Today

- Resolution

- Wavefront modulation

Resolution

Coherent imaging
as a linear, shift-invariant system

Thin transparency $t(x,y)$

$g_1(x,y)$

illumi
nation
(field)

$g_2(x,y) =$
$= g_1(x,y)\,t(x,y)$

impulse response $\Longrightarrow$ **convolution**

output amplitude

$g_3(x',y') =$
$= g_2(x,y) * h(x,y)$

$\Updownarrow$ Fourier transform

$\Updownarrow$ Fourier transform

(≡plane wave spectrum)

$G_2(u,v)$

transfer function $\Longrightarrow$ **multiplication**

$G_3(u,v) =$
$= G_2(u,v)H(u,v)$

transfer function of coherent system $H(u\,,v)$: aka **amplitude transfer function**

Incoherent imaging

as a linear, shift-invariant system

Thin transparency $t(x,y)$

$I_1(x,y)$ illumi nation (intensity) $\Rightarrow$ thin transparency $\Rightarrow$

$$I_2(x,y) = I_1(x,y)\left|t(x,y)\right|^2$$

incoherent impulse response $\Rightarrow$ **convolution**

output intensity

$$I_3(x',y') = I_2(x,y) * \left|h(x,y)\right|^2$$

$\Updownarrow$ Fourier transform (for I_2) $\qquad$ $\Updownarrow$ Fourier transform (for I_3)

$$\hat{I}_2(u,v)$$

transfer function $\Rightarrow$ **multiplication**

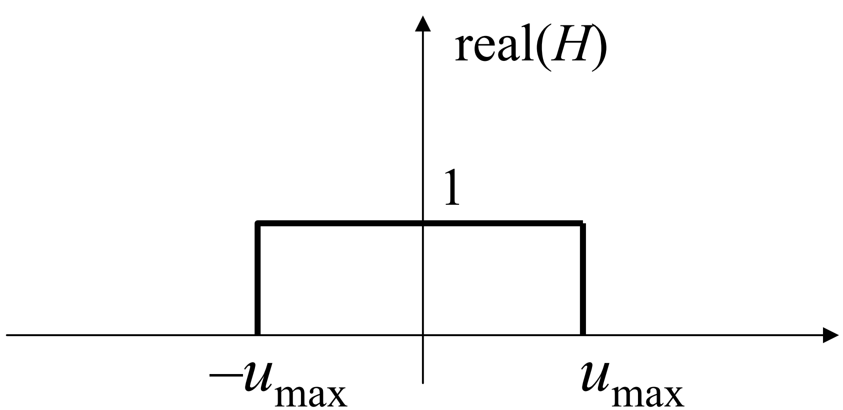
$$\hat{I}_3(u,v) = \hat{I}_2(u,v)\widetilde{H}(u,v)$$

transfer function of incoherent system: $\widetilde{H}(u,v)$ **optical transfer function (OTF)**

Transfer Functions of Clear Aperture

Coherent

$$H(u,v)\equiv\Im\{\underbrace{h(x,y)}_{\text{PSF}}\}$$

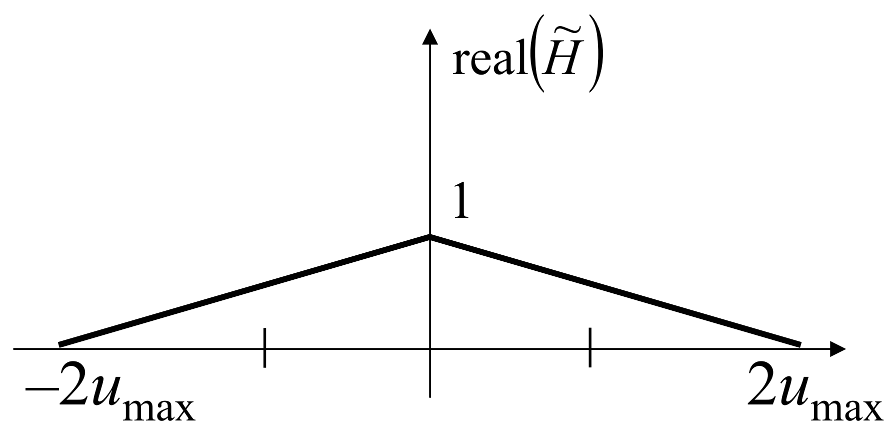


1D amplitude transfer function (ATF)

Incoherent

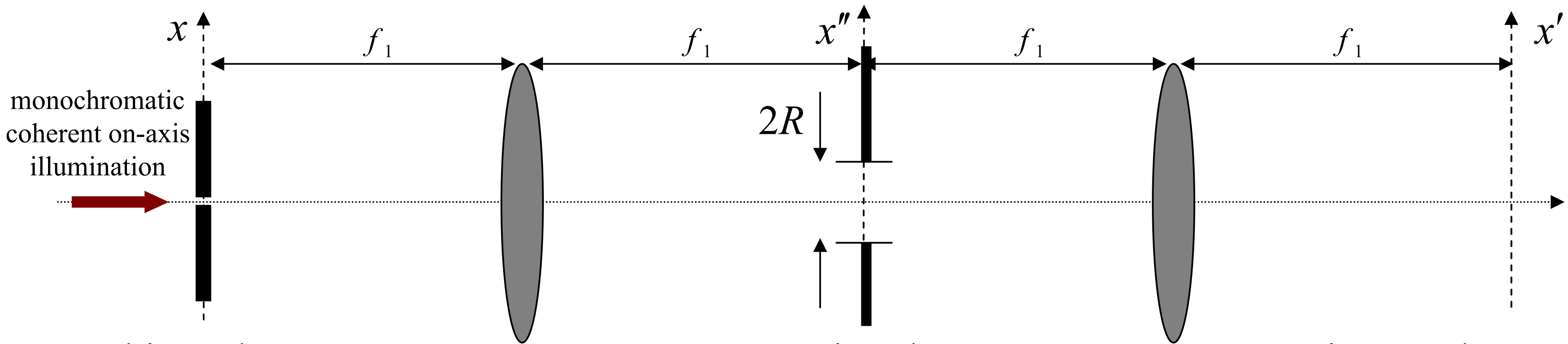
$$\tilde{H}(u,v)\equiv\Im\left\{\left|h(x,y)\right|^2\right\}\quad\text{normalized to 1}$$

$$=\frac{\iint H(u',v')H^*(u'-u,v'-v)v\,\mathrm{d}u'\mathrm{d}v'}{\iint\left|H(u',v')\right|^2\mathrm{d}u'\mathrm{d}v'}$$



1D optical transfer function (OTF)

Connection between PSF and NA



object plane
impulse

Fourier plane
circ-aperture

image plane
observed field
(PSF)

$$g_{\text{in}}(x,y)=\delta(x)\delta(y)$$

$$H(x'',y'')=\text{circ}\left(\frac{r''}{R}\right)$$

$\mathfrak{I}$ Fourier transform

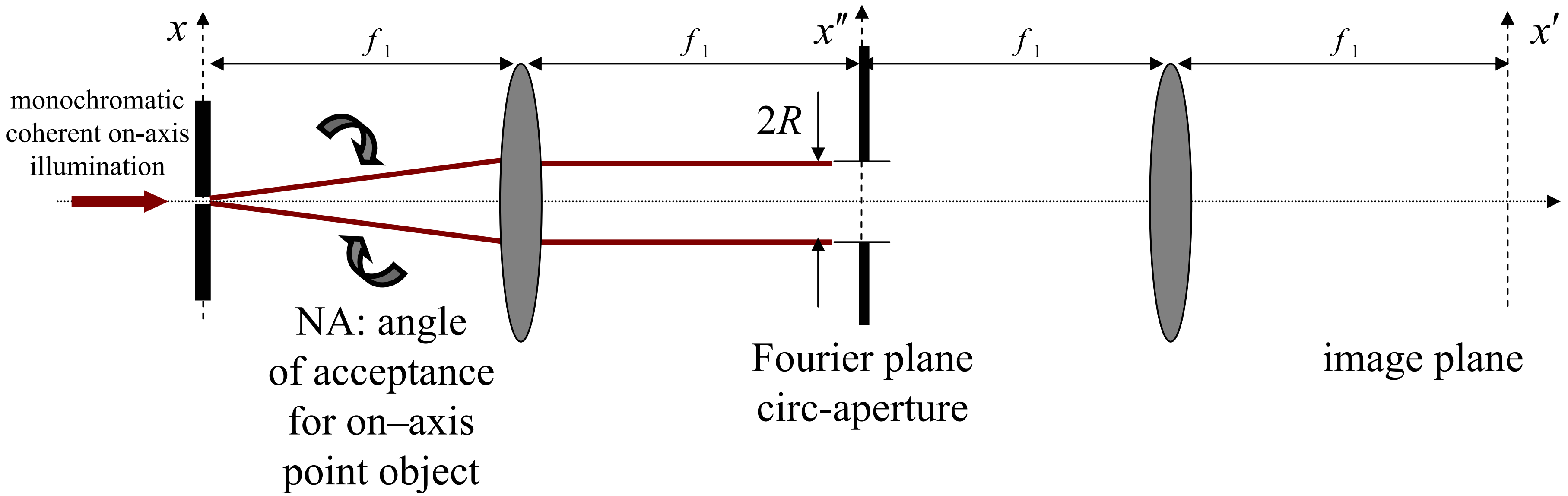
$$r''=\sqrt{x''^2+y''^2}$$ radial coordinate @ Fourier plane

$$r'=\sqrt{x'^2+y'^2}$$ radial coordinate @ image plane

$$\text{jinc}(.,.)\equiv 2\frac{\text{J}_1\left(2\pi\frac{R}{f_1}\frac{r'}{\lambda}\right)}{2\pi\frac{R}{f_1}\frac{r'}{\lambda}}$$

(unit magnification)

Connection between PSF and NA

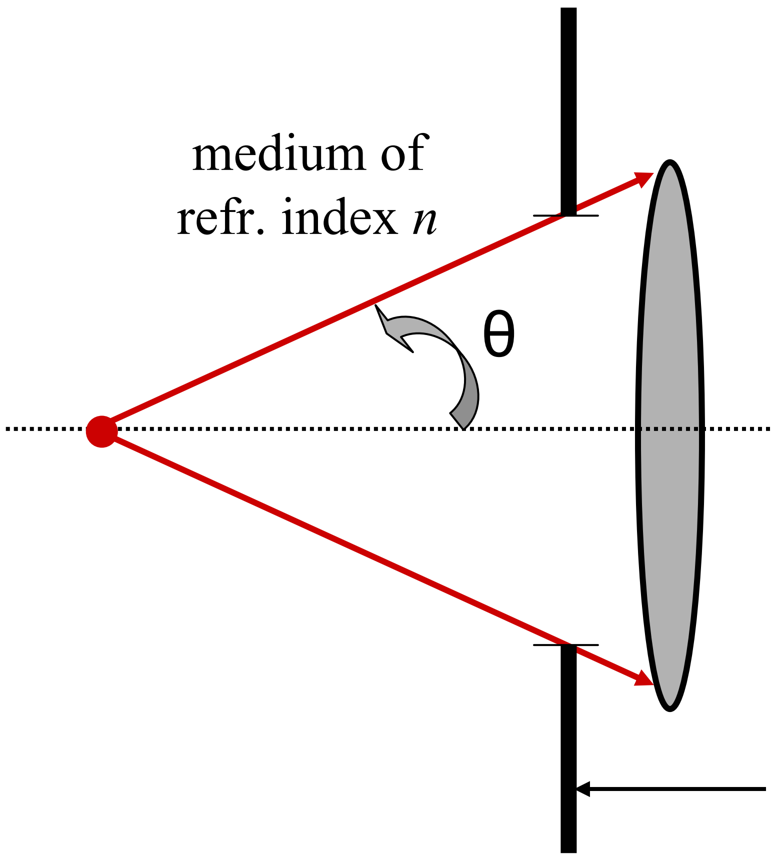


$$\mathrm{jinc}\left(-2\frac{R}{f_1}\frac{x'}{\lambda},-2\frac{R}{f_1}\frac{y'}{\lambda}\right)\equiv 2\frac{\mathrm{J}_1\left(2\pi\frac{R}{f_1}\frac{r'}{\lambda}\right)}{2\pi\frac{R}{f_1}\frac{r'}{\lambda}}=2\frac{\mathrm{J}_1\left(2\pi(\mathrm{NA})\frac{r'}{\lambda}\right)}{2\pi(\mathrm{NA})\frac{r'}{\lambda}}$$

Numerical Aperture (NA)
by definition:

$$(\mathrm{NA})\equiv\frac{R}{f_1}$$

Numerical Aperture and Speed (or F–Number)



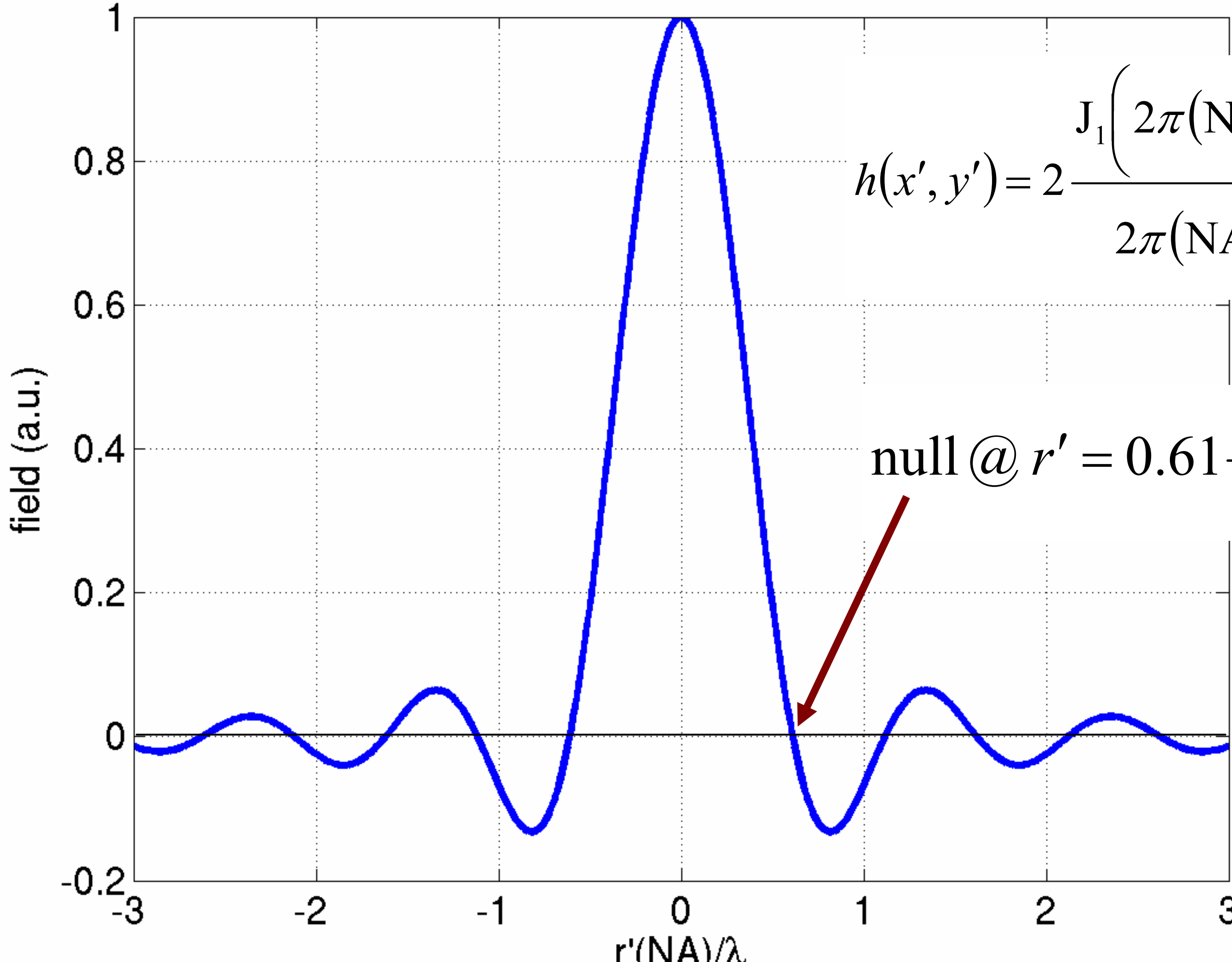
θ: half-angle subtended by the imaging system from an *axial* object

Numerical Aperture
(NA) = n sinθ

Speed (f/#)=1/2(NA)
pronounced f-number, e.g. f/8 means (f/#)=8.

Aperture stop
the physical element which limits the angle of acceptance of the imaging system

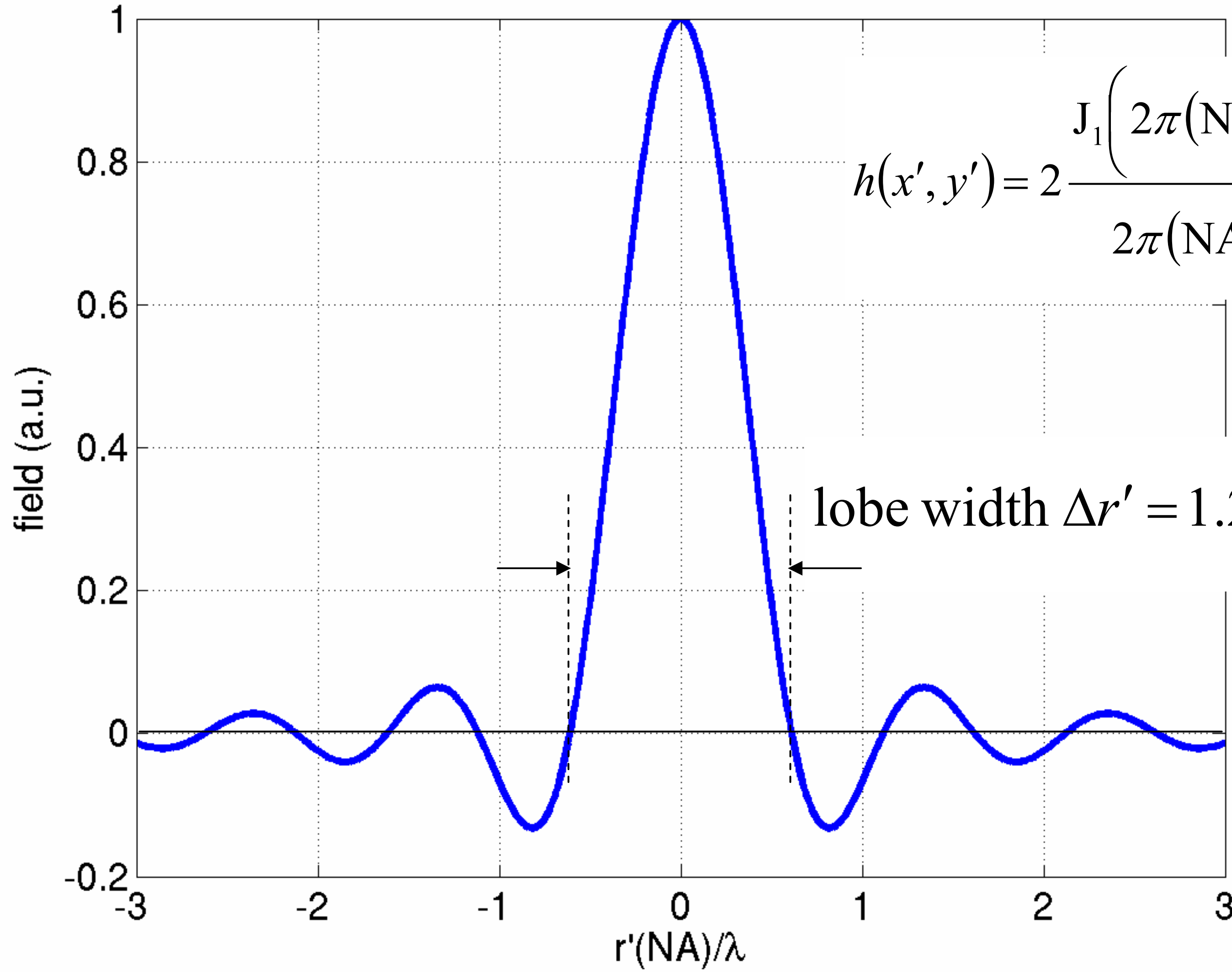
Connection between PSF and NA



$$h(x', y') = 2\frac{J_1\left(2\pi(\text{NA})\frac{r'}{\lambda}\right)}{2\pi(\text{NA})\frac{r'}{\lambda}}$$

