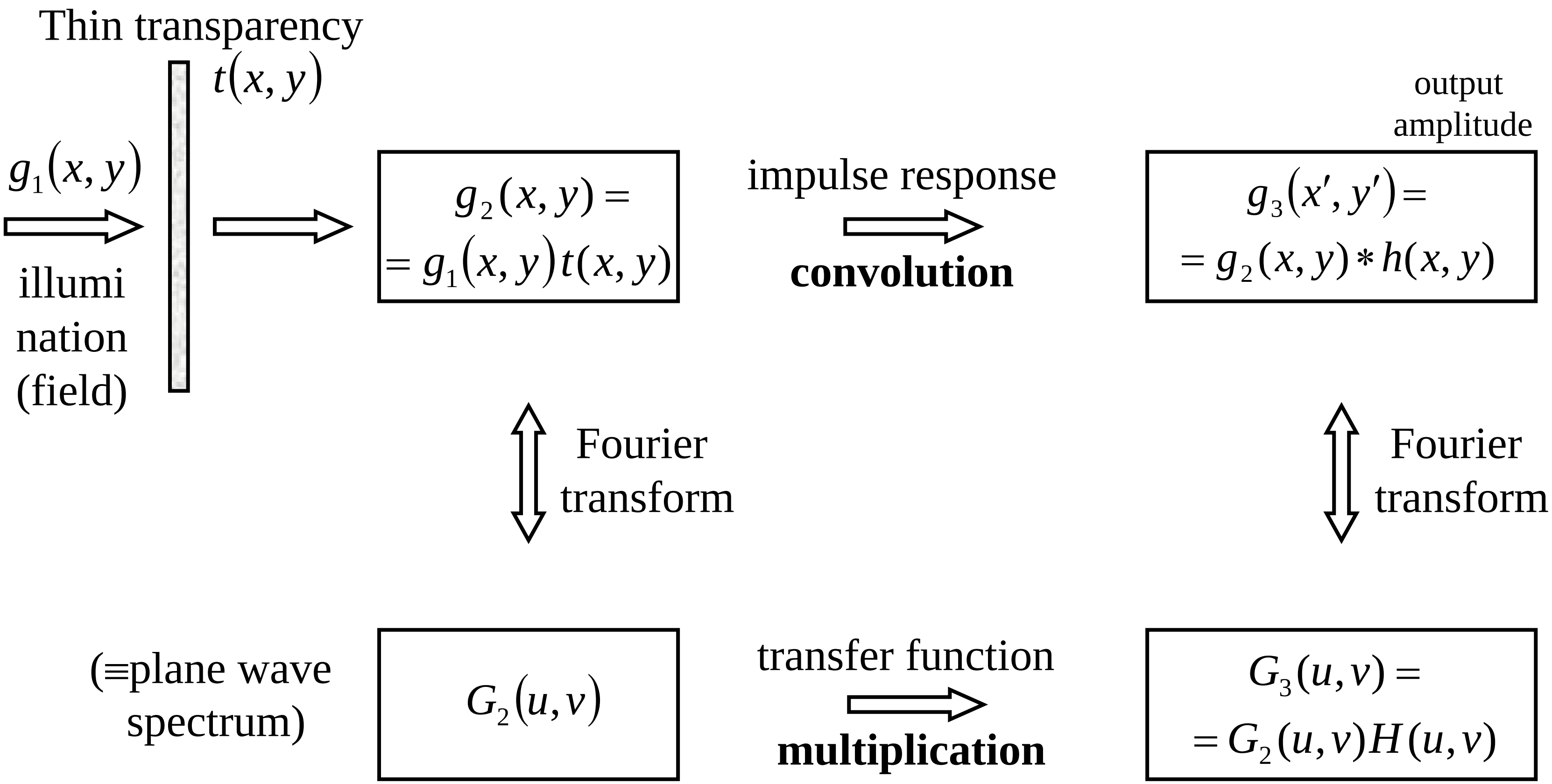
Resolution

(cont'd)

Coherent imaging

as a linear, shift-invariant system



Thin transparency $t(x,y)$

$g_1(x,y)$

illumi nation (field)

$g_2(x,y) =$
$= g_1(x,y)\,t(x,y)$

impulse response

convolution

output amplitude

$g_3(x',y') =$
$= g_2(x,y) * h(x,y)$

Fourier transform

Fourier transform

(≡plane wave spectrum)

$G_2(u,v)$

transfer function

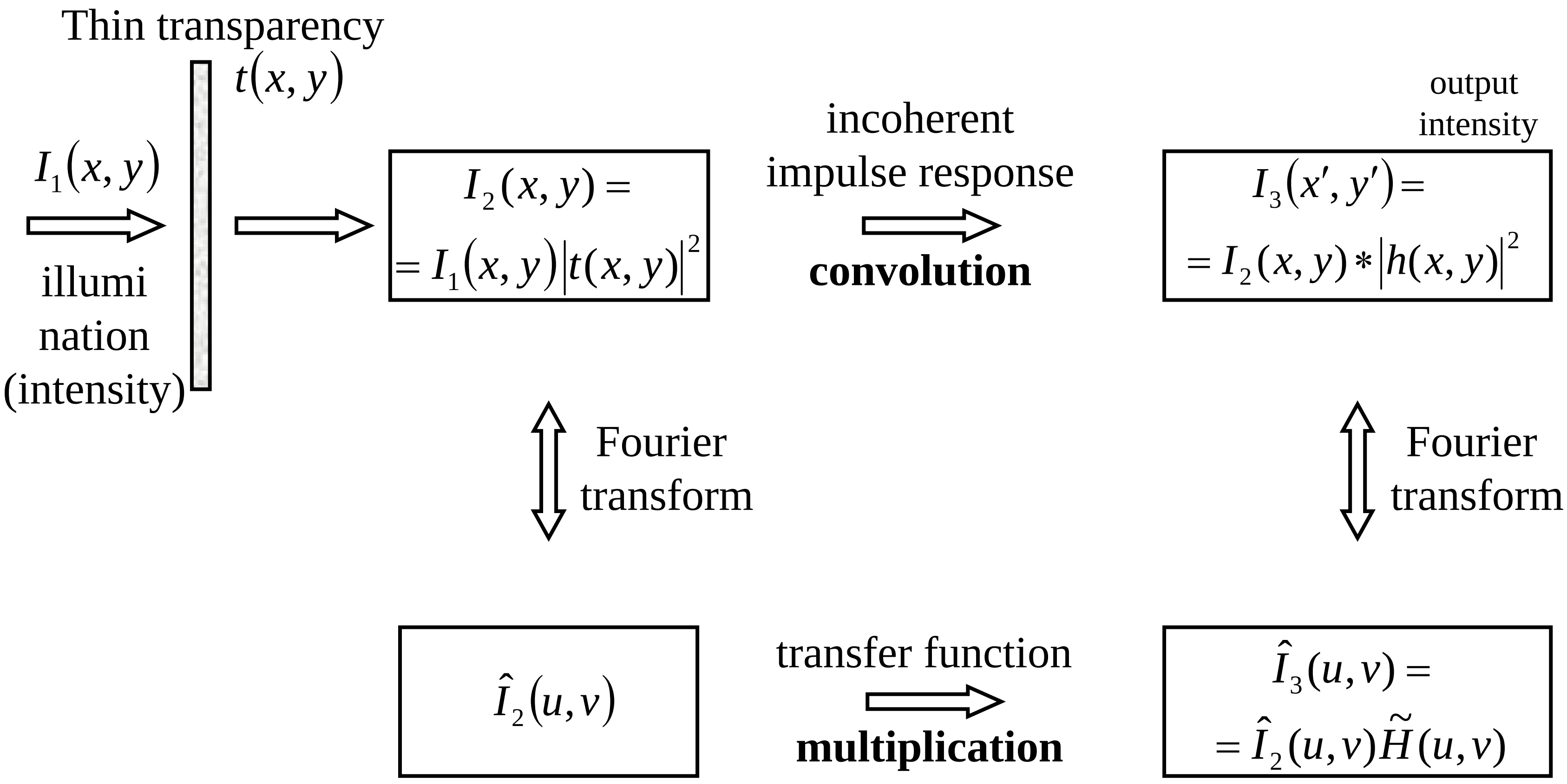
multiplication

$G_3(u,v) =$
$= G_2(u,v)H(u,v)$

transfer function of coherent system $H(u,v)$: aka **amplitude transfer function**

Incoherent imaging

as a linear, shift-invariant system



Thin transparency $t(x,y)$

$I_1(x,y)$

illumi
nation
(intensity)

$$I_2(x,y) = \\ = I_1(x,y)\left|t(x,y)\right|^2$$

incoherent
impulse response

convolution

output
intensity

$$I_3(x',y') = \\ = I_2(x,y) * \left|h(x,y)\right|^2$$

Fourier
transform

Fourier
transform

$$\hat{I}_2(u,v)$$

transfer function

multiplication

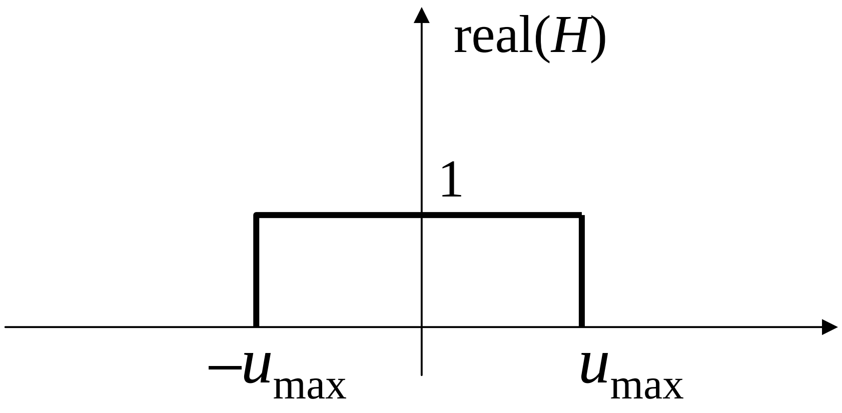
$$\hat{I}_3(u,v) = \\ = \hat{I}_2(u,v)\tilde{H}(u,v)$$

transfer function of incoherent system: $\tilde{H}(u,v)$ **optical transfer function (OTF)**

