Wind Effect on Long Span Bridge

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Abstract

This thesis has studied different types of reactions of long span bridge under wind load, such as vortex shedding, flutter and buffeting. Since all of these conditions have the chance to damage bridge structure, we calculate the particular wind speed and magnitude that will maximize the reaction. To check if the theory works well, a bridge model is set up and analyzed in SAP2000 and the results are compared with one from mechanics theory. A cable-stayed bridge from Hong Kong is selected for analysis. After we compare the bridge reaction by different methods, we find that theory of aerodynamics can provide reasonable result. However, we're still looking for a better performance from the bridge and improved motion control. Thus, to optimize the structure, design of a tuned mass damper is included as well. Summarily, wind effect on long span bridge is predictable and could be well controlled by various means based on the analysis in this thesis.

Thesis Supervisor: Jerome J. Connor

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Chapter 1 Project Preview

Introduction

Due to its variation on magnitude and direction, wind load is considered as the most uncertain factor that influences bridge's behavior. In the history of bridge engineering, lack of knowledge on bridge's reaction under wind load has led to a series of catastrophes, such as the Tacoma Bridge's collapse in 1940. Therefore, a complete theory on wind engineering is indispensible for bridge design. In the past half century, many methods are developed to meet the requirement. In this thesis, I will review and summarize these analysis theories and apply them on long span bridge.

In the first part of the project, I will summarize some possible vibration models of bridge under wind effect and explain the mechanism. The theory in this part will lay a foundation for further analysis. Along with computer software, we can check if result is reasonable.

Besides, what I will pay attention to is that Hong Kong is a city suffers from hurricane every summer and wind design for bridge seems to be more significant. Thus, instead of smooth and steady wind, we will apply wind with extremely large strength on our model to see what will happen to the bridge and if the result is reasonable. With the theory ready, we will try it on Kap Shui Mun Bridge, which is a cable bridge in Hong Kong. During the project, software like Matlab, SAP2000 will be utilized.

Finally, after we obtain data from analysis, we will try to find a way to optimize the design by introduce tuned mass damper on the system.

Methodology

Since what we concern about is dynamic reaction of the bridge, basic mechanics laws lay the foundation for us theories. And because air is a kind of fluid, most theories are also derived from fluid mechanics equations. At very beginning, wind is views as smooth flow when analyzed, which is of moderate strength with low speed. Under this circumstance, reaction like flutter excited by shedding of vortices is relatively simpler. First we will apply equation of motion and combine it with Theodorsen's function. Solution for the differential equation will give us the behavior of bridge. Later on, wind engineering develops rapidly with importance of wind turbulence taken into account then buffeting becomes the reaction that engineers try to avoid in future design. Details of the theory will be illustrated in Methodology.

The objective for the project is to come up with a summary of the wind engineering theory and to see how well they will predict the behavior of bridge under wind load. If it fit the reality well, then we will consider it successful, otherwise I will try to make adjustment. Extension of the theory can also be made to fit suspension bridge, cable bridge and other type of bridges.

Knowledge Requirement

To successfully complete the project, knowledge in the following list is needed

- Structural dynamics, basic mechanics
- Fluid mechanics
- Basic Maths of Linear Algebra, differential equation and numerical method
- Finite element method
- Software of Matlab, SAP2000, Abaqus etc.

Literature Survey

To get familiar with the knowledge required, I search literature both in library and online. Details of my adoption are listed below. [1][2]

Structural dynamics is the most important part regard to the topic I review some books relate to this field.[3] I also collect website of dynamics analysis courses of some universitys [4] [5] etc, from where I can find additional knowledge and helpful software.

Since fluid mechanics and knowledge from mathematics are covered, some textbooks[6] will be used. To get familiar with fundamental finite element method, I choose the book "Fundamentals of Finite Element Analysis" by David V.Hutton[7]. And to prepare for software application in my project, I will study it by exploring the tutorial by myself. Other paper are also reviewed [8][9][10][11]

A brief introduction is demonstrated above. In the rest of my progress report, I will clarify methodology for the thesis in Chapter 2. In Chapter 3, I will include the result generated in software and analyze it. Finally in the last Chapter 4, I will summarize everything I've done and make conclusion.

Chapter 2 Methodology

Causes of Oscillation

Oscillation of bridge under wind load can be different due the variation of wind's magnitude, direction and steadiness. Essentially, wind is flow of air, therefore theories of fluid mechanics should be applied to analysis this kind of problem. However, the real flow pattern of wind is not always easy to describe by a graph and or a simple equation because most of the time wind is not steady. Usually we use mean wind speed along with fluctuating wind to represent the flow. For a certain problem, we make different assumptions.

Three common models of bridge vibration under wind effect are vortex-shedding, flutter and buffeting. Fundamental knowledge for aerodynamics will be introduced first. Then detail of mechanism for each vibration patterns are illustrated afterwards.

Aerodynamics

When wind passes through a bridge, it will react in vertical direction, wind direction along with torsional deflection. Therefore, the bridge deck should follow equations in aerodynamics. Concerning problem of dynamics, equation of motion is the basic rule for all the analysis.

When it comes to problem of bridge, often we pick up a cross section of the bridge and let wind flow from one direction. The wind can viewed separately into two part, which are mean wind speed U and fluctuation u(t) and w(t). The fluctuation part of the wind varies with time. Reactions that induced by wind are horizontal force D, lift force L and moment M. All of above have been represented in Figure 2.1 below. [12]

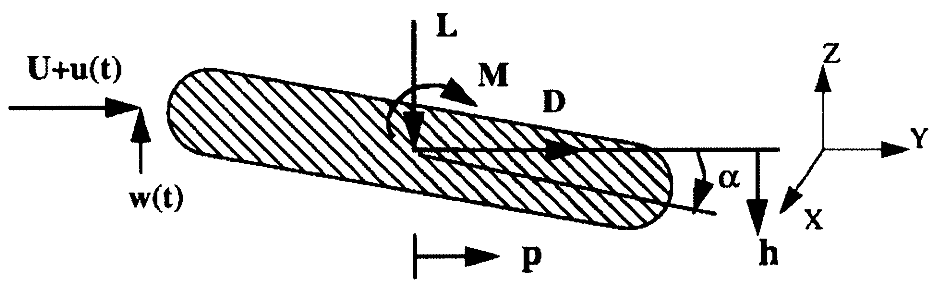


Figure 2.1

Apply equation of motion to the model in vertical direction, wind direction and rotation [1]

$$m\ddot{z} + c_z\dot{z} + s_z z = L \qquad (2.1a)$$

$$m\ddot{y} + c_y\dot{y} + s_y y = D \quad (2.1b)$$

$$I\ddot{\alpha} + c_\alpha\dot{\alpha} + s_\alpha\alpha = M \quad (2.1c)$$

Where z = vertical deflection, positive upwards,

y = horizontal deflection, positive in wind direction

α = torsional deflection, positive when windward edge of deck is above leeward edge

m = mass per unit length

I = moment of inertia per unit length

c_z, c_y, c_α = damping constants in bending and torsion

s_z, s_y, s_α = stiffness constants in bending and torsion

L = aerodynamic lift force per unit length

D = horizontal wind force per unit length

M = aerodynamic pitching moment per unit length

We can also view equations of motion in an alternative way

$$m(\ddot{z} + 2\zeta_z\omega_z\dot{z} + \omega_z^2 z) = L \quad (2.2a)$$

$$m(\ddot{y} + 2\zeta_y\omega_y\dot{y} + \omega_y^2 y) = L \quad (2.2b)$$

$$I(\ddot{\alpha} + 2\zeta_\alpha\omega_\alpha\dot{\alpha} + \omega_\alpha^2\alpha) = L \quad (2.2c)$$

