Isolation and Transmissibility of Shipboard Equipment with Carbon Fiber

Reinforced Polymer Mount

by

Rachel Taylor Lewis

Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degree of

Bachelor of Science in Mechanical Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2012

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Abstract

The isolation and transmissibility of shipboard equipment is important to the function of a ship. The transmission of vibration from an engine to its surroundings can be devastating to sensitive equipment and disrupt normal operations. Isolator pads can be used to dampen transmissions from equipment to the ship and vice versa. In this thesis isolator pads of three different materials were considered: carbon fiber reinforced material (CFRP), steel, and rubber. These isolator pads were paired with two pieces of equipment. The first was a marine diesel engine with a relatively large mass and internal rotation. The second piece of equipment was an electronic chart display and information system (ECDIS) with a relatively small mass and no rotating parts. The rubber isolator pad was not a good isolating pad compared to either CFRP or steel, which had comparable responses to impulse and step inputs as well as transmissibility or isolation. For the marine diesel engine the steel isolator pad was marginally better, while the CFRP was best for the ECDIS.

Thesis Advisor: James H. Williams, Jr. Title: Professor of Mechanical Engineering and Writing and Humanistic Studies

Acknowledgements

There are a number of people I would like to thank. The first person is Professor James H. Williams, Jr. (who is supported by the DDG-1000 Program Manager/NAVSEA PMS 500 and the DDG-1000 Ship Design Manager/NAVSEA 05D). Professor Williams' patience and knowledge as my thesis advisor has allowed me to truly understand all of my research and provide the best interpretation I can of my findings. I am also thankful of fellow undergraduate students Anna Haas and Joanna Faulk for their combined research efforts; their diligence has guided my thoughts and provided a path for my work. I would also like to thank David Hawes for developing related ideas in his project with Professor Williams.

On a personal note, I would like to thank my family for supporting my efforts while at school and a special thanks to Jeremy deGuzman for his encouragement during my studies.

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Introduction

An important part of how a ship works is how sensitive or "noisy" equipment is isolated from the rest of the ship. The engine of a ship creates vibrations as it runs transmitting these vibrations to the ship's surroundings. A sensitive piece of equipment onboard a ship must be stabilized as much as possible to obtain accurate readings. The equipment, whether a marine diesel engine or an electronic chart display, must be supported by a structural mount that isolates it from the rest of the ship.

The ideas of isolation and transmissibility are very similar, but with a subtle distinction. On a ship a large engine creates vibrations that radiate outwards to the ship's hull and passengers onboard, an example of transmissibility. On that same ship a monitor that displays charts is sensitive to vibrations transmitted from the ship, an example of isolation. The ship in both cases is more massive than either piece of equipment—the engine transmits vibrations to the ship while the monitor is vibrated by the ship [1].

Three isolator pads of different materials were chosen to damp vibrations and other forces for this study. All three materials and their properties are listed below in Table 1. The first material is carbon fiber reinforced polymer, which will hereafter be called CFRP. It is woven with carbon fibers and epoxy into a strong, low-density material. The second isolator pad is made from steel and the third from rubber. Each material is listed below with its strength, in the form of Young's modulus, and density [2] [3].

	Young's Modulus [GPa]	Density $\left[\frac{\text{kg}}{\text{m}^3}\right]$
Carbon Fiber Reinforced Polymer (CFRP)	150	1600
Steel	210	7800
Rubber	0.1	1100

Table 1: Material Properties of Isolator Pads.

Two pieces of equipment were chosen to get a basic understanding of how these isolator pads actually work. Table 2 contains the characteristics for both pieces including mass, isolator pad area, and the rotating frequency (where applicable).

	Marine Diesel Engine	Electronic Chart Display and Information System (ECDIS)
Mass [kg]	2000	200
Area in contact with mount [m ²]	3	2
Rotating Frequency [rpm]	2000	N/A

Table 2: Equipment Characteristics.

The first is a mid-sized marine diesel engine. The engine is typical of other mid-sized engines in size and mass with a rotating frequency of 2000 rpm [4]. The electronic chart display and information system, hereafter known as ECDIS, is used for planning, executing, and documenting a ship's route using tide and current information [5]. The ECDIS has fragile components that would benefit from isolation of vibrations and sudden shocks.

Analytical Approach

Modeling a system is the first key to understanding how that system truly works. The easiest way of doing this is by separating a system into its base components. In the case of a structural mount for a shipboard instrument, this model is a mass-spring-damper system which is depicted below.



Figure 1. Mass-spring-damper model of ship, structural mount, and engine or ECDIS.

In this case, the large secondary mass is that of the ship which is so great compared to the mass of the structural mount and the equipment that it will not be considered a moving object, but a stationary mass.

Now that the ship is considered stationary it is possible to achieve a mathematical representation of the mass-spring-damper system. Using Newton's second law of motion the following equation is achieved.

$$M_{eff}\ddot{x} + c\dot{x} + kx = 0\tag{1}$$

The variable x represents the displacement of the engine or ECDIS as a function of time. The coefficient for acceleration of x is M_{eff} , the effective mass of the system. The coefficient for velocity of x is c, the damping coefficient of the structural mount. The coefficient of x is k, the spring constant of the structural mount. All of these coefficients are constants for the system and do not change.

