SMA 6304 / MIT 2.853 / MIT 2.854
Manufacturing Systems
Lecture 21: Single-stage, multiple part type, MTS Production

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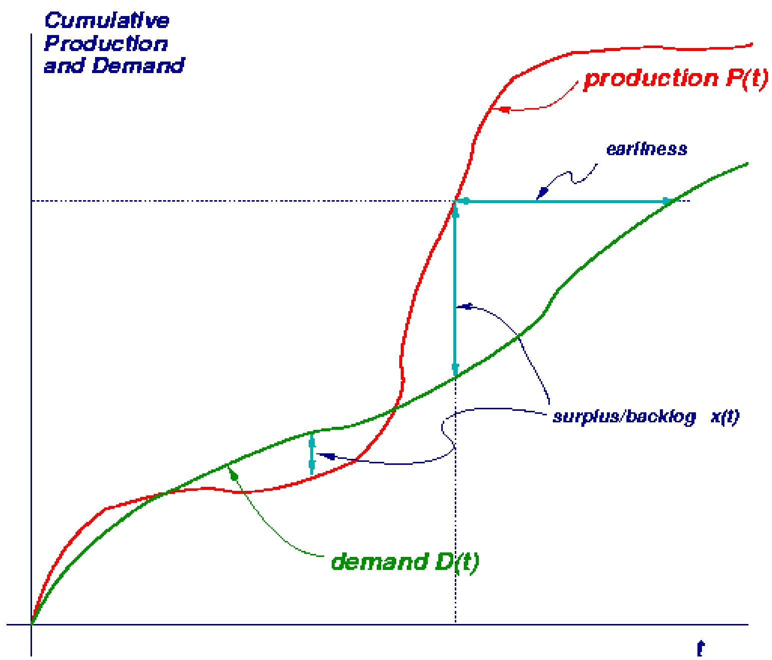
Make to Stock

- *Ideal:* The product is available *(in stock)* when the order arrives.
- *Service rate:* the fraction of orders that are fulfilled with no waiting.
- *Costs:* Inventory, lost sales.

Make to Stock

- *Trade-off:*
  - ⋆ The higher the service rate, the happier the customers *(so sales go up)* .
  - ⋆ The lower the service rate, the less inventory is required *(so costs go down)* .
- *The Quantification Dilemma:* Costs are easy to quantify. Benefits are not.

Objective



Objective is to keep the cumulative production line close to the cumulative demand line.

Setups

- *Setup:* It costs less to make a Type i part after making a Type i part than after making a Type j part.
- Examples:
  - ⋆ Tool change
  - ⋆ Paint color change

Setups

Setup costs can include

- Money costs, especially in labor. Also materials.
- Time, in loss of capacity and delay.
- Setup motivates *lots* or *batches:* a set of parts that are processed without interruption by setups.

Setups

Economic Order Quantity

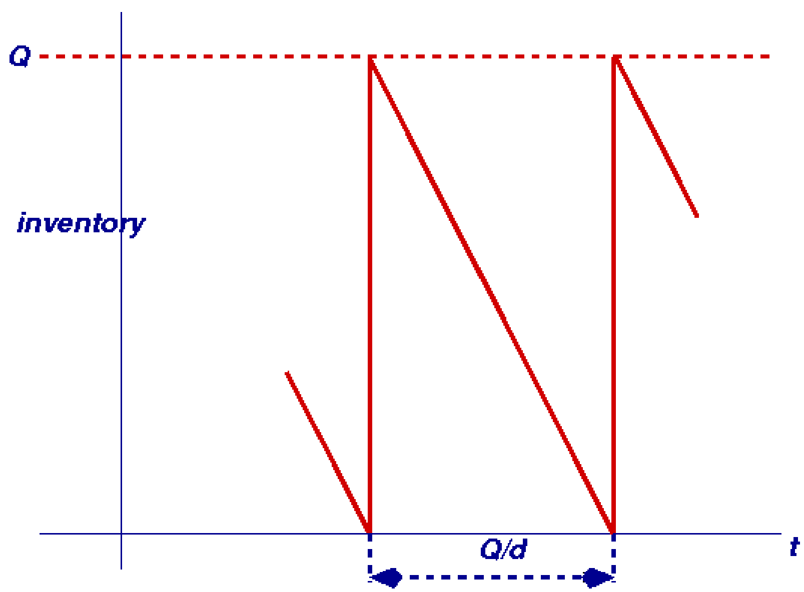
(EOQ)

- Money cost only.
- Each time the firm obtains a lot of Q items, it must pay $A + cQ$ dollars for that lot.
  - ⋆ The unit cost is c, and the cost of ordering is A.
- The firm orders Q items when its inventory level is 0, and it receives them instantly.