Where $\zeta_z, \zeta_y, \zeta_\alpha$ = mechanical damping ratios to critical in bending and torsion

$\omega_z, \omega_y, \omega_\alpha$ = natural circular frequencies in bending and torsion

Then apply Theodorsen's expression for the lift and moment on flat plate. Deflection in wind direction will not be considered here. Theodorsen's expression is derived from basis solution of Laplace equation and modeling the wing as a circle that can be mapped onto a flat plate through a conformal transformation. Three assumptions are made. 1) The flow is always attached than the motion's amplitude is small. 2) The wing is a flat plate. 3) The wake is flat. The final Theodorsen's expression for lift and moment on a flat plate is [12]

$$L = \pi\rho UBC(h)\left(\dot{z} + U\alpha + \frac{B}{4}\dot{\alpha}\right) - \frac{\pi}{4}\rho B^2(\ddot{z} + U\dot{\alpha}) \qquad (2.3)$$

$$M = \frac{\pi}{4}\rho UB^2C(h)\left(\dot{z} + U\alpha + \frac{B}{4}\dot{\alpha}\right) - \frac{\pi}{16}\rho B^3(\frac{\pi}{16}\ddot{\alpha} + U\dot{\alpha}) \qquad (2.4)$$

Where B = half-width of plate

U = mean wind speed

$h = \omega B/U$ is reduced frequency

$C(h)$ =is the Theodorsen's function and it's a complex number

Since effect of virtual mass and virtual moment of inertia are neglected for the reason that air density is so low, the equation about is reduced as

$$L = \pi\rho UBC(h)\left(\dot{z} + U\alpha + \frac{B}{4}\dot{\alpha}\right) \tag{2.5}$$

$$M = \frac{\pi}{4}\rho UB^2C(h)\left(\dot{z} + U\alpha + \frac{B}{4}\dot{\alpha}\right) \tag{2.6}$$

The form of these two equations is adopted by Scanlan as

$$L = \rho U^2B\left(hH_1^*\frac{\dot{z}}{U} + hH_2^*\frac{B}{U}\dot{\alpha} + h^2H_3^*\alpha\right) \tag{2.7}$$

$$M = \rho U^2B^2\left(hA_1^*\frac{\dot{z}}{U} + hA_2^*\frac{B}{U}\dot{\alpha} + h^2A_3^*\alpha\right) \tag{2.8}$$

Where H_i^*, A_i^* $i = 1,2,3$, are non-dimensional self-excited aerodynamic coefficients

These coefficients sometimes referred to as flutter or aerodynamic derivatives, which are functions of the reduced frequency h. And they are evaluated in the following manner. We make the assumption that a bridge deck is in a wind tunnel and the torsional motion is restrained. Then $\alpha = 0$. Combine equation 2.5 and 2.6, we will have [13]

$$m(\ddot{z} + 2\zeta_z\omega_z\dot{z} + \omega_z^2z) = \rho U^2B(hH_1^*\frac{\dot{z}}{U}) \tag{2.9}$$

If we assume the vertical displacement z is in the form of

$$z = z_0e^{at}\sin\omega t \tag{2.10}$$

Substituting in equation and simplifying leads to the expression

$$\left(a^2 - \omega^2 + 2\zeta_z\omega_z a + \omega_z^2 - \frac{a\rho UBhH_1^*}{m}\right)\sin\omega t + 2a\cos\omega t = 0 \qquad (2.11)$$

For this to be valid at all times, the bracketed terms must both be zero. Then we can derive expression for H_1^* and H_2^*

$$H_1^* = \frac{m}{\rho\omega B^2}2(a + \zeta_z\omega_z) \qquad (2.12)$$

By varying U and determining a and ω from the resulting oscillation each time, H_1^* can be obtained as a function of h

$$H_1^* = \frac{m}{\rho\omega a B^2}2(a^2 - \omega^2 + 2\zeta_z\omega_z a + \omega_z^2) \qquad (2.13)$$

Similarly by restraining vertical motion to allow torsional motion only and then removing all constraints to permit coupled motion, the remaining coefficients $H_2^*, H_3^*, A_1^*, A_2^*, A_3^*$will be determined. Then expression is too complicated and will not be shown here. The other method to obtain the coefficient is referring to graphs below.