The effective mass of the system is the mass of the equipment plus a fraction of the structural mount's mass. The value of effective mass can be discovered by evaluating the energy of the system—the sum of the kinetic and potential energies [6]. Kinetic energy, T, is integrated over the length of the spring, L, and potential energy, V, comes only from the spring.

$$T = \frac{1}{2} \left(m + \frac{m_s}{3} \right) \dot{x}^2 \tag{2}$$

$$V = \frac{1}{2}kx^2\tag{3}$$

The effective mass for this system comes from Equation 2 and is defined in Equation 4. The mass of the equipment (engine or ECDIS) is m and the mass of the structural mount is m_s .

$$M_{eff} = m + \frac{m_s}{3} \tag{4}$$

If Equation 1 is divided by M_{eff} , the equation of motion can be simplified to Equation 5 [7]. The coefficients for this equation are defined below.

$$\ddot{x} + 2\zeta \omega_N \dot{x} + \omega_N^2 x = 0 \tag{5}$$

The damping ratio of the system, ζ , is shown in Equation 6. It is dependent on the structural mount's damping coefficient and spring constant as well as the system's effective mass. For the rest of this paper, the damping ratio will be set to a predetermined value, $\zeta = 0.3$.

$$\zeta = \frac{c}{2\sqrt{kM_{eff}}} \tag{6}$$

The natural frequency of the system, ω_N , is shown in Equation 7 as the square root of the structural mount's spring constant divided by the effective mass. It represents the natural undamped frequency at which the system oscillates.

$$\omega_N = \sqrt{\frac{k}{M_{eff}}} \tag{7}$$

The damped frequency of the system, ω_D , is the frequency at which the system oscillates when a damper is applied. It is dependent on the system's natural frequency and the system's damping ratio as can be seen in Equation 8.

$$\omega_D = \omega_N \sqrt{1 - \zeta^2} \tag{8}$$

There are two equations of motion for this system, the motion after an impulse is applied and after a step is applied. Using Equation 5 and Laplace transforms, the equations of motion for these two situations can be solved. The equations of motion are set to the displacement of the equipment, earlier defined as x. This displacement is dependent on time, damping ratio, natural frequency, and damped frequency. Equation 9 is the response to an impulse input.

$$x_{impulse}(t) = \frac{1}{\omega_D} e^{-\zeta \omega_N t} \sin(\omega_D t)$$
(9)

Equation 10 is the response to a step input, where u(t) is the step input. For the purposes of this paper, this step will be a constant value of 0.005 m.

$$x_{step}(t) = u(t) - \frac{1}{\omega_N^2} e^{-\zeta \omega_N t} \left[\cos(\omega_D t) - \frac{\zeta \omega_N}{\omega_D} \sin(\omega_D t) \right]$$
(10)

In addition to displacement, transmissibility provides a way to judge the system's response to a structural mount. Transmissibility, T, is defined in Equation 11 in terms of damping ratio, natural frequency, and excitation frequency, ω .

$$T = \sqrt{\frac{1 + \left(2\zeta \frac{\omega}{\omega_N}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_N}\right)^2\right]^2 + \left(2\zeta \frac{\omega}{\omega_N}\right)^2}}$$
(11)

There is one last equation that will help make sense of the graphs later in this paper. One of the ways to judge whether an isolator pad is working well is how long the equipment takes to settle back to its original state, in this case to within 2% of the original displacement. The

equation below, Equation 12, provides a way to find that time using the damping ratio and the system's natural frequency [8].

$$t_{sett} = \frac{-ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta\omega_N} \tag{12}$$

Results

Using MATLAB software, equations from the previous section, Analytical Approach, and information from tables in Introduction, the following plots and tables were created. These visualizations show the structural mount-equipment response to each input and the system's transmissibility. The amplitude has been multiplied by a factor of 1000 to better visualize the response of the system in Figures 2 through 9.

Impulse Response

Figures 2 through 5 are plots of the spring-mass-damper response to an impulse input, an infinitely large force of an infinitely small duration, like a fast jab. Each plot depicts the system's return to its original state, the first peak representing the maximum displacement of the equipment.



Figure 2. Impulse Response of Marine Diesel Engine for Mounts of Various Materials.



Figure 3. Close-up of Impulse Response of Marine Diesel Engine for Mounts of Various

Materials.



Figure 4. Impulse Response of Electronic Chart Display Information System (ECDIS) for

Mounts of Various Materials.



Figure 5. Close-up of Impulse Response of Electronic Chart Display Information System (ECDIS) for Mounts of Various Materials.

Table 3 shows the maximum displacement and settling time for each of the mounting materials using the parameters for the marine diesel engine and the ECDIS.

Table 3: Maximum Displacement and Settling Time for Mounts of Various Materials for

Equipment	Mounting Material	Maximum Displacement [m]	Settling Time [s]
Marina Discol	CFRP	0.1465	2.8782e-4
Fngine	Steel	0.1404	2.7601e-4
Engine	Rubber	4.7905	0.0094
ECDIS	CFRP	0.0679	1.3343e-4
	Steel	0.0879	1.7279e-4
	Rubber	2.4826	0.0049

Impulse Response.

Step Response

Figures 6 through 9 are plots of the step response for both pieces of equipment. The axis of time has been normalized by dividing by the settling time of CFRP. The step used for these plots is 0.005 m. As with the impulse response, the amplitude of response is multiplied by a factor of 1000 in the following graphs to see the response more easily.



Figure 6. Step Response of Marine Diesel Engine for Mounts of Various Materials.



Figure 7. Close-Up of Step Response of Marine Diesel Engine for Mounts of Various Materials.



Figure 8. Step Response of Electronic Chart Display Information System (ECDIS) for Mounts

of Various Materials.



Figure 9. Close-Up of Step Response of Electronic Chart Display Information System (ECDIS) for Mounts of Various Materials.

The maximum displacement and settling time, normalized using the settling time of CFRP, are listed below in Table 4.

Table 4: Maximum Displacement and Settling Time for Mounts of Various Materials for Step

Response.

Equipment	Mounting Material	Maximum Displacement [m]	Settling Time [s]
Marina Diasal	CFRP	5.0019e-3	1.0000
Finding	Steel	5.0021e-3	0.9590
Lingine	Rubber	7.2203e-3	32.7093
	CFRP	5.004e-3	1.0000
ECDIS	Steel	5.008e-3	1.2950
	Rubber	5.5963e-3	36.5644

Transmissibility and Isolation

Figures 10 through 13 are plots of the transmissibility as a function of excitation

frequency. Of interest is the excitation frequency at peak transmissibility, which is the resonance frequency of each system.