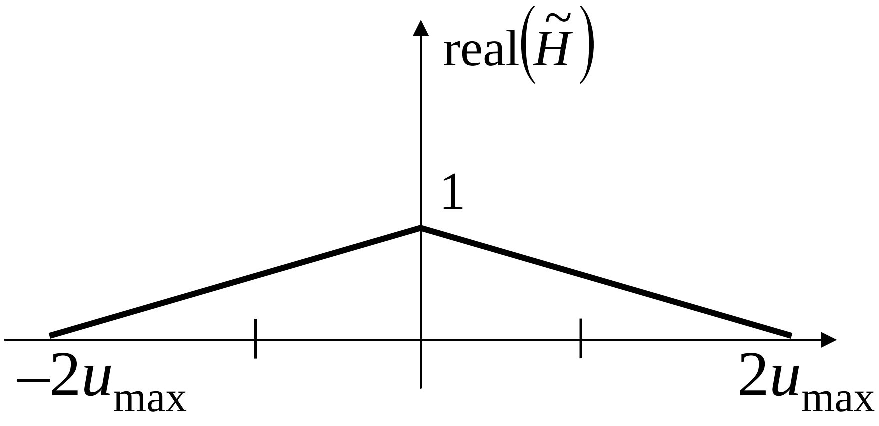
The Optical Transfer Function

$$\tilde{H}(u,v) \equiv \Im\left\{ \left| h(x,y) \right|^2 \right\} \quad \text{normalized to } 1$$

$$= \frac{\iint H(u',v') H^*(u'-u, v'-v) v \, \mathrm{d}u' \mathrm{d}v'}{\iint \left| H(u',v') \right|^2 \mathrm{d}u' \mathrm{d}v'}$$

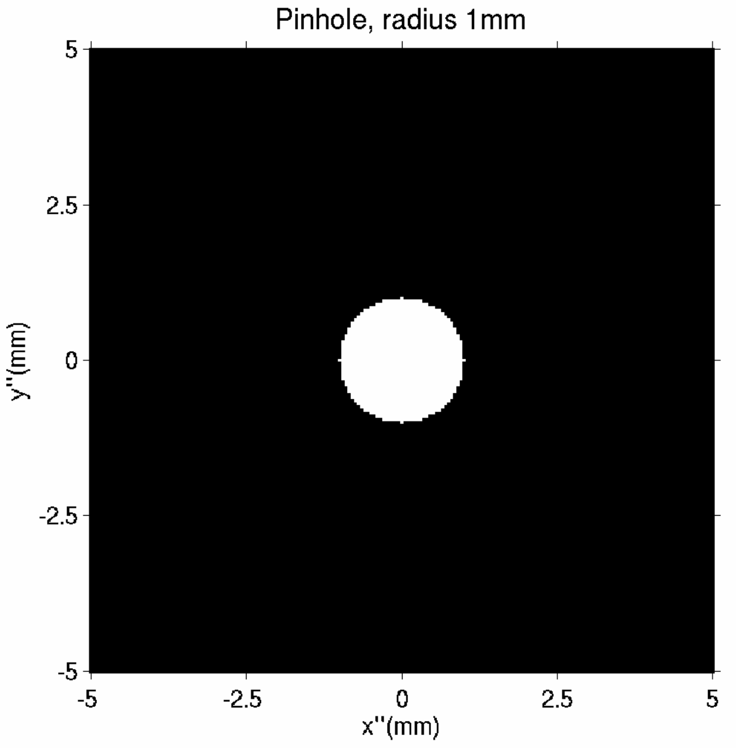


1D amplitude transfer function



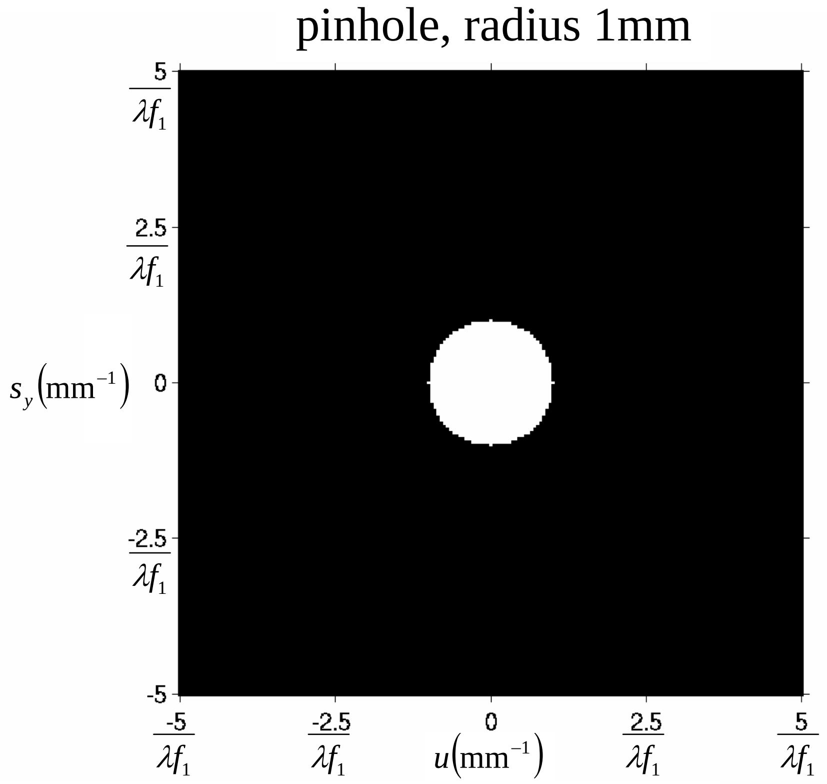
1D OTF

Amplitude transfer function and MTF of circular aperture in a 4F system

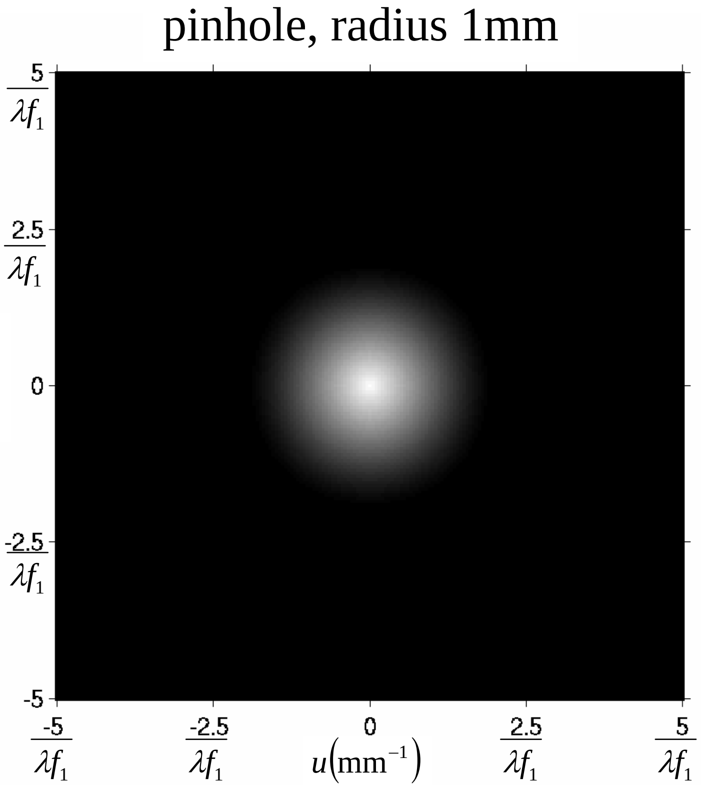


physical aperture
(pupil function)

