Solutions for the Short Range Shallow Water Channel:

1) Direct path => \( d_0 = 100 \) m  
   Bottom reflection => \( d_1 = \frac{2h}{\cos(\theta_1)} = 107.7 \) m  
   \( \theta_1 = \arctan(r/2h) \)  
   Surface reflection => \( d_2 = \frac{2d}{\cos(\theta_2)} = 188.7 \) m  
   \( \theta_2 = \arctan(r/2d) \)  
   SBS reflection => \( d_3 = \frac{2(2d/\cos(\theta_3)+ h/\cos(\theta_3))}{\cos(\theta_3)} = 260 \) m  
   BSB reflection => \( d_4 = \frac{(2d/\cos(\theta_4)+ 2*(h/\cos(\theta_4)))}{\cos(\theta_4)} = 399.5 \) m

And so forth. See the simple_ray_trace1.m file for calculating the rest of the length of each propagation path.

2) Assuming that the sound speed is constant at \( c=1500 \) m/s, we can calculate the delay of each path using the relationship \( \tau_i = \frac{d_i}{c} \). Then,

- \( \tau_0 = 66.7 \) msec  
- \( \tau_1 = 71.8 \) msec  
- \( \tau_2 = 125.8 \) msec  
- \( \tau_3 = 173.3 \) msec  
- \( \tau_4 = 266.3 \) msec

3) We can calculate the transmission loss along each path as

\[
TL_i = TL_s + TL_a + RL_s + RL_b
\]

\[
TL_i = 20\log(d_i) + ad_i + RL_s + RL_b
\]

- \( TL_0 = 40.3 \) dB  
- \( TL_1 = 42.0 \) dB  
- \( TL_2 = 49.1 \) dB  
- \( TL_3 = 54.0 \) dB  
- \( TL_4 = 60.2 \) dB

where \( a = \frac{(A fm f^2)}{(f^2+fm^2)+Bf^2} \), \( fm \) for 15 C is 100 kecycles/sec, \( A \) is 6e-4, \( B \) is 2.4e-7. The unit of \( a \) is dB/m. We chose \( RL_s \) as 1 dB and \( RL_b \) as 3 dB. (See transmission_loss.m)

We can obtain the impulse response of the channel based on this information. The simple definition of impulse response is the output of a system (in this case the acoustic channel) to an excitation with a unit impulse. A unit impulse is a mathematical signal that has infinite amplitude, zero width, and has an area of unity (1). In discrete time, (after we sample the continues time signals), we can define an impulse as

\[
\delta(n) = \begin{cases} 
1 & n = 0 \\
0 & \text{otherwise} 
\end{cases}
\]
With this in mind, let's assume that we excite the channel with a very short pulse of 0 dB re $1\mu$Pa @ 1m, the received SIL at the receiver will represent the impulse response of the channel. Figure 1 shows the impulse response for this channel.

(see `construct_impulse_response.m`)

![Impulse Response estimate](image)

**Figure 1** The impulse response of the channel at 100 m range.

4) From the TVR graph, at 22 kHz, the SIL level is 144 dB re $\mu$Pa / 1V at 1m. Deriving the transducer with 400 Vrms, the SIL level will be

$$\text{SIL}_{s} = 144 + 10 \log(400) = 170 \text{ dB re } \mu\text{Pa at 1m}$$

Note that for VdB calculation, we used 1 V as our reference.