Setups

Economic Order Quantity

(EOQ)



- Inventory is depleted at a constant rate d until it is 0, at time Q/d.
- The average inventory level is $Q/2$.

Holding cost

- If h is the dollar cost per time unit of holding an item in inventory, then $hQ/2$ is the average inventory holding cost per time unit.
- Over a long time interval of length T, the total holding cost is $hQT/2$.

Acquisition cost

- Over that interval, Td units of material is acquired, in lot sizes of Q. The *number* of times it is ordered is Td/Q.
- The *total* cost of acquiring material is $(Td/Q)(A + cQ) = TdA/Q + Tdc$

Setups

Economic Order Quantity

Cost Minimization

- The total cost over the interval is

$$\frac{TdA}{Q} + Tdc + \frac{ThQ}{2} = T\left\{\frac{dA}{Q} + dc + \frac{hQ}{2}\right\}.$$

- The minimizing lot size is therefore

$$Q^\star = \left(\frac{2dA}{h}\right)^{\frac{1}{2}}.$$

In a random environment, EOQ can be converted into a real-time policy:

- When the inventory goes to 0, order $Q^\star$ units.

When delivery is not instantaneous, a variation is

- When the inventory goes below some level Q_{min}, order enough to bring it up to $Q^\star$ units.

Setups

Assume

- there is one setup for every Q parts (Q=lot size),
- the setup time is S,
- the time to process a part is τ.

Then the time to process Q parts is $S + Q\tau$. The average time to process one part is $\tau + S/Q$.

If the demand rate is d parts per time unit, then the demand is feasible only if

$$\tau + S/Q < 1/d \qquad \text{or} \qquad d < \frac{Q}{S + Q\tau} < \frac{1}{\tau}$$

This is not satisfied if S is too large or Q is too small.

