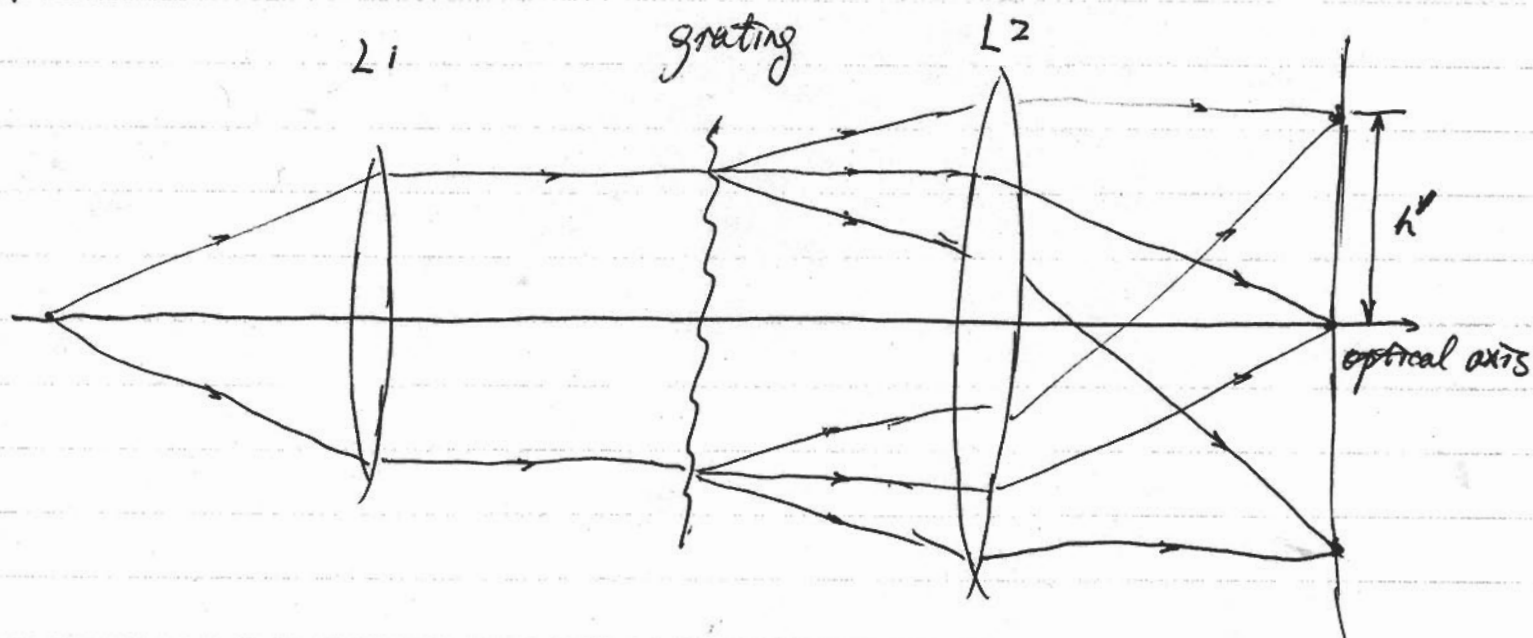


There are three ways to think about this problem:

1. We can think that the grating is illuminated by a plane wave and the plane where the grating is placed is "object plane" (Fourier plane in the plot of the problem) while the image plane in the plot of the problem is actually the "Fourier plane". So at the image place, we should obtain the Fourier Transform of the transparency and aperture.
2. At the input plane, it's a delta function (point source). So at the Fourier plane, we should get 1 everywhere, before the transparency. After the transparency we can have 1 ~~at the~~ multiplied by the function of the transparency and aperture. Then at the image plane we should observe the Fourier transform of the transparency and aperture.
3. Because a delta function is input into this system, at image plane we should get the PSF of this system. We know that the Amplitude transform function is just the transparency and aperture which are put at the Fourier plane. So the PSF is exactly the Fourier transform of the transfer function.

1.



When the aperture of the grating is ignored, at the image plane we will obtain 3 peaks (δ function):

$$\delta(x'') + \frac{1}{2}\delta(x'' - h'') + \frac{1}{2}\delta(x'' + h'')$$

where $h'' = \lambda f u = \frac{1 \mu\text{m} \cdot 10\text{cm}}{0.01\text{mm}} = 10\text{mm}$