Amplitude transfer function and MTF of circular aperture in a 4F system

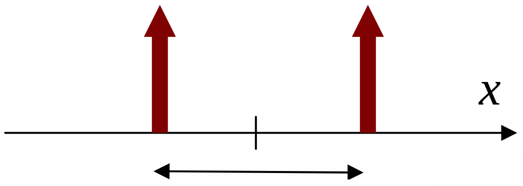

pinhole, radius 1mm

amplitude transfer function

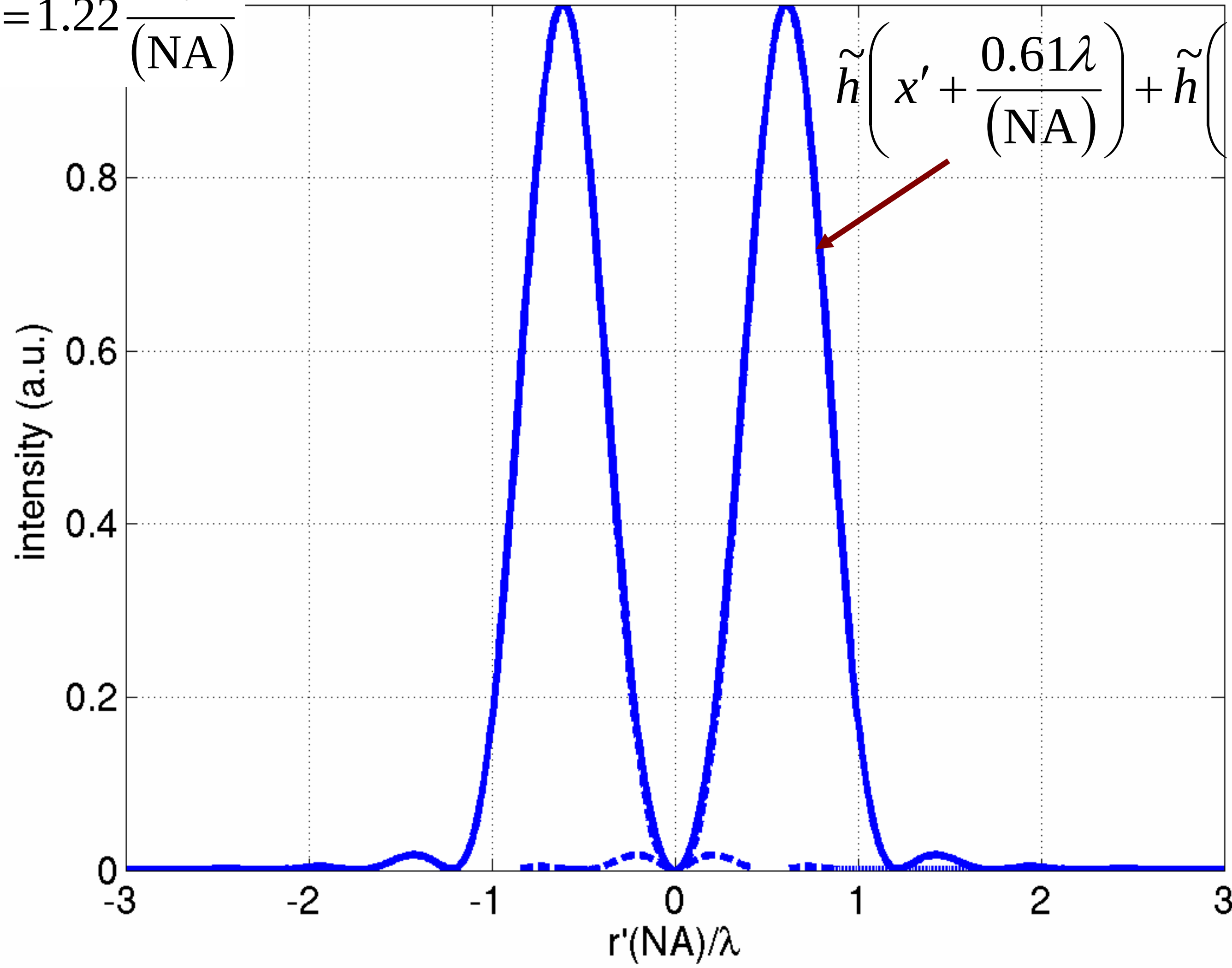

pinhole, radius 1mm

OTF/MTF

“Safe” resolution in optical systems



$$\Delta r = 1.22\frac{\lambda}{(\mathrm{NA})}$$



$$\tilde{h}\left(x' + \frac{0.61\lambda}{(\mathrm{NA})}\right) + \tilde{h}\left(x' - \frac{0.61\lambda}{(\mathrm{NA})}\right)$$

Diffraction–limited resolution (safe)

Two point objects are "**just resolvable**" (limited by diffraction only)
if they are separated by:

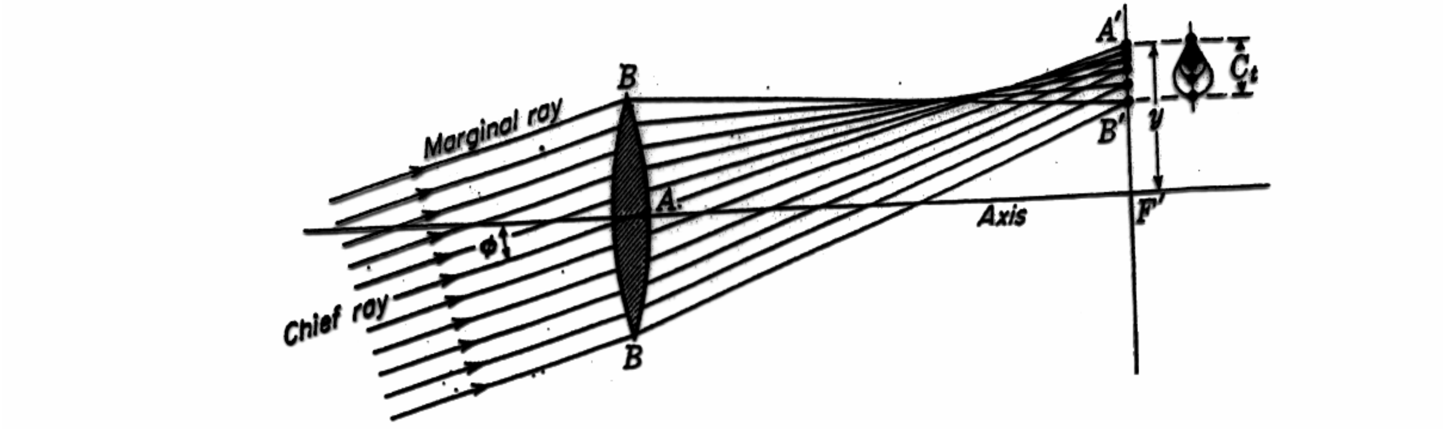
Two–dimensional systems (rotationally symmetric PSF)	One–dimensional systems (e.g. slit–like aperture)
Safe definition: (one–lobe spacing) $\Delta r' = 1.22 \frac{\lambda}{(\mathrm{NA})}$	$\Delta x' = \frac{\lambda}{(\mathrm{NA})}$
Pushy definition: (1/2–lobe spacing) $\Delta r' = 0.61 \frac{\lambda}{(\mathrm{NA})}$	$\Delta x' = 0.5 \frac{\lambda}{(\mathrm{NA})}$

You will see different authors giving different definitions.
Rayleigh in his original paper (1879) noted the issue of noise
and warned that the definition of "just–resolvable" points
is system– or application–dependent

Also affecting resolution: aberrations

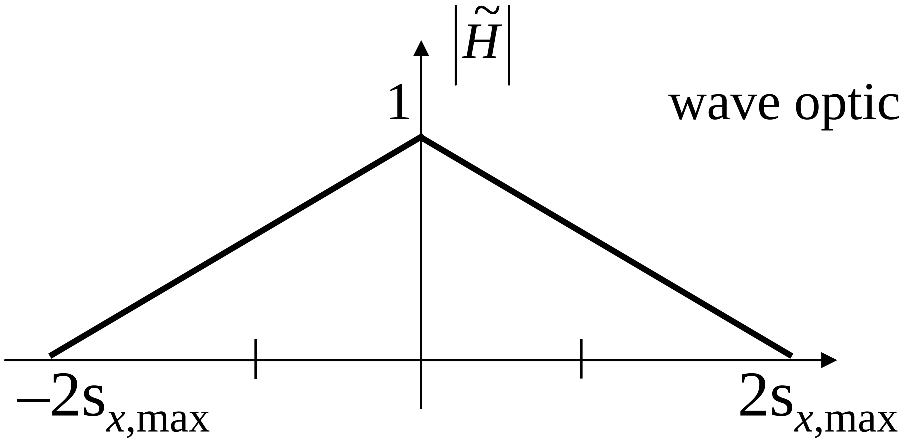
All our calculations have assumed “geometrically perfect” systems, i.e. we calculated the wave–optics behavior of systems which, in the paraxial geometrical optics approximation would have imaged a point object onto a perfect point image.

The effect of aberrations (calculated with non–paraxial geometrical optics) is to blur the “geometrically perfect” image; including the effects of diffraction causes additional blur.

