2.71 Problem 2 solution.

Follow the 2.710 Problem 2 solution,

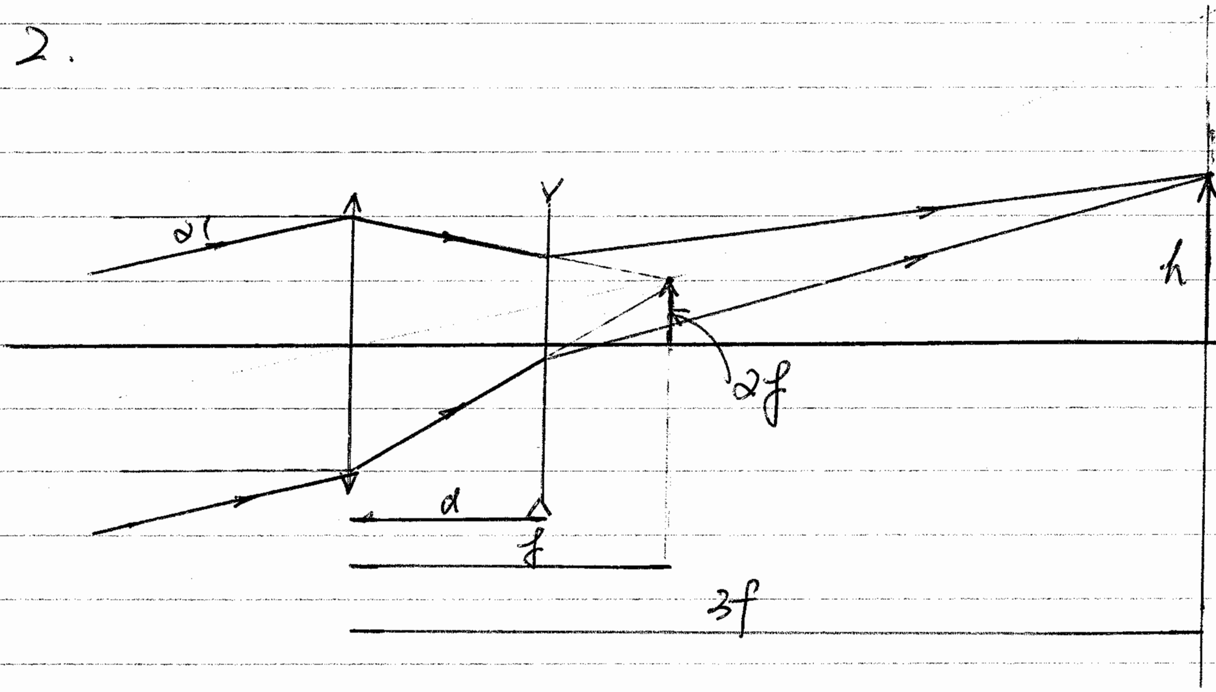
substituting numerical value $f = 10$ cm.

Answers:

$d = 5$ cm

$f_0 = -6.25$ cm

Problem 2.



a)

From requirements (i), (ii)

$$h = 5\alpha f$$

the magnification is

$$-\frac{S_i}{S_o} = -\frac{3f-d}{d-f} = \frac{h}{\alpha f} \Rightarrow \frac{3f-d}{f-d} = 5$$