2. If the aperture of the grating is 10 mm, the field after transparency is

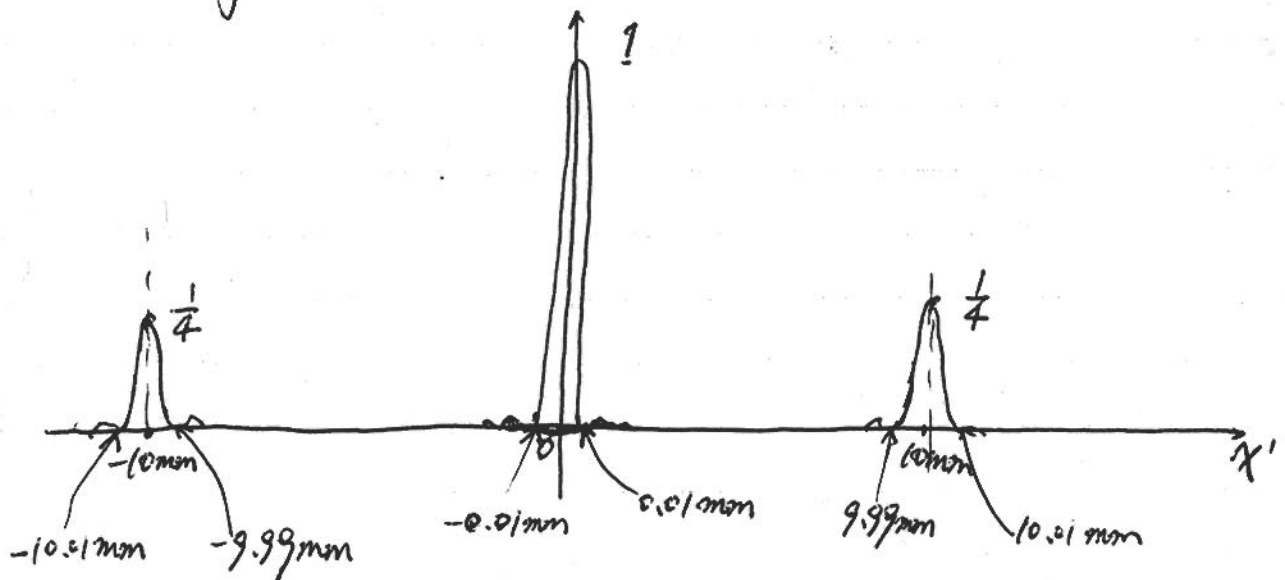
$$t^+(x'') = t(x'') \cdot \text{rect}\left(\frac{x''}{10\text{mm}}\right)$$

Here, we can think that the grating ~~is~~ illuminated by a plane wave and the plane where the transparency is placed is object plane while the image plane in the problem is actually the Fourier plane.

So at the image plane, we will get the Fourier transform of $t^*(x'')$

$$\begin{aligned}
 \mathcal{F}\{t^*(x'')\} &= \mathcal{F}\{t(x'')\} \Big|_{u=\frac{x'}{xf}} * \mathcal{F}\left\{\text{rect}\left(\frac{x''}{10\text{mm}}\right)\right\} \Big|_{u=\frac{x'}{xf}} \\
 &= \left[\delta(x') + \frac{1}{2}\delta(x'-10\text{mm}) + \frac{1}{2}\delta(x'+10\text{mm}) \right] \\
 &\quad * \text{sinc}\left(10\text{mm} \cdot \frac{x'}{1\mu\text{m} \times 10\text{cm}}\right) \\
 &= \text{sinc}\left(\frac{x'}{0.01\text{mm}}\right) + \frac{1}{2}\text{sinc}\left(\frac{x'-10\text{mm}}{0.01\text{mm}}\right) \\
 &\quad + \frac{1}{2}\text{sinc}\left(\frac{x'+10\text{mm}}{0.01\text{mm}}\right)
 \end{aligned}$$

The intensity pattern will look like



3. If the point source is shifted by 10mm, the grating is actually illuminated by a tilted plane wave

$$\exp \left\{ -i \cdot \frac{2\pi}{\lambda} \cdot \frac{h x''}{f} \right\} = \exp \left\{ -i \cdot 2\pi \cdot \frac{x''}{0.01 \text{mm}} \right\}$$

The field after the grating is

$$t^+(x'') = t(x'') \cdot \text{rect} \left(\frac{x''}{10 \text{mm}} \right) \cdot \exp \left\{ -i \cdot 2\pi \cdot \frac{x''}{0.01 \text{mm}} \right\}$$

According to the shift theorem of Fourier transform, at the image plane we will obtain

$$F \{ t^+(x'') \} = \text{sinc} \left(\frac{x' + 10 \text{mm}}{0.01 \text{mm}} \right) + \frac{1}{2} \text{sinc} \left(\frac{x'}{0.01 \text{mm}} \right) + \frac{1}{2} \text{sinc} \left(\frac{x' + 20 \text{mm}}{0.01 \text{mm}} \right)$$

The intensity pattern looks like