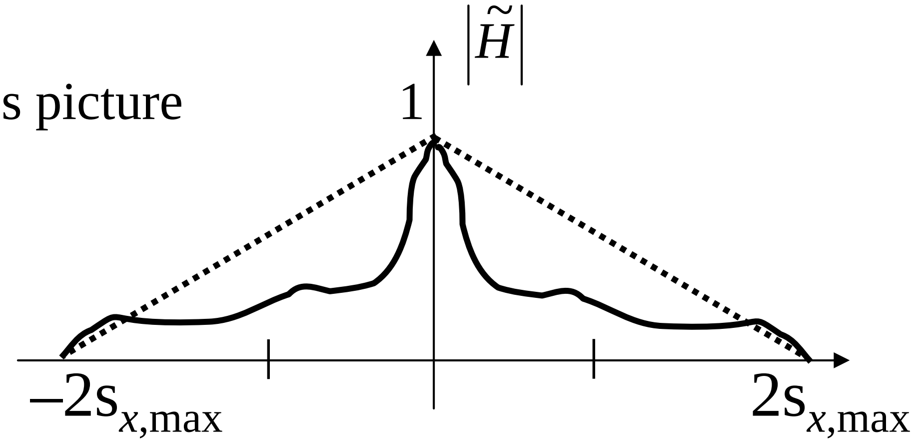


geometrical optics picture

Also affecting resolution: aberrations

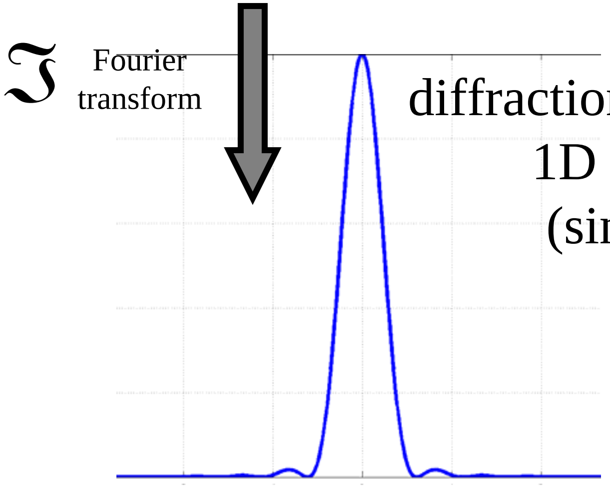


wave optics picture

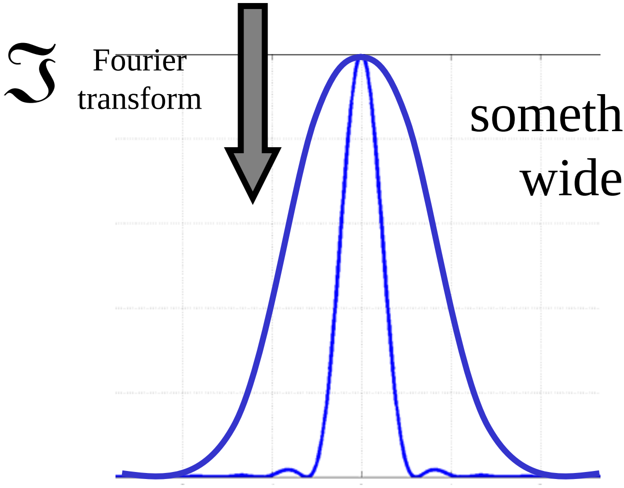


“**diffraction–limited**”
(aberration–free) 1D MTF

1D MTF with aberrations

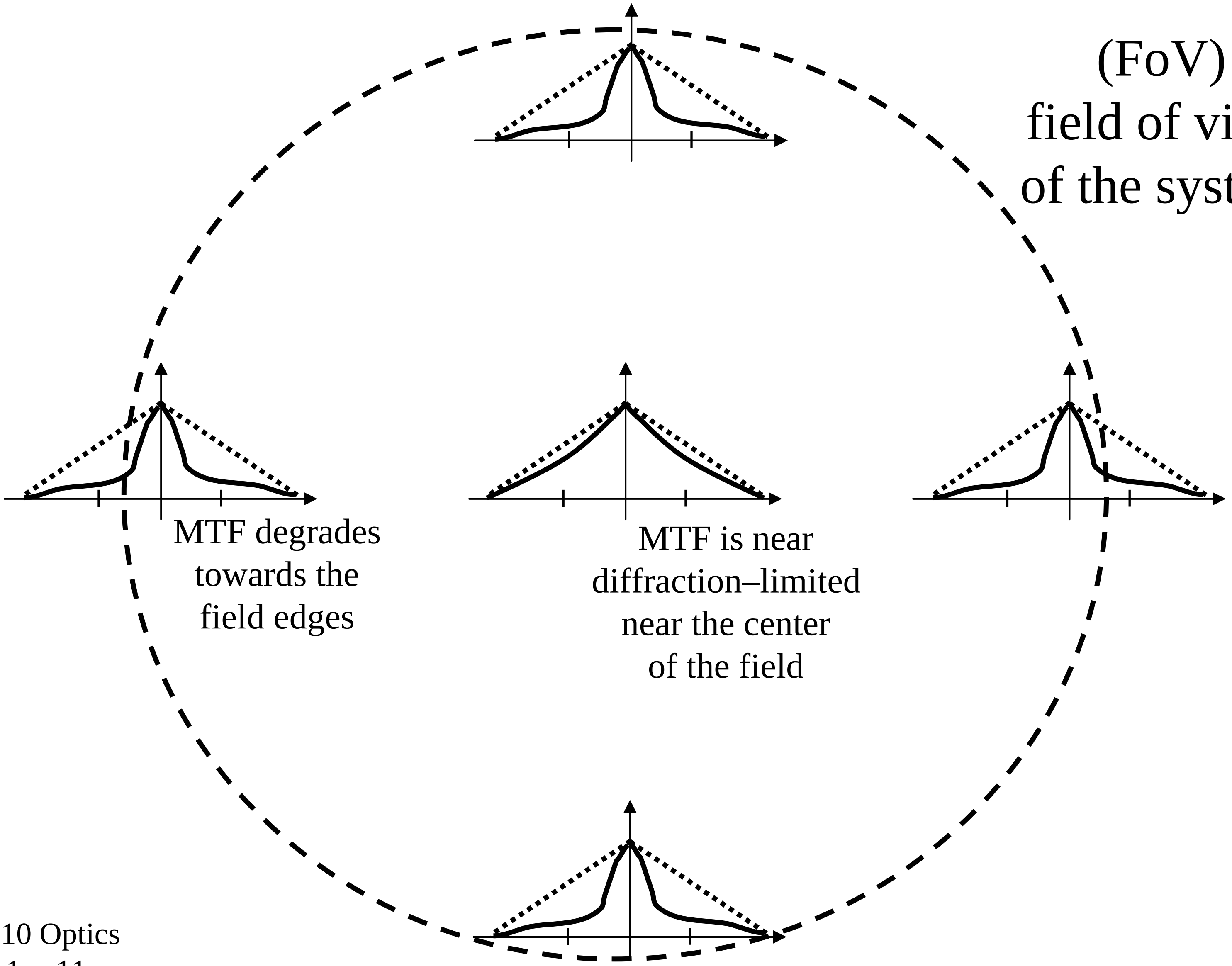


diffraction–limited
1D PSF
(sinc^2)



something
wider

Typical result of optical design



(FoV)
field of view
of the system

MTF degrades
towards the
field edges

MTF is near
diffraction–limited
near the center
of the field

The limits of our approximations

- Real–life MTFs include aberration effects, whereas our analysis has been "diffraction–limited"
- Aberration effects on the MTF are FoV (field) location–dependent: typically we get more blur near the edges of the field (narrower MTF ⇔ broader PSF)
- This, in addition, means that real–life optical systems are not shift invariant either!
- ⇒ the concept of MTF is approximate, near the region where the system is approximately shift invariant (recall: transfer functions can be defined only for shift invariant linear systems!)

The utility of our approximations

- Nevertheless, within the limits of the paraxial, linear shift–invariant system approximation, the concepts of PSF/MTF provide
  - a useful way of *thinking* about the behavior of optical systems
  - an upper limit on the performance of a given optical system (diffraction–limited performance is the best we can hope for, in paraxial regions of the field; aberrations will only make worse non–paraxial portions of the field)

Common misinterpretations

Attempting to resolve object features smaller than the "resolution limit" (e.g. 1.22λ/NA) is hopeless.

NO:

Image quality degradation as object features become smaller than the resolution limit ("exceed the resolution limit") is noise dependent and gradual.

Common misinterpretations

Attempting to resolve object features smaller than the “resolution limit” (e.g. $1.22\lambda/\mathrm{NA}$) is hopeless.

NO: Besides, digital processing of the acquired images (e.g. methods such as the CLEAN algorithm, Wiener filtering, expectation maximization, etc.) can be employed.

Common misinterpretations

Super-resolution

By engineering the pupil function ("apodizing") to result in a PSF with narrower side–lobe, one can "beat" the resolution limitations imposed by the angular acceptance (NA) of the system.

NO:

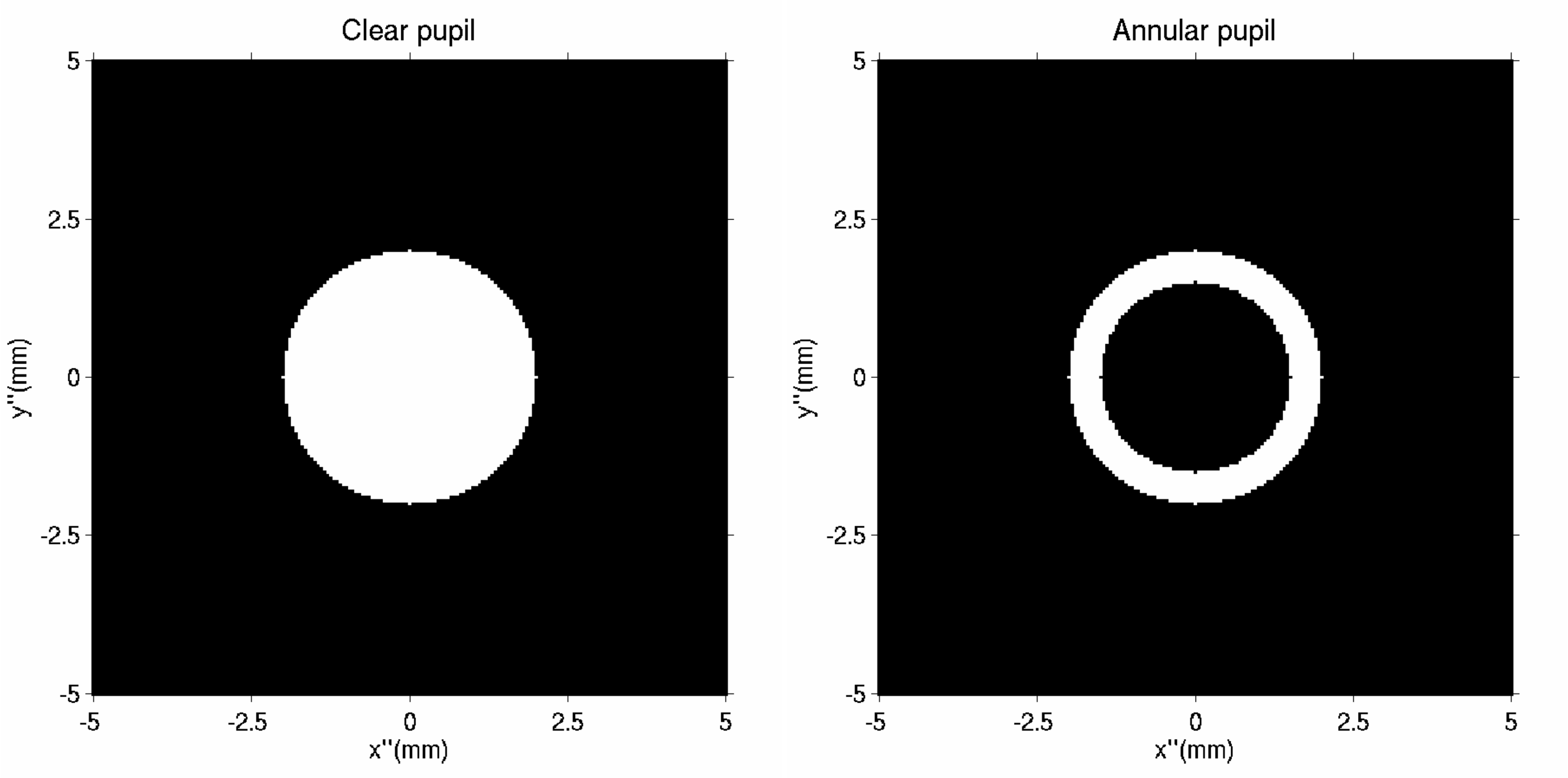
Pupil function design always results in

(i) narrower main lobe but accentuated side–lobes

(ii) lower power transmitted through the system

Both effects are **BAD** on the image

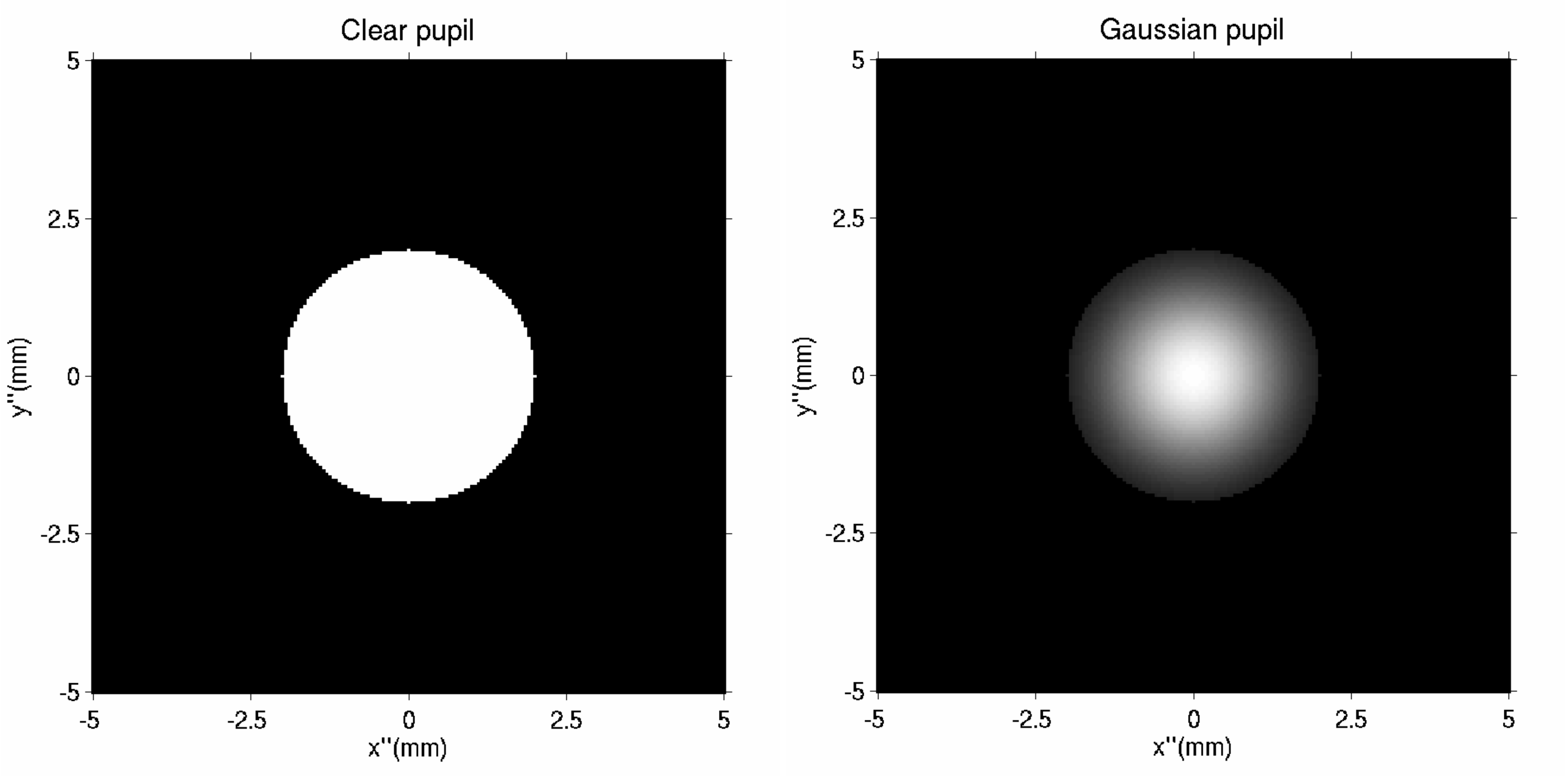
Apodization



f_1=20cm
λ=0.5µm

$$H(r'') = \text{circ}\left(\frac{r''}{R}\right) - \text{circ}\left(\frac{r''}{R_2}\right)$$

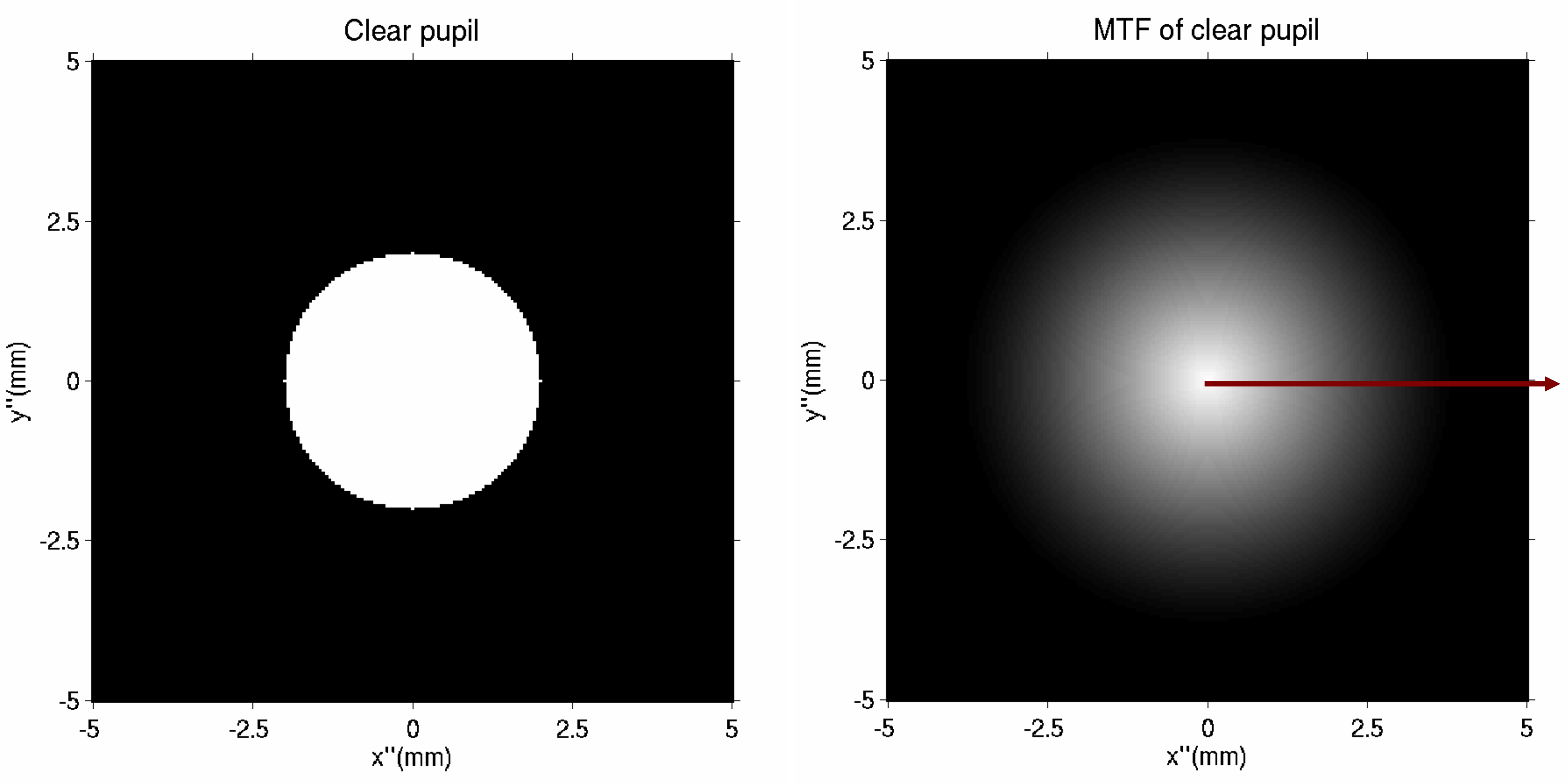
Apodization



f_1=20cm
λ=0.5µm

$$H(r'') = \mathrm{circ}\left(\frac{r''}{R}\right) \times \exp\left(-\frac{r''^2}{2R_0^2}\right)$$