Frequency vs. Transmissibility for Marine Diesel Engine

Figure 10. Transmissibility of Marine Diesel Engine for Mounts of Various Materials.



Figure 11. Close-Up of Transmissibility of Marine Diesel Engine for Mounts of Various

Materials.



Figure 12. Isolation of Electronic Chart Display Information System (ECDIS) for Mounts of

Various Materials.



Figure 13. Close-Up of Isolation of Electronic Chart Display Information System (ECDIS) for Mounts of Various Materials.

Table 5 contains the resonance excitation frequency, which also converts to rpm, of both systems using each mounting material. The resonance frequency of a system is the frequency at which a system's displacement is the greatest it will ever be.

Table 5: Resonance Equipment Excitation Frequencies and Revolutions/Minute for Mounts of

Equipment	Mounting Material	Excitation Frequency [Hz]	Revs/Minute [rpm]
Marine Diesel	CFRP	4.1735e4	3.9854e5
	Steel	4.3520e4	4.1558e5
Engine	Rubber	1.2759e4	1.2184e4
ECDIS	CFRP	4.1544e4	3.9672e5
	Steel	4.3329e4	4.1376e5
	Rubber	1.0853e3	1.0364e4

Various Materials.

Discussion and Conclusions

The marine diesel engine and the ECDIS have both been modeled using three mounts of different materials: CFRP, steel, and rubber. To judge whether a mount is good means looking at the Results section and drawing conclusions from the plots. As a reminder, the amplitude for all of the plots has been multiplied by a factor of 1000 to better visualize the response of the system.

Impulse Response

An impulse is an infinitely large force applied over a brief time, so small as to be immeasurable. For the marine diesel engine, Figures 2 and 3 show a response to an impulse. The amplitude of the mass-spring- damper response is greatest for rubber. The response for the steel and CFRP mounts is an order of magnitude smaller, the CFRP having slightly greater amplitude of response than that of steel. The shape of each of the responses is the same, an initial peak followed by consecutively smaller peaks until the response has been damped. Because the shape is the same, the settling time for each mounting material follows the same order as the amplitude. The shortest settling time was for the steel mounting pad, followed closely by CFRP, while the time for rubber was an order of magnitude longer the other two.

Figures 4 and 5 show the impulse responses for the ECDIS. The amplitude responses of the different material mounts fall in the following order, from greatest to smallest: rubber, steel, and CFRP. The initial peak for rubber is two orders of magnitude larger than either initial peak for steel and CFRP. Steel has the second smallest peak, CFRP achieving the smallest peak. The settling time for each of the responses from largest to smallest is rubber, then steel, and finally CFRP.

Step Response

A step input of 0.005 m was applied to the mass-spring-damper of both the marine diesel engine and ECDIS using MATLAB software to plot the response of the systems. The response of both systems should settle around the initial step value of 0.005 m. The time plotted for each response has been normalized about the settling time for CFRP and is actually a ratio.

The response of the marine diesel engine to the step input can be seen in Figures 6 and 7. The greatest amplitude response belongs to rubber; the initial peak is 3 orders of magnitude larger than initial peak for either steel or CFRP. The close-up of the plot shows steel has a lower peak than CFRP, but the overall response of both is very similar. The normalized settling time for the different mounting materials from largest to smallest is as follows: rubber, CFRP, steel.

Figures 8 and 9 show the step responses of different materials for the ECDIS. The response of the system for the rubber mounting pad was 3 orders of magnitude greater than for steel or CFRP, with the response of steel greater than that of CFRP. The normalized settling time follows the same pattern as of the amplitude of response, rubber having the greatest time and CFRP having the smallest.

Transmissibility and Isolation

The transmissibility of the marine diesel engine vs. the excitation frequency can be seen in Figures 10 and 11. The excitation frequency at peak transmissibility is of interest as the resonance frequency for the system. These resonance frequencies have been computed from the plots and are listed in Table 5 for each mounting material. The smallest resonance frequency belonged to rubber, and the resonance frequencies for steel and CFRP are similar—steel had a slightly larger frequency than CFRP. Isolation of the ECDIS vs. excitation frequency is plotted in Figures 12 and 13. The excitation frequency at peak isolation is the resonance frequency for the mass-spring-damper system of ECDIS and mounting pad. Table 5 contains the resonance frequency for each material. The resonance frequency for rubber is the smallest, more than 3 times smaller than either steel or CFRP. The resonance frequency for CFRP is slightly smaller than that of steel.

Final Conclusions

The marine diesel engine showed comparable results between the two mounting materials CFRP and steel. The amplitudes of response for impulse and step inputs were smallest for steel. This means that for those two inputs, steel is the better mounting material. The resonance frequency for the steel mount, from the transmissibility plots, is over 40000 Hz which is 20 times the frequency of the engine itself. CFRP and steel have very similar plots of transmissibility vs. frequency, but steel has a slightly higher resonance frequency for the marine diesel engine. Looking at these three results, the steel mounting pad is the best material for the marine diesel engine.

The ECDIS showed similar results for the CFRP and steel mounts. For both the impulse and step input the plots of responses were similar. The CFRP mount had smaller amplitudes of displacement for the system and shorter settling times, indicating the CFRP mount decreased the transmission of movement to the ECDIS the most. The plots of isolation vs. excitation frequency for ECDIS are comparable for the two mounts of CFRP and steel, with resonance frequencies of over 40000 Hz. Steel has a slightly higher resonance frequency, 1800 Hz higher than CFRP. With all three of these tests, the CFRP mount is the best for the ECDIS.