5) Received sound intensity level, SILr given by $\text{SILr} = \text{SIL}_s - \text{Tli}$

- $\text{SILr}_0 = 129.7 \text{ dB}$
- $\text{SILr}_1 = 128.0 \text{ dB}$
- $\text{SILr}_2 = 120.9 \text{ dB}$
- $\text{SILr}_3 = 116.0 \text{ dB}$
- $\text{SILr}_4 = 109.8 \text{ dB}$
6) For a hydrophone with OCRR -162 dB re 1V/\mu Pa, the output voltage in dB will be

\[ V^h_{\text{dB}} = \text{SILri} - 162 \]

\[ V^h = 10^{(V_{\text{dB}}/10)} \]

The amplifier will add 40 dB voltage gain. Since our reference is Volts, we can write the amplifier gain in dB as

\[ G(\text{dB}) = 10 \log_{10}(V_{\text{out}}/V_{\text{in}}) \]

Then

\[ V_{\text{out}}(\text{dB}) = V_{\text{in}}(\text{dB}) + G \]

or

\[ V_{\text{out}}(\text{dB}) = V_{\text{in}}(\text{dB}) + G \]

where \( V(\text{dB}) = 10 \log_{10}(V) \). Then,

\[ V^a_{\text{dB}} = 10 \log_{10}(V^h) + G = V^h_{\text{dB}} + G = \text{SILri} - 162 + G \]

Then voltage will be

\[ V = 10^{(V_{\text{dB}}/10)} = 10^{(\text{SILri} - 162 + G)/10} \]

\[ V_0 = 5.9 \text{ V} \]
\[ V_1 = 4.0 \text{ V} \]
\[ V_2 = 0.8 \text{ V} \]
\[ V_3 = 0.2 \text{ V} \]
\[ V_4 = 0.1 \text{ mV} \]

If we excite this channel with a pulse of length 8 msec at frequency 22 kHz, we can estimate the received signal as given in Figure 2.

(see construct_rcvd_signal.m)
Figure 2 Estimate of the received signal at 100 m range.

7)

1) Direct path => $d_0 = 1000m$
   Bottom reflection => $d_1 = \frac{2h}{\cos(\theta_1)} = 1000.8 m$
   $\theta_1 = \tan(\frac{r}{2h})$
   Surface reflection => $d_2 = \frac{2d}{\cos(\theta_2)} = 1012.7 m$
   $\theta_2 = \tan(\frac{r}{2d})$
   SBS reflection => $d_3 = 2(\frac{2d}{\cos(\theta_3)} + \frac{h}{\cos(\theta_3)}) = 1028.4 m$
   BSB reflection => $d_4 = (\frac{2d}{\cos(\theta_4)} + 2*(\frac{h}{\cos(\theta_4)})) = 1129.9 m$

2) Assuming that the sound speed is constant at $c=1500 m/s$, we can calculate the delay of each path using the relationship $\tau_i = \frac{d_i}{c}$. Then,

$\tau_0 = 666.7$ msec
$\tau_1 = 667.2$ msec
$\tau_2 = 675.1$ msec
$\tau_3 = 685.6$ msec
$\tau_4 = 753.3$ msec
3) We can calculate the transmission loss along each path as

\[ TL = TL_s + TL_a + RL_s + RL_b \]

\[ TL_i = 20 \log(\text{d}_i) + a \text{d}_i + RL_s + RL_b \]

\[ TL_0 = 62.9 \text{ dB} \]
\[ TL_1 = 63.9 \text{ dB} \]
\[ TL_2 = 66.0 \text{ dB} \]
\[ TL_3 = 68.2 \text{ dB} \]
\[ TL_4 = 71.3 \text{ dB} \]

where \( a = \frac{(A \cdot \text{f}_m \cdot f^2)}{(f^2 + \text{f}_m^2)} + Bf^2 \), \( \text{f}_m \) for 15 C is 100 kcycles/sec, \( A \) is 6e-4, \( B \) is 2.4e-7. We chose \( RL_s \) as 1 dB and \( RL_b \) as 3 dB.

If assume that we excite the channel with a very short pulse of 0 dB re 1mPa @ 1m, the received SIL at the receiver will represent the impulse response of the channel. Figure 3 shows the impulse response for this channel.