Unapodized (clear–aperture) MTF

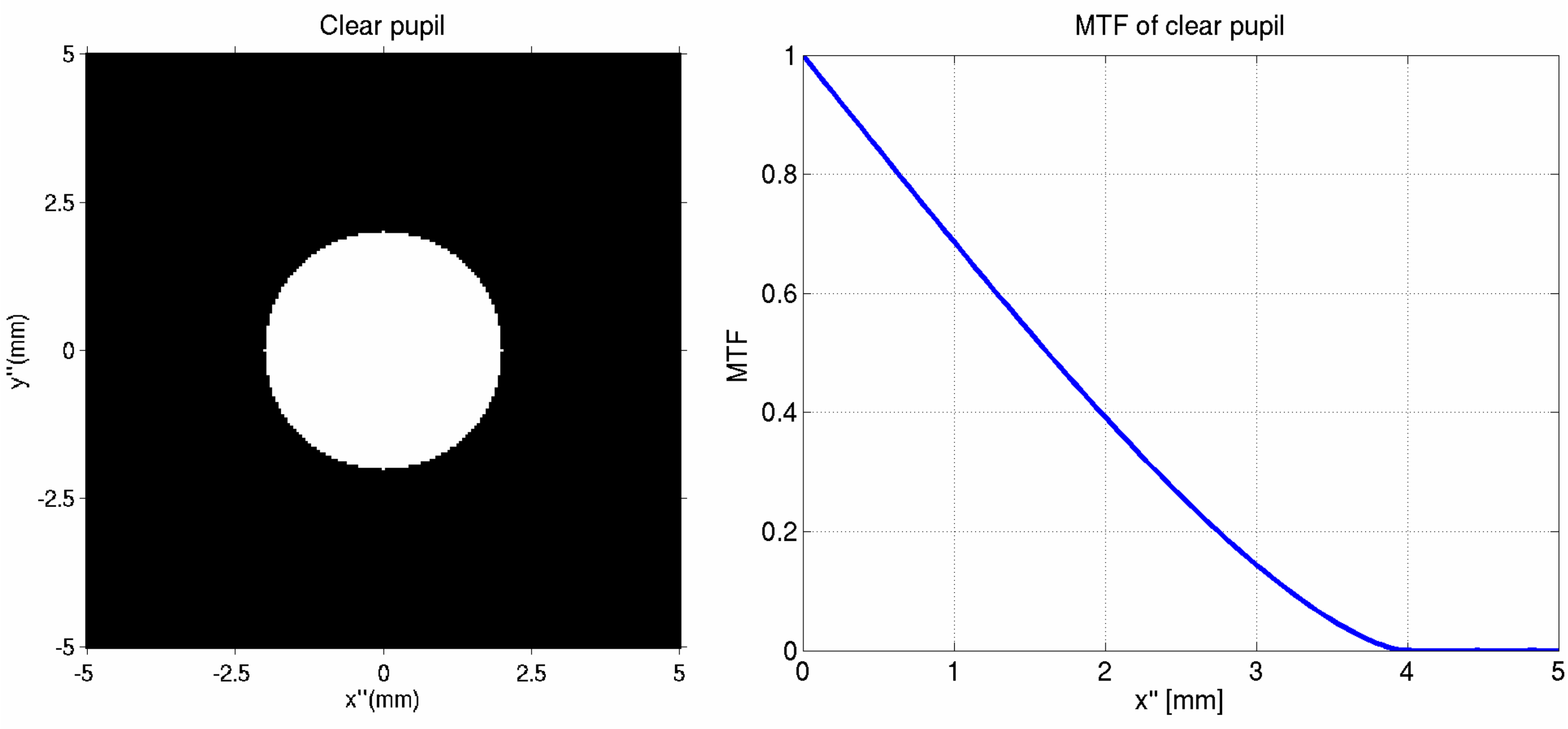


f_1=20cm
λ=0.5μm

$$\tilde{H}(r'') = \mathrm{circ}\left(\frac{r''}{R}\right) \otimes \mathrm{circ}\left(\frac{r''}{R}\right)$$