$$\text{null} @ r' = 0.61\frac{\lambda}{(\text{NA})}$$

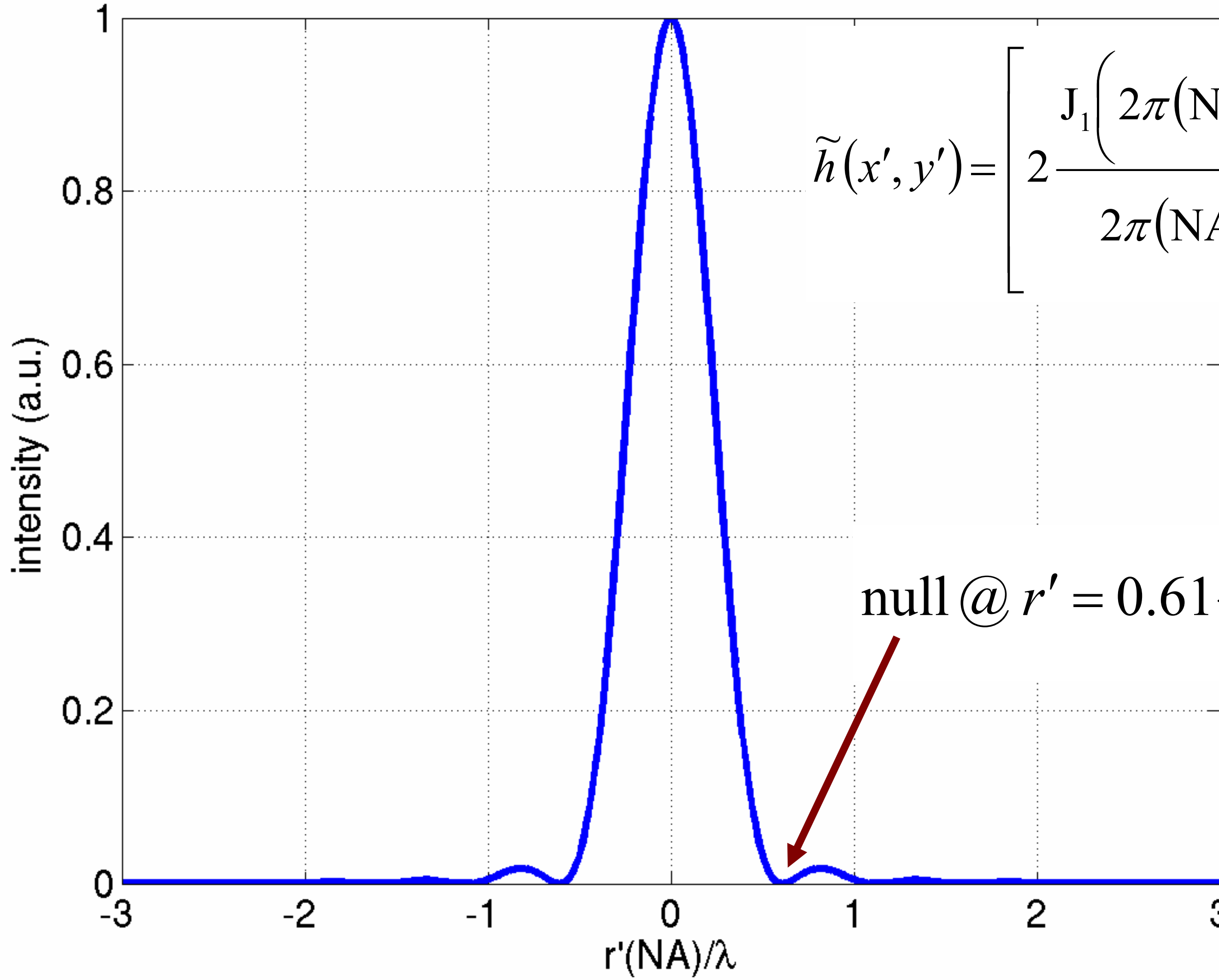
Connection between PSF and NA



$$h(x', y') = 2\frac{J_1\left(2\pi(\mathrm{NA})\frac{r'}{\lambda}\right)}{2\pi(\mathrm{NA})\frac{r'}{\lambda}}$$

lobe width $\Delta r' = 1.22\frac{\lambda}{(\mathrm{NA})}$

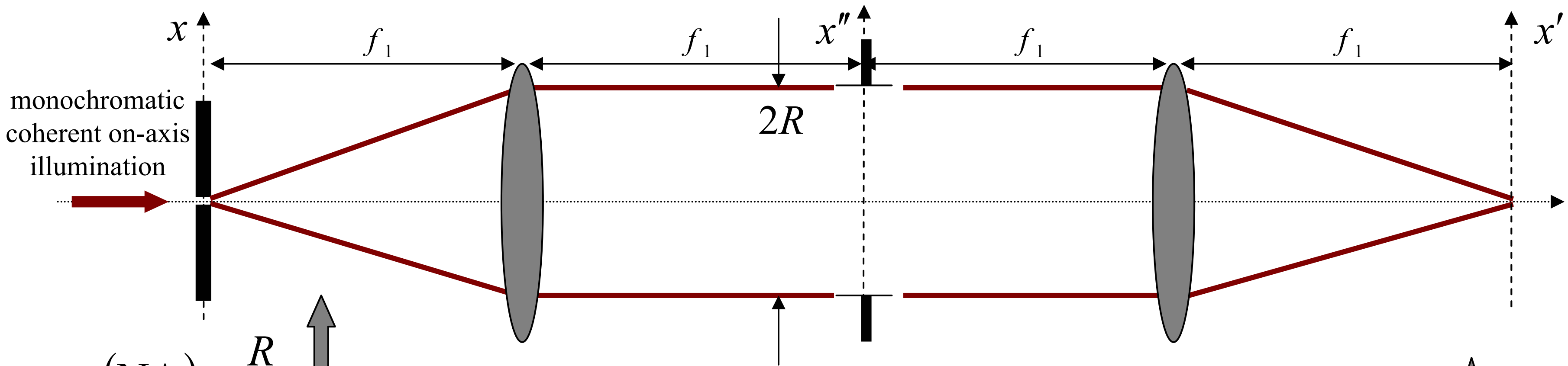
The incoherent case: $\tilde{h}(x', y') = |h(x', y')|^2$



$$\tilde{h}(x', y') = \left[2\frac{J_1\left(2\pi(\mathrm{NA})\frac{r'}{\lambda}\right)}{2\pi(\mathrm{NA})\frac{r'}{\lambda}} \right]^2$$

$$\text{null} \,@\, r' = 0.61\frac{\lambda}{(\mathrm{NA})}$$

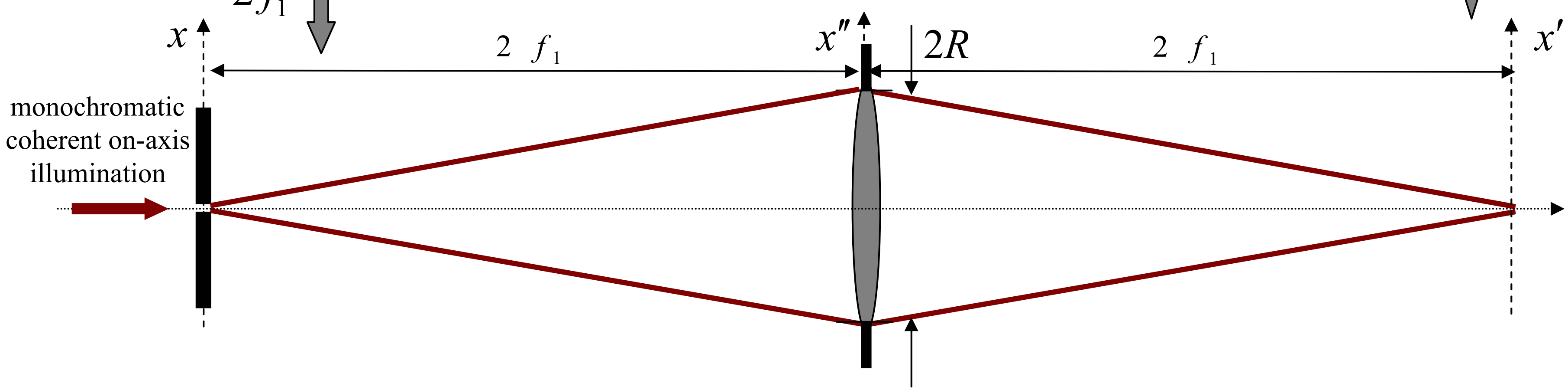
NA in unit–mag imaging systems



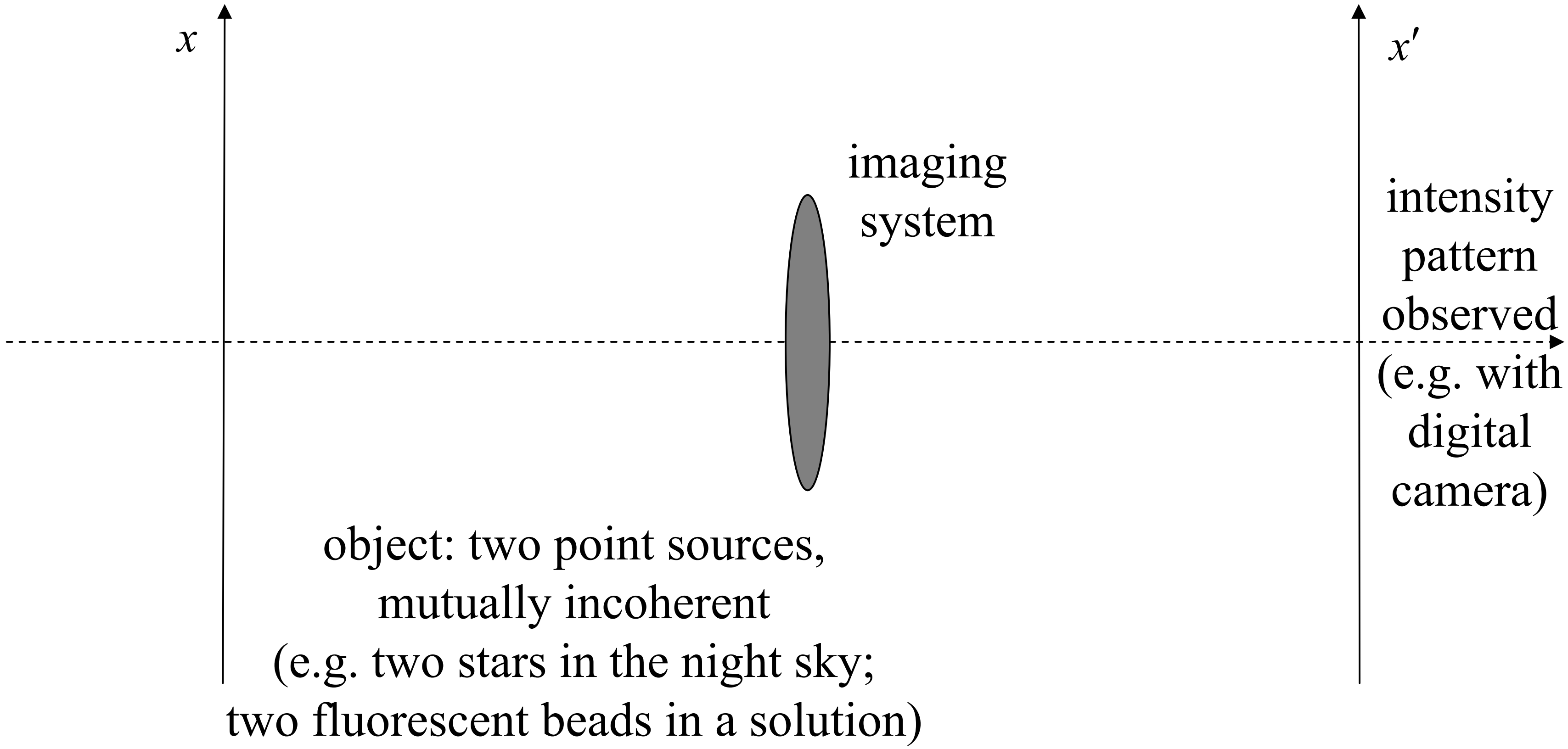
$$(\mathrm{NA}) \equiv \frac{R}{f_1}$$

in both cases, $\mathrm{PSF} = h(x', y') = h(r') = \mathrm{jinc}\left(2(\mathrm{NA})\frac{r'}{\lambda}\right)$

$$(\mathrm{NA}) \equiv \frac{R}{2f_1}$$



The two–point resolution problem



<u>The resolution question</u> [Rayleigh, 1879]: when do we cease to be able to <u>resolve</u> the two point sources (*i.e.*, tell them apart) due to the blurring introduced in the image by the finite (NA)?

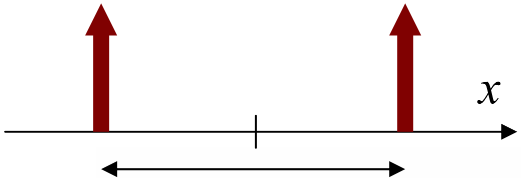
The meaning of “resolution”

[from the New Merriam-Webster Dictionary, 1989 ed.]:

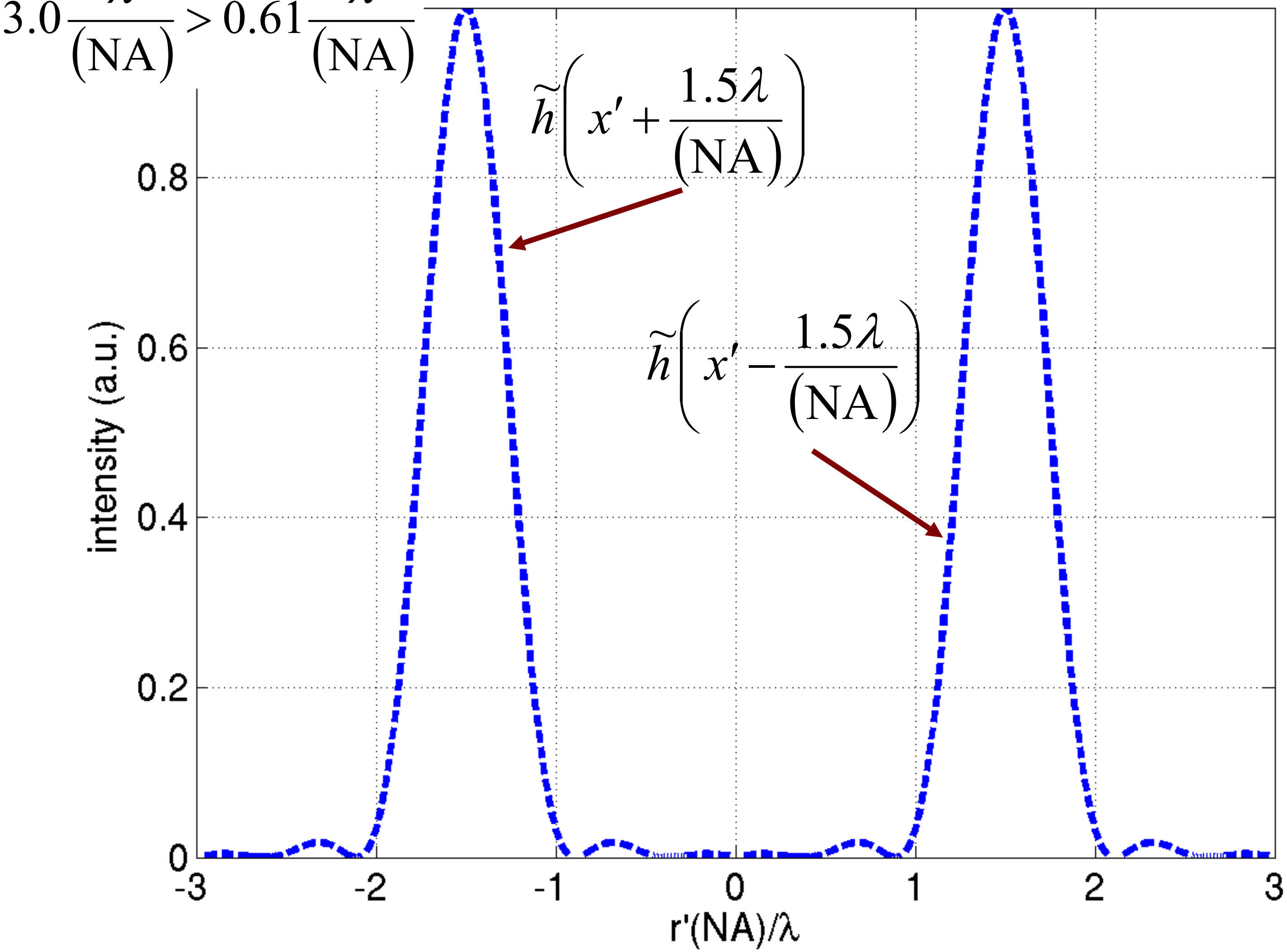
resolve *v* : **1** to break up into constituent parts: ANALYZE; **2** to find an answer to : SOLVE; **3** DETERMINE, DECIDE; **4** to make or pass a formal resolution

resolution *n* : **1** the act or process of resolving **2** the action of solving, *also* : SOLUTION; **3** the quality of being resolute : FIRMNESS, DETERMINATION; **4** a formal statement expressing the opinion, will or, intent of a body of persons

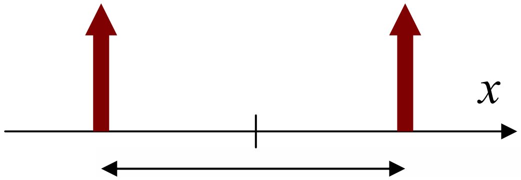
Resolution in optical systems



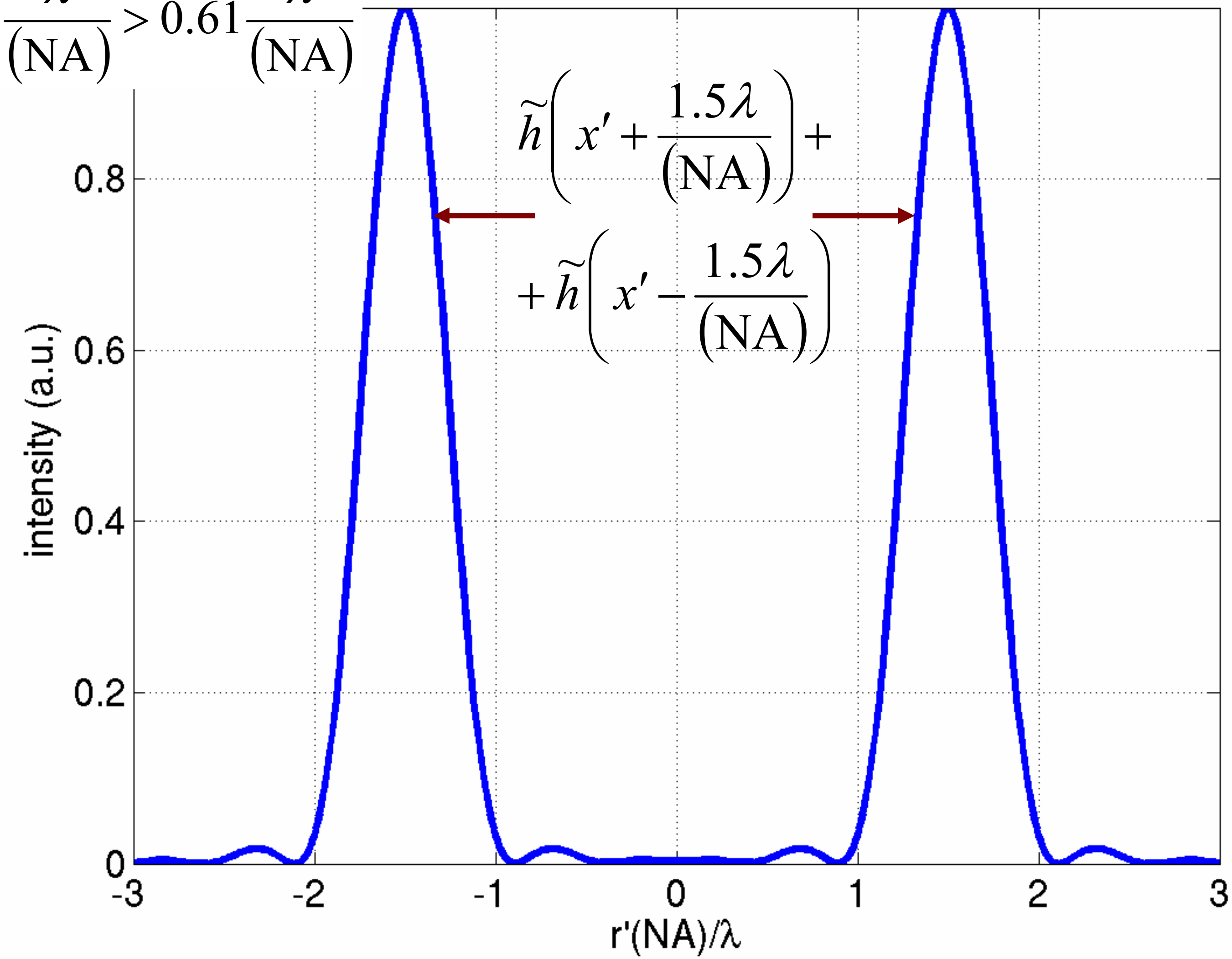
$$\Delta r = 3.0\frac{\lambda}{(\mathrm{NA})} > 0.61\frac{\lambda}{(\mathrm{NA})}$$