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The results for both the marine diesel engine and the ECDIS were very close. The best mount was different for each piece of equipment. This was because the marine diesel engine had an internal source of movement which added to the natural frequency of the system.

For this study, all that has been addressed are the responses to two specific inputs. There are of course other contributing factors that have not been taken into consideration for the above conclusions. A large consideration for this problem is the weight of the mounting material. A ship must always be as light as possible for maneuverability and weight constraints. The CFRP and steel mounts showed similar results for different inputs and transmissibility/isolation, but a CFRP mount weighs almost 1/5th of a steel mount of the same size. Other factors such as cost, electrical and thermal conductivity, and durability have not been taken into account, but should be considered towards the decision of which is the best mounting material.

Recommendations

Further study might lead to more definitive results as to the best mounting material for isolating a ship from engine vibrations or sensitive equipment from the ship. As was stated in the Analytical Approach, certain system properties, isolator pad dimensions and damping ratio, were set to constants. Changing the size and even the shape of the isolator pad or the damping ratio might yield different results. Other constants for this thesis were the pieces of equipment. Testing engines with different weights and inherent rotating frequencies would give more information for selecting an isolator pad. CFRP is made through interweaving carbon fibers and an epoxy. The orientation of these fibers and the epoxy chosen can have a great effect on the strength of the material. Comparing different kinds of CFRP could lead to a more definitive conclusion as to mounting material for shipboard equipment. Any one of these recommendations could provide more comparison between mounting materials and a more definitive conclusion as to the best mounting material.

Appendix A: MATLAB Code of Impulse Input for Marine Diesel Engine

% Impulse Input Function and Mass-Spring-Damper Response for Marine Diesel % Engine % Problem Constants % Isolator Pad Area [m^2] Engine Area = 3; L spring = 0.1;% Isolator Pad Thickness [m] M Engine = 2000;% Mass of Engine [kg] damp ratio = 0.3;% Damping Ratio [] freq_engine = (2000/60)*(2*pi); % Engine Frequency [Hz] % Carbon Fiber Reinforced Polymer CFRP E = 150e9;% Young's Modulus [Pa] CFRP_k = CFRP_E*Engine_Area/L_spring; % Spring Constant [N/m] CFRP density = 1600; % Density [kg/m^3] CFRP M = CFRP density*Engine Area*L spring; % Spring Mass [kg] % Effective Mass [kg] CFRP M eff = M Engine + CFRP M/3; $CFRP w n = (CFRP_k/CFRP_M_eff)^{0.5};$ % Natural Frequency [Hz] CFRP_w_sys = CFRP_w_n + freq_engine; % System Frequency [Hz] CFRP_w_d = CFRP_w_sys*(1-damp_ratio^2)^0.5; % Damped Frequency [Hz] CFRP t sett = -reallog(0.02*(1-damp ratio^2)... % Settling Time [s] ^0.5)/(damp ratio*CFRP w sys); t = 0:1/1000/CFRP w n:750/CFRP w n; % Time [s] % Amplitude of displacement for CFRP isolator pad CFRP_x = (1e4/CFRP_w_d)*(exp(-damp_ratio*CFRP_w_sys*t)).*sin(CFRP_w_d*t); % Steel % Young's Modulus [Pa] Steel E = 210e9;Steel k = Steel E*Engine Area/L spring; % Spring Constant [N/m] Steel density = 7800; % Density [kg/m^3] Steel M = Steel density*Engine Area*L spring; % Spring Mass [kg] Steel M eff = M Engine + Steel M/3; % Effective Mass [kg] % Natural Frequency [Hz] Steel w n = (Steel k/Steel M eff)^0.5; Steel_w_sys = Steel_w_n + freq_engine; % System Frequency [Hz] Steel_w_d = Steel_w_sys*(1-damp_ratio^2)^0.5; % Damped Frequency [Hz] Steel_t_sett = -reallog(0.02*(1 - damp_ratio... % Settling Time [s] ^2)^0.5)/(damp ratio*Steel w sys); % Amplitude of displacement for steel isolator pad Steel x = (1e4/Steel w d)*(exp(-damp ratio*Steel w sys*t)).*sin(... Steel w d*t); % Rubber % Young's Modulus [Pa] Rubber E = 0.1e9;Rubber k = Rubber E*Engine Area/L spring; % Spring Constant [N/m] Rubber density = 1100;% Density [kg/m^3] Rubber M = Rubber density*Engine Area*L spring; % Spring Mass [kg] Rubber M eff = M Engine + Rubber M/3; % Effective Mass [kg] Rubber w n = (Rubber k/Rubber M eff)^0.5; % Natural Frequency [Hz] Rubber_w_sys = Rubber_w_n + freq_engine; % System Frequency [Hz] Rubber w d = Rubber w sys*(1-damp_ratio^2)^0.5; % Damped Frequency [Hz] Rubber t sett = -reallog(0.02*(1 - damp ratio...% Settling Time ^2)^0.5)/(damp ratio*Rubber w sys); % Amplitude of displacement for rubber isolator pad

Rubber_x = (1e4/Rubber_w_d)*(exp(-damp_ratio*Rubber_w_sys*t)).*sin(... Rubber_w_d*t);

% Plot Impulse Response for ECDIS
plot(t, CFRP_x, '-', t, Steel_x, '--', t, Rubber_x, ':')
xlabel('Time [s]')
ylabel('Amplitude [m]')
title('Impulse Response for Marine Diesel Engine')
legend('CFRP', 'Steel', 'Rubber')