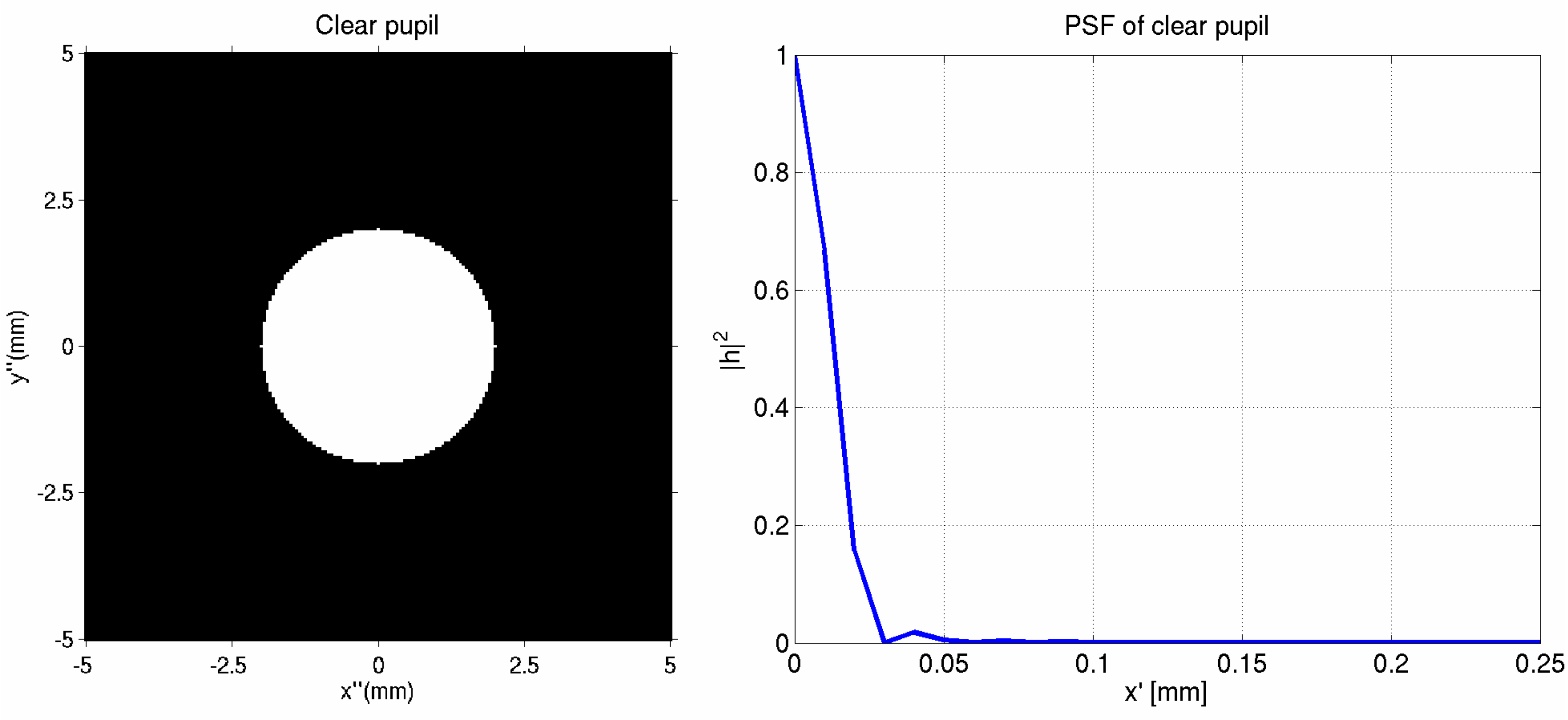
auto-correlation

Unapodized (clear–aperture) MTF



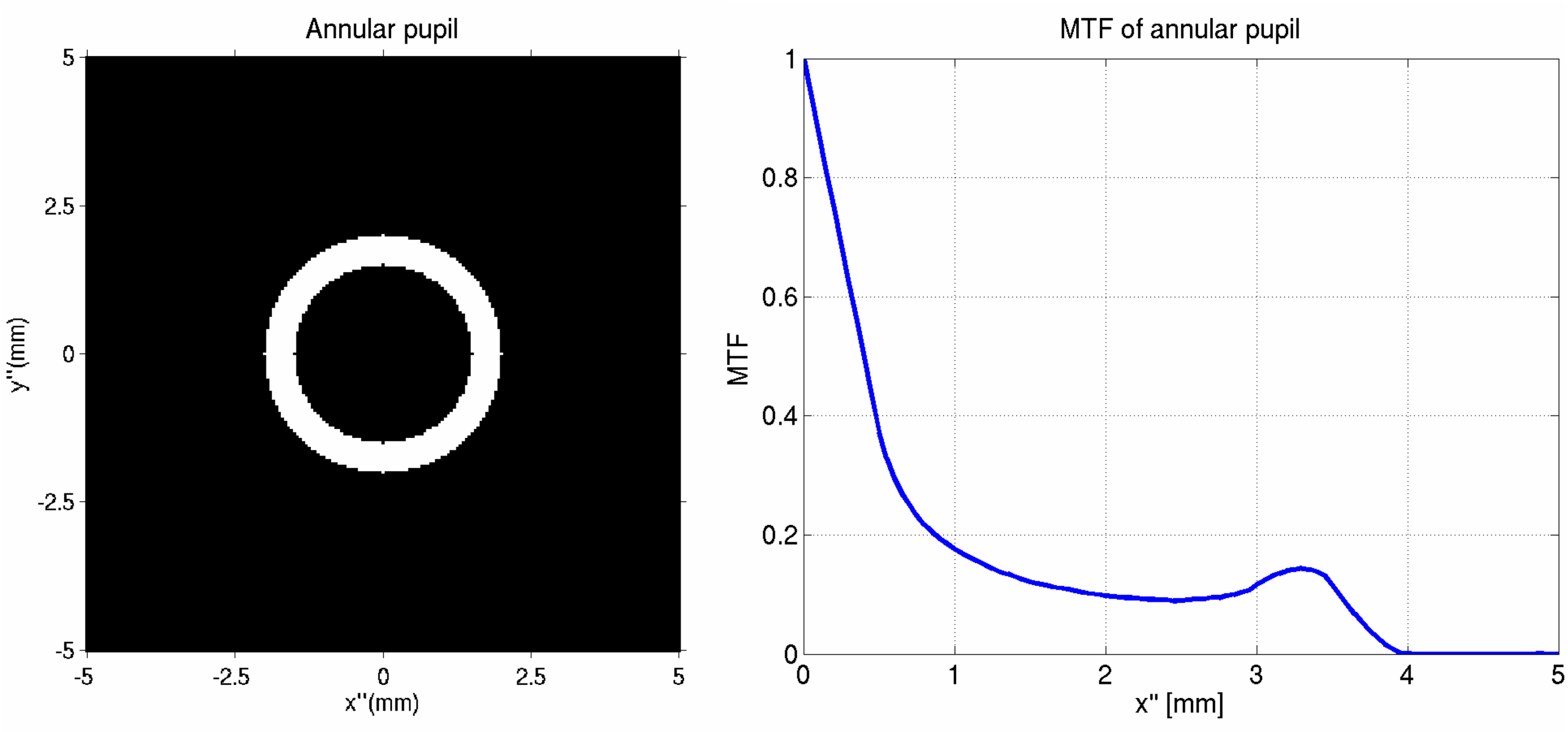
f_1=20cm
λ=0.5µm

Unapodized (clear–aperture) PSF



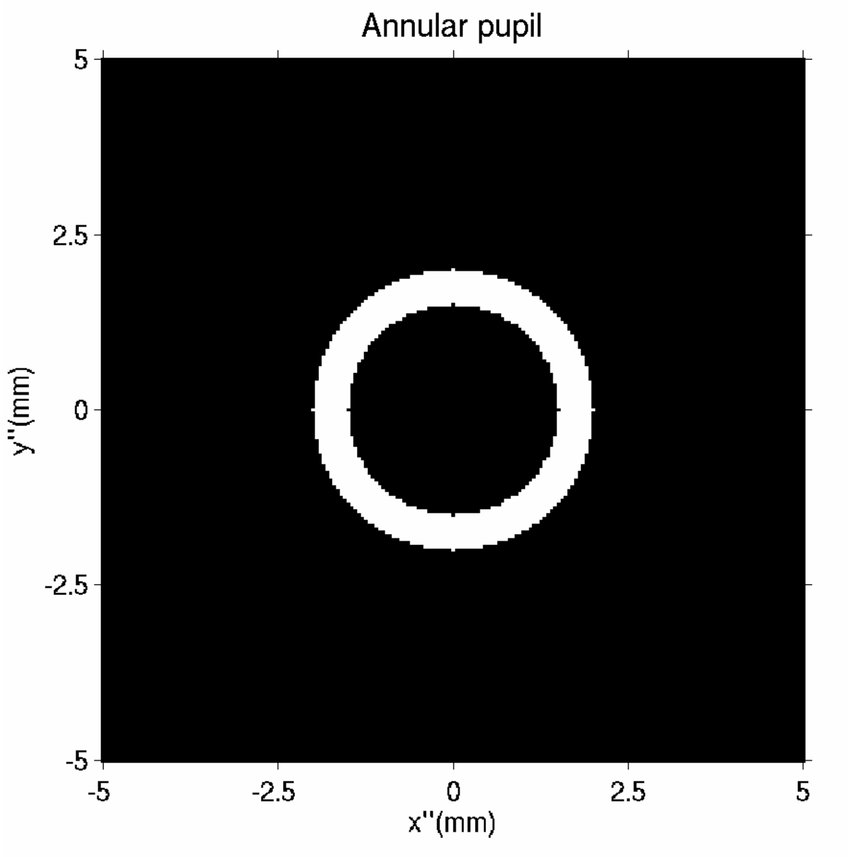
f_1=20cm
λ=0.5μm

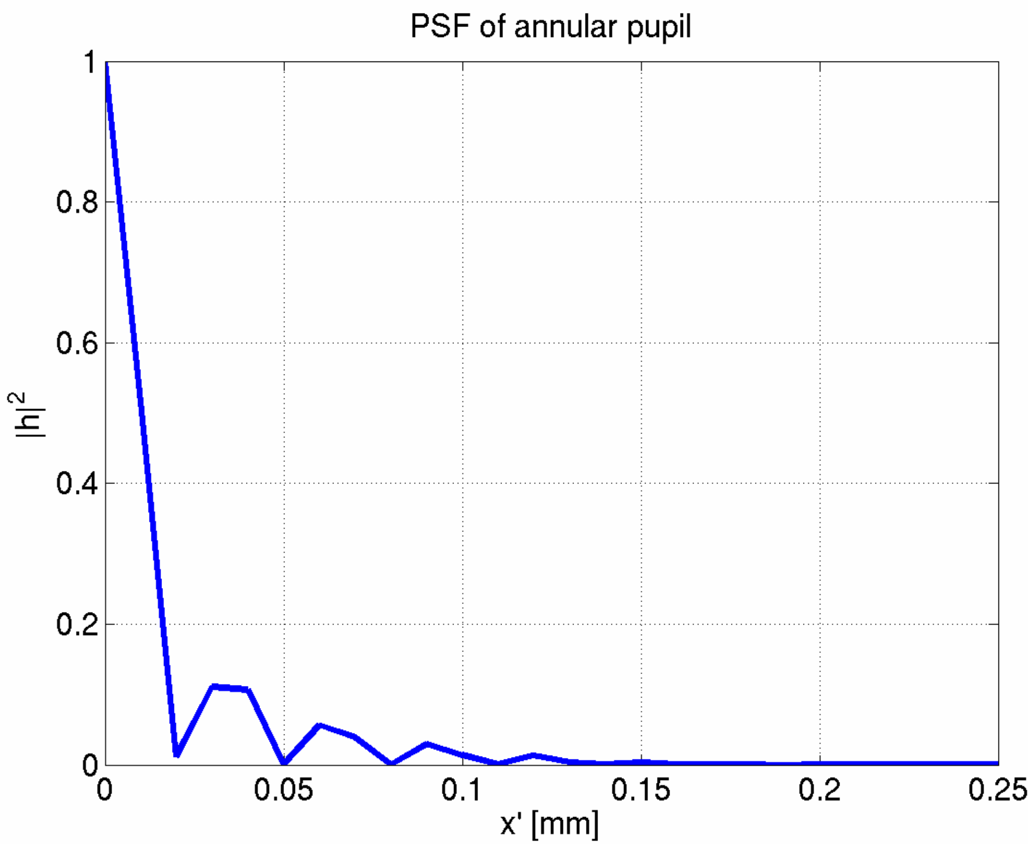
Apodized (annular) MTF



f_1=20cm
λ=0.5μm

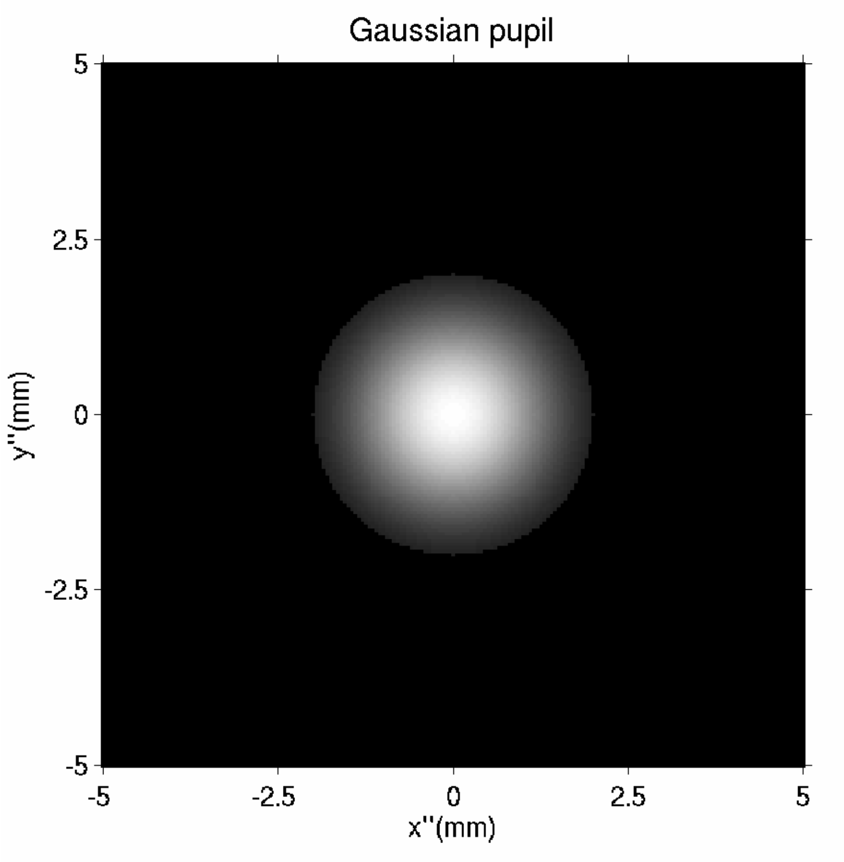
Apodized (annular) PSF

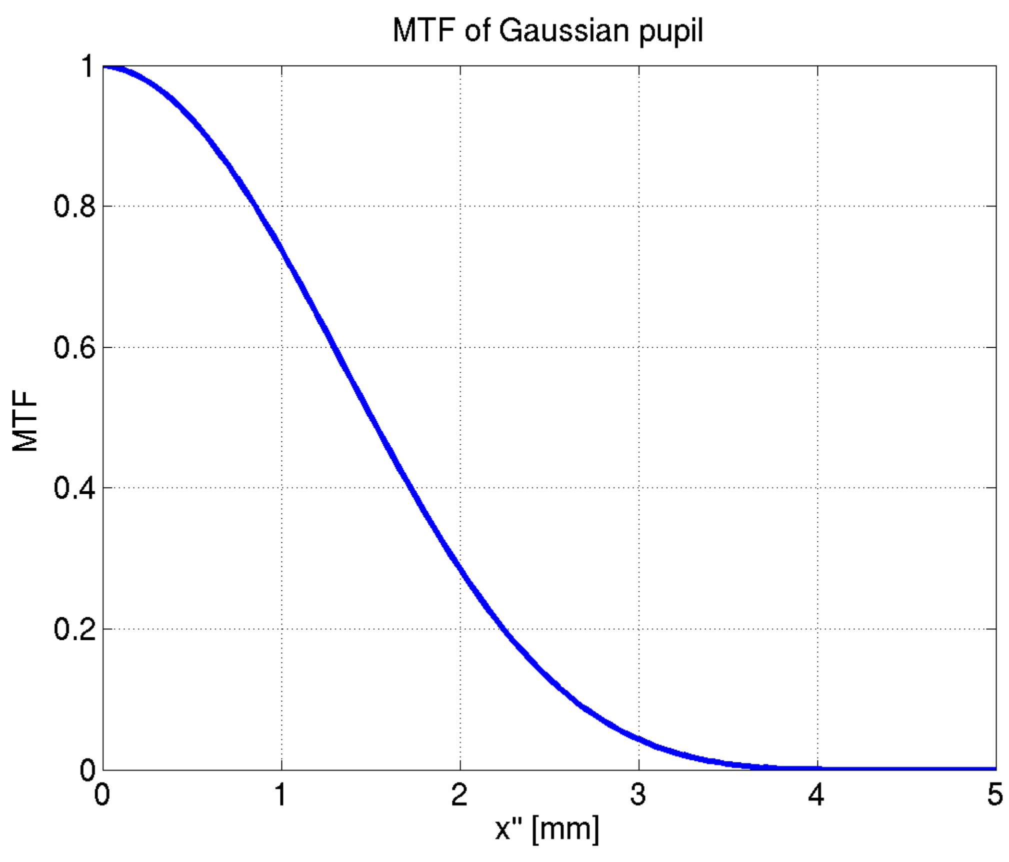




f_1=20cm
λ=0.5µm

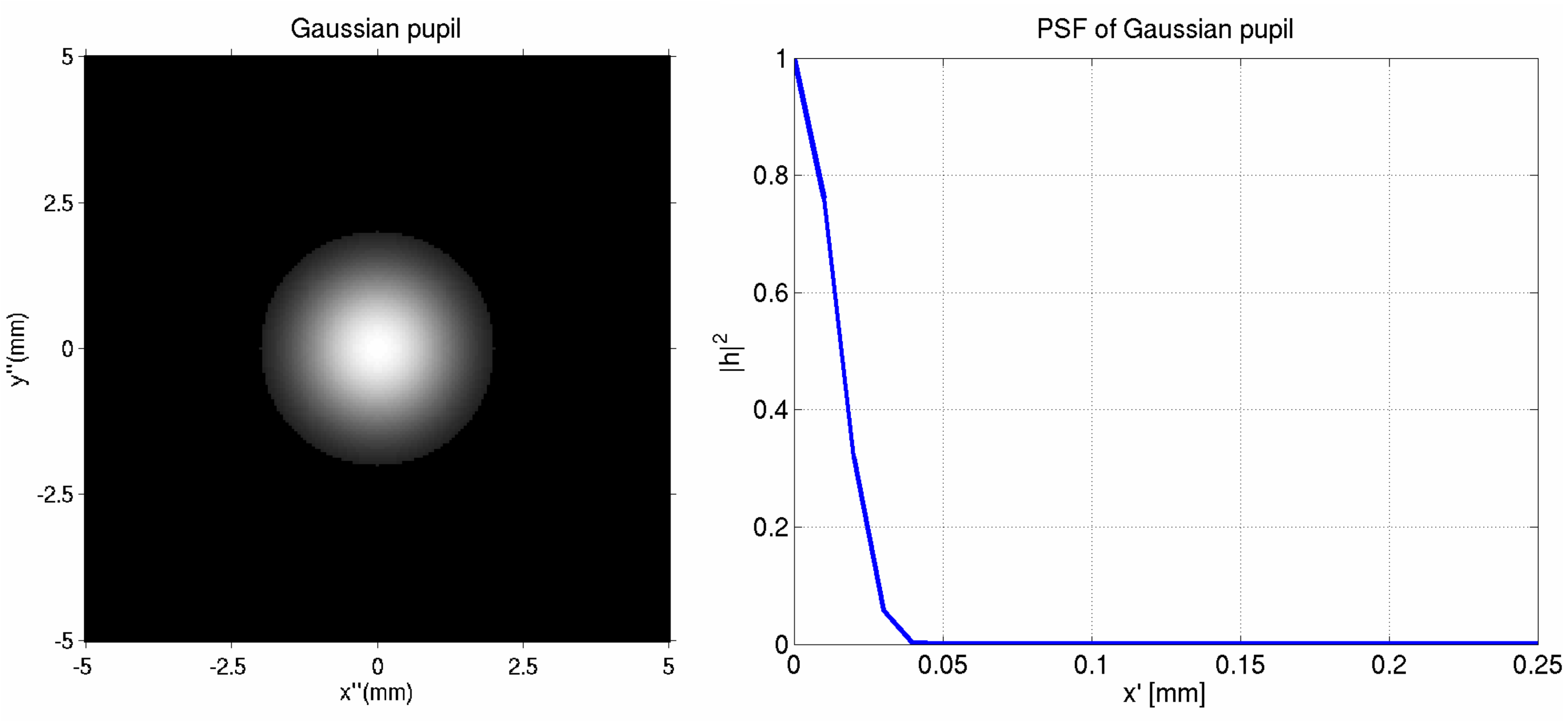
Apodized (Gaussian) MTF





f_1=20cm
λ=0.5µm

Apodized (Gaussian) PSF



f_1=20cm
λ=0.5µm

Conclusions (?)

- Annular–type pupil functions typically narrow the main lobe of the PSF at the expense of higher side lobes
- Gaussian–type pupil functions typically suppress the side lobes but broaden the main lobe of the PSF
- Compromise? → application dependent
  - for point–like objects (e.g., stars) annular apodizers may be a good idea
  - for low–frequency objects (e.g., diffuse tissue) Gaussian apodizers may image with fewer artifacts
- Caveat: Gaussian amplitude apodizers very difficult to fabricate and introduce energy loss ⇒ binary phase apodizers (lossless by nature) are used instead; typically designed by numerical optimization

Common misinterpretations

Super-resolution

By engineering the pupil function ("apodizing") to result in a PSF with narrower side–lobe, one can "beat" the resolution limitations imposed by the angular acceptance (NA) of the system.