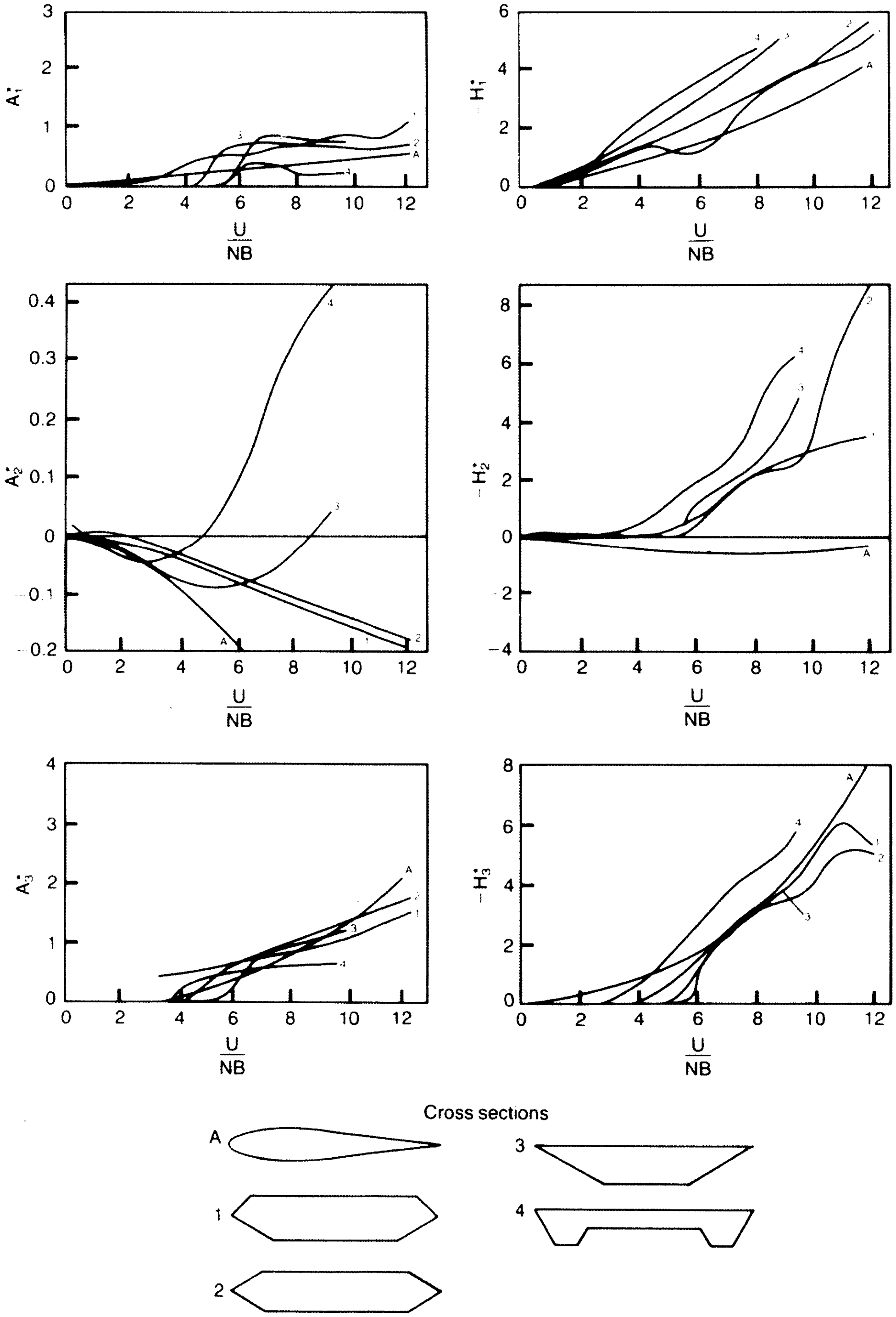


Figure 2.2 Aerodynamics self-excited coefficients

The basic aerodynamics rules have been all introduced, next we will classify wind reaction and explain details of each type in turns. There are differences between each scenario but the main point would always set up equation of motion and solve it.

Vortex-Shedding

As we all known, an object with certain mass and stiffness has its own natural frequency, which is defined as the square root of stiffness over mass. When this object A interfaces with an other object B which is vibrating at A's natural frequency, resonance will occur. The phenomenon is very common in nature and is familiar to everybody. Also, this is the basic mechanism of vortex shedding.

When wind flows through a bridge deck at a particular speed, vortex shedding will be generated at downwind side. Figure 2.3 shows the vortex when flow passes through a cylinder. A low pressure region is generated at downstream side of cylinder. As upper and down flow move to the low pressure region alternatively, vortex is created. Then certain vortex pattern here is called Von Karman Vortex Street.

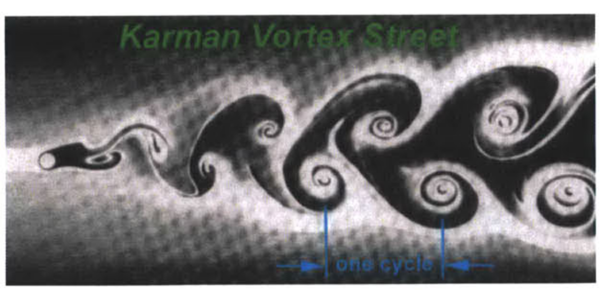


Figure 2.3 Vortex Street

Time for one cycle to occur according to the graph is defined as period of vortex. Frequency of vortex shedding can be determined from following equation. [13]

$$St = \frac{fD}{U} \quad (2.13)$$

Where,

St = Strouhal number, which shows the flow property. Normally, it's between 0.05-0.2, according to table 1.1 [9]

f = frequency of vortex

D = diameter of the cylinder or dimension in vertical direction of the object

U = flow velocity

To determine Strouhal number, we need some experiments on bridge model in wind tunnel. From the measured data, we can calculate Strouhal number for bridge of real size by multiplying scale factor. Then if we know wind velocity, we can easily obtain frequency of vortex.

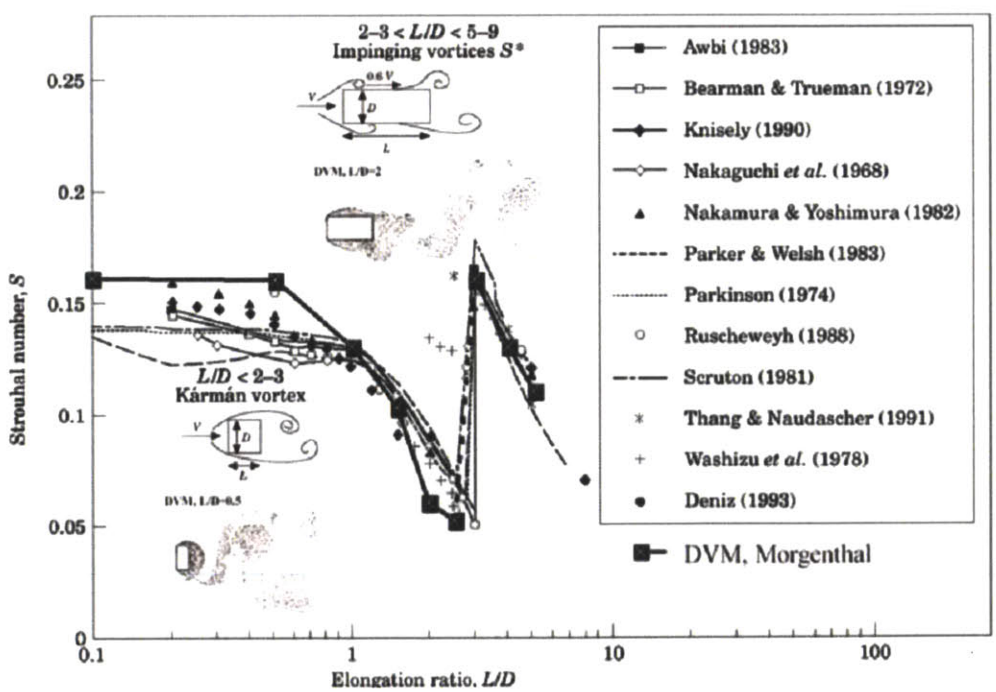


Figure 2.4 Strouhal Number [9]

The harmonic sinusoidal force induced by vortex shedding on bridge is expressed as,[14]

$$F(t) = \rho U^2 D C_L \sin(2\pi f t) \qquad (2.14)$$

Where

ρ is density of air

C_L is lift coefficient which can be found on table 1.2, method to calculate it would be provided later

Cross-section	*S*	$\hat{C}_L$	*B*	*L*
Circular				
Subcritical Re<3x10^5	0.18	0.50	0.10	2.5
Supercritical and transcritical Re>3x10^5	0.25	0.20	0.30	1.0
Square	0.11	0.60	0.25	3.0
Multisided members, rolled structural shapes	0.15	0.60	0.25	2.75

Table 2.1

When it comes to bridge, diameter of object D should be substitute as bridge deck width B. Then we need to apply the load onto equation of motion.