Appendix B: MATLAB Code of Impulse Input for Electronic Chart Display and Information System (ECDIS)

```
% Impulse Input Function and Mass-Spring-Damper Response for Electronic
% Chart Display Information System (ECDIS)
% Problem Constants
                                                 % Isolator Pad Area [m^2]
ECDIS Area = 2;
                                                 % Isolator Pad Thickness [m]
L spring = 0.1;
                                                 % Mass of ECDIS [kg]
M ECDIS = 200;
                                                 % Damping Ratio [ ]
damp ratio = 0.3;
% Carbon Fiber Reinforced Polymer
                                                  % Young's Modulus [Pa]
CFRP E = 150e9;
CFRP_k = CFRP_E*ECDIS_Area/L_spring;
                                                  % Spring Constant [N/m]
CFRP_density = 1600;
                                                  % Density [kg/m^3]
CFRP_M = CFRP_density*ECDIS_Area*L_spring;
                                               % Spring Mass [kg]
CFRP_M_eff = M_ECDIS + CFRP_M/3;
                                                  % Effective Mass [kg]
CFRP_w n = (CFRP_k/CFRP_M_eff)^{0.5};
                                                  % Natural Frequency [Hz]
CFRP_w_n = (CFRP_k/CFRP_M_eff)^0.5; % Natural Frequency [Hz]
CFRP_w_d = CFRP_w_n*(1-damp_ratio^2)^0.5; % Damped frequency [Hz]
CFRP t sett = -reallog(0.02*(1 - damp_ratio... % Settling Time [s]
    ^2)^0.5)/(damp_ratio*CFRP_w_n);
t = 0:1/1000/CFRP w n:750/CFRP w n;
                                                  % Time [s]
% Amplitude of displacement for CFRP isolator pad
CFRP x = (1e4/CFRP w d)*(exp(-damp ratio*CFRP w n*t)).*sin(CFRP w d*t);
% Steel
                                                  % Young's Modulus [Pa]
Steel E = 210e9;
Steel k = Steel E*ECDIS Area/L spring;
                                                  % Spring Constant [N/m]
Steel density = 7800;
                                                  % Density [kg/m^3]
Steel M = Steel density*ECDIS Area*L spring; % Spring Mass [kg]
Steel M eff = M ECDIS + Steel M/3;
                                                % Effective Mass [kg]
Steel_w_n = (Steel_k/Steel M eff)^0.5;
                                                  % Natural Frequency [Hz]
Steel_w_d = Steel_w_n*(1-damp_ratio^2)^0.5; % Damped Frequency [Hz]
Steel t_sett = -reallog(0.02*(1 - damp_ratio... % Settling Time [s]
    ^2)^0.5)/(damp_ratio*Steel_w_n);
% Amplitude of displacement for steel isolator pad
Steel x = (1e4/Steel w d)*(exp(-damp ratio*Steel w n*t)).*sin(Steel_w_d*t);
% Rubber
Rubber E = 0.1e9;
                                                  % Young's Modulus [Pa]
Rubber k = Rubber E*ECDIS Area/L spring;
                                                  % Spring Constant [N/m]
Rubber density = 1100;
                                                  % Density [kg/m^3]
Rubber M = Rubber density*ECDIS Area*L spring; % Spring Mass [kg]
Rubber_M_eff = M_ECDIS + Rubber_M/3;% Effective Mass [kg]Rubber_w_n = (Rubber_k/Rubber_M_eff)^0.5;% Natural Frequency [Hz]
Rubber w d = Rubber w n*(1-damp_ratio^2)^0.5; % Damped Frequency [Hz]
Rubber t sett = -reallog(0.02*(1 - damp ratio...% Settling Time [s]
    ^2)^0.5)/(damp_ratio*Rubber_w_n);
% Amplitude of displacement for rubber isolator pad
Rubber x = (1e4/Rubber w d)*(exp(-damp_ratio*Rubber w n*t)).*sin( ...
    Rubber w d*t);
% Plot Impulse Response for ECDIS
plot(t, CFRP x, '-', t, Steel x, '--', t, Rubber x, ':')
```

xlabel('Time [s]')
ylabel('Amplitude [m]')
title('Impulse Response for Electronic Chart Display Information System')
legend('CFRP', 'Steel', 'Rubber')