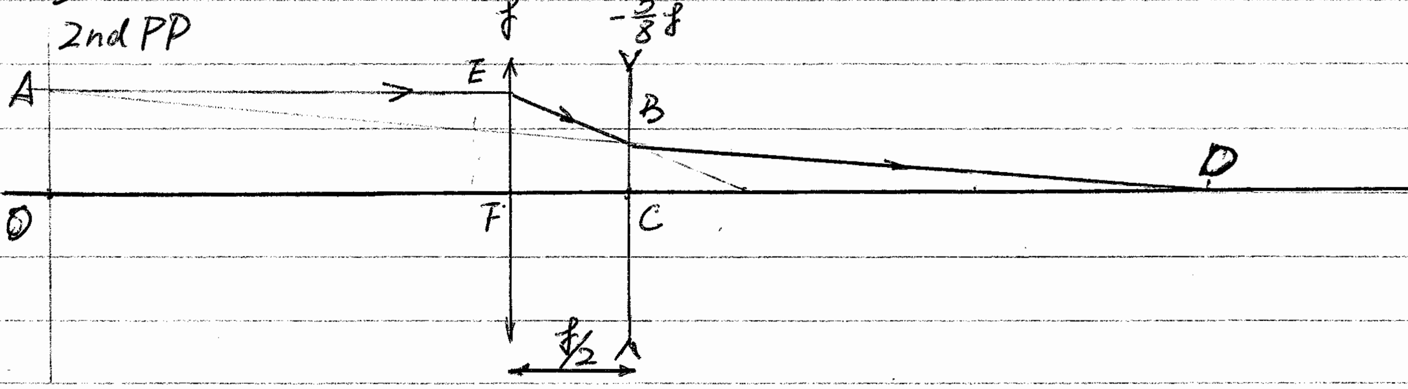
$$\Rightarrow d = \frac{f}{2}$$

So we have the imaging condition as

$$S_o = d - f = -\frac{f}{2} \qquad S_i = 3f - d = \frac{5}{2}f$$

$$\frac{1}{f_o} = \frac{1}{S_o} + \frac{1}{S_i} \Rightarrow f_o = -\frac{5}{8}f$$

b>



$$\overline{BC} = \frac{1}{2}\overline{EF} = \frac{1}{2}\overline{AO}$$

So we have

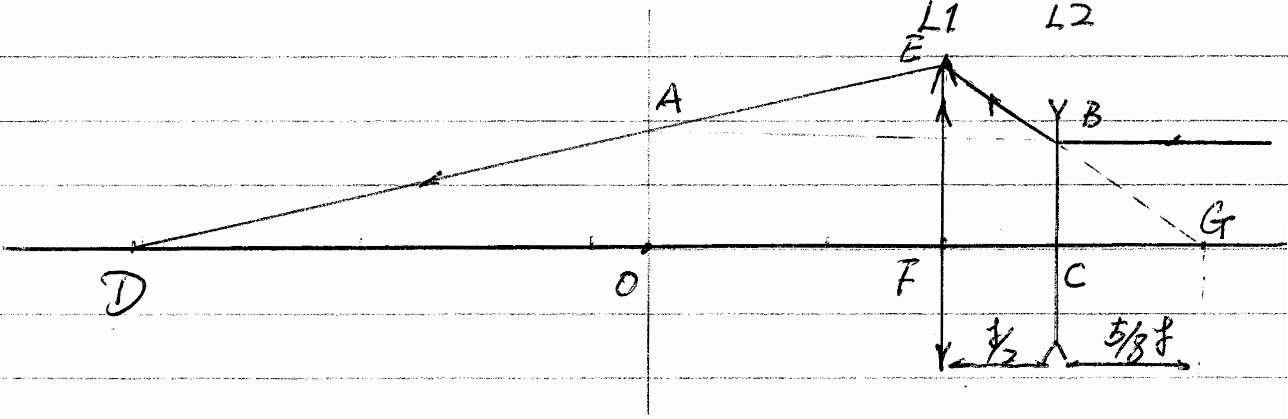
$$\overline{OD} = 2\,\overline{CD} = 2\cdot\left(3f - \frac{f}{2}\right) = 5f$$

$$\overline{OF} = 5f - 3f = 2f.$$

2nd PP is located to the left of L1 at distan

$2f$.

Now continue with the 1st PP.



The parallel rays from left will have a virtual fo
point at G after passing through Lens L2.

Use lens law, we know that the ~~front~~ focal point (FF
of this telephoto system is

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

where $S_o = \frac{1}{2}f + \frac{5}{8}f = \frac{9}{8}f$.