The form of EOM illustrated in previous session is a complete version of EOM for aerodynamics analysis. However, we will select the vortex load as the force on RHS of the equation to meet situation of vortex shedding, which is, [15]

$$m(\ddot{z} + 2\zeta_z\omega_z\dot{z} + \omega_z^2 z) = \rho U^2 B\left(hH_0^* \frac{\dot{z}}{U} + C_L \sin\omega_s t\right) \qquad (2.15)$$

In this equation, second term on RHS stands for periodic force generated by vortex shedding. We use H_0^* instead of H_1^* to signify vortex excitation. Since $\omega = \omega_z = \omega_s$ for vortex excitation, by transferring first term on RHS to left, equation would be rewritten as,

$$\ddot{z} + 2\gamma\omega_z\dot{z} + \omega_z^2 z = \frac{\rho U^2 B C_L}{m}\sin\omega_s t \tag{2.16}$$

where $\gamma = \zeta_z - \frac{\rho B^2 H_0^*}{2m}$

Solve the equation, we will get

$$z = \frac{\rho U^2 B C_L}{2m\gamma\omega_z^2}\sin(\omega_s t - \theta) \tag{2.17}$$

where θ is a phase angle. If the steady state amplitude is measured as z_0, the lift coefficient is

$$C_L = \frac{2m\gamma\omega_z^2}{\rho U^2 B} \tag{2.18}$$

And this is how table 2.1 obtained.

To solve equation of motion, we can use finite element method by denoting vibration mode shape $\emptyset(y)$, vertical deflection at a distance y along the span may be represented by

$$z(y,t) = z_e(t)\emptyset(y) \tag{2.19}$$

Substituting it into equation of motion, multiplying through by $m\emptyset(y)$ and integrating across the span L, it will found that

$$M(\ddot{z_e} + 2\gamma\omega_z\dot{z_e} + \omega_z^2 z_e) = P\sin\omega_s t \tag{2.20}$$

where

$$M = \int_0^L m\emptyset^2(y)dy$$

$$P = \rho U^2 B \int_0^L \emptyset(y)dy$$

Values of parameters γ and ω_z are those appropriate to prototype. Solve the equation and we will get amplitude of oscillation at any point y on the span,

$$z_e(y) = \frac{P}{2M\gamma\omega_z^2}\emptyset(y) \tag{2.21}$$

Now we are able to make a prediction on the behavior of bridge in vertical direction under vortex shedding with equations above. Torsional deflection can be interpreted in same way. In my project, I will analysis Kap Shui Mun Bridge under wind load with the help of SAP 2000. Details will be shown later on. Next, we are going to explain the mechanism of flutter buffeting, which are also wind induced reaction on bridge.

Flutter and Buffeting

Flutter and buffering effect also follows the rule of aerodynamics so the equations introduced above will be used but with some adjustment. Force on the right hand side of equation of motion will be illustrated in two items, which are self-excited force and buffeting force.

Concerning phenomenon like flutter and buffeting, usually we apply finite element method to conduct analysis. Mass, damping ratio, stiffness and force will be transformed into matrix form to enable computer's calculation. The basic equation of motion will be in the form, [16]

$$\mathbf{M\ddot{Z}} + \mathbf{C\dot{Z}} + \mathbf{KZ} = \mathbf{F_{se}} + \mathbf{F_b} \qquad (2.22)$$

Where

M,C,K = mass, damping ratio and stiffness matrices

Z = nodal displacement vector

F = nodal force vector, subscript *se* and *b* represent self-excited and turbulence-induced buffeting

Self-excited force induced by wind can be expressed in the form [16]

$$L_{se}(t) = \rho U^2 B\left(hH_1^*\frac{\dot{z}}{U} + hH_2^*\frac{B}{U}\dot{\alpha} + h^2H_3^*\alpha + h^2H_4^*\frac{z}{B} + hH_5^*\frac{\dot{p}}{U} + h^2H_6^*\frac{p}{B}\right) \qquad (2.23a)$$

$$D_{se}(t) = \rho U^2 B\left(hP\frac{\dot{p}}{U} + hP_2^*\frac{B}{U}\dot{\alpha} + h^2P_3^*\alpha + h^2P_4^*\frac{p}{B} + hP_5^*\frac{\dot{z}}{U} + h^2P_6^*\frac{z}{B}\right) \qquad (2.23b)$$

$$M_{se}(t) = \rho U^2 B^2 \left(hA_1^* \frac{\dot{z}}{U} + hA_2^* \frac{B}{U}\dot{\alpha} + h^2 A_3^* \alpha + h^2 A_4^* \frac{z}{B} + hA_5^* \frac{\dot{P}}{U} + h^2 A_6^* \frac{P}{B} \right) \quad (2.23c)$$

The flutter derivatives are shown in Figure 2.2.Since arbitrary bridge motion is also considered in this situation, I would like to include the equations expressed in terms of convolution integrals as below. [16]

$$L_{se}(t) = \frac{1}{2}\rho U^2 \int_{-\infty}^{t} \begin{pmatrix} I_{L_{seh}}(t-\tau)h(\tau) + I_{L_{sep}}(t-\tau)p(\tau) + \\ I_{L_{se\alpha}}(t-\tau)\alpha(\tau) \end{pmatrix} d\tau \quad (2.24a)$$

$$D_{se}(t) = \frac{1}{2}\rho U^2 \int_{-\infty}^{t} \begin{pmatrix} I_{D_{seh}}(t-\tau)h(\tau) + I_{D_{sep}}(t-\tau)p(\tau) + \\ I_{D_{se\alpha}}(t-\tau)\alpha(\tau) \end{pmatrix} d\tau \quad (2.24b)$$

$$M_{se}(t) = \frac{1}{2}\rho U^2 \int_{-\infty}^{t} \begin{pmatrix} I_{M_{seh}}(t-\tau)h(\tau) + I_{M_{sep}}(t-\tau)p(\tau) + \\ I_{M_{se\alpha}}(t-\tau)\alpha(\tau) \end{pmatrix} d\tau \quad (2.24c)$$

Where I indicates the impulse function of the self-excited forces, which is related to indicial aerodynamics functions. The relationship between this impulse function and flutter derivatives can be obtained by taking the Fourier Transform of 2.1 and compare to the force indicated by Scanlan above,

$$\bar{I}_{L_{seh}} = 2k^2(H_4^* + iH_1^*) \quad (2.25a)$$

$$\bar{I}_{L_{sep}} = 2k^2(H_6^* + iH_5^*) \quad (2.25b)$$

$$\bar{I}_{L_{se\alpha}} = 2k^2 B(H_3^* + iH_2^*) \quad (2.25c)$$

$$\bar{I}_{D_{seh}} = 2k^2(P_6^* + iP_5^*) \quad (2.25d)$$

$$\bar{I}_{D_{sep}} = 2k^2(P_4^* + iP_1^*) \quad (2.25e)$$

$$\bar{I}_{D_{se\alpha}} = 2k^2 B(P_3^* + iP_2^*) \quad (2.25f)$$

$$\bar{I}_{M_{seh}} = 2k^2 B(A_4^* + iA_1^*) \quad (2.25g)$$

$$\bar{I}_{M_{seh}} = 2k^2B(A_6^* + iA_5^*) \tag{2.25h}$$

$$\bar{I}_{M_{seh}} = 2k^2B^2(A_3^* + iA_2^*) \tag{2.25i}$$

Where the overbar denotes Fourier Transform. In light of(3),the self-excited forces can be described in the frequency domain as a product of the bridge displacement and the corresponding transfer function, which has been expressed in terms of flutter derivatives. Because the flutter derivatives are normally known only at discrete values of the reduced frequency k, approximate expressions are used to develop these as continuous functions of the reduced frequency, for future analysis. The rational function approximation approach, known as Roger's approximation, can be utilized for this purpose. With regard to the term corresponding to the lift induced by vertical motion $L_{se}(t)$. The aerodynamic transfer function can be expressed as [16]

$$\bar{I}_{L_{seh}}(i\omega) = 2k^2(H_4^* + iH_1^*) = A_1 + A_2\left(\frac{i\omega B}{U}\right) + A_3\left(\frac{i\omega B}{U}\right)^2 + \sum_{i=1}^{m}\frac{A_{l+3}i\omega}{i\omega + \frac{d_l U}{B}} \tag{2.26}$$

where $A_1,\ A_2,\ A_3, A_{l+3}$ and d_l $(d_l \geq 0; l = 1 \text{ to } m)$ are frequency independent coefficients;