Resolution in optical systems

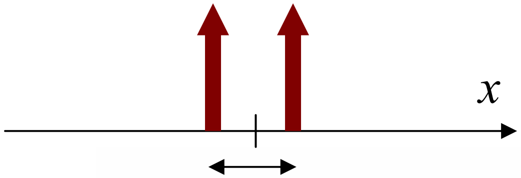


$$\Delta r = 3.0\frac{\lambda}{(\mathrm{NA})} > 0.61\frac{\lambda}{(\mathrm{NA})}$$

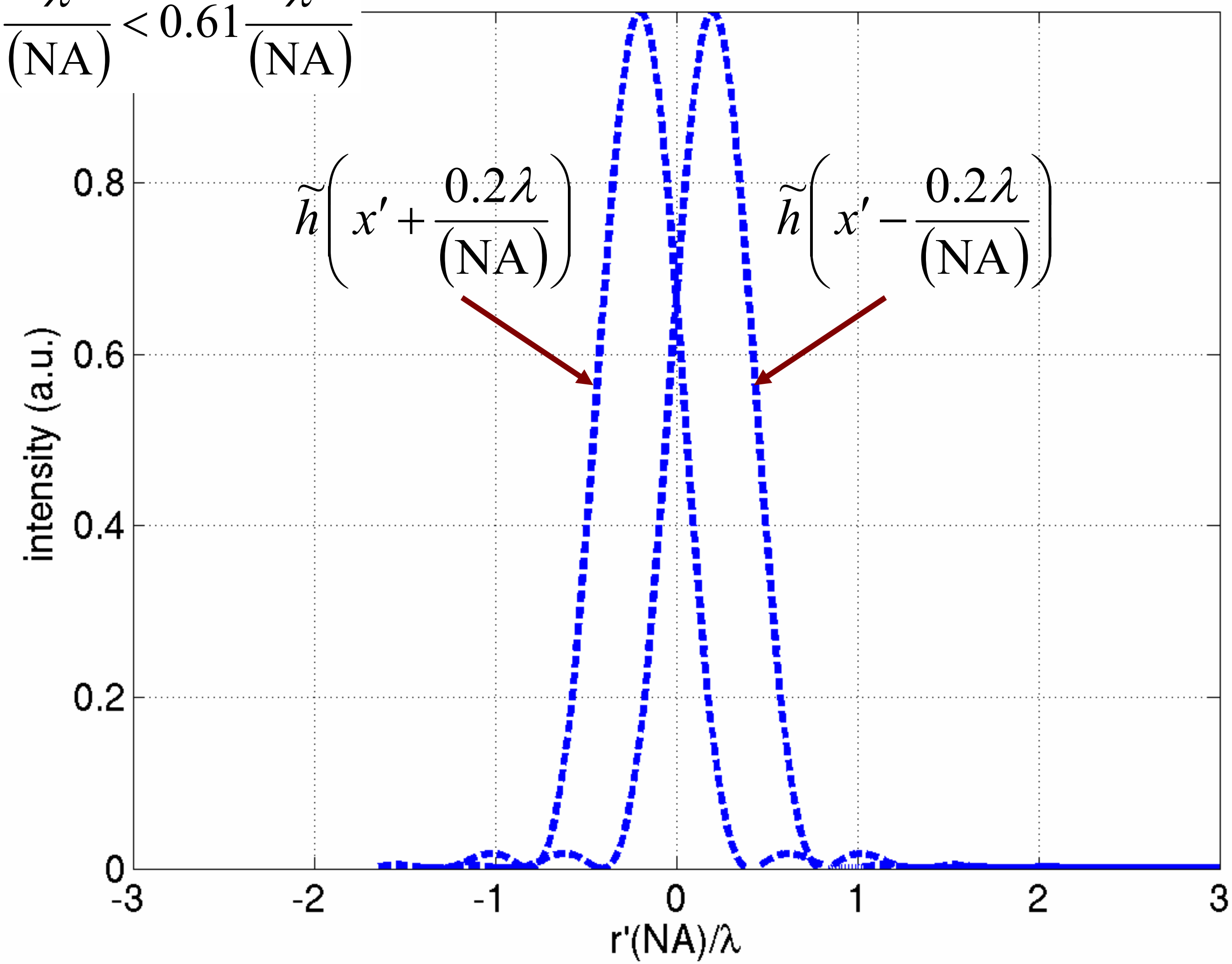


$$\tilde{h}\left(x' + \frac{1.5\lambda}{(\mathrm{NA})}\right) + \tilde{h}\left(x' - \frac{1.5\lambda}{(\mathrm{NA})}\right)$$

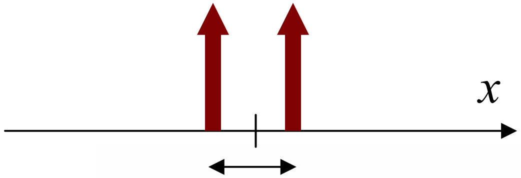
Resolution in optical systems



$$\Delta r = 0.4\frac{\lambda}{(\mathrm{NA})} < 0.61\frac{\lambda}{(\mathrm{NA})}$$

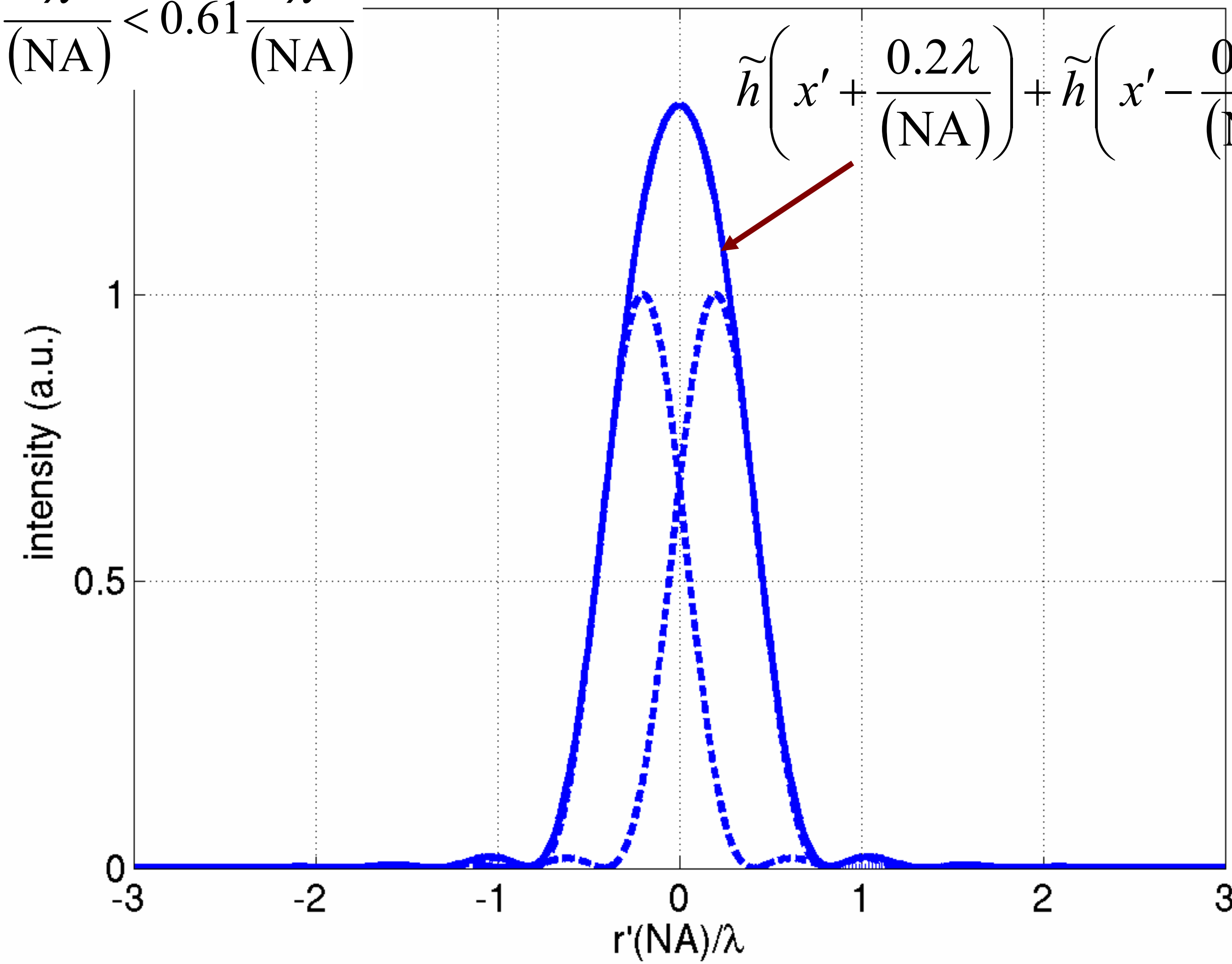


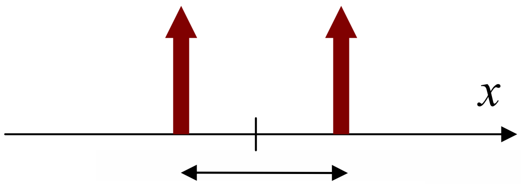
Resolution in optical systems



$$\Delta r = 0.4\frac{\lambda}{(\mathrm{NA})} < 0.61\frac{\lambda}{(\mathrm{NA})}$$

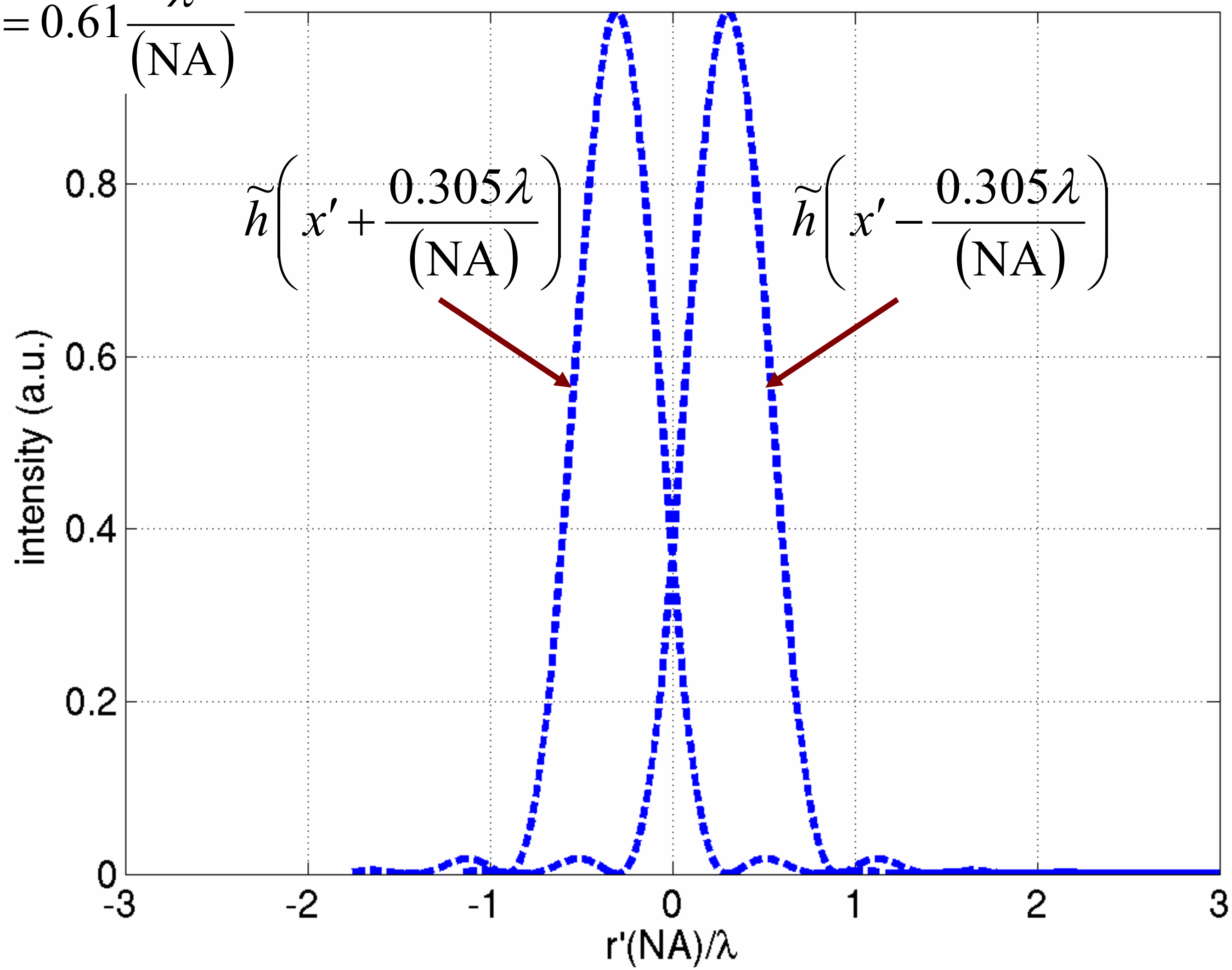
$$\tilde{h}\left(x' + \frac{0.2\lambda}{(\mathrm{NA})}\right) + \tilde{h}\left(x' - \frac{0.2\lambda}{(\mathrm{NA})}\right)$$



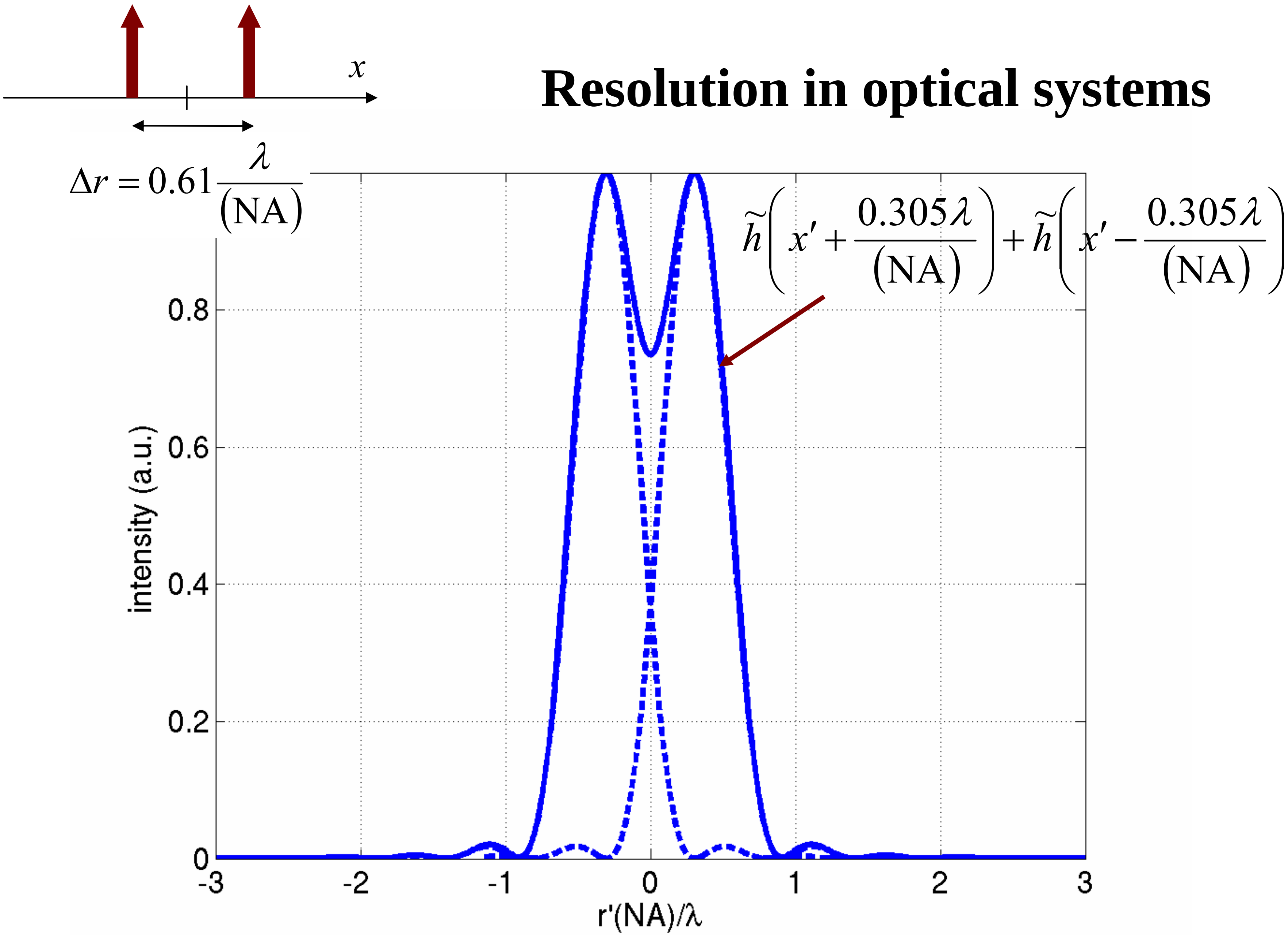


Resolution in optical systems

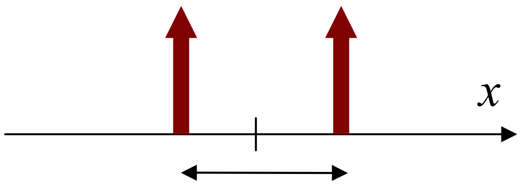
$$\Delta r = 0.61 \frac{\lambda}{(\mathrm{NA})}$$



