3.021J / 1.021J / 10.333J / 18.361J / 22.00J Introduction to Modeling and Simulation
Spring 2008

# 1.021/3.021/10.333/18.361/22.00
# Introduction to Modeling and Simulation
# Homework assignment # 1
# Handed out: 2/16/08
# Due: 2/26/07

February 16, 2008

1. Consider the following differential equation and boundary conditions governing the equilibrium response of a bar with and end spring subject to a distrubuted and a tip axial load:
$$\frac{d}{dx}\left(a\frac{du}{dx}\right) = q \quad 0 < x < L$$
$$u(0) = 0, \quad \left(a\frac{du}{dx} + ku\right)\Big|_{x=L} = P$$
where $a, q$ are functions of $x$, and $k$ and $P$ are constants. Derive the weak formulation of this problem.

2. Derive the weak form for the following nonlinear equation:
$$-\frac{d}{dx}\left(u\frac{du}{dx}\right) + f = 0 \quad 0 < x < 1$$
$$u(1) = \sqrt{2}, \quad \left(u\frac{du}{dx}\right)\Big|_{x=0} = 0$$

3. A steel rod of diameter $D = 2$ cm, length $L = 25$ cm, and thermal conductivity $k = 50$ W m$^{-1}$ °C$^{-1}$ is exposed to ambient air at $T_\infty = 20$°C with a heat transfer coefficient $\beta = 64$ W m$^{-2}$ °C$^{-1}$. Given the left of the rod is maintained at a temperature $T_0 = 120$°C and the other end is exposed to the ambient temperature, determine the equilibrium temperature distribution in the rod using a two-parameter $(c_1, c_2)$ Galerkin approximation with polynomial approximation functions. The governing differential equation and boundary conditions for this problem are:
$$-\frac{d^2\theta}{dx^2} + c\theta = 0 \quad 0 < x < L$$
$$\theta(0) = T(0) - T_\beta, \quad \left(k\frac{d\theta}{dx} + \beta\theta\right)\Big|_{x=L} = 0$$

where $\theta = T - T_\infty$, $T$ is the temperature, $c$ is given by:

$$c = \frac{\beta P}{Ak} = \frac{\beta \pi D}{\frac{1}{4}\pi D^2 k} = \frac{4\beta}{kD}$$

where $P$ is the perimeter and $A$ the area of the cross section of the rod.

4. Obtain a two-parameter Galerkin approximation of the following equations associated with a simply supported beam subjected to a uniform transverse load:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) = q_0 \quad 0 < x < L$$

$$w = EI\frac{d^2w}{dx^2} = 0 \quad \text{at } x = 0, L$$

   using:

   (a) polynomial functions

   (b) trigonometric functions

5. Consider the differential equation:

$$-\frac{d^2u}{dx^2} = \cos \pi x \quad 0 < x < 1$$

   subject to the following three sets of boundary conditions:

   (a) $u(0) = 0$, $u(1) = 0$

   (b) $u(0) = 0$, $\left(\frac{du}{dx}\right)\Big|_{x=1} = 0$

   (c) $\left(\frac{du}{dx}\right)\Big|_{x=0} = 0$ $\left(\frac{du}{dx}\right)\Big|_{x=1} = 0$

   Determine a 3-parameter solution with trigonometric functions, using:

   (a) the Galerkin method

   (b) the least squares method

   (c) the collocation method

   Compare with the analytical solutions:

   (a) $u = \frac{1}{\pi^2}(\cos \pi x + 2x - 1)$

   (b) $u = \frac{1}{\pi^2}(\cos \pi x - 1)$

   (c) $u = \frac{1}{\pi^2}\cos \pi x$