Appendix C: MATLAB Code of Step Input for Marine Diesel Engine

% Step Input Function and Mass-Spring-Damper Response for Marine Diesel % Engine

```
% Problem Constants
                                                % Isolator Pad Area [m^2]
Engine area = 3;
L spring = 0.1;
                                                % Isolator Pad Thickness [m]
M engine = 2000;
                                                % Mass of Engine [kg]
damp_ratio = 0.3;
                                               % Damping Ratio [ ]
freq_engine = (2000/60)*(2*pi);
                                                % Engine Frequency [Hz]
% Carbon Fiber Reinforced Polymer
CFRP E = 150e9;
                                                % Young's Modulus [Pa]
                                                % Spring Constant [N/m]
CFRP k = CFRP E*Engine_area/L_spring;
                                                % Density [kg/m^3]
CFRP density = 1600;
CFRP M = CFRP density*Engine area*L spring;
                                               % Spring Mass [kg]
CFRP M eff = M engine + CFRP M/3;
                                               % Effective Mass [kg]
CFRP w n = (CFRP k/CFRP M eff)^{0.5};
                                               % Natural Frequency [Hz]
CFRP_w_sys = CFRP_w_n + freq_engine;% Natural Frequency [Hz]
CFRP_w_d = CFRP_w_sys*(1 - damp_ratio^2)^0.5; % Damped Frequency [Hz]
CFRP t sett = -reallog(0.02*(1-damp_ratio^2)... % Settling Time [s]
    ^0.5)/(damp_ratio*CFRP_w_sys);
CFRP norm t sett = CFRP t sett/CFRP t sett;
                                                 % Normalized Settling Time
t = \overline{0:1/1000}/CFRP w n:750/CFRP w n;
                                                 % Time [s]
% Amplitude of displacement for CFRP isolator pad
CFRP_x = 0.005 - (le4/CFRP_w_sys^2)*(exp(-damp_ratio*CFRP_w_sys*t).*( ...
    cos(CFRP w d*t) - (damp ratio*CFRP w sys/CFRP w d)*sin(CFRP w sys*t)));
% Steel
Steel E = 210e9;
                                                 % Young's Modulus [Pa]
Steel k = Steel E*Engine area/L spring;
                                                % Spring Constant [N/m]
Steel density = 7800;
                                                % Density [kg/m^3]
Steel M = Steel density*Engine area*L spring; % Spring Mass [kg]
                                                % Effective Mass [kg]
Steel M eff = M engine + Steel M/3;
                                               % Natural Frequency [Hz]
Steel w n = (Steel k/Steel M eff)^0.5;
Steel_w_sys = Steel_w_n + freq_engine; % System Frequency [Hz]
Steel_w_d = Steel_w_sys*(1 - damp_ratio^2)^0.5; % Damped Frequency [Hz]
Steel t sett = -reallog(0.02*(1 - damp ratio... % Settling Time [s]
    ^2)^0.5)/(damp ratio*Steel w sys);
Steel norm_t_sett = Steel_t_sett/CFRP_t_sett;
                                                % Normalized Settling Time
% Amplitude of displacement for steel isolator pad
Steel_x = 0.005 - (le4/Steel_w_sys^2)*(exp(-damp_ratio*Steel_w_sys*t).* ...
     (cos(Steel_w_d*t) - (damp_ratio*Steel_w_sys/Steel_w_d)*sin( ...
    Steel w sys*t)));
% Rubber
Rubber E = 0.1e9;
                                                 % Young's Modulus [Pa]
Rubber k = Rubber E*Engine area/L spring;
                                                 % Spring Constant [N/m]
Rubber density = 1100;
                                                 % Density [kg/m^3]
Rubber M = Rubber density*Engine area*L spring; % Spring Mass [kg]
Rubber M eff = M engine + Rubber M/3;
                                             % Effective Mass [kg]
Rubber_w_n = (Rubber_k/Rubber_M_eff)^0.5;% Natural Frequency [Hz]Rubber_w_sys = Rubber_w_n + freq_engine;% System Frequency [Hz]
Rubber_w_d = Rubber_w_sys*(1-damp_ratio^2)^0.5; % Damped Frequency [Hz]
```

Appendix D: MATLAB Code of Step Input for Electronic Chart Display and Information System (ECDIS)

% Step Input Function and Mass-Spring-Damper Response for Electronic Chart % Display Information System (ECDIS) % Problem Constants % Isolator Pad Area [m^2] ECDIS area = 2; L spring = 0.1;% Isolator Pad Thickness [m] M ECDIS = 200;% Mass of ECDIS [kg] damp ratio = 0.3;% Damping Ratio [] % Carbon Fiber Reinforced Polymer CFRP E = 150e9;% Young's Modulus [Pa] CFRP_k = CFRP_E*ECDIS_area/L_spring; % Spring Constant [N/m] CFRP density = 1600; % Density [kg/m^3] CFRP_M = CFRP_density*ECDIS_area*L_spring; % Spring Mass [kg] CFRP_M_eff = M_ECDIS + CFRP_M/3; % Effective Mass [kg] CFRP_w_n = (CFRP_k/CFRP_M_eff)^0.5;% Natural Frequency [Hz]CFRP_w_d = CFRP_w_n*(1 - damp_ratio^2)^0.5;% Damped Frequency [Hz] CFRP_t_sett = -reallog(0.02*(1 - damp_ratio... % Settling Time [s] ^2)^0.5)/(damp_ratio*CFRP_w_n); CFRP_norm_t_sett = CFRP_t_sett/CFRP_t_sett; % Normalized Settling Time t = 0:1/2000/CFRP_w_n:750/CFRP_w_n; % Time [s] % Amplitude of dsiplacement for CFRP isolator pad CFRP x = $0.005 - (1e4/CFRP w n^2)*(exp(-damp ratio*CFRP w n*t).*(cos(...$ CFRP w d*t) - (damp ratio*CFRP w n/CFRP w d)*sin(CFRP w n*t))); % Steel Steel E = 210e9;% Young's Modulus [Pa] Steel k = Steel E*ECDIS area/L spring; % Spring Constant [N/m] Steel_density = 7800; % Density [kg/m^3] Steel M = Steel density*ECDIS area*L spring; % Spring Mass [kg] .5; Sping Mass [kg] % Effective Mass [kg] Steel M eff = M ECDIS + Steel M/3; % Natural Frequency [Hz] Steel w n = (Steel k/Steel M eff)^0.5; Steel_w_d = Steel_w_n*(1 - damp_ratio^2)^0.5; % Damped frequency [Hz] Steel_t_sett = -reallog(0.02*(1 - damp_ratio... % Settling Time [s] ^2)^0.5)/(damp ratio*Steel w n); Steel_norm_t_sett = Steel_t_sett/CFRP_t_sett; % Normalized Settling Time % Amplitude of displacement for steel isolator pad Steel_x = 0.005 - (le4/Steel_w_n^2)*(exp(-damp_ratio*Steel_w_n*t).*(cos(... Steel_w_d*t) - (damp_ratio*Steel_w_n/Steel_w_d)*sin(Steel_w_n*t))); % Rubber Rubber E = 0.1e9;% Young's Modulus [Pa] Rubber_k = Rubber E*ECDIS area/L spring; % Spring Constant [N/m] Rubber density = 1100;% Density [kg/m^3] Rubber M = Rubber density*ECDIS area*L spring; % Spring Mass [kg] Rubber_M_eff = M_ECDIS + Rubber_M/3; % Effective Mass [kg] Rubber_M_eff = M_ECDIS + Rubber_M/3; % Effective Mass [kg] Rubber_w_n = (Rubber_k/Rubber_M_eff)^0.5; % Natural Frequency [Hz] Rubber_w_d = Rubber_w_n*(1 - damp_ratio^2)^0.5; % Damped Frequency [Hz] Rubber t sett = -reallog(0.02*(1 - damp_ratio...% Settling Time [s] ^2)^0.5)/(damp ratio*Rubber_w_n); Rubber_norm_t_sett = Rubber_t_sett/CFRP_t_sett; % Normalized Settling Time % Amplitude of displacement for rubber isolator pad