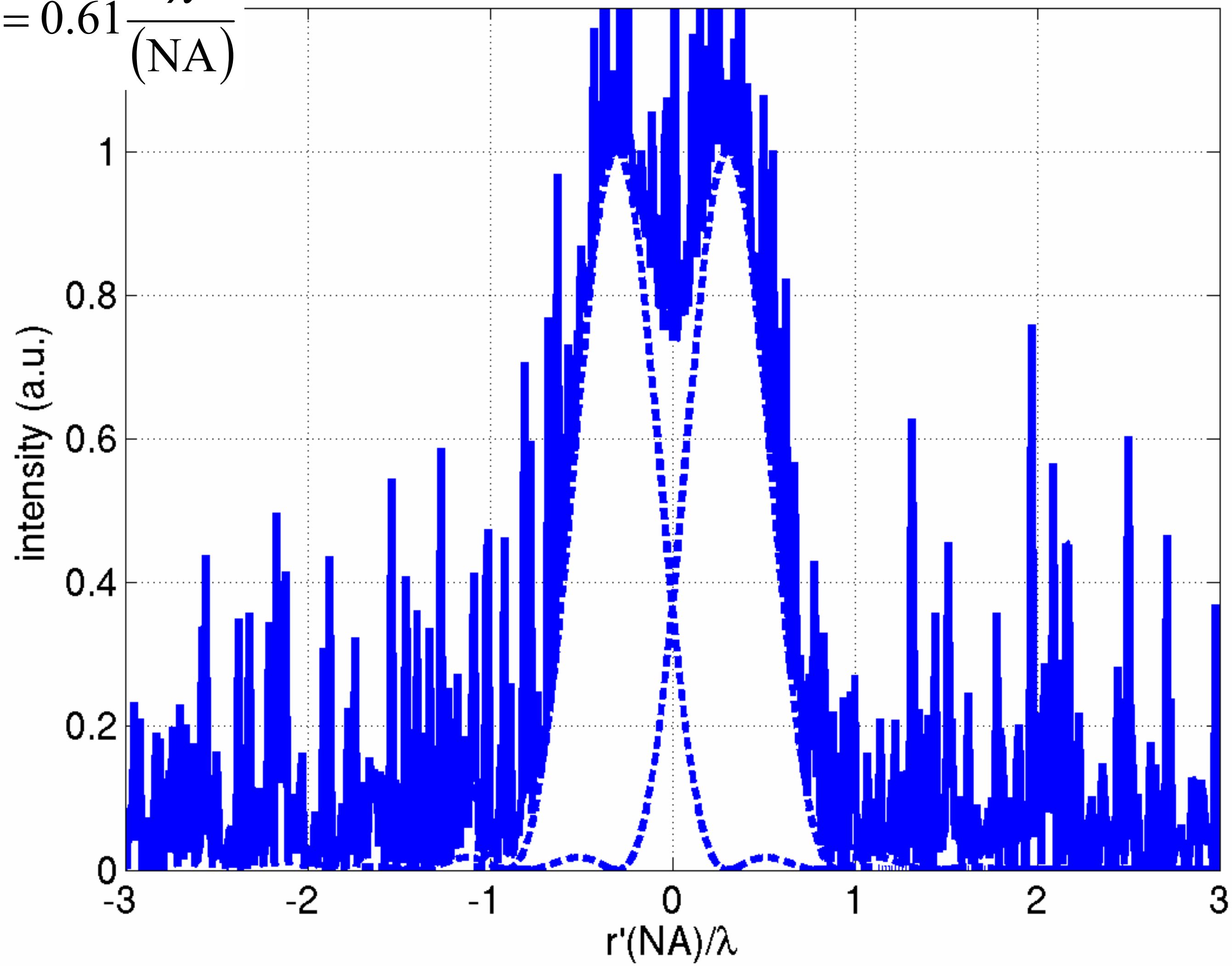
Resolution in optical systems



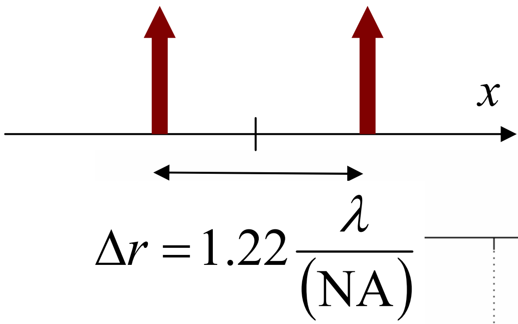
Resolution in **noisy** optical systems

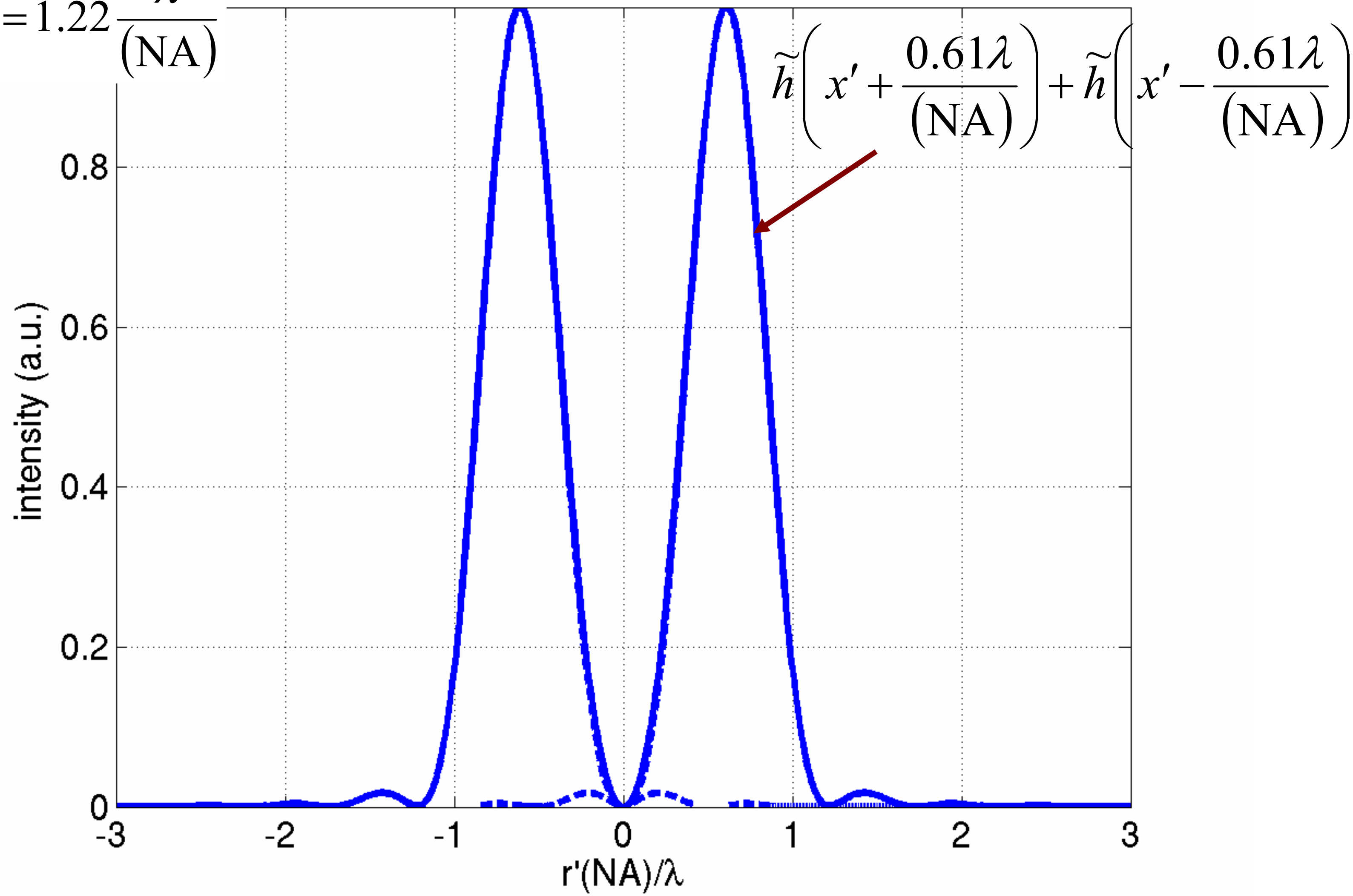


$$\Delta r = 0.61\frac{\lambda}{(\mathrm{NA})}$$



"Safe" resolution in optical systems





Diffraction–limited resolution (safe)

Two point objects are "**just resolvable**" (limited by diffraction only) if they are separated by:

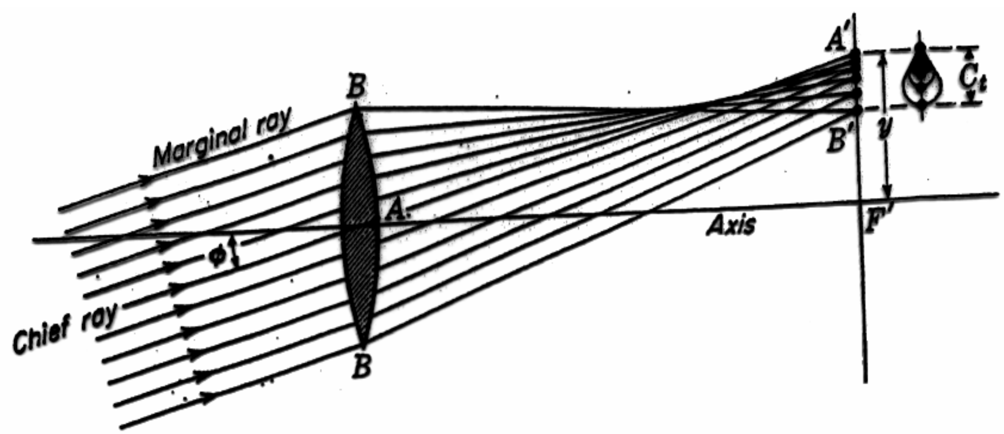
Two–dimensional systems (rotationally symmetric PSF)	One–dimensional systems (e.g. slit–like aperture)
Safe definition: (one–lobe spacing) $\Delta r' = 1.22 \frac{\lambda}{(\mathrm{NA})}$	$\Delta x' = \frac{\lambda}{(\mathrm{NA})}$
Pushy definition: (1/2–lobe spacing) $\Delta r' = 0.61 \frac{\lambda}{(\mathrm{NA})}$	$\Delta x' = 0.5 \frac{\lambda}{(\mathrm{NA})}$

You will see different authors giving different definitions.
Rayleigh in his original paper (1879) noted the issue of noise and warned that the definition of "just–resolvable" points is system– or application –dependent

Also affecting resolution: aberrations

All our calculations have assumed "geometrically perfect" systems, i.e. we calculated the wave–optics behavior of systems which, in the paraxial geometrical optics approximation would have imaged a point object onto a perfect point image.

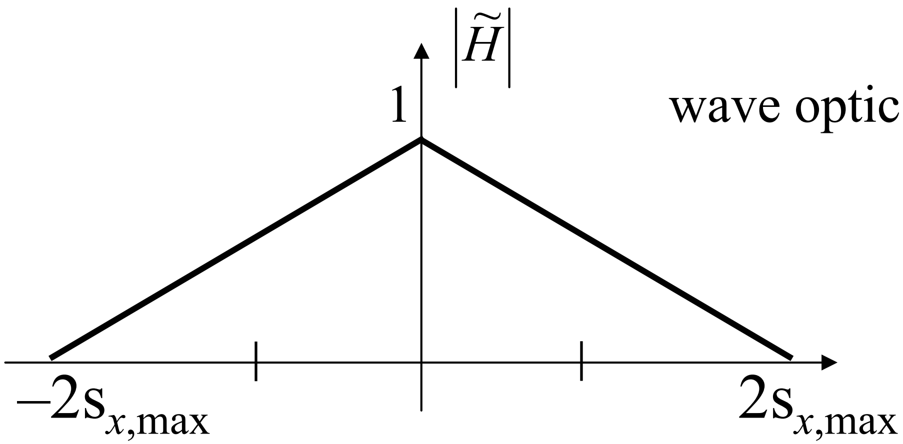
The effect of aberrations (calculated with non–paraxial geometrical optics) is to blur the "geometrically perfect" image; including the effects of diffraction causes additional blur.



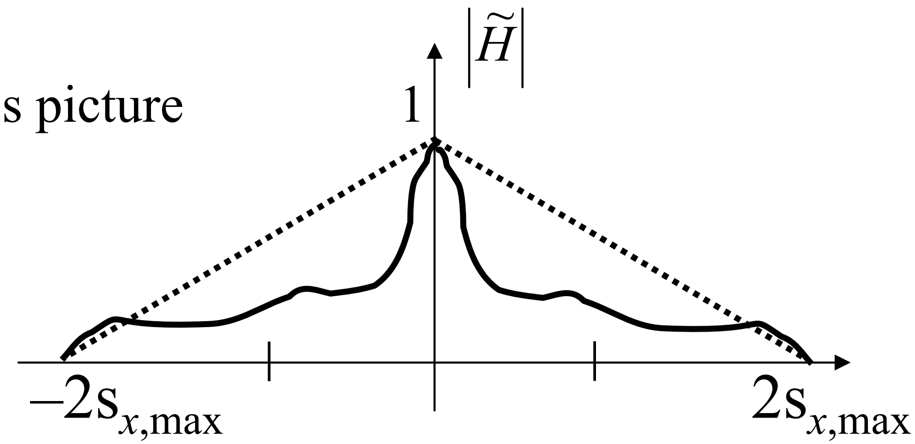
geometrical optics description

Also affecting resolution: aberrations

wave optics picture

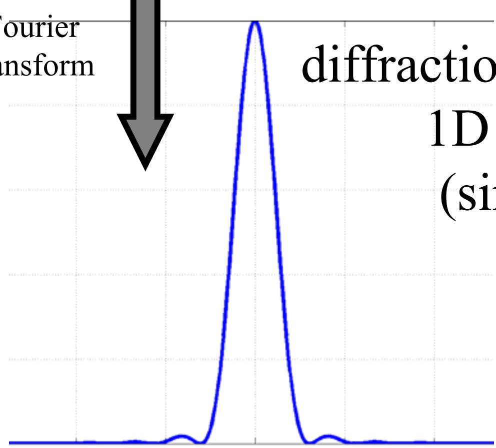


“**diffraction–limited**”
(aberration–free) 1D MTF



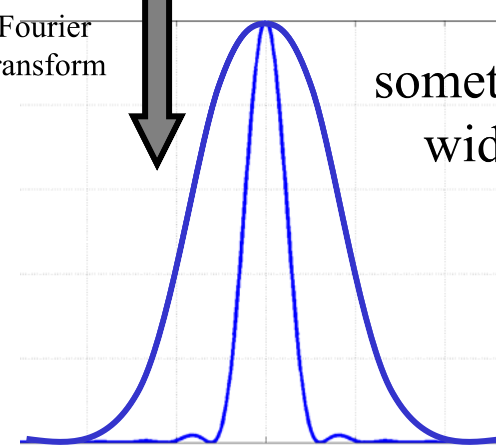
1D MTF with aberrations

$\mathfrak{I}$ Fourier transform



diffraction–limited
1D PSF
(sinc^2)

$\mathfrak{I}$ Fourier transform

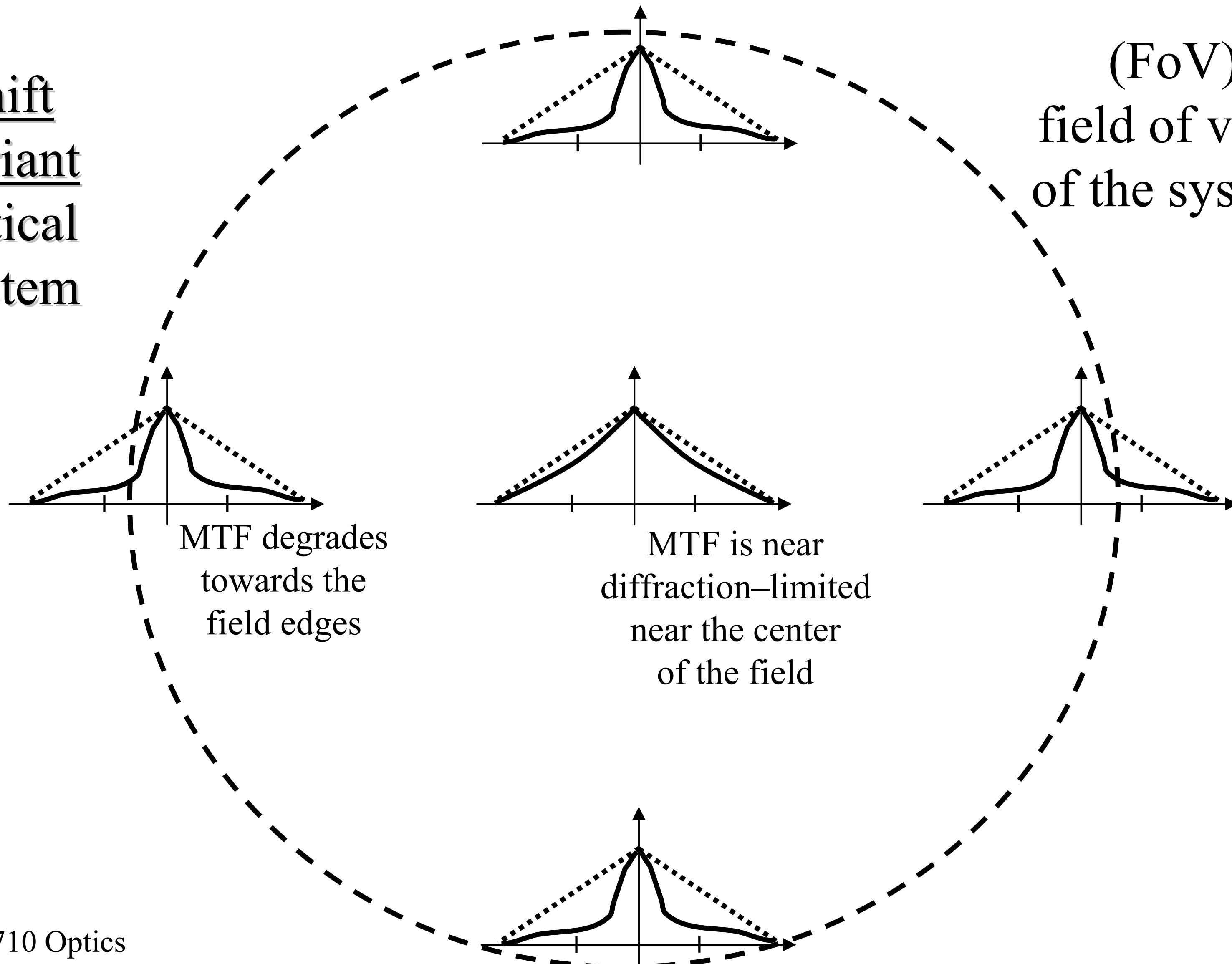


something
wider

Typical result of optical design

<u>shift</u>
<u>variant</u>
optical
system

(FoV)
field of view
of the system



The limits of our approximations

- Real–life MTFs include aberration effects, whereas our analysis has been “diffraction–limited”
- Aberration effects on the MTF are FoV (field) location–dependent: typically we get more blur near the edges of the field (narrower MTF $\Leftrightarrow$ broader PSF)
- This, in addition, means that real–life optical systems are not shift invariant either!
- $\Rightarrow$ the concept of MTF is approximate, near the region where the system is approximately shift invariant (recall: transfer functions can be defined only for shift invariant linear systems!)

The utility of our approximations

- Nevertheless, within the limits of the paraxial, linear shift–invariant system approximation, the concepts of PSF/MTF provide
  – a useful way of *thinking* about the behavior of optical systems
  – an upper limit on the performance of a given optical system (diffraction–limited performance is the best we can hope for, in paraxial regions of the field; aberrations will only make worse non–paraxial portions of the field)

Common misinterpretations

Attempting to resolve object features smaller than the "resolution limit" (e.g. $1.22\lambda/\mathrm{NA}$) is hopeless.

NO: Image quality degradation as object features become smaller than the resolution limit ("exceed the resolution limit") is noise dependent and gradual.

Common misinterpretations

Attempting to resolve object features smaller than the "resolution limit" (e.g. $1.22\lambda/\mathrm{NA}$) is hopeless.

NO: Besides, digital processing of the acquired images (e.g. methods such as the CLEAN algorithm, Wiener filtering, expectation maximization, etc.) can be employed.

Common misinterpretations

Super-resolution

By engineering the pupil function ("apodizing") to result in a PSF with narrower side–lobe, one can "beat" the resolution limitations imposed by the angular acceptance (NA) of the system.

NO:

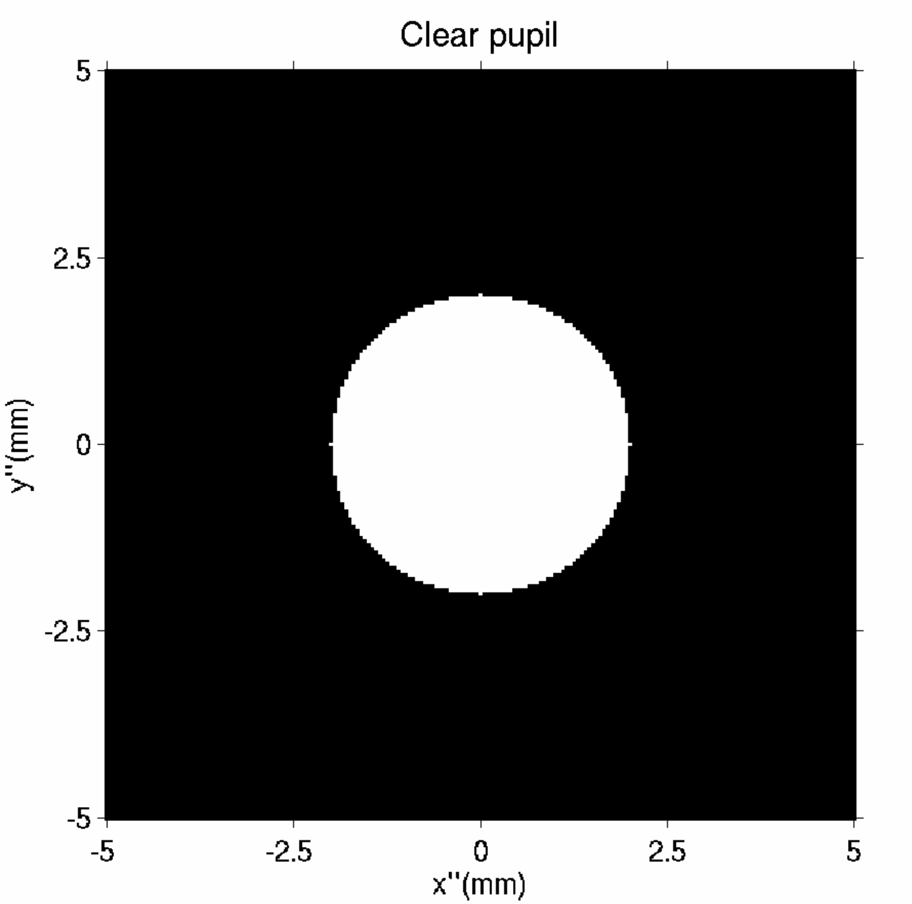
Pupil function design always results in

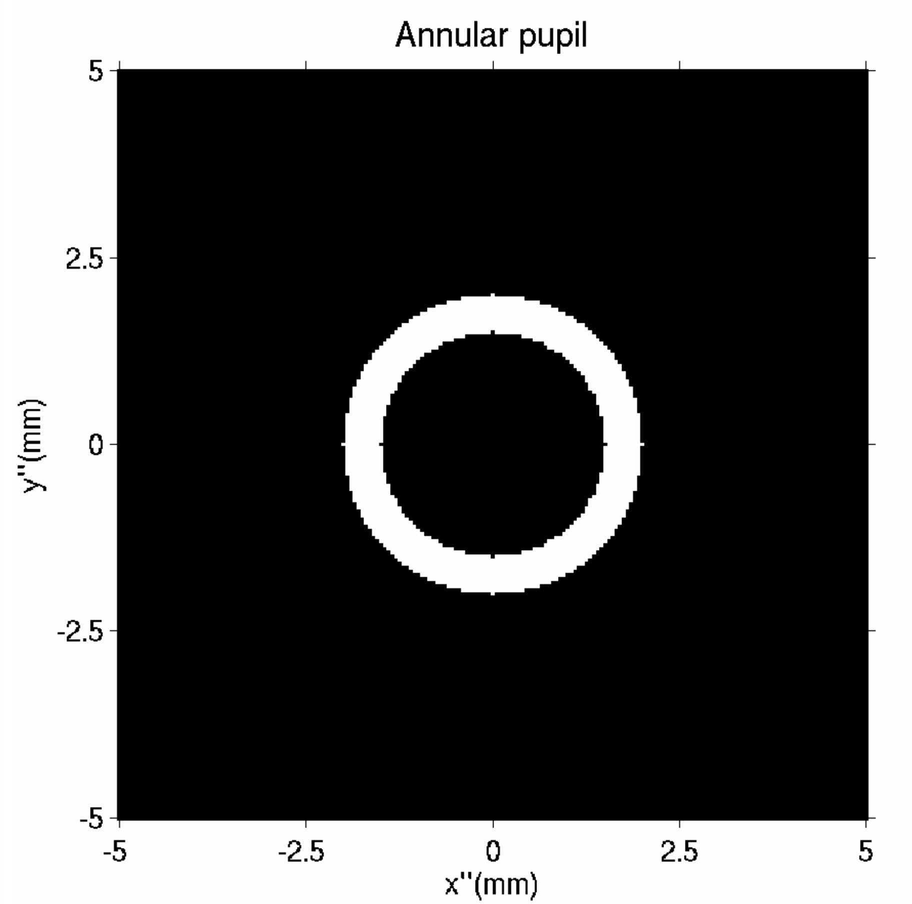
(i) narrower main lobe but accentuated side–lobes