![Impulse Response estimate](image)

**Figure 3** Impulse response of the channel at 1000m range.

4) From the TVR graph, at 22 kHz, the SIL level is 144 dB re \( \mu \text{Pa}/\text{V} \) at 1m. Deriving it at 100 V, the SIL level will be
SILs = 144 + 10\log(400) = 170 \text{ dB re } \mu\text{Pa at 1m}

5) Received sound intensity level, SILr given by SILri = SILs – Tli

SILr0 = 107.1 \text{ dB}
SILr1 = 106.1 \text{ dB}
SILr2 = 104.0 \text{ dB}
SILr3 = 101.8 \text{ dB}
SILr4 = 98.7 \text{ dB}

6) For a hydrophone with OCRR -162 dB re 1V/\mu\text{Pa}, the output voltage in dB will be

\[ V_{\text{dB}} = \text{SILri} - 162 \]

The preamplifier will add 40 dB gain. Then voltage will be

\[ V = 10^{(V_{\text{dB}}/10)} = 10^{((\text{SILri} - 162 + G)/10)} \]

\[ V_0 = 32 \text{ mV} \]
\[ V_1 = 25 \text{ mV} \]
\[ V_2 = 15 \text{ mV} \]
\[ V_3 = 10 \text{ mV} \]
\[ V_4 = 5 \text{ mV} \]

If we excite this channel with a pulse of length 8 msec at frequency 22 kHz, we can estimate the received signal as given in Figure 4.
Solutions for determining the range of a source:

1) The first two paths arrive at 66.7 and 71.8 msec. Therefore, the maximum pulse shape should be 5.1 msec. If the pulse becomes longer than this time, the two replicas of the transmitted pulse will overlap in time and cause interference. This interference is called the multipath interference.

2) If we correlate the received signal and the transmitted pulse, we will obtain a peak in the correlation function for each multipath arrival. By determining the time of the direct path’s peak location, we can estimate the propagation time. In the previous section, we assumed that the pulse duration is 8 msec, which causes interference. If we correlate the incoming signal with the original pulse, we obtain the signals given in Figure 5 and Figure 6. We observe that even though there is interference, we can estimate the propagation delay. (see find_delay.m)
Figure 5 The correlation between the received signal and the transmitted pulse.
3) We can send a pulse from the surface craft and listen for the reply of the target. The duration between the transmitted pulse and the reply will be

\[ \tau = 2T_p + \tau_t \]

where \( T_p \) is the propagation time and \( \tau_t \) is the delay of the transponder.
One problem that can arise in this configuration is that we may confuse the bottom and/or surface reflections with the reply of the AUV. To overcome this problem, we may assign different frequencies to downlink (surface craft to AUV) and uplink (AUV to surface craft).

**Determining the Direction of the Target**
As a simple solution, we can measure the propagation time at each hydrophone. By comparing the propagation times, we can determine in which quadrant the target resides.

A more complicated method is to determine the angle $\theta$ for each pair of hydrophones. By averaging the four readings, we can come up with an angle for the target direction. We can calculate the angle as

$$\theta = \text{sign}(r_1-r_4) \arccos\left(\frac{|r_1-r_4|}{d}\right)$$
Program Listings

*simple_ray_trace.m*:  

```matlab
function [delay, transLoss] = simple_ray_trace1(d, h, r, c, f, T, RLs, RLb)

%d = 20;
%h = 80;
%r = 1000;
%c = 1500;

Lcnt = 1;

% Direct path
L(Lcnt) = r;
transLoss(Lcnt) = transmission_loss(L(Lcnt), f, T, 0);
Lcnt = Lcnt + 1;

for a=1:10
    % Rays first bouncing from the surface
    a1 = a;
    a2 = a1-1;
    r1=(d*r)/(2*(d*a1+h*a2));
    r2=h*r1/d;
    L1 = sqrt(r1^2+d^2);
    L2 = sqrt(r2^2+h^2);
    L(Lcnt) = 2*(a1*L1+a2*L2);
    transLoss(Lcnt) = transmission_loss(L(Lcnt), f, T, 0) + a1*RLs + a2*RLb;
    Lcnt = Lcnt + 1;

    % Rays first bouncing from the bottom
    a2 = a;
    a1 = a2-1;
    r1=(h*r)/(2*(h*a1+d*a2));
    r2=d*r1/h;
    L1 = sqrt(r1^2+d^2);
    L2 = sqrt(r2^2+h^2);
    L(Lcnt) = 2*(a1*L1+a2*L2);
    transLoss(Lcnt) = transmission_loss(L(Lcnt), f, T, 0) + a1*RLs + a2*RLb;
    Lcnt = Lcnt + 1;
end;

delay = L / c;
```
transmission_loss.m:

function TL = transmission_loss(d, f, T, verbose)
%
% TL = transmission_loss(d, f, T, verbose)
%
% Calculates transmission loss due to spherical spreading
% and absorption losses. The absorption loss is calculated
% for either T=5°C or T=15°C. For absorption, we use equation
% (9.33) in Fundamentals of Acoustics.
%
% Usage:
% d: distance in meters
% f: center frequency of signals in kilocycles/sec
% T: sea water temperature in Celsius
% verbose: (1) on, (0) off
%
% Decide on the relaxation frequency and B based on the temperature
switch (T)
  case 5
    fm = 60; %(kilocycles/sec)
    B = 3.2e-7;
  case 15
    fm = 100; %(kilocycles/sec)
    B = 2.4e-7;
  otherwise
    error('Temperature can be either 5 or 15 C!!!');
end;
A = 6e-4;

a = (A*fm*f^2)/(f^2+fm^2)+B*f^2;

% Spreading loss
TLs = 20*log10(d);

% Absorption loss
TLa = a * d;

% Transmission loss
TL = TLs + TLa;

if ( verbose )
  disp(['Absorption coefficient a is ' num2str(a)]);
  disp(['Absorption loss is ' num2str(TLa) 'dB']);
  disp(['Spreading loss is ' num2str(TLs) 'dB']);
  disp(['Transmission loss is ' num2str(TL) 'dB']);
end;
**construct_impulse_response.m:**

```matlab
function [impResp, t] = construct_impulse_response(delay, TL, fsamp)

    t = 0:1/fsamp:max(delay);
    impResp = zeros(size(t));

    for cnt=1:length(delay)
        t1 = round(delay(cnt)*fsamp);
        impResp(t1) = 10^(-TL(cnt)/10);
    end;
```

**construct_rcvd_signal.m:**

```matlab
function [rcvd, t] = construct_rcvd_signal(pulse, delay, TL, xmtSIL, rcvrSens, preAmp, fsamp)

Tpulse = length(pulse)/fsamp;
t = 0:1/fsamp:(max(delay)+Tpulse+100e-3);
rcvd = zeros(size(t));

for cnt=1:length(delay)
    t1 = round(delay(cnt)*fsamp);
    t2 = t1 + length(pulse) - 1;
    rcvd(t1:t2) = rcvd(t1:t2) + pulse * 10^((xmtSIL-TL(cnt)+rcvrSens+preAmp)/10);
end;
```

find_delay.m:

function [delay corr] = find_delay(rcvd, fc, Tpulse, fsamp, fsamp1)

t = 0:1/fsamp1:Tpulse;
pulseS = sin(2*pi*fc*t);
pulseC = cos(2*pi*fc*t);

deltaT = fsamp/fsamp1;
t = 1;
cnt=1;
% downsample
while ((t+1) < length(rcvd))
    t1 = floor(t);
    t2 = t1 + 1;
    rcvd1(cnt) = rcvd(t1)*(t-t2) - rcvd(t2)*(t-t1);
    cnt = cnt + 1;
    t = t + deltaT;
end;

corr = xcorr(rcvd1, pulseS).^2 + xcorr(rcvd1, pulseC).^2;
[val, ind] = max(corr);
delay = (ind-length(rcvd1))/fsamp1;
corr = corr(length(rcvd1):end);