The preceding rational function representation of the aerodynamics transfer functions can be extended into the Laplace domain with s [where s=(-ζ+i) ω, ζ is the damping ratio of the motion] in (2.26) substituted for $i\omega$. The inverse Laplace transform yields the aerodynamics impulse function [16]

$$I_{L_{seh}}(t) = A_1\delta(t) + A_2\frac{B}{U}\dot{\delta}(t) + A_3\frac{B^2}{U}\ddot{\delta}(t) + \sum_{l=1}^{m}\int_{-\infty}^{t} A_{l+3}\exp\left(-\frac{d_l U}{B}(t-\tau)\dot{\delta}(t)\right)d\tau \tag{2.27}$$

Where $\delta(t)$=Dirac delta function

Thus, self-excited lift induced by arbitrary vertical motion can be expressed as [15]

$$L_{se}(t) = \frac{1}{2}\rho U^2\left(A_1 h(t) + A_2\frac{B}{U}\dot{h}(t) + A_3\frac{B^2}{U}\ddot{h}(t) + \sum_{l=1}^{m}\emptyset_l(t)\right) \tag{2.28}$$

Where $\emptyset_l(t)$ $(l = 1 \text{ to } m)$ are new variables that are introduced to express the aerodynamics phase lag and satisfy the following equation [16]

$$\dot{\emptyset}_l(t) = -\frac{d_l U}{B}\emptyset_l(t) + A_{l+3}\dot{h}(t), (l = 1 \text{ to } m) \tag{2.29}$$

Similar formulations for other self-excited force components can be given with analogous definitions and are omitted here for the sake of brevity.

The buffeting forces per unit span corresponding to arbitrary wind fluctuations are expressed in terms of convolution integrals involving the aerodynamics impulse functions and fluctuating wind velocities

$$L_b(t) = -\frac{1}{2}\rho U^2\int_{-\infty}^{t}\left(I_{L_{bu}}(t-\tau)\frac{u(\tau)}{U} + I_{L_{bw}}(t-\tau)\frac{w(\tau)}{U}\right)d\tau \tag{2.30a}$$

$$D_b(t) = -\frac{1}{2}\rho U^2\int_{-\infty}^{t}\left(I_{D_{bu}}(t-\tau)\frac{u(\tau)}{U} + I_{D_{bw}}(t-\tau)\frac{w(\tau)}{U}\right)d\tau \tag{2.30b}$$

$$M_b(t) = -\frac{1}{2}\rho U^2\int_{-\infty}^{t}\left(I_{M_{bu}}(t-\tau)\frac{u(\tau)}{U} + I_{M_{bw}}(t-\tau)\frac{w(\tau)}{U}\right)d\tau \tag{2.30c}$$

Where I indicates the aerodynamics impulse functions of buffeting forces; the subscript represents the corresponding component; and u and w = longitudinal and vertical components of the fluctuating wind velocity, respectively.

Therefore, from the loading expression above, we can solve equation of motion and get the result.

Chapter 3 Test on Software

Vortex-Shedding test

1. Background

By applying the method indicated in previous chapter of this paper, we are going to stimulate wind induce load on Kap Shui Mun Bridge and analyze the result to see if it make sense.

Kap Shui Mun Bridge is built in Hong Kong in year 1997 and is the longest cable bridge that transport both road and railway in the world. The main span of the bridge is 430 meters with overall length of 750 meters. Width of Kap Shui Mun Bridge is 32.5 meters, so half width B that will be frequently used in calculation should be 16.2 meters. The structure of cross section of bridge deck is design as Vierendeel truss to take load both from rail and traffic. [17]

SAP 2000 v12 is the software that I conduct all the analysis with. I will show all the procedure and result of my analysis as follows.

2. Procedure

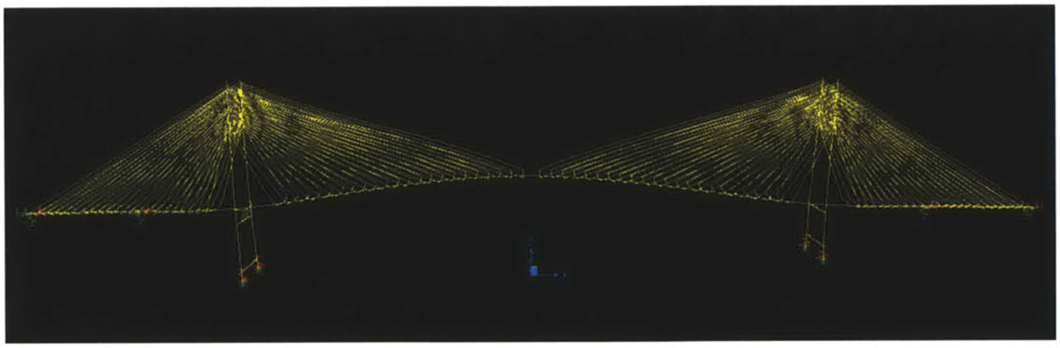
In the model, dead load and load in cables are defined. However, the model is not exactly same as real bridge, some simplifications and assumptions are made to reduce calculation. The bridge deck is transferred into a reinforced concrete beam with same cross section moment of inertia as real one. The connection between bridge and land is defined as some spring. This will give us a relatively better approximation.

Firstly, we need some basic properties of the bridge, like natural frequency and period. By running the Eigen value model, we can get natural frequency for different mode shape. The number of mode shape is set as 10. Result can be found as follows.

Table 3.1

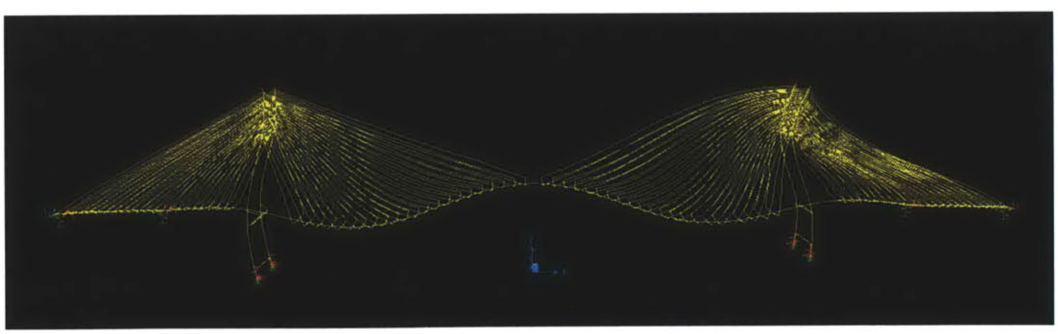
Mode Shape	**Period**	**ω**	**Eigenvalue**
Unit	sec	rad/sec	rad2/sec2
1	2.391549	2.6272	6.9024
2	1.883392	3.3361	11.13
3	1.62744	3.8608	14.906
4	1.573965	3.9919	15.936
5	1.455379	4.3172	18.638
6	1.225917	5.1253	26.269
7	1.052359	5.9706	35.648
8	0.732985	8.5721	73.48
9	0.700958	8.9637	80.348
10	0.601704	10.442	109.04

Mode shapes that deflect in vertical direction are shown below.