(ii) lower power transmitted through the system

Both effects are **BAD** on the image

Apodization

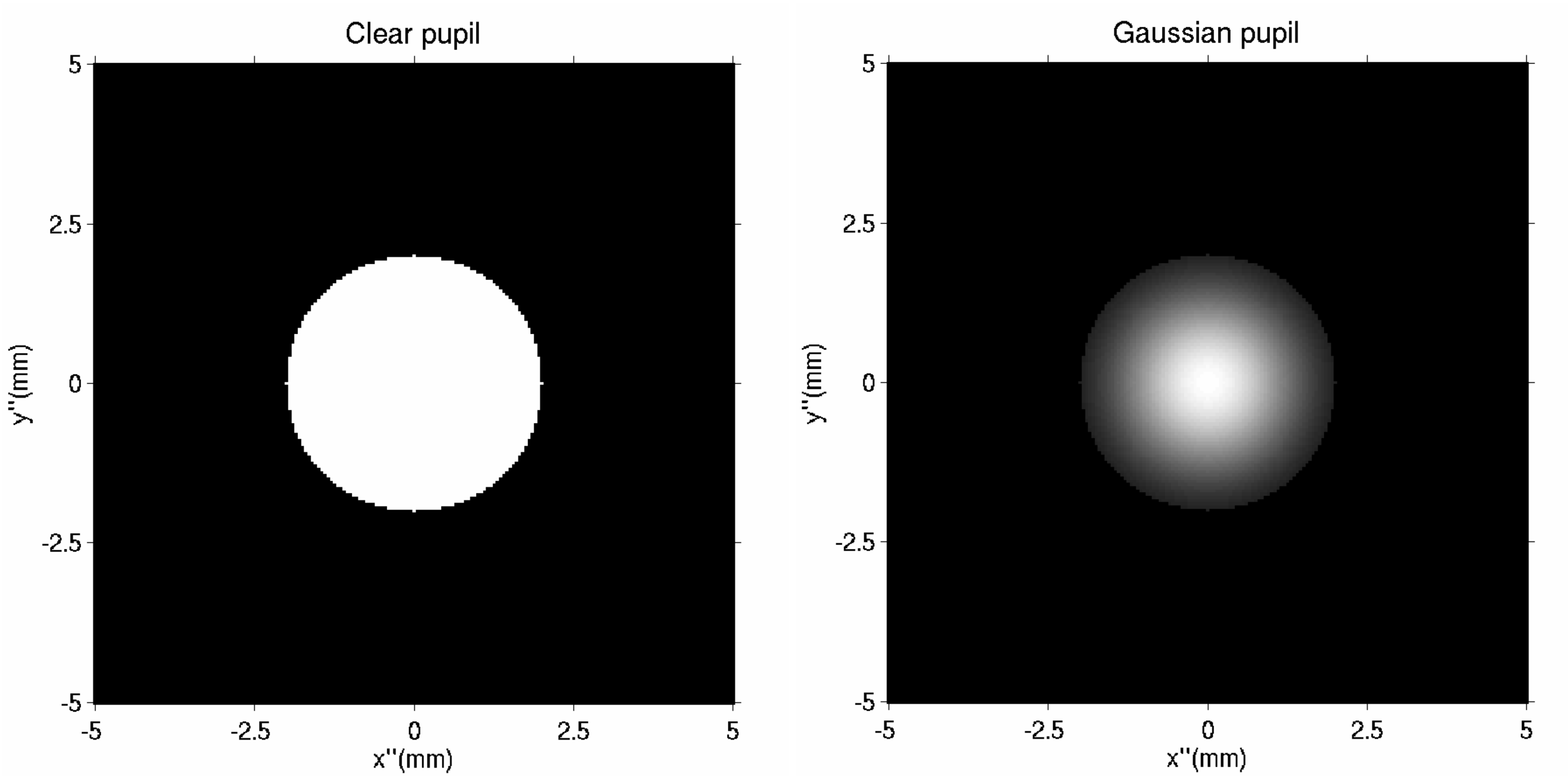




f_1=20cm
λ=0.5μm

$$H(r'') = \operatorname{circ}\left(\frac{r''}{R}\right) - \operatorname{circ}\left(\frac{r''}{R_2}\right)$$

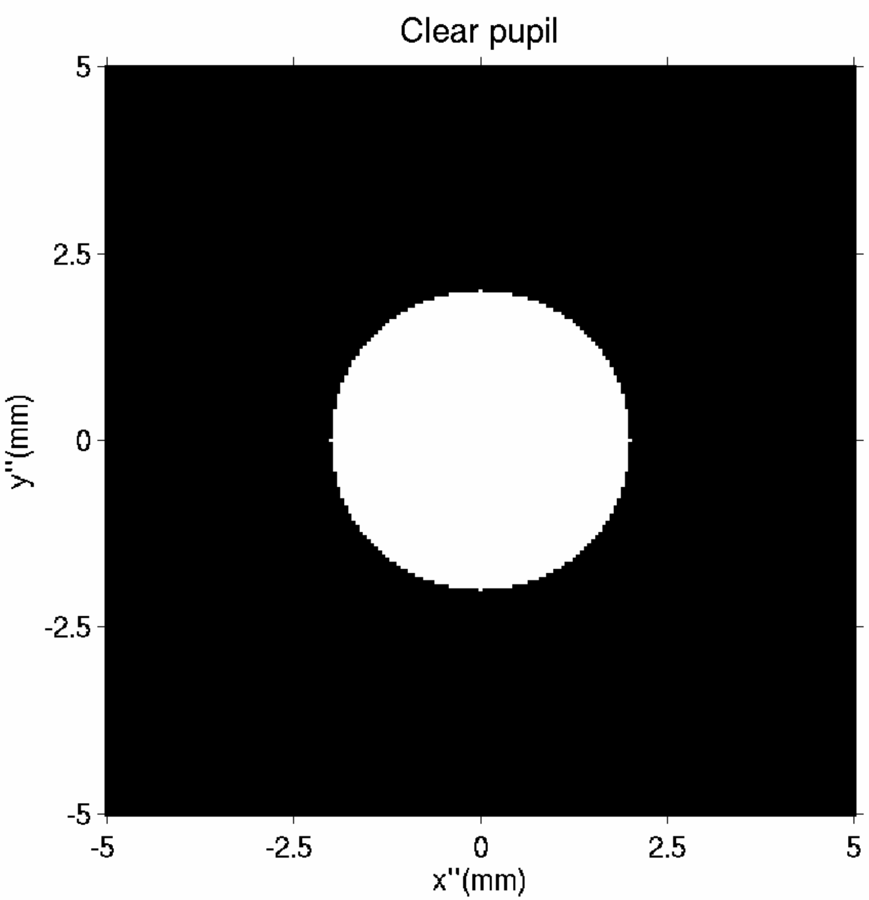
Apodization

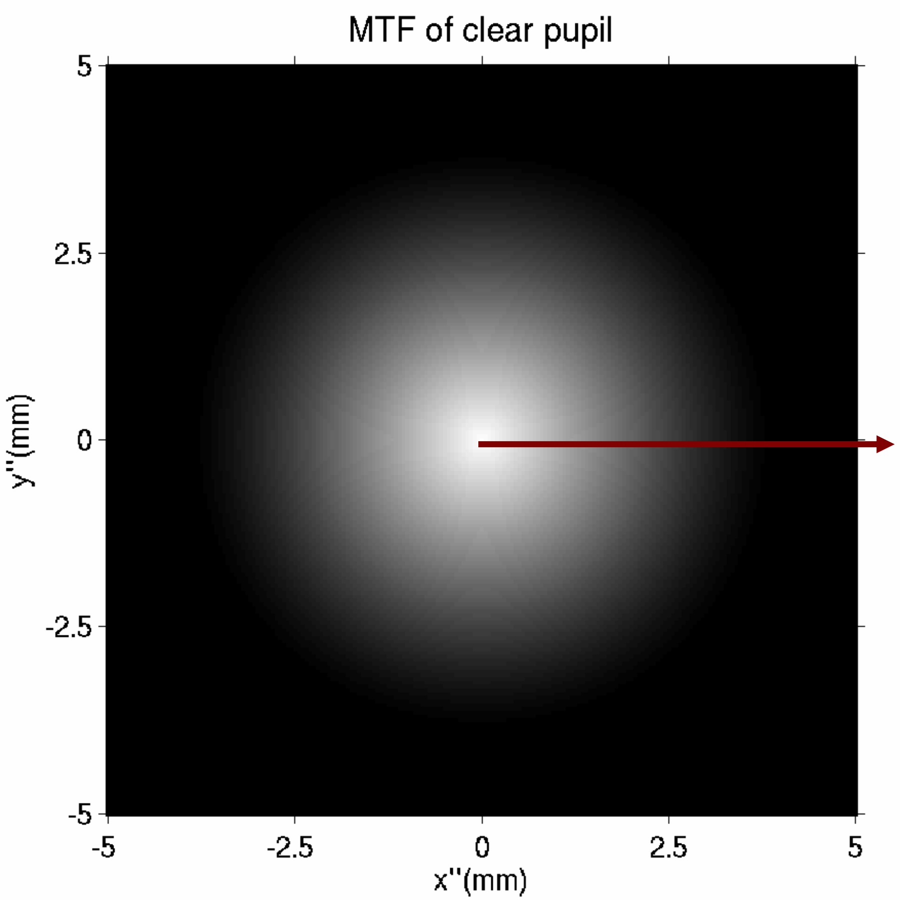


f_1=20cm
λ=0.5μm

$$H(r'') = \mathrm{circ}\left(\frac{r''}{R}\right) \times \exp\left(-\frac{r''^2}{2R_0^2}\right)$$

Unapodized (clear–aperture) MTF



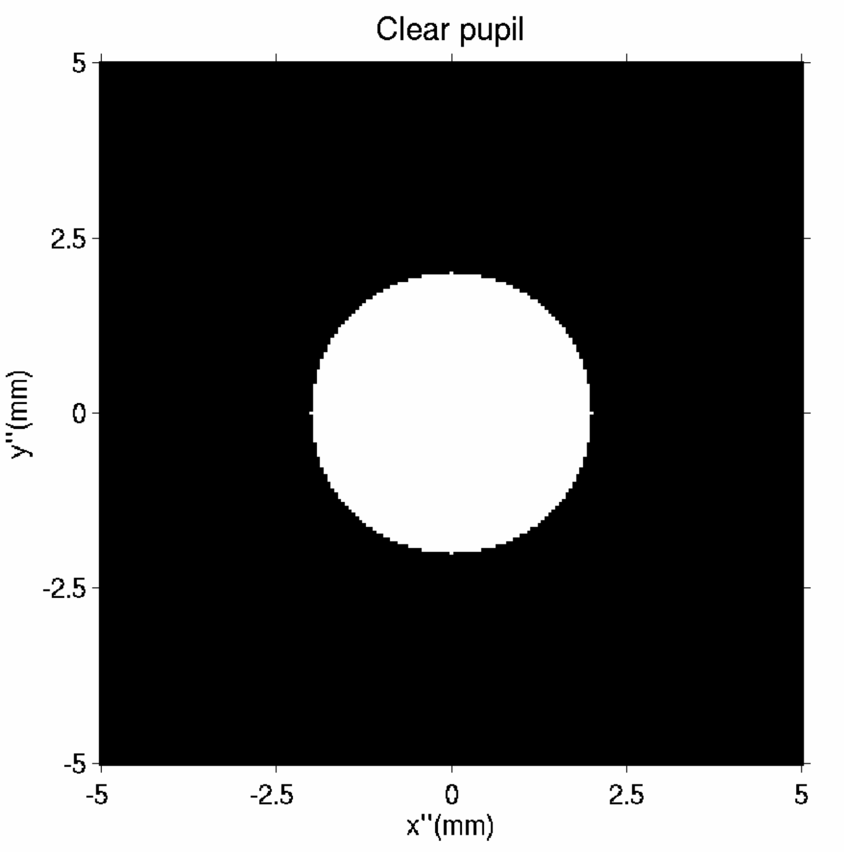


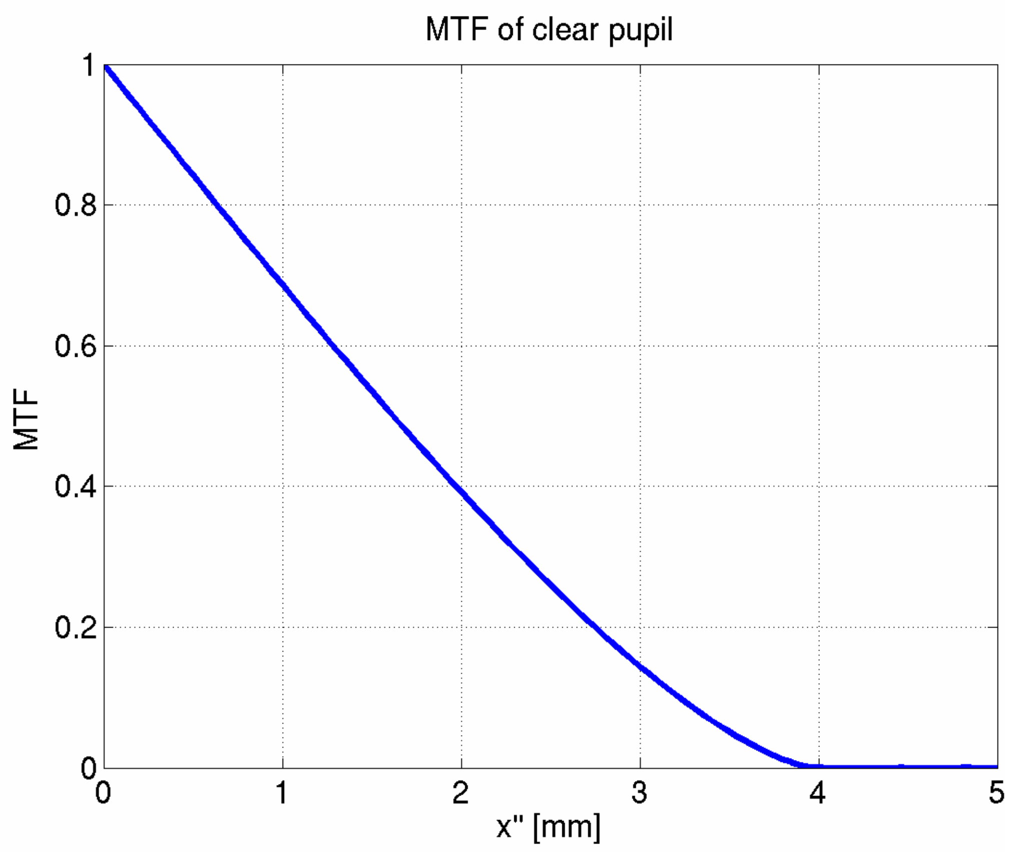
f_1=20cm
λ=0.5μm

$$\tilde{H}(r'') = \mathrm{circ}\left(\frac{r''}{R}\right) \otimes \mathrm{circ}\left(\frac{r''}{R}\right)$$