$\Rightarrow S_i = 9f$. to the left of Lens L1

$$\frac{\overline{BC}}{\overline{EF}} = \frac{\frac{5}{8}f}{\frac{9}{8}f} = \frac{5}{9} = \frac{\overline{AO}}{\overline{EF}}$$

$$\Rightarrow \frac{\overline{DO}}{\overline{DF}} = \frac{5}{9} \qquad \Rightarrow \overline{DO} = 5f$$

$$\Rightarrow \overline{OF} = 4f.$$

1st PP is located to the left of Lens L2
at distance 4f.

) From part (b), we know

$EFL = \overline{DO} = 5f.$

Or even from the problem we can know the EFL without any calculation:

Because ~~[illegible]~~ the parallel rays bundle with angle α passing through the system will focus at the focal plane with height $h = 5\alpha f$, so the effective focal length is equal to

$$(EFL) = \frac{h}{\alpha} = 5f.$$

which is consistent with the results we got from ray tracing.

d) $S_o = 24f - 4f = 20f$

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{5f} \Rightarrow S_i = \frac{20}{3}f.$$

The image plane is to the right of Lens L1 at distance $\frac{20}{3}f - 2f = \frac{14}{3}f.$

Matrix method (complicated):

$$\begin{bmatrix} \alpha_{out} \\ x_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3f-d & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{f_0} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{f} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_{in} \\ x_{in} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 3f-d & 1 \end{bmatrix} \begin{bmatrix} 1-\frac{d}{f_0} & -\left(\frac{1}{f}+\frac{1}{f_0}-\frac{d}{ff_0}\right) \\ d & 1-\frac{d}{f} \end{bmatrix} \begin{bmatrix} \alpha_{in} \\ x_{in} \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{d}{f_0}, & -\left(\frac{1}{f}+\frac{1}{f_0}-\frac{d}{ff_0}\right) \\ (3f-d)\left(1-\frac{d}{f_0}\right)+d, & -(3f-d)\left(\frac{1}{f}+\frac{1}{f_0}-\frac{d}{ff_0}\right)+\left(1-\frac{d}{f}\right) \end{bmatrix} \begin{bmatrix} \alpha_{in} \\ x_{in} \end{bmatrix}$$

$$x_{out} = \left[(3f-d)\left(1-\frac{d}{f_0}\right)+d\right]\alpha_{in} + \left[-(3f-d)\left(\frac{1}{f}+\frac{1}{f_0}-\frac{d}{ff_0}\right)+\left(1-\frac{d}{f}\right)\right]x_{in}$$

Because all the parallel rays with angle α will focus at the point at the image plane with $h = 5\alpha f$, so we have

x_{out} is independent of x_{in}. $\quad x_{out} = h = 5\alpha f$.

$$\begin{cases} (3f-d)\left(1-\frac{d}{f_0}\right)+d = 5f & (1) \\ -(3f-d)\left(\frac{1}{f}+\frac{1}{f_0}-\frac{d}{ff_0}\right)+\left(1-\frac{d}{f}\right) = 0 & (2) \end{cases}$$

From Equation (1), we have

$$\frac{1}{f_0} = \left(1 - \frac{5f-d}{3f-d}\right) \cdot \frac{1}{d}$$

Substitute it into (2).

$$-(3f-d)\left(\frac{1}{f} + \left(1-\frac{5f-d}{3f-d}\right)\cdot\frac{1}{d}\left(1-\frac{d}{f}\right)\right) + \left(1-\frac{d}{f}\right) = 0$$

$$\Rightarrow -3 + \frac{d}{f} + 2\frac{f}{d}\left(1-\frac{d}{f}\right) + \left(1-\frac{d}{f}\right) = 0$$

$$\Rightarrow \quad 2\frac{f}{d} - 4 = 0 \quad \Rightarrow \frac{f}{d} = 2$$

$$\Rightarrow d = \frac{f}{2}.$$

which is consistent with the result that I got before.