Mode 1

Mode 7



Mode 10

The mode shapes are exaggerated so not to scale.

If more modes, for example 20 or 30, we can obtain more mode shapes. However, most of the time we only consider mode shape 1 because this is the vibration shape with lowest energy and is easiest to induce. Since natural frequency is 0.41814, period is 2.391549. Therefore, if vortex shedding generated by wind is also at this frequency, resonance may force the bridge to vibrate at extreme high level. To obtain vortex shedding with frequency of 0.41814, we should apply equation 2.13. For Kap Shui Mun bridge, value of L/D is about 3-5, so we choose 0.16 for Strauhal number. Then at f=0.41814 and D=7.6m, we can get the critical wind speed that will generate vortex-shedding with same natural frequency as bridge,

$$U= 49.87 \text{ m/s}$$

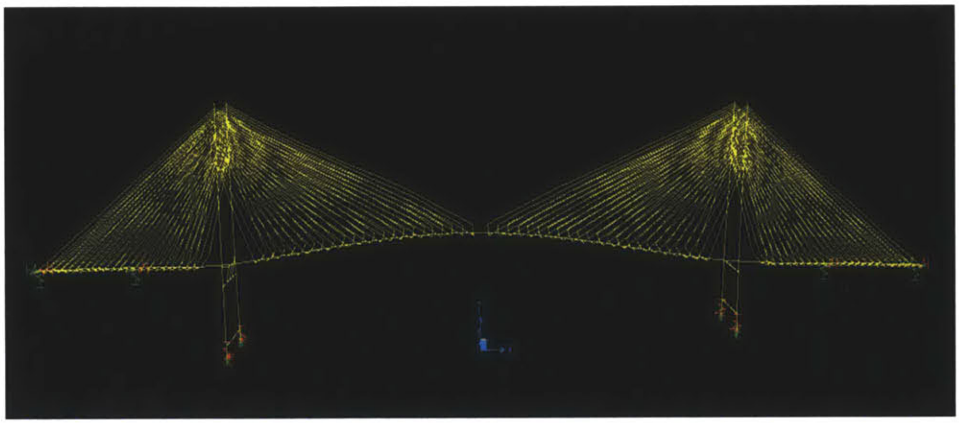
Then substitute the value into equation 2.13 to calculate the sinusoidal force exerted on bridge. Wind density is chosen to be 1.2 kg/m^3 at temperature of 20 at Celcius Degree. Half-width of bridge deck is B = 16.25 m. Lift coefficient is 0.6 according to table 2.1. So the sinusoidal force should be

$$F= 14.88 \sin(2.63t) \text{ (kN/m)}$$

Now we have the data required ready, to input the information, we need to use time history function to deal with dynamics problem. Since the force is sine function, we input a time history function with period of 2.39 second, which has ω = 2.63 rad/s. Then define a load case in form of this time history function. Result would be shown and analyzed as follows.

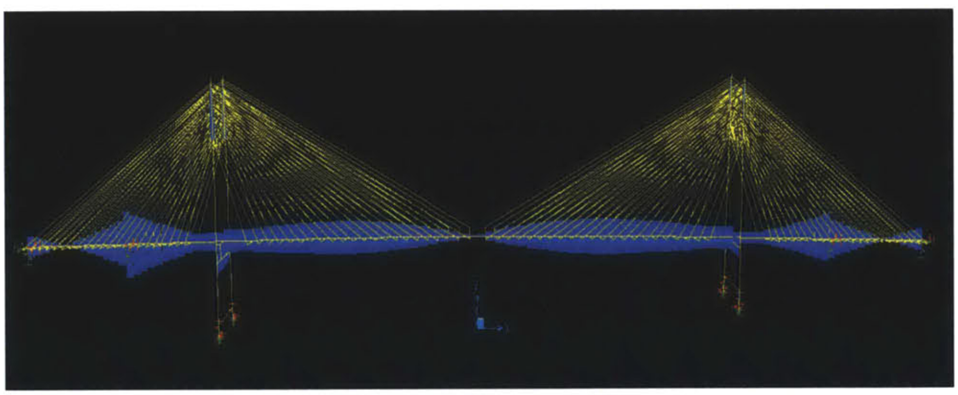
3. Result

The deformed shape according to SAP 2000 is,

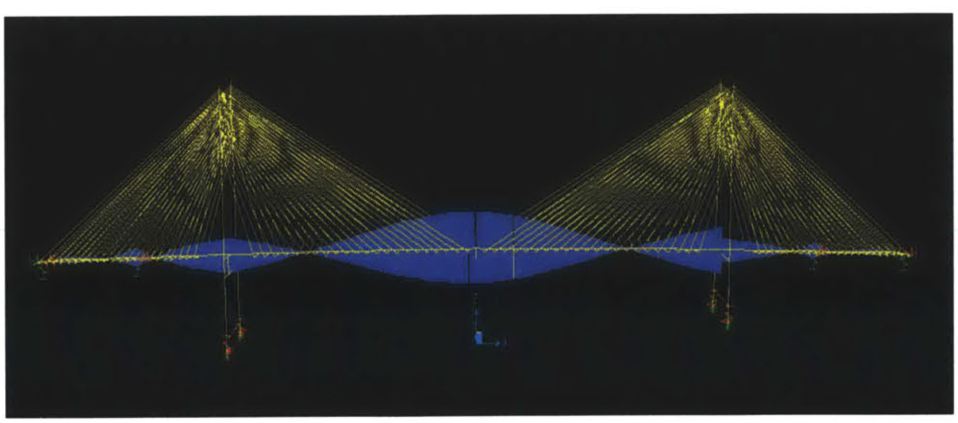


Also, this is not to scale. We can see that it matches mode shape 1 above.