auto-correlation

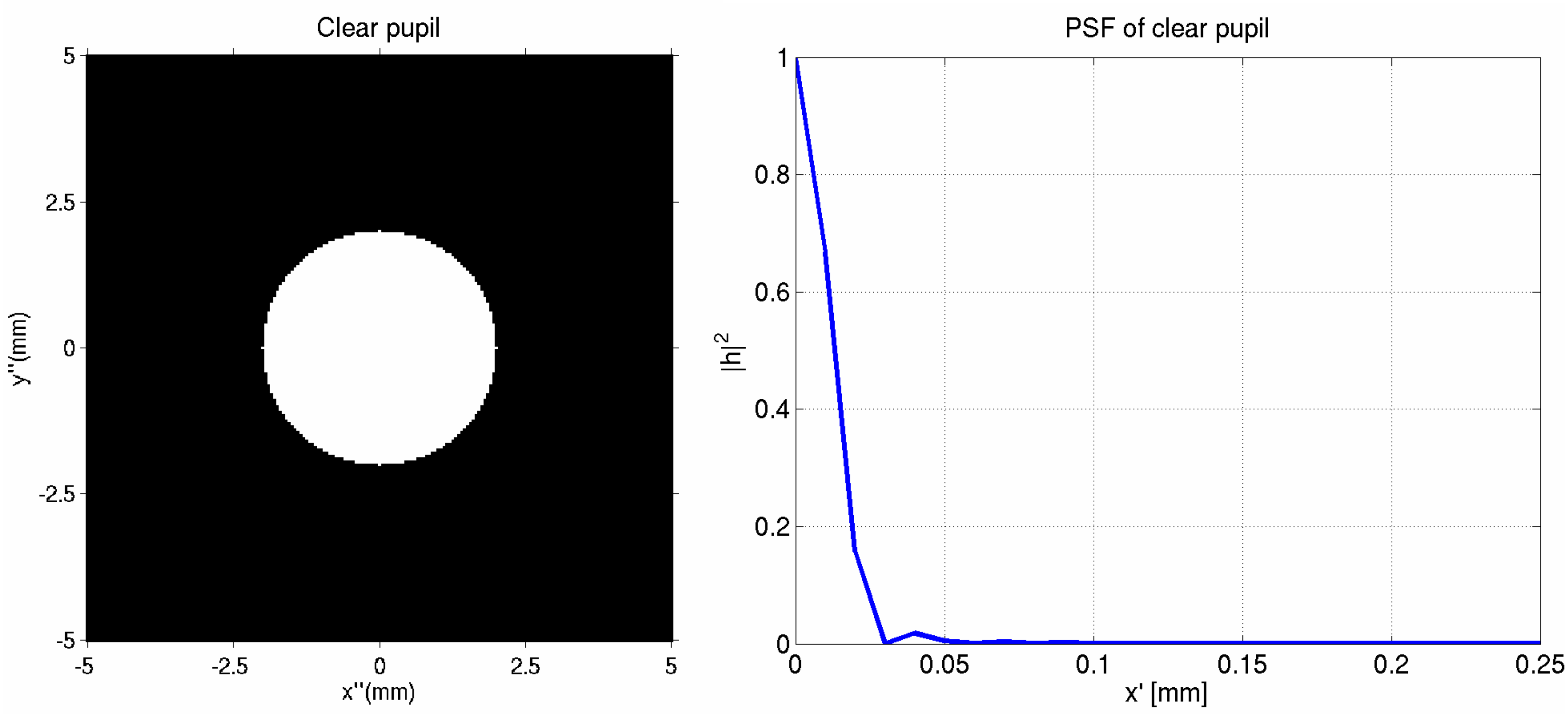
Unapodized (clear–aperture) MTF





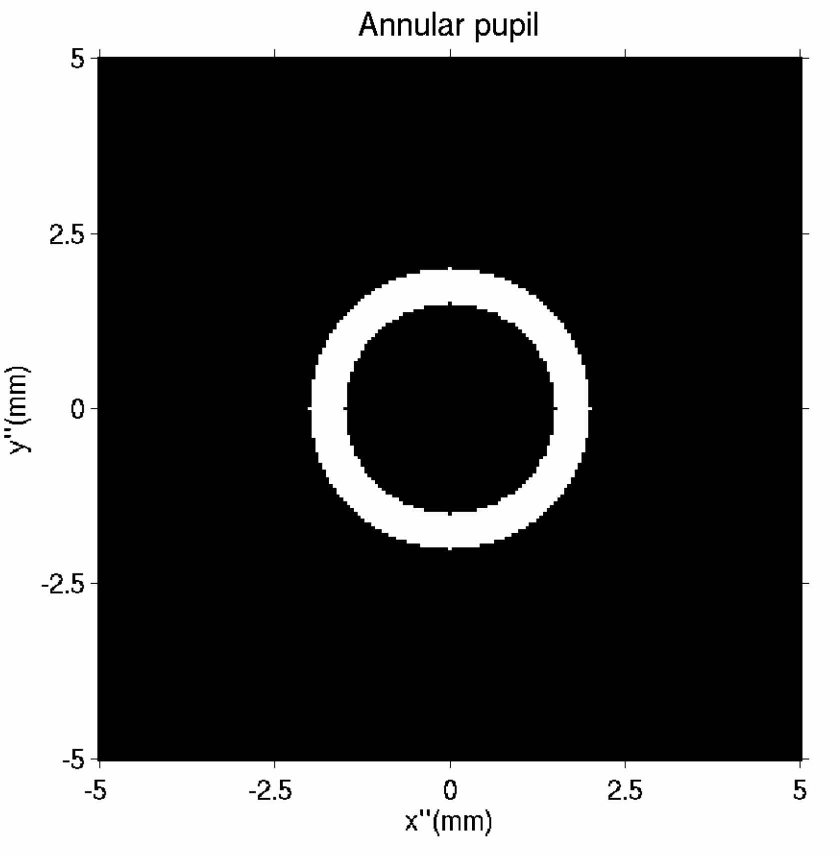
f_1=20cm
λ=0.5µm

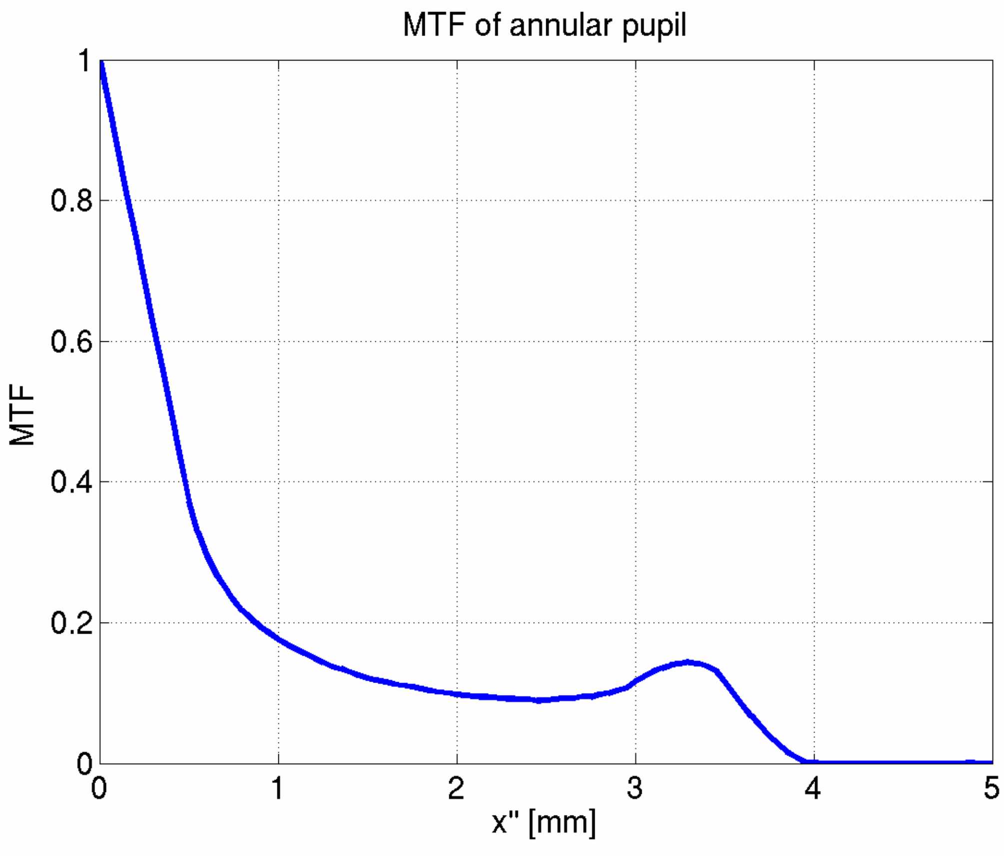
Unapodized (clear–aperture) PSF



f_1=20cm
λ=0.5µm

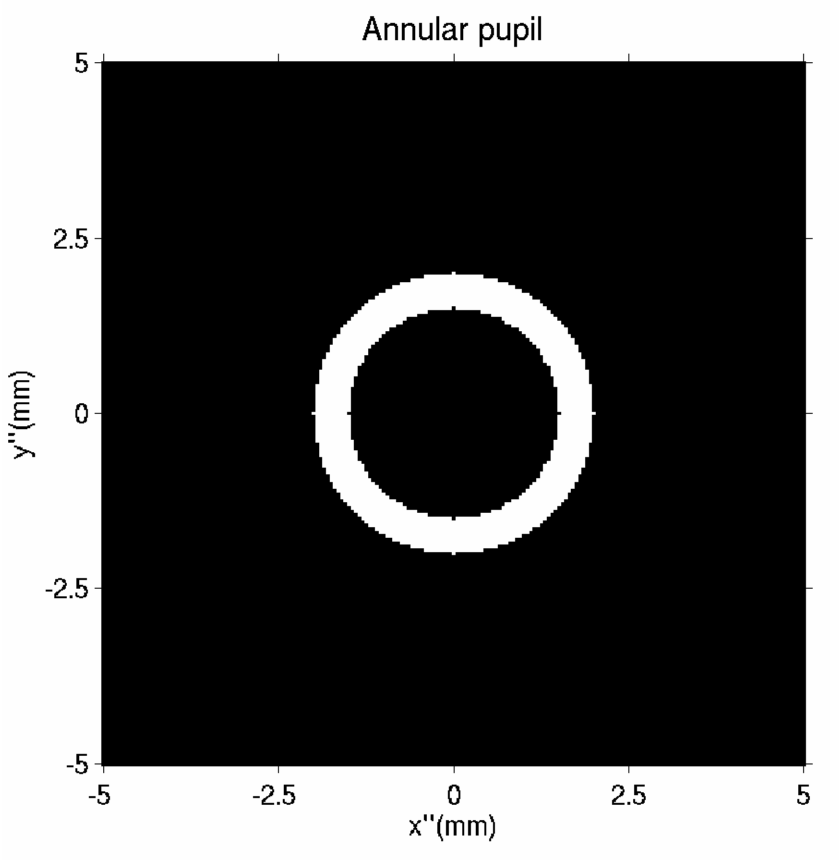
Apodized (annular) MTF

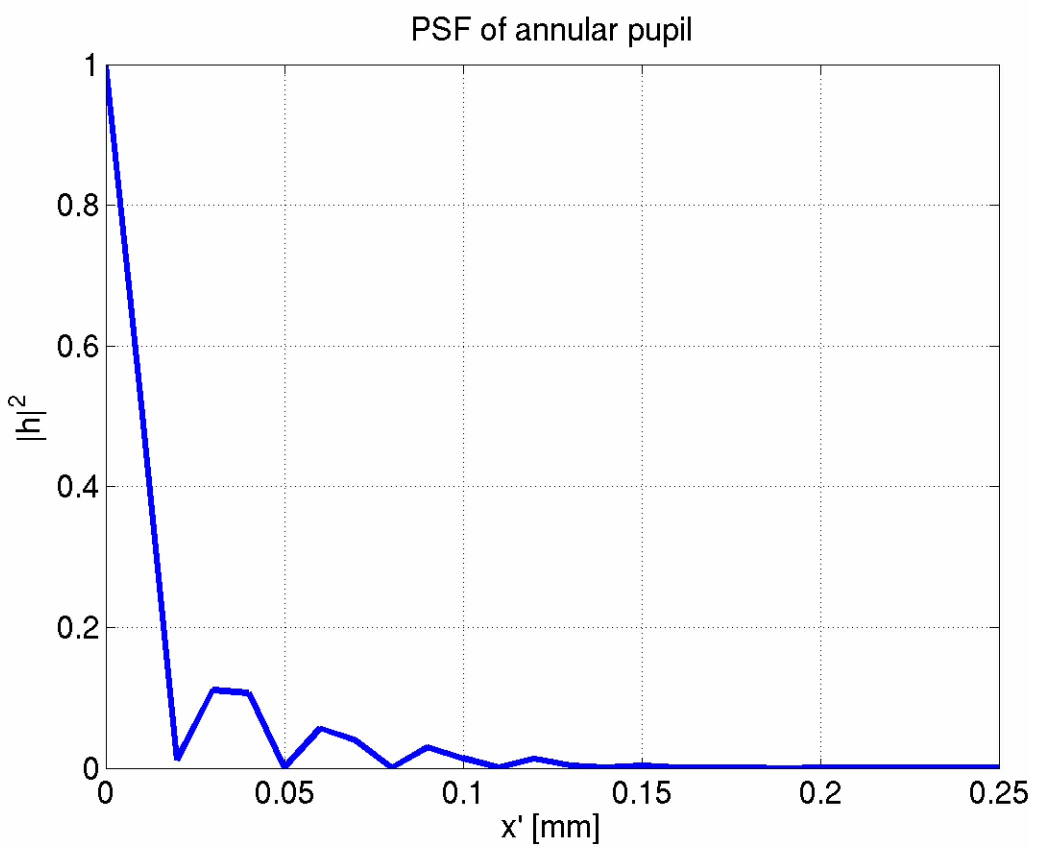




f_1=20cm
λ=0.5μm

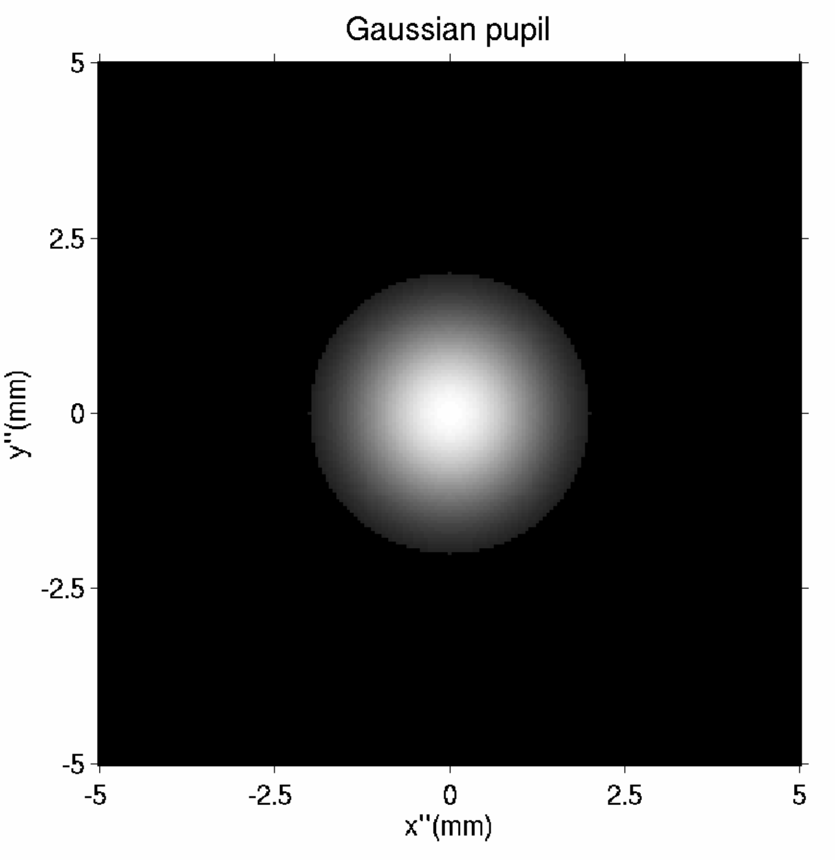
Apodized (annular) PSF

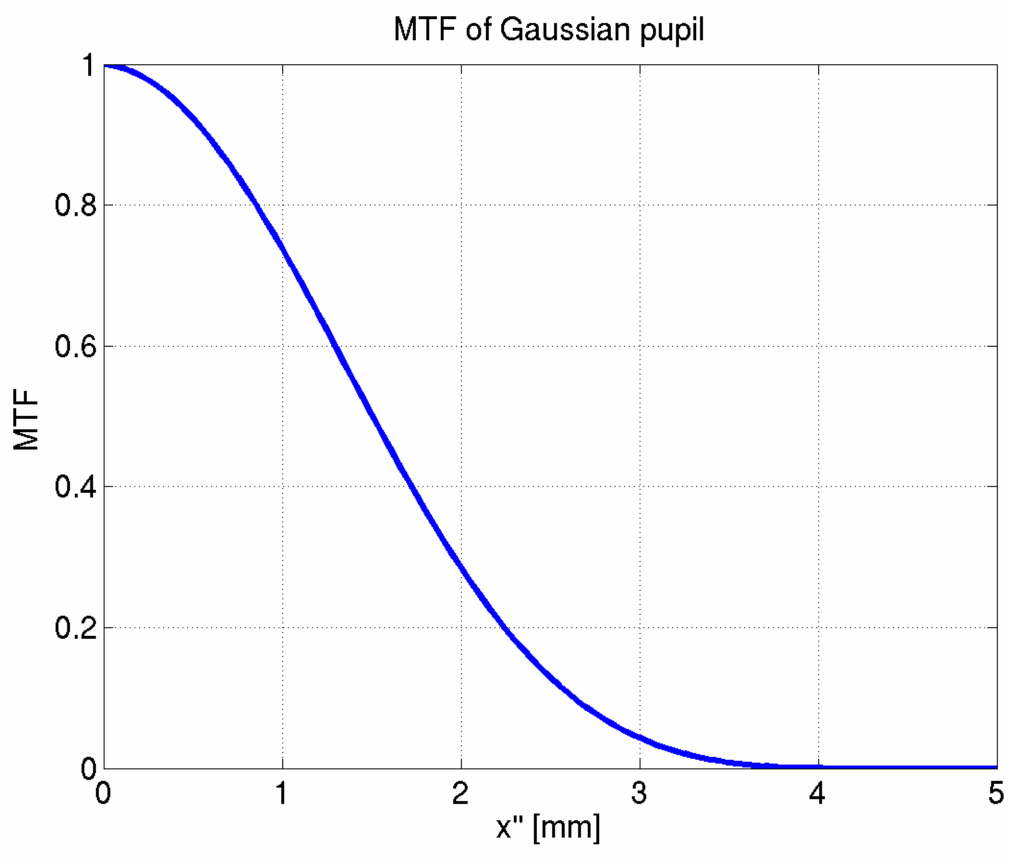




f_1=20cm
λ=0.5μm

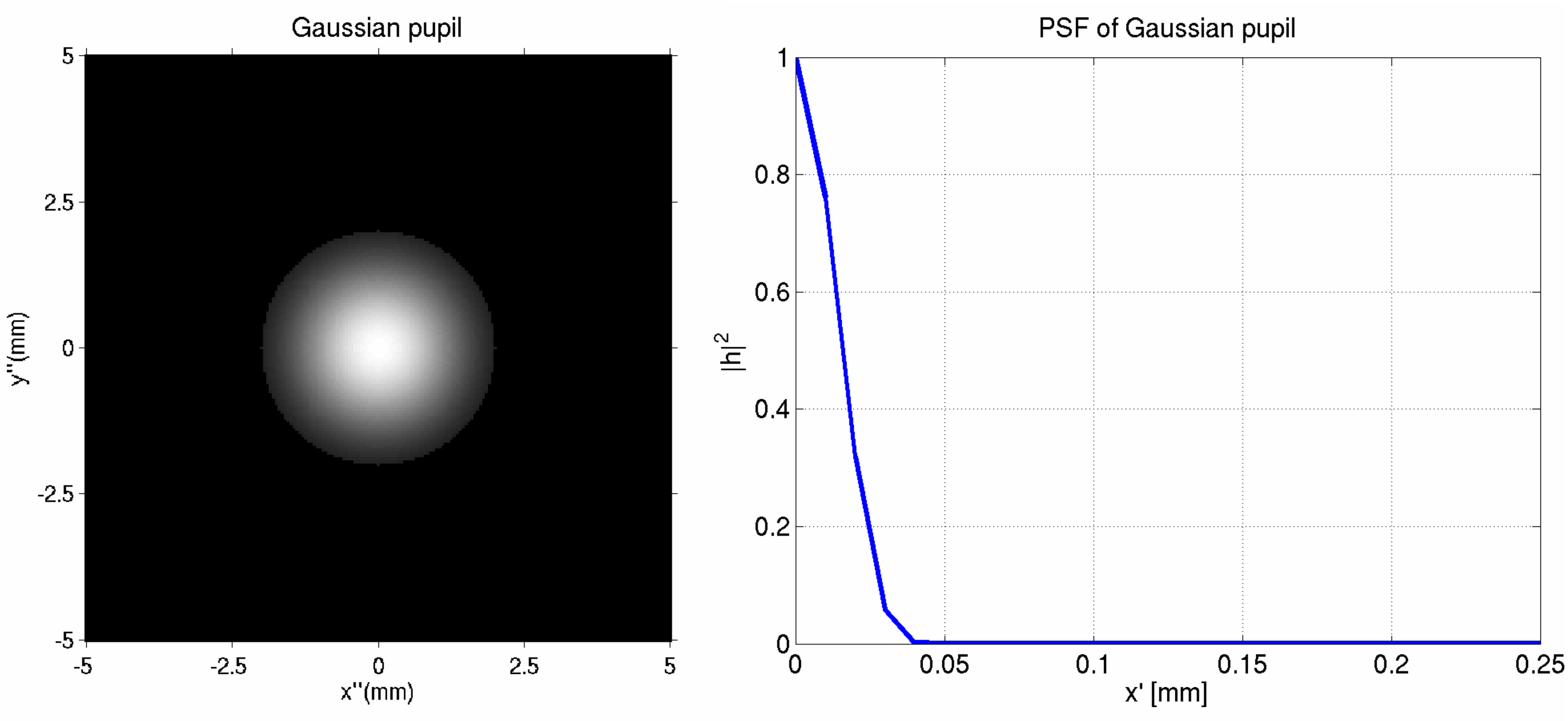
Apodized (Gaussian) MTF





f_1=20cm
λ=0.5μm

Apodized (Gaussian) PSF



f_1=20cm
λ=0.5µm

Conclusions (?)

- Annular–type pupil functions typically narrow the main lobe of the PSF at the expense of higher side lobes
- Gaussian–type pupil functions typically suppress the side lobes but broaden the main lobe of the PSF
- Compromise? $\rightarrow$ application dependent
  - for point–like objects (e.g., stars) annular apodizers may be a good idea
  - for low–frequency objects (e.g., diffuse tissue) Gaussian apodizers may image with fewer artifacts
- Caveat: Gaussian amplitude apodizers very difficult to fabricate and introduce energy loss $\Rightarrow$ binary phase apodizers (lossless by nature) are used instead; typically designed by numerical optimization

Common misinterpretations

Super-resolution

By engineering the pupil function ("apodizing") to result in a PSF with narrower side–lobe, one can "beat" the resolution limitations imposed by the angular acceptance (NA) of the system.

NO:

main lobe size $\downarrow \Leftrightarrow$ sidelobes $\uparrow$
and *vice versa*
main lobe size $\uparrow \Leftrightarrow$ sidelobes $\downarrow$

power loss an important factor

compromise application dependent

Common misinterpretations

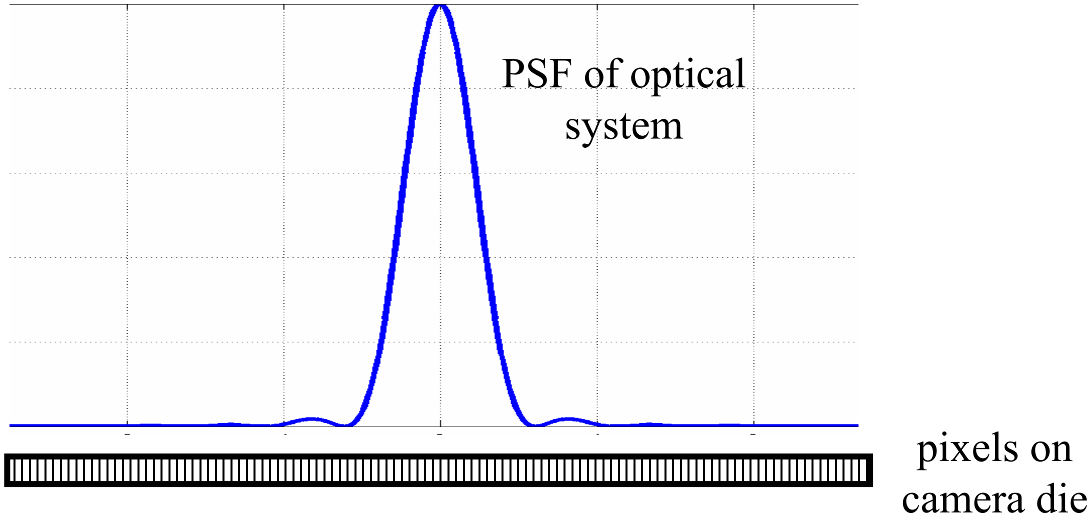
"This super cool digital camera has resolution of 5 Mega pixels (5 million pixels)."