NO:

main lobe size ↓ ⇔ sidelobes ↑
and *vice versa*
main lobe size ↑ ⇔ sidelobes ↓

power loss an important factor

compromise application dependent

Common misinterpretations

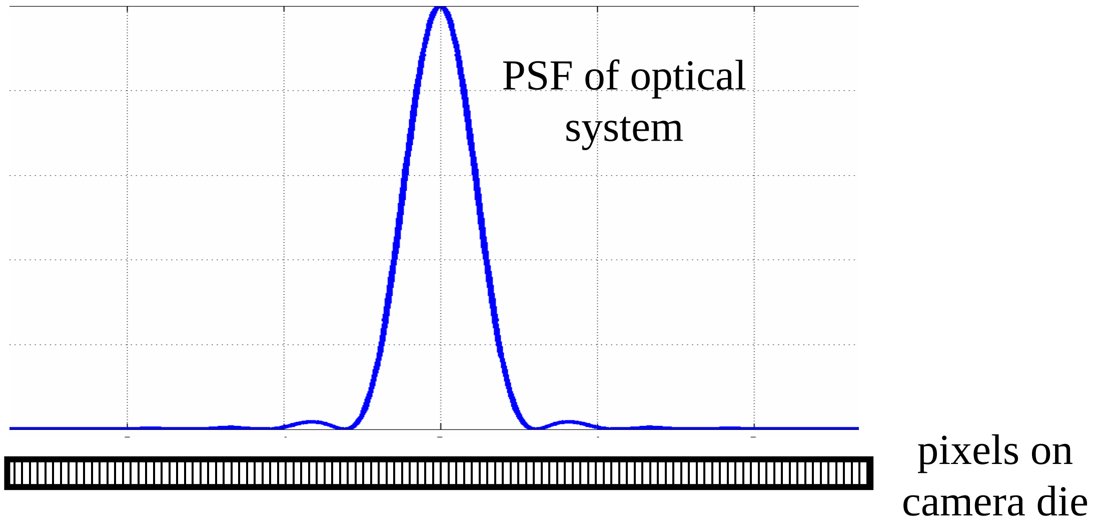
"This super cool digital camera has resolution
of 5 Mega pixels (5 million pixels)."

NO:

This is the most common and worst misuse of the term "resolution." They are actually referring to the **space–bandwidth product (SBP)** of the camera

What *can* a camera resolve?

Answer depends on the magnification and
PSF of the optical system attached to the camera



Pixels significantly smaller than the system PSF
are somewhat underutilized (the effective SBP is reduced)

Summary of misinterpretations of “resolution” and their refutations

- It is pointless to attempt to resolve beyond the Rayleigh criterion (however defined)
  - NO: difficulty increases gradually as feature size shrinks, and difficulty is noise dependent
- Apodization can be used to beat the resolution limit imposed by the numerical aperture
  - NO: watch sidelobe growth and power efficiency loss
- The resolution of my camera is $N \times M$ pixels
  - NO: the maximum possible SBP of your system may be $N \times M$ pixels but you can easily underutilize it by using a suboptimal optical system