Shear force Envelope of bridge deck



Moment Envelope of bridge deck

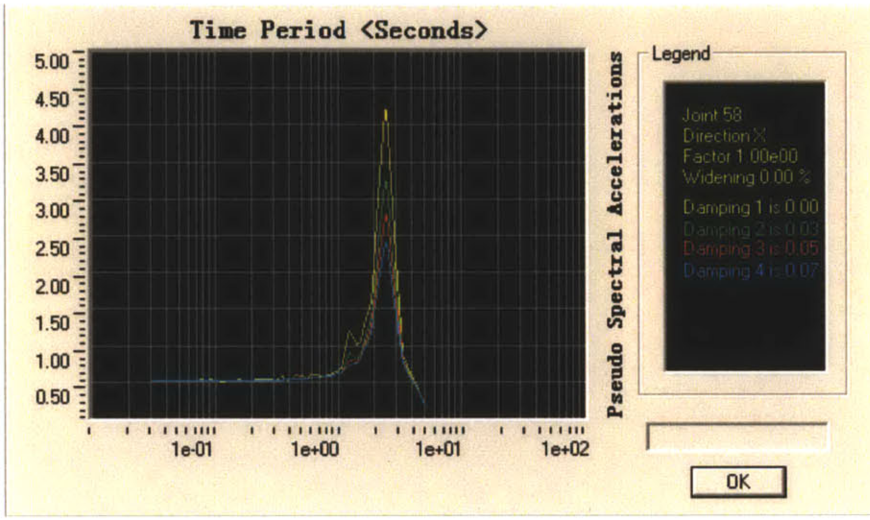


Displacement at mid-span of the bridge

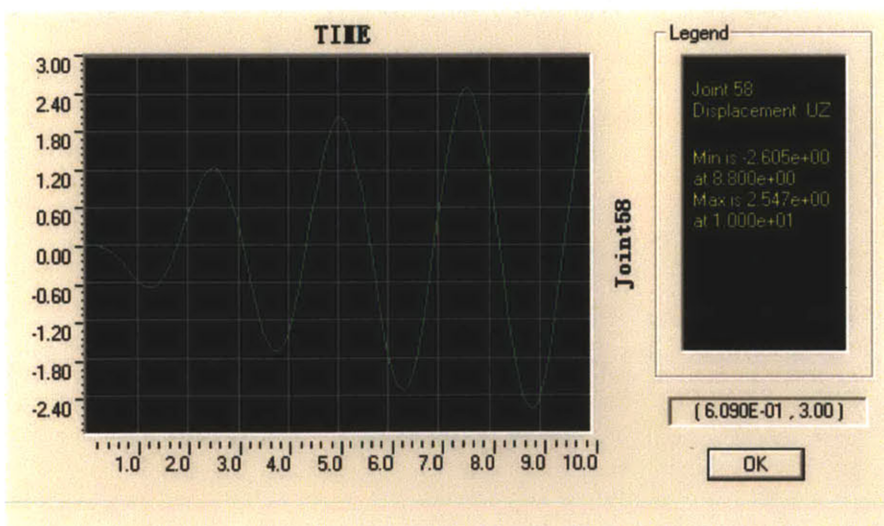
Joint Text	OutputCase Text	CaseType Text	StepType Text	StepNum Unitless	U1 m	U2 m	U3 m
58	PDELTA	NonStatic	Max		-.005966	.000022	-.355714
58	PDELTA	NonStatic	Min		-.005966	.000022	-.355714
58	EIGENMODES	LinModal	Mode	1	-.000249	.0000001432	-.007158
58	EIGENMODES	LinModal	Mode	2	.000001492	-.003397	-.0000004862
58	EIGENMODES	LinModal	Mode	3	.002215	.000001635	-.000115
58	EIGENMODES	LinModal	Mode	4	-.0000002741	-.004734	.000003622
58	EIGENMODES	LinModal	Mode	5	.001523	.000002999	-.001891
58	EIGENMODES	LinModal	Mode	6	-.000372	.000005882	-.000152
58	EIGENMODES	LinModal	Mode	7	.000569	.00039	.005155
58	EIGENMODES	LinModal	Mode	8	-.001064	.00038	.000089
58	EIGENMODES	LinModal	Mode	9	.000154	-.000767	.000199
58	EIGENMODES	LinModal	Mode	10	-.000659	-.000387	-.000172
58	Time History	LinModHist	Max		.081033	.000806	2.547265
58	Time History	LinModHist	Min		-.084755	-.000736	-2.604701

Also refer to joint 58 at mid span, we can see that maximum displacement is about 2.6 meters. Since the problem we consider here is elastic, the program will give us an exact result. However, the 2.6 meter is much more than we could tolerant and the deformation may reach nonlinear region. Whether the bridge will fail or not depends on further calculation. The conclusion is that when resonance occurs, the bridge has the chance of losing control. Therefore, we will introduce methods to mitigate the effect in following chapter.

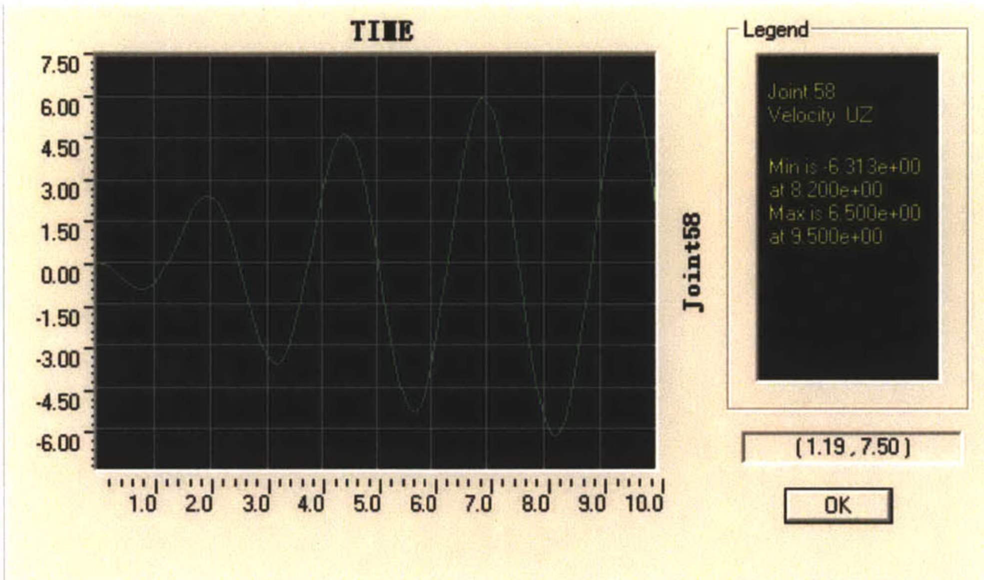
At mid-span joint 58, we can also generate the Response Spectrum Curve and Time history function for displacement, velocity and acceleration. We can see that maximum acceleration occurs at between 2 and 3 second.



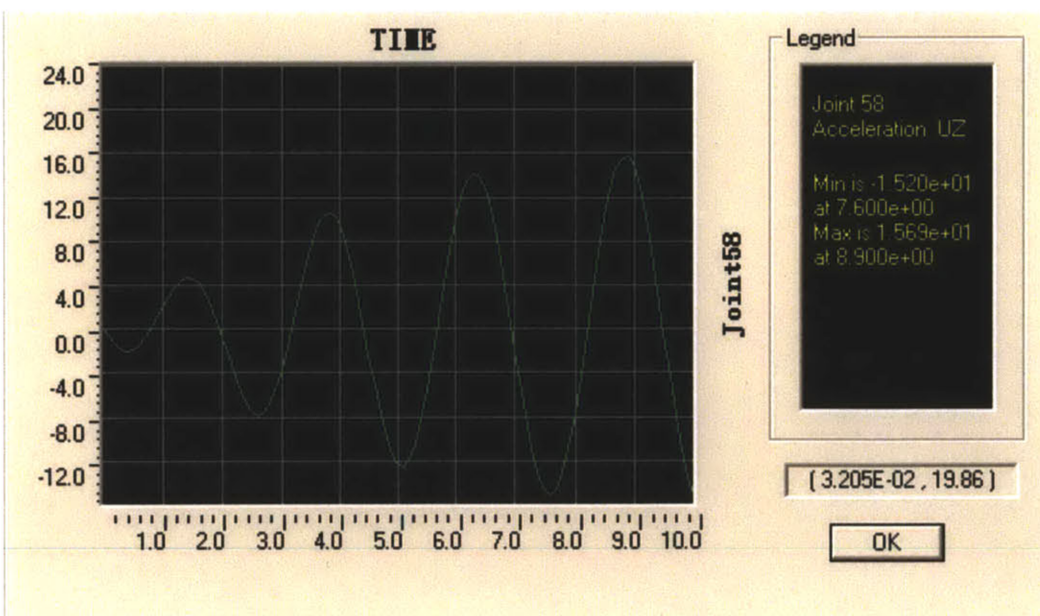
Displacement (m)