NO:

This is the most common and worst misuse of the term "resolution." They are actually referring to the **space–bandwidth product (SBP)** of the camera

What *can* a camera resolve?

Answer depends on the magnification and
PSF of the optical system attached to the camera



Pixels significantly smaller than the system PSF
are somewhat underutilized (the effective SBP is reduced)

Summary of misinterpretations of "resolution" and their refutations

- It is pointless to attempt to resolve beyond the Rayleigh criterion (however defined)
  - NO: difficulty increases gradually as feature size shrinks, and difficulty is noise dependent
- Apodization can be used to beat the resolution limit imposed by the numerical aperture
  - NO: watch sidelobe growth and power efficiency loss
- The resolution of my camera is $N \times M$ pixels
  - NO: the maximum possible SBP of your system may be $N \times M$ pixels but you can easily underutilize it by using a suboptimal optical system

So, what *is* resolution?

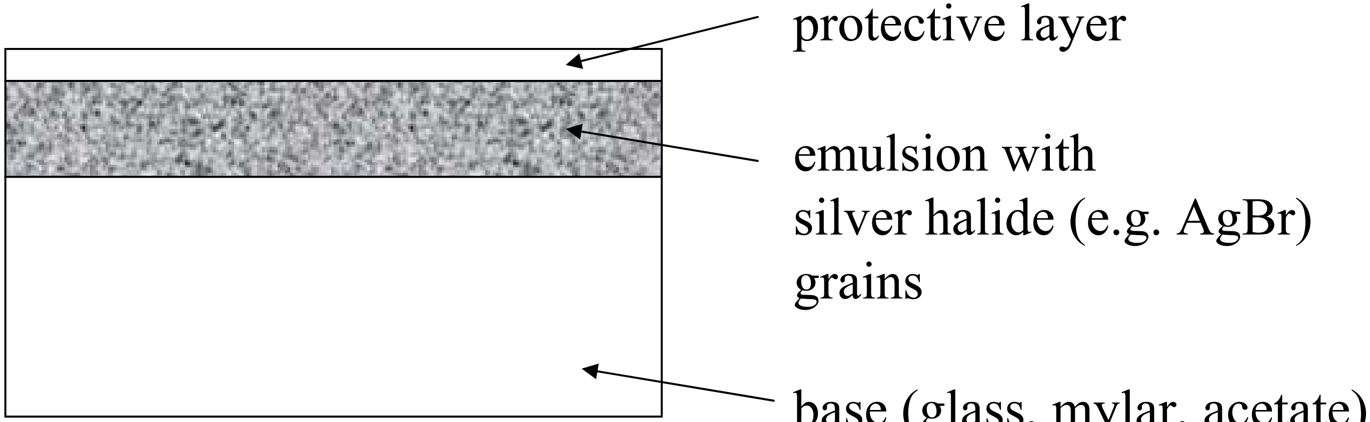
- Our ability to resolve two point objects (in general, two distinct features in a more general object) based on the image
- It is *related* to the NA but *not exclusively* limited by it
- Resolution, as it relates to NA:
  - **Resolution improves as NA increases**
- Other factors affecting resolution:
  - **aberrations / apodization** (i.e., the exact shape of the PSF)
  - **NOISE!**
- Is there an easy answer?
  - No ……

  but when in doubt quote $0.61\lambda/(\mathrm{NA})$ as an *estimate* (not as an exact limit).

Wavefront modulation

- Photographic film
- Spatial light modulators
- Binary optics

Photographic films / plates



Exposure: $Ag^+ + e^- \rightarrow Ag$ (or $2Ag+ + 2e^- \rightarrow Ag_2$)

development speck

Collection of development specks = ***latent image***

Development (1st chemical bath): converts specks to metallic silver

Fixing the emulsion (2nd chemical bath): removal of unexposed silver halide

Photographic film / plates

- Exposure (energy) : energy incident per unit area on a photographic emulsion during the exposure process (units: mJ/cm^2)

  Exposure = incident intensity × exposure time $E = I_{\text{expose}} \times T$

- Intensity transmittance : average ratio of intensity transmitted over intensity incident after development

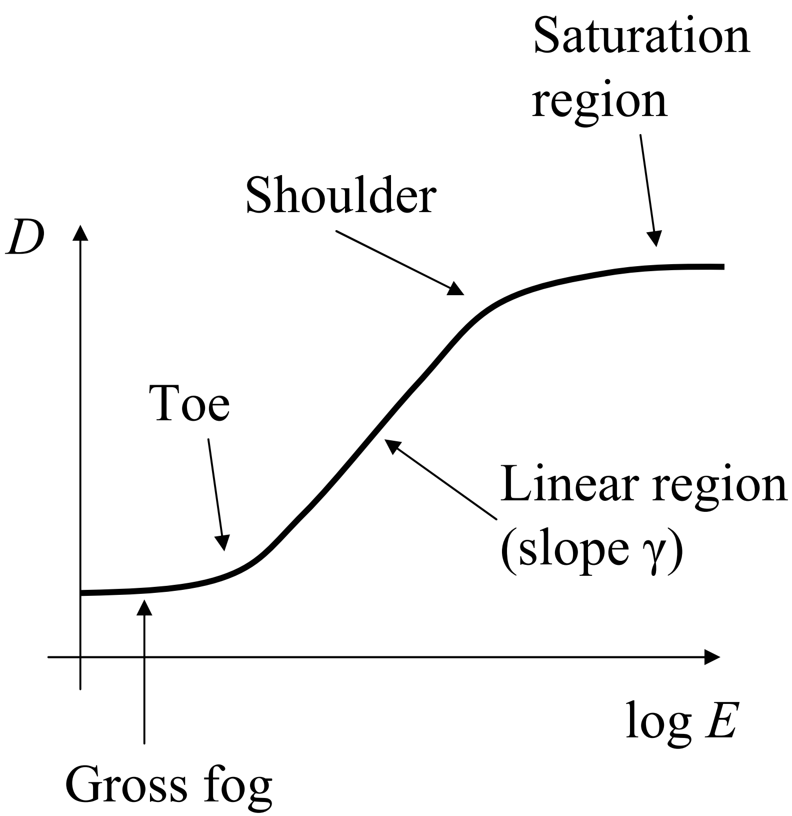
$$\tau(x, y) = \underset{\text{average}}{\text{local}} \left\{ \frac{I \text{ transmitted at } x, y}{I \text{ incident at } x, y} \right\}$$

- Photographic density

$$D = \log_{10}\left(\frac{1}{\tau}\right) \quad \Leftrightarrow \quad \tau = 10^{-D}$$

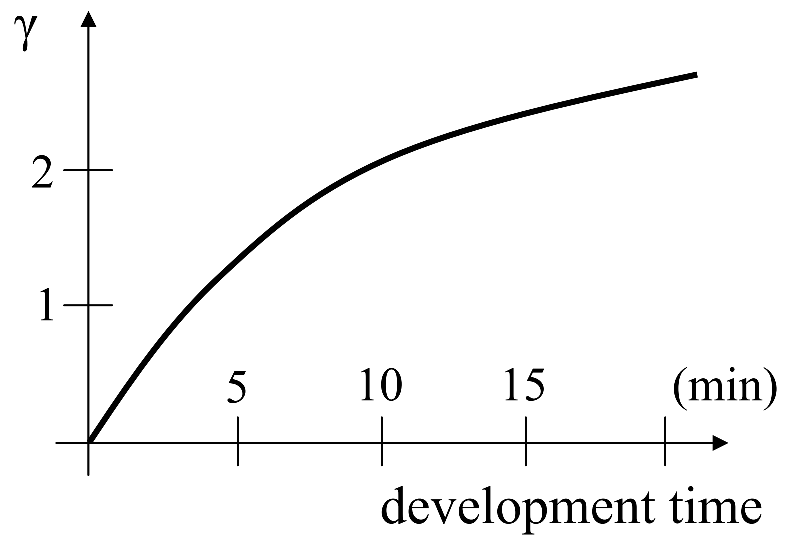
Photographic film / plates

- Hurter-Driffield curve



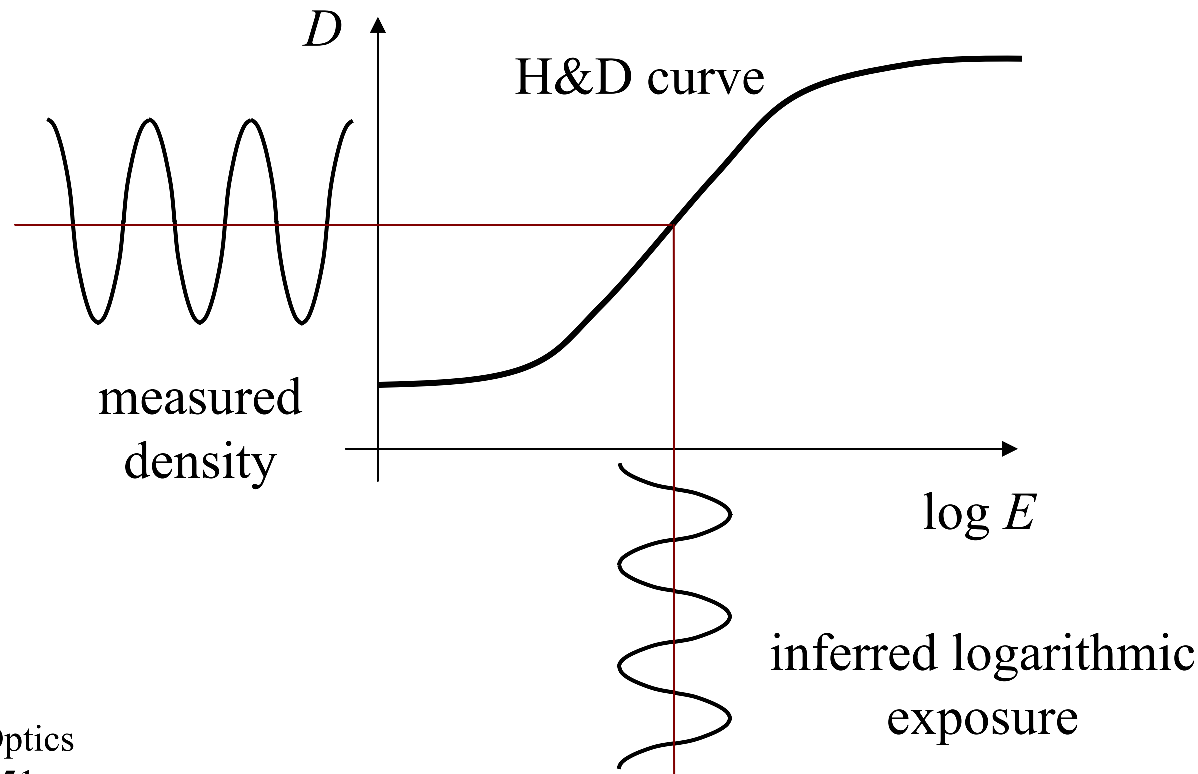
- Gamma curve

γ high/low : high/low contrast film

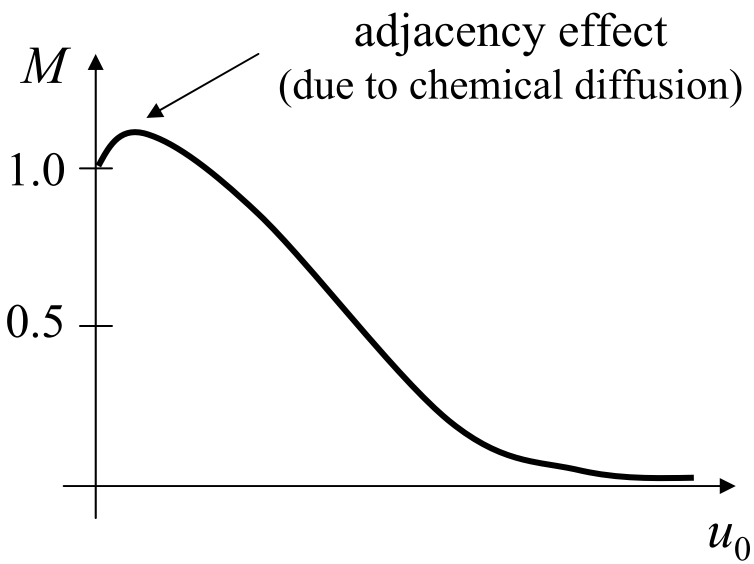


Kelley model of photographic process

Optical imaging during exposure (linear)	→	H&D curve (nonlinear)	→	additional blur due to chemical diffusion (linear)



The Modulation Transfer Function



Exposure:

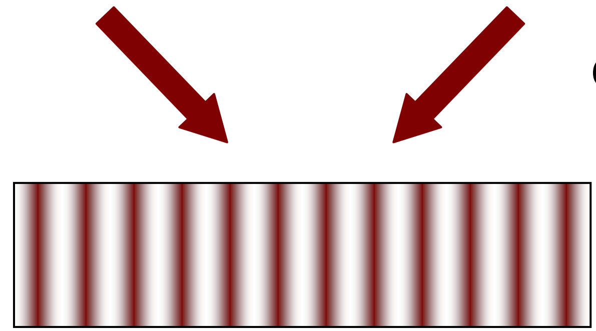
$$E = E_0 + E_1 \cos 2\pi u_0 x$$

"Effective exposure":

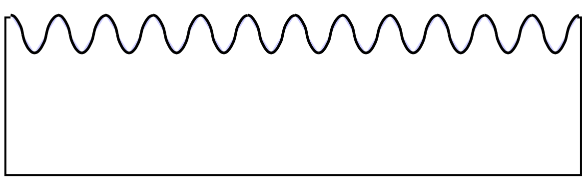
$$E' = E_0 + M(u_0) E_1 \cos 2\pi u_0 x$$

Bleaching / phase modulation

emulsion

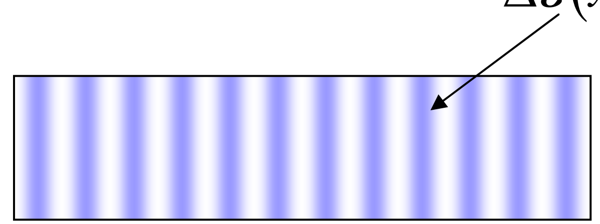


exposure



tanning bleach
(relief grating)

$\Delta\varepsilon(x, y, z)$



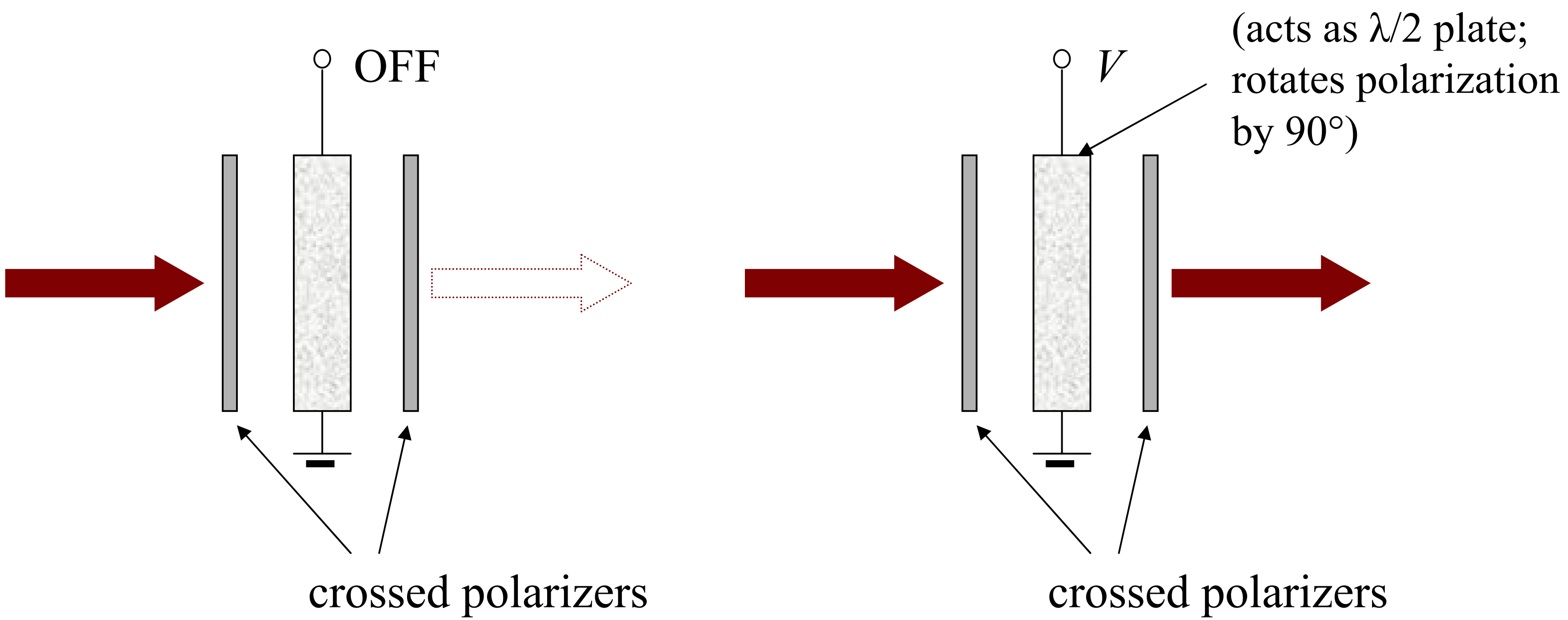
non-tanning bleach
(index grating)

Spatial Light Modulators

- Liquid crystals
- Magneto-Optic
- Micro-mirror
- Grating Light Valve
- Multiple Quantum Well
- Acousto-Optic

Liquid crystal modulators

- Nematic
- Smectic (smectic-C* phase: ferroelectric)
- Cholesteric



Micro-mirror technology

Images removed due to copyright concerns

Lucent (Bell Labs)

Texas Instruments DMD/DLP

Sandia

Micro-mirror display

Image removed due to copyright concerns

http://www.howstuffworks.com

Micromirrors for adaptive optics

Image removed due to copyright concerns

Véran, J.-P. & Durand, D. 2000, ASP Conf. Ser 216, 345 (2000).