Rubber_x = 0.005 - (le4/Rubber_w_n^2)*(exp(-damp_ratio*Rubber_w_n*t).* ... (cos(Rubber_w_d*t) - (damp_ratio*Rubber_w_n/Rubber_w_d)*sin(... Rubber_w_n*t)));

% Plot Step Response for ECDIS
plot(t/CFRP_t_sett, CFRP_x, '-', t/CFRP_t_sett, Steel_x, '--', t/ ...
 CFRP_t_sett, Rubber_x, ':')
xlabel('Time/Settling Time of CFRP []')
ylabel('Amplitude [m]')
title('Step Response for Electronic Chart Display Information System')
legend('CFRP', 'Steel', 'Rubber')

Appendix E: MATLAB Code of Transmissibility for Marine Diesel Engine

```
% Transmissibilty of Mass-Spring-Damper for Marine Diesel Engine
% Problem Constants
Engine area = 3;
                                                   % Isolator Pad Area [m^2]
L spring = 0.1;
                                                   % Isolator Pad Thickness [m]
M engine = 2000;
                                                   % Mass of Engine [kg]
damp ratio = 0.3;
                                                  % Damping ratio [ ]
freq_engine = (2000/60)*(2*pi);
                                                  % Engine Frequency [Hz]
Trans Peak = 2.2914;
                                                  % Peak Transmissibility [ ]
% Carbon-Fiber Reinforced Polymer (CFRP)
CFRP E = 150e9;
                                                    % Young's Modulus [Pa]
CFRP_k = CFRP_E*Engine_area/L_spring;
                                                    % Spring Constant [N/m]
CFRP density = 1600;
                                                    % Density [kg/m^3]
CFRP_M = CFRP_density*Engine_area*L_spring;
                                                  % Spring Mass [kg]
CFRP_M_eff = CFRP_M/3 + M_engine;
CFRP_w_n = (CFRP_k/CFRP_M_eff)^0.5;
                                                   % Effective Mass [kg]
                                                   % Natural Frequency [Hz]
                                               % System Frequency [Hz]
CFRP_w_sys = CFRP_w_n + freq_engine;
w = 0:CFRP w n/10000:10*CFRP w n;
                                                   % Excitation Frequency [Hz]
% Transmissibility of CFRP
CFRP Transmissibilty = (1 + (2*damp ratio*(w/CFRP w sys)).^2)./ ...
    ((1 - (w/CFRP_w_sys).^2).^2 + (2*damp_ratio*(w/CFRP_w_sys)).^2).^0.5;
% Resonance Excitation Frequency of CFRP
CFRP A = (CFRP \ w \ sys^2) * (damp \ ratio * (1 - Trans \ Peak^{-2}) - 1);
CFRP B = (CFRP \ w \ sys^4) * (1 - Trans \ Peak^{-2});
CFRP \ w \ ex = ((CFRP \ A.^2 - CFRP \ B).^0.5 - CFRP \ A).^0.5;
CFRP rpm = CFRP w ex*60/(2*pi);
% Steel
Steel E = 210e9;
                                                    % Young's Modulus [Pa]
Steel_k = Steel_E*Engine_area./L spring;
                                                    % Spring Constant [N/m]
Steel_density = 7800;
                                                    % Density [kg/m^3]
Steel_M = Steel_density*Engine_area*L_spring; % Mass [kg]
Steel_M_eff = Steel_M/3 + M_engine; & Effective Mass [kg]
Steel_w_n = (Steel_k/Steel_M_eff).^0.5; & Natural Frequency [Hz]
Steel w sys = Steel w n + freq engine;
                                                   % System Frequency [Hz]
% Transmissibility of Steel
Steel Transmissibilty = (1 + (2*damp ratio.*(w/Steel w sys)).^2)./((1 ...
    - (w/Steel w sys).^2).^2 + (2*damp ratio.*(w/Steel w sys)).^2).^0.5;
% Resonance Excitation Frequency of Steel
Steel A = (Steel w sys<sup>2</sup>)*(damp ratio*(1 - Trans Peak<sup>-2</sup>) - 1);
Steel B = (Steel w sys^4) * (1 - Trans Peak^{-2});
Steel_w_ex = ((Steel A.^2 - Steel B).^0.5 - Steel A).^0.5;
Steel rpm = Steel w ex*60/(2*pi);
% Rubber
Rubber E = 0.1e9;
                                                    % Young's Modulus [Pa]
Rubber k = Rubber E*Engine area./L spring;
                                                    % Spring Constant [N/m]
Rubber density = 1100;
                                                    % Density [kg/m^3]
Rubber_M = Rubber_density*Engine_area*L_spring; % Mass [kg]
Rubber_M_eff = Rubber_M/3 + M_engine; % Effective Mass [kg]
Rubber_w_n = (Rubber_k/Rubber_M_eff).^0.5; % Natural Frequency [Hz]
```

```
% System Frequency [Hz]
Rubber w sys = Rubber w n + freq engine;
% Transmissibility of Rubber
Rubber_Transmissibilty = (1 + (2*damp_ratio.*(w/Rubber_w_sys)).^2)./((1 ...
    - (w/Rubber w sys).^2).^2 + (2*damp ratio.*(w/Rubber w sys)).^2).^0.5;
% Resonance Excitation Frequency of Rubber
Rubber A = (Rubber_w_sys^2)*(damp_ratio*(1 - Trans_Peak^-2) - 1);
Rubber B = (Rubber w sys^4) * (1 - Trans Peak^{-2});
Rubber w ex = ((Rubber A.^2 - Rubber B).^{0.5} - Rubber A).^{0.5};
Rubber rpm = Rubber w ex*60/(2*pi);
% Plot Transmissibility for Marine Diesel Engine
plot(w, CFRP_Transmissibilty, '-', w, Steel_Transmissibilty, '--', w, ...
    Rubber_Transmissibilty, ':')
xlabel('Excitation Frequency [Hz]')
ylabel('Transmissibility [ ]')
title('Frequency vs. Transmissibility for Marine Diesel Engine')
```

```
legend('CFRP', 'Steel', 'Rubber')
```