Velocity (m/s)



Acceleration (m/s^2)



4. Analysis

From the result we can see that the maximum displacement at mid-span of the bridge is about 1/500 of total span. And total displacement is less than 800mm. So the result is acceptable, which means Kap Shui Mun Bridge should be out of vortex shedding problem under wind load in vertical direction. However, we do not take displacement in wind load direction into consideration so now we will estimate the effect.

From running result of SAP2000, we find that mode 2 related to horizontal movement in wind load direction. Period for mode 2 is 1.88s, which is 0.53Hz for natural frequency.

Horizontal wind load on bridge deck, F = 0.5*C*A*v2* ρ = 70 kN

Since ω_2 = 3.34 rad/s, M = 3*10^5 kg, we have k = 5584 kN/m. Therefore displacement under wind load would be about 0.05m. Compare with total length of the bridge, the displacement is acceptable.

For other vibration mode, they can't be easily induced by wind load. Most of them will occur under seismic load, traffic load or other load with higher magnitude. So they won't be considered in this report.

Besides, errors exist in the result and they origin from different aspects. Firstly, the model we set up in SAP2000 doesn't reflect all the properties of real model. We simplified the deck of the bridge to several bars with same weight and stiffness. We substitute the support with several springs. The cables, which has significant influence on the bridge's behavior, is modeled as bars. Secondly, as wind has high variance, there is error when we calculating the wind, either magnitude or direction. Lastly, the interaction between wind load and bridge is not perfectly modeled. We exert equivalent load on bridge, however, reality is far more complicated. Therefore, the result we obtain is acceptable only under certain assumption. To optimize the analysis, more precise model should be introduced.

For further analysis, we plan to compare the real observation data with the virtual experiment. From that we can check whether the behavior of the bridge is under control and if the monitoring system works well. However, as real data from monitor can't be obtained, we have to give up this analysis.

Fluttering and Buffeting

To analyze fluttering and buffeting effect, we're going to follow similar procedure. Firstly, we have to estimate the magnitude of wind exerting on bridge. Since fluttering and buffeting are response to wind speed change in certain amount of time, we make the assumption that maximum increase rate of wind speed is 0 to 50m/s in 3s, which is about 15m/s^2. Properties of air stay the same. After we plug all the numbers in, we get the estimated equivalent force,

$$F_f + F_b = 9.86\sin(5.6t)\,kN/m$$

Input the force into our model, we find that maximum displacement is about 0.3m, which is far lower than that in vortex shedding. So we assume the bridge wouldn't be dominate by fluttering and buffeting effect.

In the following chapter, we will optimize the design by introducing tuned mass damper.

Chapter 4 Optimization

As we have already understand how the bridge would behave under different type and different magnitude of wind load, we still wonder if there are methods that can improve the property of this structure. One candidate is to add tuned mass damper on the structure. In the following paragraph, I will design a tuned mass damper to reduce vertical displacement of the bridge.

Tuned Mass Damper Design

From the analysis result we find that vibration due to vortex shedding is acceptable but still exceed 0.5m in vertical direction. We would like to reduce the vibration by introducing tuned mass damper. From the data we obtained from SAP2000, we will design the tuned mass damper by specify m_d, k_d and ξ_d. The procedure will follow the guide in professor Jerome J. Connor's book Introduction to Structural Motion Control to come up with a reasonable design. [18] According to Chapter 4 in the book, we need basic data to start our analysis, which are,

Natural Frequency of first mode: ω_1 = 2.63 rad/s

Stiffness of the bridge: k = $3.46*10^3$ kN/m

Maximum allowable vertical displacement: $\hat{u}$ = 0.2m

Amplitude of force on bridge deck: $\hat{p}$ = 74.4 kN

Damping ratio bridge deck : ξ = 0

Following equation 4.19, we get,

$$\xi_e = \frac{\hat{p}}{2k\hat{u}} \approx 5.4\%$$

Mode shape can be provides by SAP2000. As we have large amount of variable, it's not shown. Form the mode shape we can get $\tilde{m}$. Then refer to Figure 4.32, we can get optimal $\bar{m}$ = 0.018. Then we can get ξ_d, f_{opt} from Figure 4.31 and Figure 4.30.

$$\xi_d = 0.088$$

$$f_{opt} = 0.975$$

Therefore,

$$m_d = \bar{m} * M = 8880\ kg$$

$$\omega_d = f_{opt} * \omega = 2.56\ rad/s$$

$$k_d = \omega_d^2 m_d = 58.4\ kN/m$$

$$c_d = 2\xi_d \omega_d m_d = 4000\ Ns/m$$

With this tuned mass damper attached on bridge deck, maximum displacement will be limited under 0.2m. However, the design is only preliminary with simplified consideration. For example, bridge deck can also vibrate in wind direction and most of the time, it will rotate. Therefore, to prevent motion in larger profile, more design should be brought out.

Chapter 5 Accomplishment

Knowledge Acquirement

During the past semester, I've studied the theory of aerodynamics and also analyze problems by applying these methods. By reading and studying the book, I've studied the method, and compare the result with that from software. Wind engineering is not a new subject, on the contrary, theories on this field have been greatly developed already. However, there will always be requirement for engineering work on the subject because bridge construction is always going on. Therefore, after handling the all the knowledge of wind effect on bridge, we can move further to design, such as optimization and damage prevention etc, instead of just making analysis.

Software

By using SAP 2000 to assist my project, I've saved a lot of time on tedious calculation and get more precise results. By setting up and running the model under SAP, I get familiar

During my project, firstly I wasn't so familiar with the software. The way I analysis is to simply Kap Shui Mun bridge as a clamped-clamped beam with two supports. Then I use finite element method to solve the problem. The MATLAB result, however, was not satisfactory, which prevent me from going on. Later, I figure out the mistake that the bridge can't be simply as only a beam because there are two pylons and cables we have to consider. The right way to solve the problem is complete utilize existed model in SAP2000 and input load to see how would bridge react. Then I get the result soon.Computer software is really a powerful tool and it's a necessary to keep studying in future work as a civil engineer.

Conclusion

In my project, I conduct structural analysis on Kap Shui Mun bridge to find wind response of vortex-shedding, flutter and buffeting. Computer softeware SAP2000 and MATLAB is used. During the past one semester, I've reviewed basic dynamics knowledge and I'm able to solve some problems of dynamics. Besides, other researching methods are taught by my supervisor. Summarily, the one year experience on the project will help me a lot in future work as well as study.

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