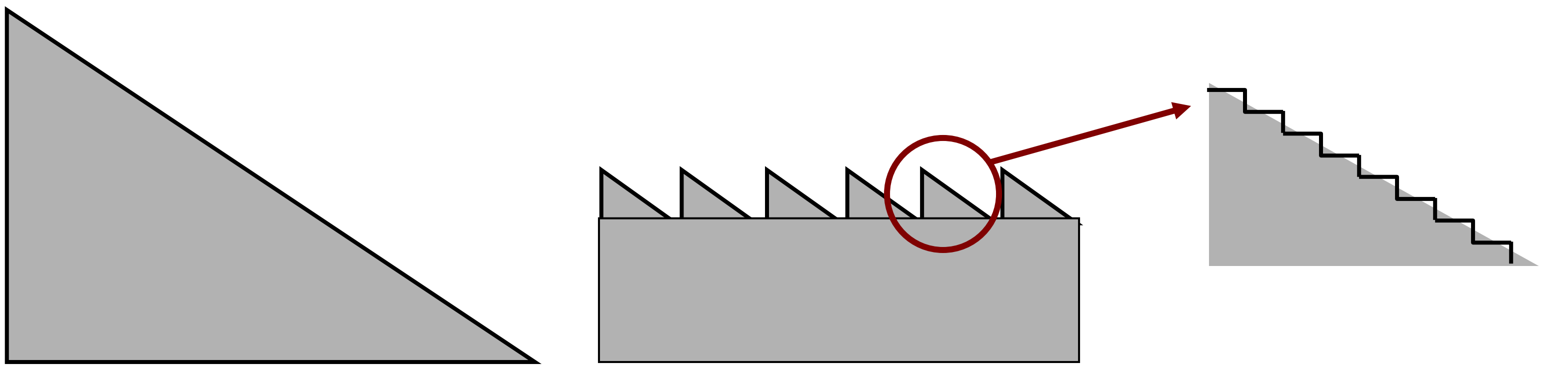
Grating Light Valve (GLV) display

Images removed due to copyright concerns

www.meko.co.uk

Silicon Light Machines, www.siliconlight.com

Binary Optics



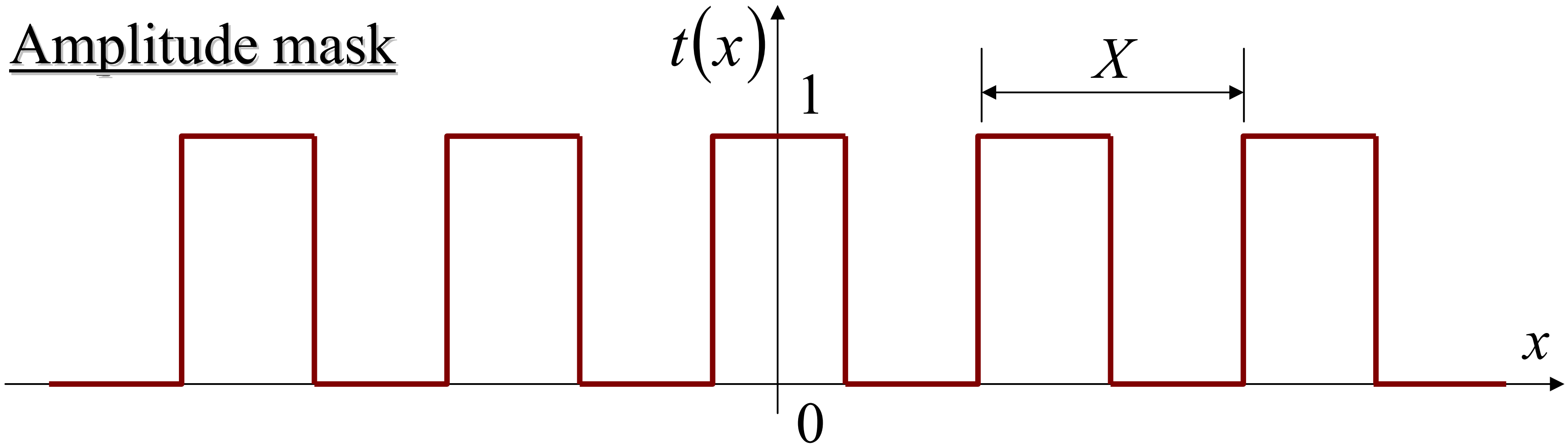
Refractive	Diffractive	Binary
(prism)	(blazed grating)	

$$\eta_1 = \mathrm{sinc}^2\left(\frac{1}{2^N}\right)$$

efficiency of 1st diffracted order from step-wise (binary) approximation to blazed grating with N steps over 2π range

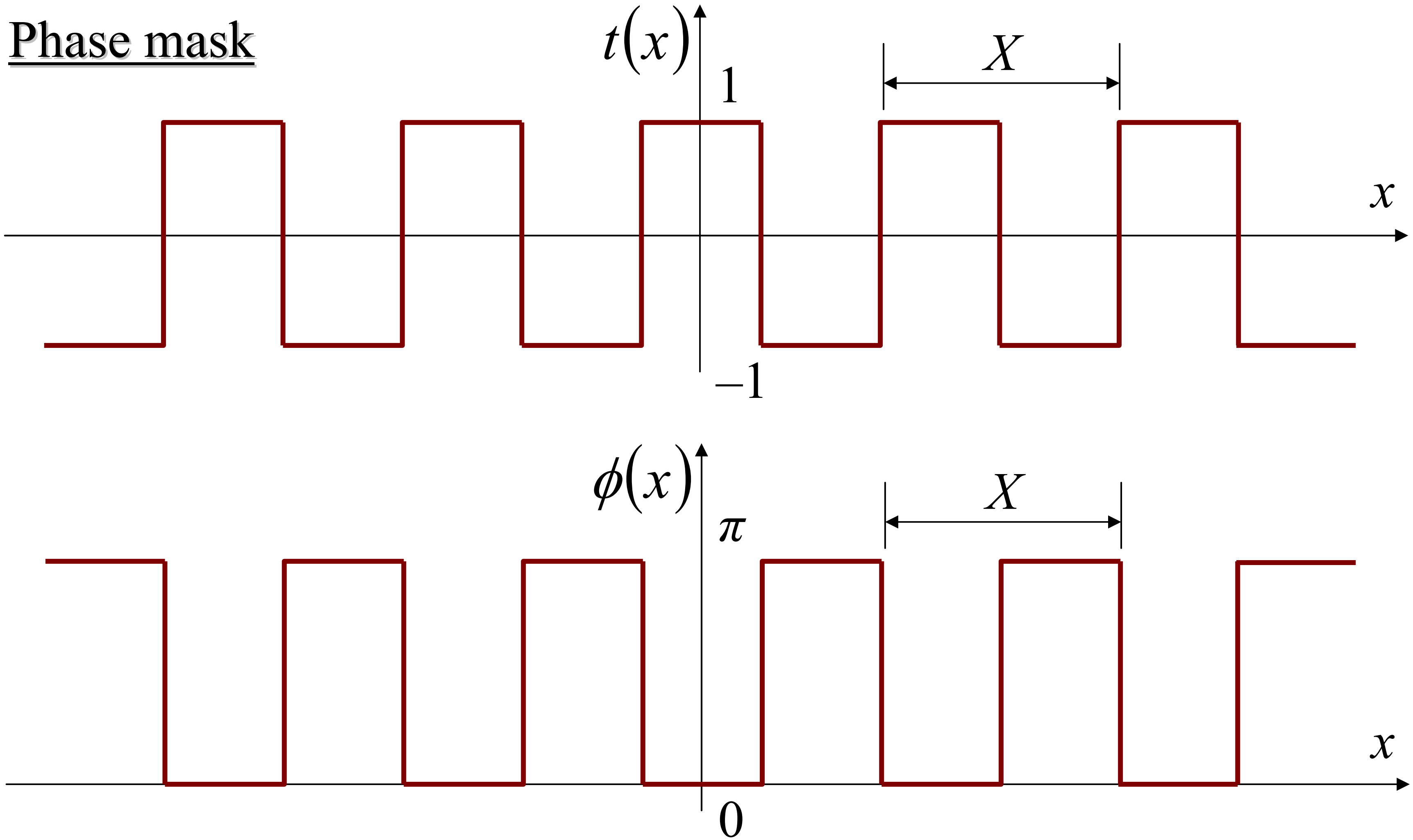
Binary Optics: binary grating

Amplitude mask



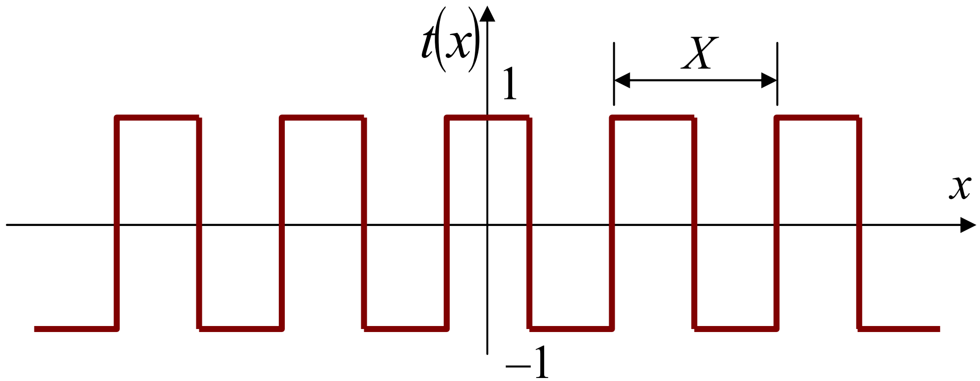
Binary Optics: binary grating

Phase mask



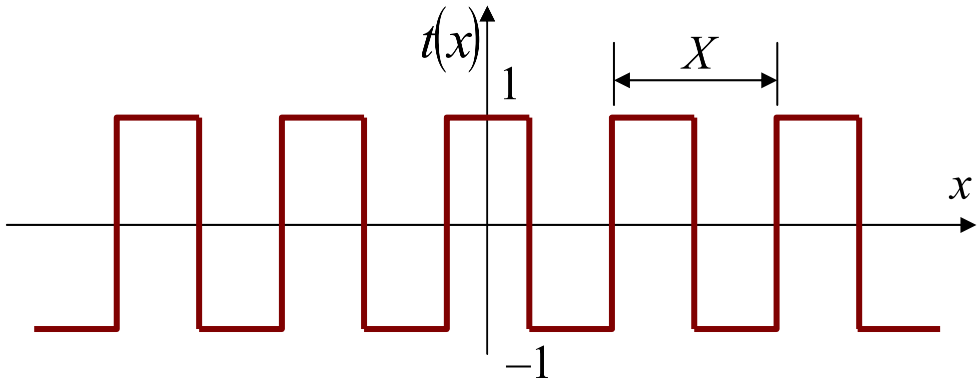
$$t(x) = \exp[i\,\phi(x)]$$

Fourier series for binary phase grating



$$= \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \exp\left(i2\pi \frac{nx}{X}\right)$$

Fourier series for binary phase grating



$$= \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \exp\left(i2\pi \frac{nx}{X}\right)$$

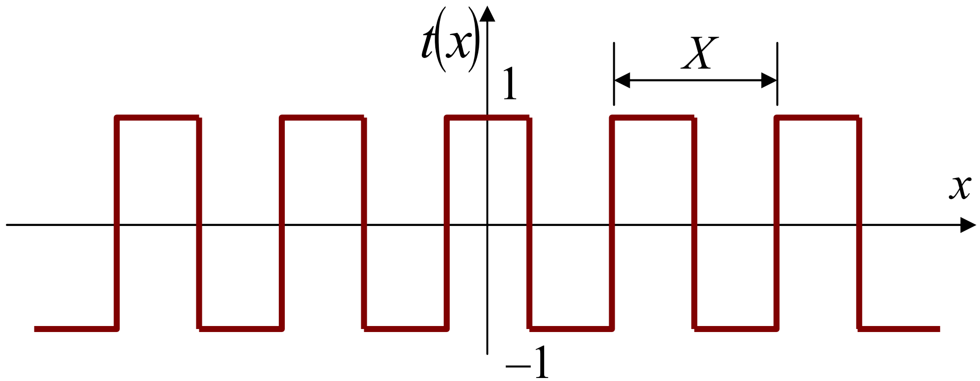
identify physical meaning:

- plane waves
- orientation of n^{th} plane wave:

  $\sin\theta_n = n\lambda / X$

- ***diffracted orders***

Fourier series for binary phase grating



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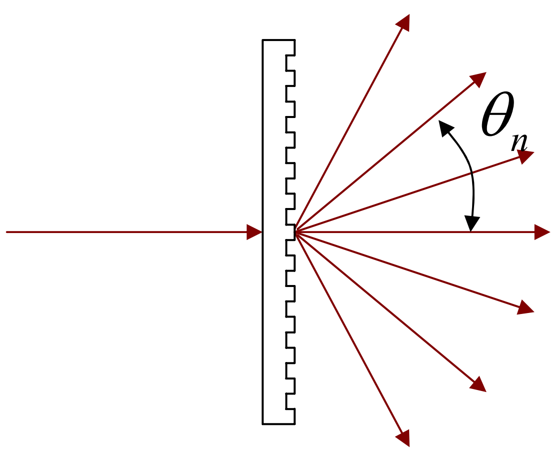

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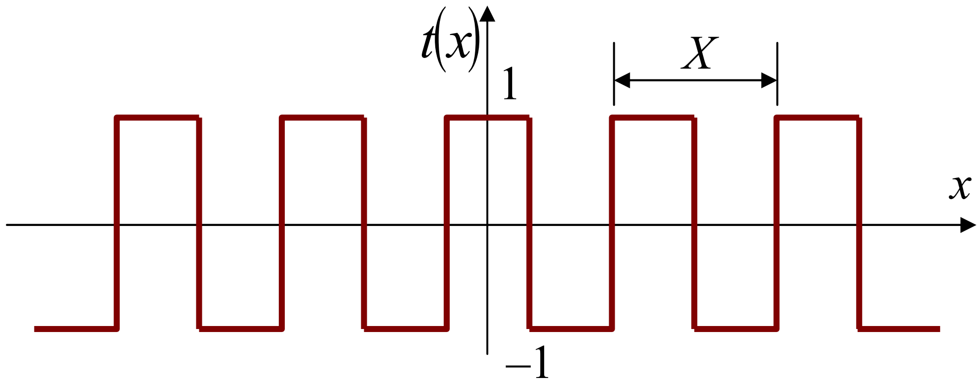
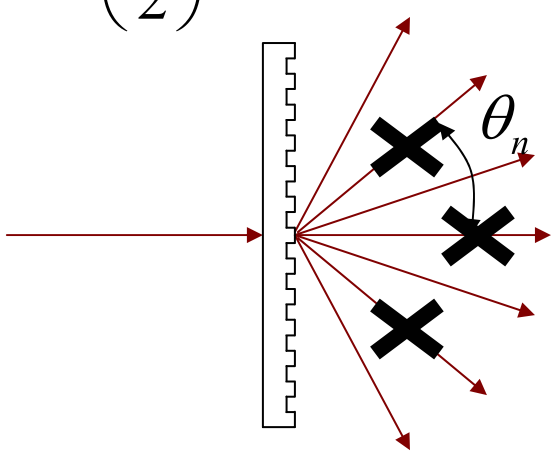
Fourier series for binary phase grating



$$= \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \exp\left(i2\pi \frac{nx}{X}\right)$$

$$\operatorname{sinc}\left(\frac{n}{2}\right) \Rightarrow \text{odd orders only!}$$



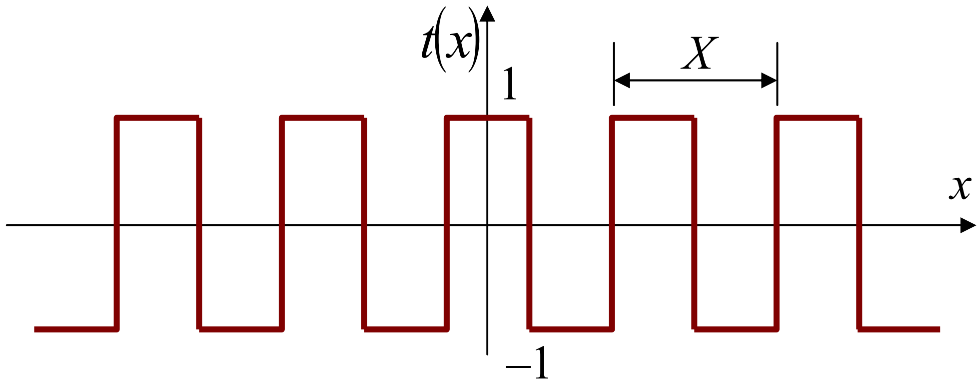
identify physical meaning:

- plane waves
- orientation of n^{th} plane wave:

  $\sin\theta_n = n\lambda / X$

- ***diffracted orders***

Fourier series for binary phase grating



$$= \sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \exp\left(i2\pi \frac{nx}{X}\right)$$

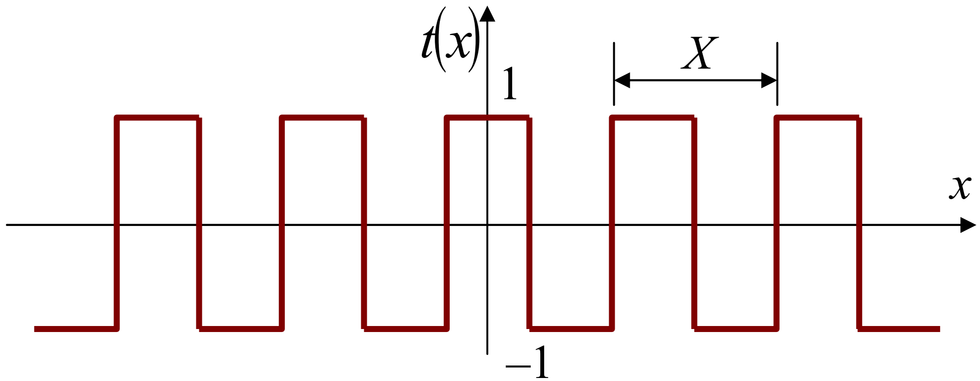
Fourier transform:

- $\exp(i2\pi u_0 x) \leftrightarrow \delta(u - u_0)$

- result is

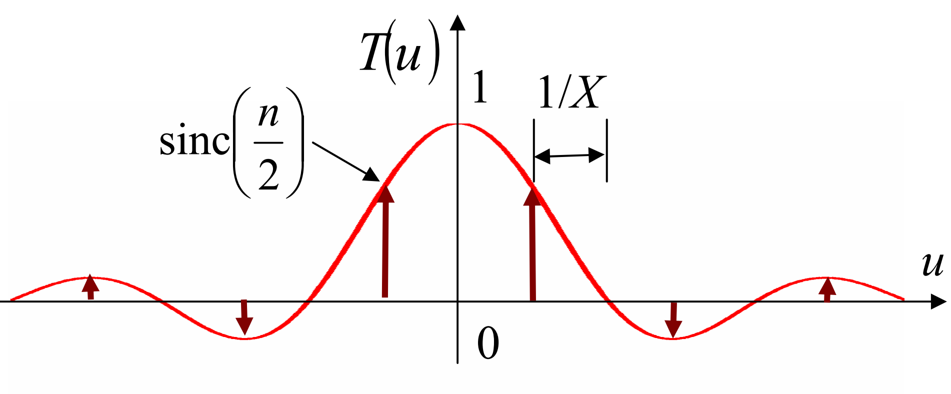
$$\sum_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \delta\left(u - \frac{n}{X}\right)$$ (Fourier series)

Diffracted spectrum from binary phase grating



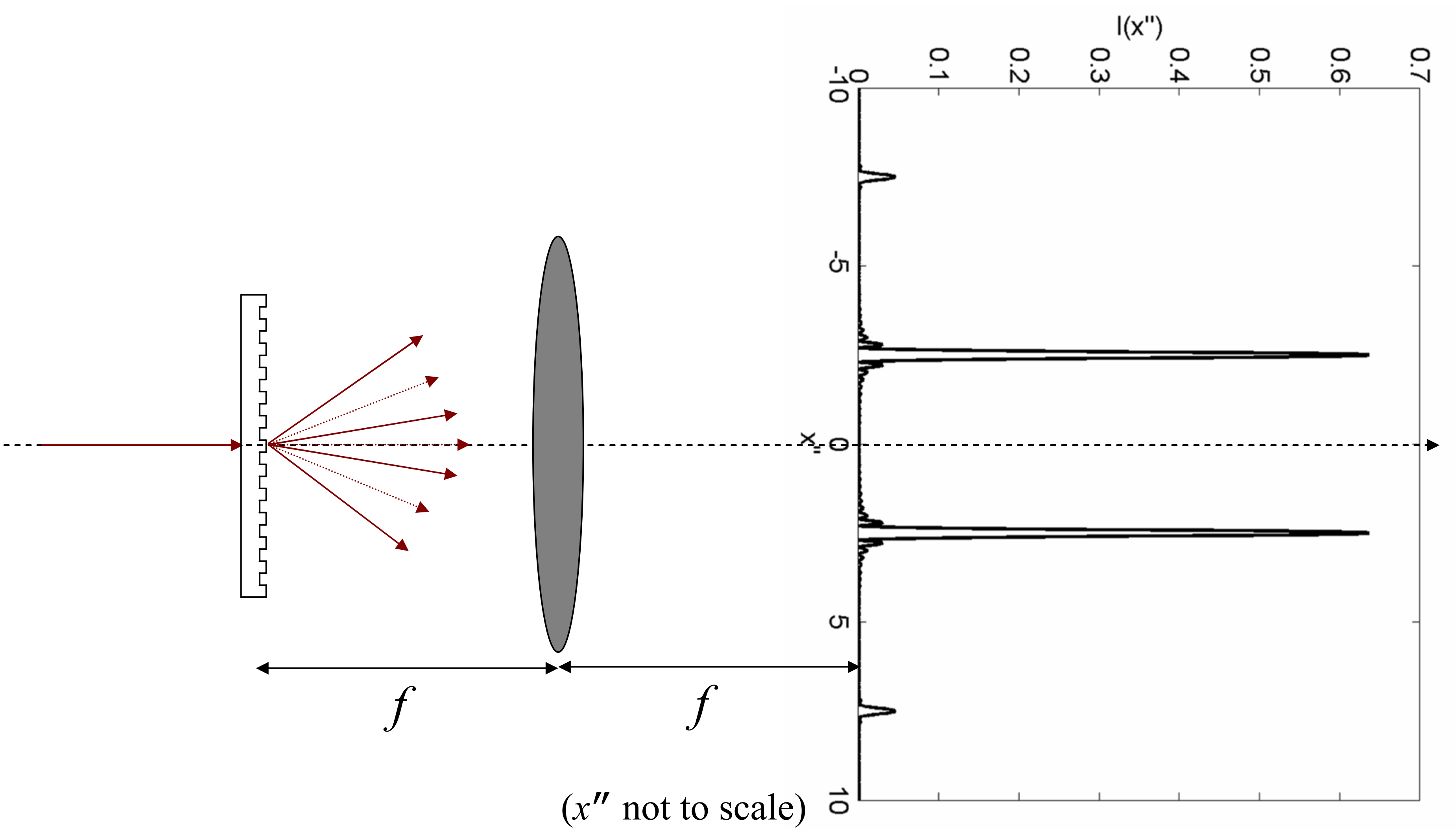
$$= \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \exp\left(i 2\pi \frac{nx}{X}\right)$$

$\Im$ $\Im$



$$= \sum_{\substack{n=-\infty \\ n \neq 0}}^{+\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \delta\left(u - \frac{n}{X}\right)$$

Fourier-plane diffraction from finite binary phase grating



(x″ not to scale)