Appendix F: MATLAB Code of Isolation for Electronic Chart Display and Information System (ECDIS)

```
% Isolation of Mass-Spring-Damper for Electronic Chart Display Information
% System (ECDIS)
% Problem Constants
ECDIS area = 2;
                                               % Isolator Pad Area [m^2]
L spring = 0.1;
                                               % Isolator Pad Thickness [m]
M ECDIS = 200;
                                               % Mass of ECDIS [kg]
damp ratio = 0.3;
                                               % Damping ratio [ ]
Iso Peak = 2.2914;
                                               % Peak Isolation [ ]
% Carbon-Fiber Reinforced Polymer (CFRP)
CFRP E = 150e9;
                                                 % Young's Modulus [Pa]
CFRP_k = CFRP_E*Engine_area/L_spring;
                                                % Spring Constant [N/m]
CFRP_density = 1600;
                                                 % Density [kg/m^3]
CFRP_M = CFRP_density*Engine_area*L_spring;
                                                % Spring Mass [kg]
CFRP M eff = CFRP M/3 + M engine;
                                                % Effective Mass [kg]
CFRP w n = (CFRP k/CFRP M eff)^0.5;
                                                % Natural Frequency [Hz]
w = 0:CFRP_w_n/10000:10*CFRP_w_n;
                                               % Excitation Frequency [Hz]
% Isolation of CFRP
CFRP Isolation = (1 + (2*damp_ratio*(w/CFRP_w_n)).^2)./((1 - (w/...
    CFRP w n).^2).^2 + (2*damp ratio*(w/CFRP w n)).^2).^0.5;
% Resonance Excitation Frequency of CFRP
CFRP A = (CFRP_w_n^2) * (damp_ratio*(1 - Iso_Peak^{-2}) - 1);
CFRP_B = (CFRP_w_n^4) * (1 - Iso_Peak^{-2});
CFRP \ w \ ex = ((CFRP \ A.^2 - CFRP \ B).^0.5 - CFRP \ A).^0.5;
CFRP rpm = CFRP w ex*60/(2*pi);
% Steel
Steel E = 210e9;
                                                 % Young's Modulus [Pa]
Steel k = Steel E*Engine area./L spring;
                                                 % Spring Constant [N/m]
Steel density = 7800;
                                                % Density [kg/m^3]
Steel M = Steel density*Engine area*L spring;
                                               % Mass [kg]
Steel M eff = Steel M/3 + M engine;
                                                % Effective Mass [kg]
Steel_w_n = (Steel_k/Steel_M_eff).^0.5;
                                                % Natural Frequency [Hz]
% Isolation of Steel
Steel Isolation = (1 + (2*damp ratio.*(w/Steel w n)).^2)./((1 - (w/...
    Steel_w_n).^2).^2 + (2*damp_ratio.*(w/Steel_w_n)).^2).^0.5;
% Resonance Excitation Frequency of Steel
Steel A = (Steel_w_n^2) * (damp_ratio*(1 - Iso_Peak^{-2}) - 1);
Steel_B = (Steel_w_n^4) * (1 - Iso_Peak^{-2});
Steel w ex = ((Steel A.^2 - Steel B).^{0.5} - Steel A).^{0.5};
Steel_rpm = Steel_w_ex*60/(2*pi);
% Rubber
Rubber E = 0.1e9;
                                                 % Young's Modulus [Pa]
Rubber k = Rubber E*Engine area./L spring;
                                                 % Spring Constant [N/m]
Rubber density = 1100;
                                                 % Density [kg/m^3]
Rubber M = Rubber_density*Engine_area*L_spring; % Mass [kg]
Rubber M eff = Rubber M/3 + M engine;
                                                 % Effective Mass [kg]
Rubber w n = (Rubber k/Rubber M eff).^0.5;
                                                % Natural Frequency [Hz]
Rubber w ratio = w/Rubber w n;
                                                 % Frequency Ratio [ ]
% Isolation of Rubber
```

```
Rubber Isolation = (1 + (2*damp ratio.*(w/Rubber w n)).^2)./((1 - (w/...
    Rubber w n).^2).^2 + (2*damp ratio.*(w/Rubber w n)).^2).^0.5;
% Resonance Excitation Frequency of Rubber
Rubber_A = (Rubber_w_n^2) * (damp_ratio*(1 - Iso_Peak^-2) - 1);
Rubber B = (Rubber w n^4) * (1 - Iso Peak^{-2});
Rubber w ex = ((Rubber A.^2 - Rubber B).^{0.5} - Rubber A).^{0.5};
Rubber_rpm = Rubber_w_ex*60/(2*pi);
% Plot Isolation for Marine Diesel Engine
plot(w, CFRP_Isolation, '-', w, Steel_Isolation, '--', w, ...
    Rubber_Isolation, ':')
xlabel('Excitation Frequency [Hz]')
ylabel('Isolation [ ]')
my_title_1 = 'Frequency vs. Isolation for Electronic';
my_title_2 = 'Chart Display Information System (ECDIS)';
my title = {my title 1, my_title 2};
title(my title)
legend('CFRP', 'Steel', 'Rubber')
```

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