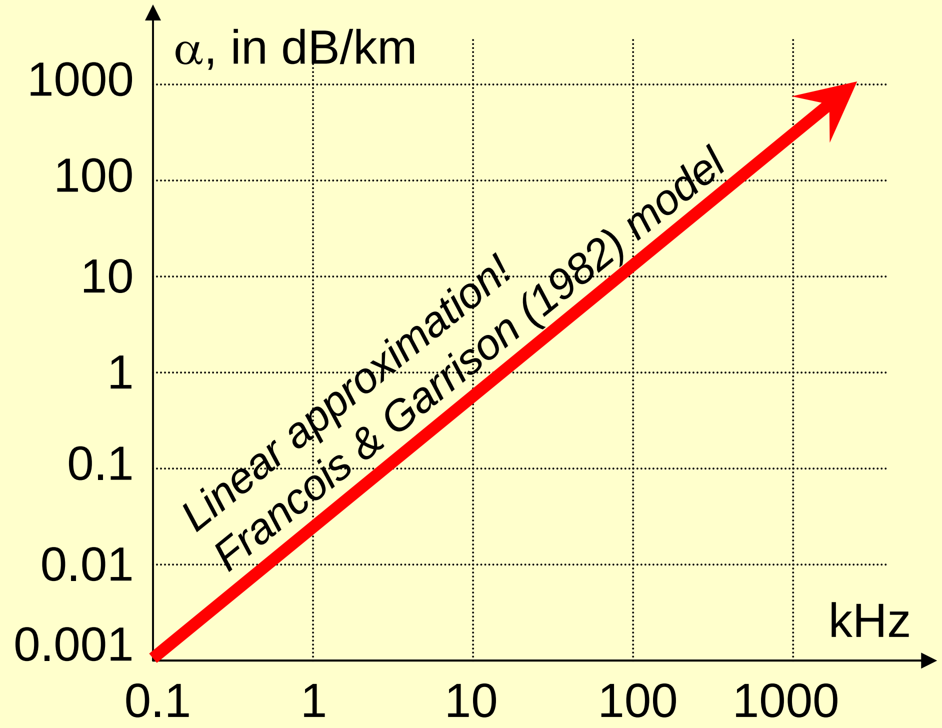
# Attenuation Losses with Range

Acoustic power does have losses with transmission distance – primarily related to relaxation of boric acid and magnesium sulfate molecules in seawater. Also bubbles, etc.

At 100 Hz, ~1dB/1000km:
OK for thousands of km, ocean-scale seismics and communications

At 10kHz, ~1dB/km:
OK for ~1-10km, long-baseline acoustics

At 1MHz, 3dB/10m:
OK for ~10-100m, imaging sonars, Doppler velocity loggers




$$TL = 20\log_{10} r + \alpha r$$

(pressure transmission loss)

# The Piezo-Electric Actuator

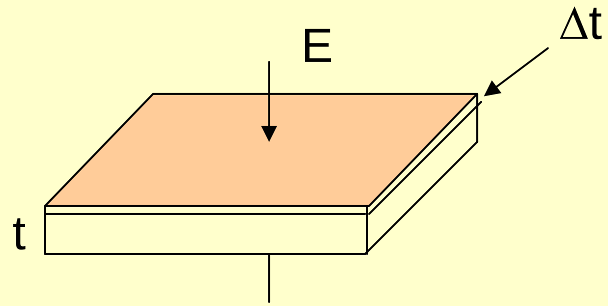
strain = constant X electric field

$\varepsilon = \mathbf{d} \times \mathbf{E}$ or

$\Delta t / t = d \times (V / t)$

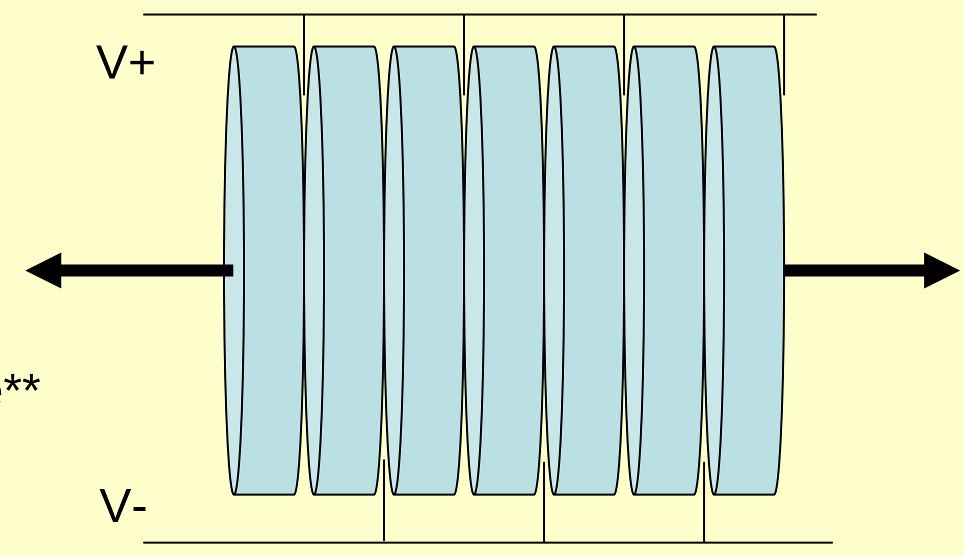
where d = 40-750 x $10^{-12}$ m / V →

Drive at 100V, we get only 4-75 nm thickness change!




Series connection amplifies displacement

**still capable of MHz performance**



# The Piezo-Electric Sensor

electric field = constant X stress

$$\boldsymbol{E = g} \; X \; \sigma \qquad \text{or}$$

$$\boldsymbol{V = t\, g} \; \sigma$$

where g = 15-30 x $10^{-3}$ V/mN

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Ideal Actuator: Assume the water does not impede the driven motion of the material

Ideal Sensor: Assume the sensor does not deform in response to the water pressure waves

Typical Transducer:

*120 to 150 dB re 1$\mu$Pa, 1m, 1V* means

$10^6 – 10^{7.5}$ $\mu$Pa at 1m for each Volt applied or

1-30 Pa at 1m for each Volt applied

Typical Hydrophone:

*-220 to -190 dB re 1$\mu$Pa, 1V* means

$10^{-11}$ to $10^{-9.5}$ V for each $\mu$Pa incident or

$10^{-5}$ to $10^{-3.5}$ V for each Pa incident

So considering a transducer with 16Pa at 1m per Volt, and a hydrophone with $10^{-4}$ V per Pa:

If V = 200V, we generate 3200Pa at 1m, or 3.2Pa at 1km, assuming spreading losses only;

The hydrophone signal at this pressure level will be 0.00032V or 320$\mu$V !