Setups

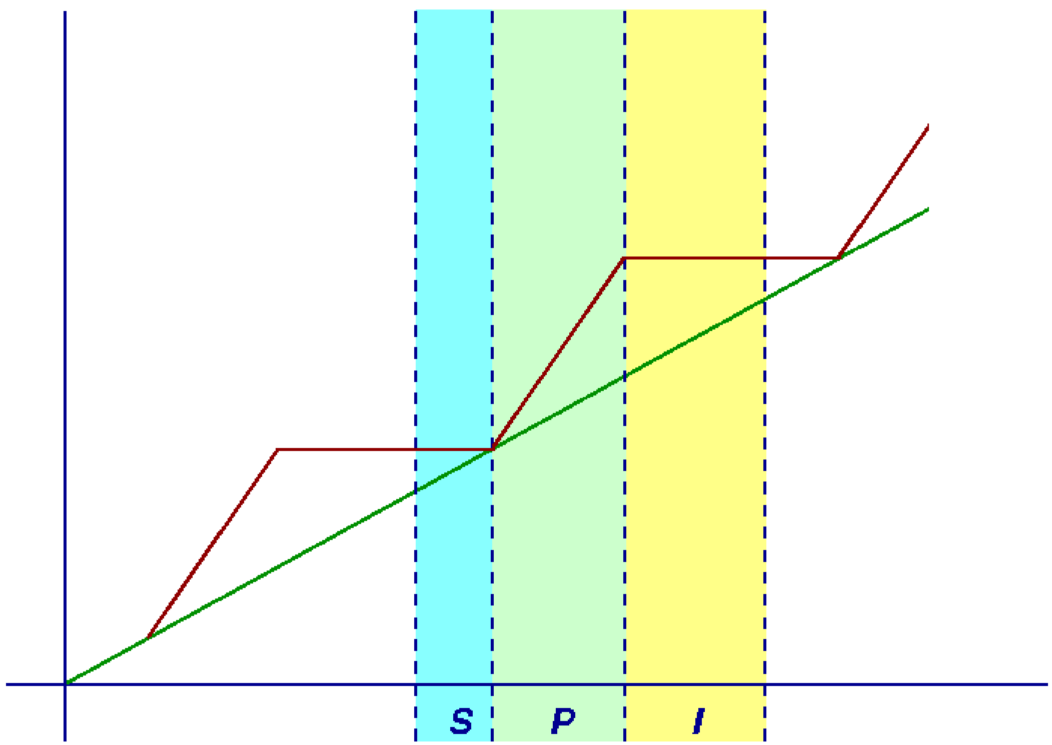
Deterministic Example

- Focus on a single part type *(simplification!)*
- Short time scale (hours or days).
- Constant demand.
- Deterministic setup and operation times.
- Setup/production/(idleness) cycles.
- *Policy:* Produce at maximum rate until the inventory is enough to last through the next setup time.

Setups

Deterministic Example

Cycle:



Setups

Cycle:

- *Setup period.* Duration: S. Production: 0. Demand: dS. Net change of *surplus* , ie of $P - D$ is $\Delta_S = -dS$.
- *Production period.* Duration: $t = Q\tau$. Production: Q. Demand: dt. Net change of $P - D$ is $\Delta_P = Q - dt = Q\tau(1/\tau - d) = Q(1 - d\tau)$.

Setups

- *Idleness period (for the part we focus on).* Duration: I. Production: 0. Demand: dI. Net change of $P - D$ is $\Delta_I = -dI$.
- Total (desired) net change over a cycle: 0.
- Therefore, net change of $P - D$ over whole cycle is $\Delta_S + \Delta_P + \Delta_I = Q(1 - d\tau) - dS - dI = 0.$

Setups

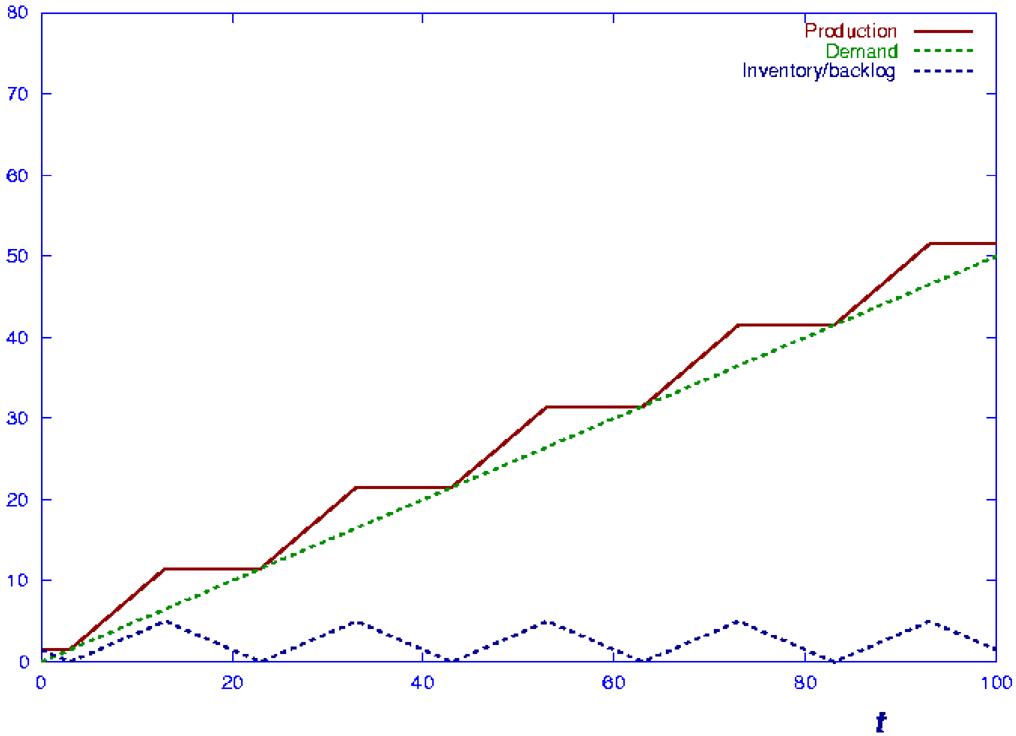
- Since $I \geq 0$, $Q(1 - d\tau) - dS \geq 0$.
- If $I = 0$, $Q(1 - d\tau) = dS$.
- If $d\tau > 1$, net change in $P - D$ will be negative.

Setups

Deterministic Example

Production & inventory history

$S = 3, Q = 10, \tau = 1, d = .5$



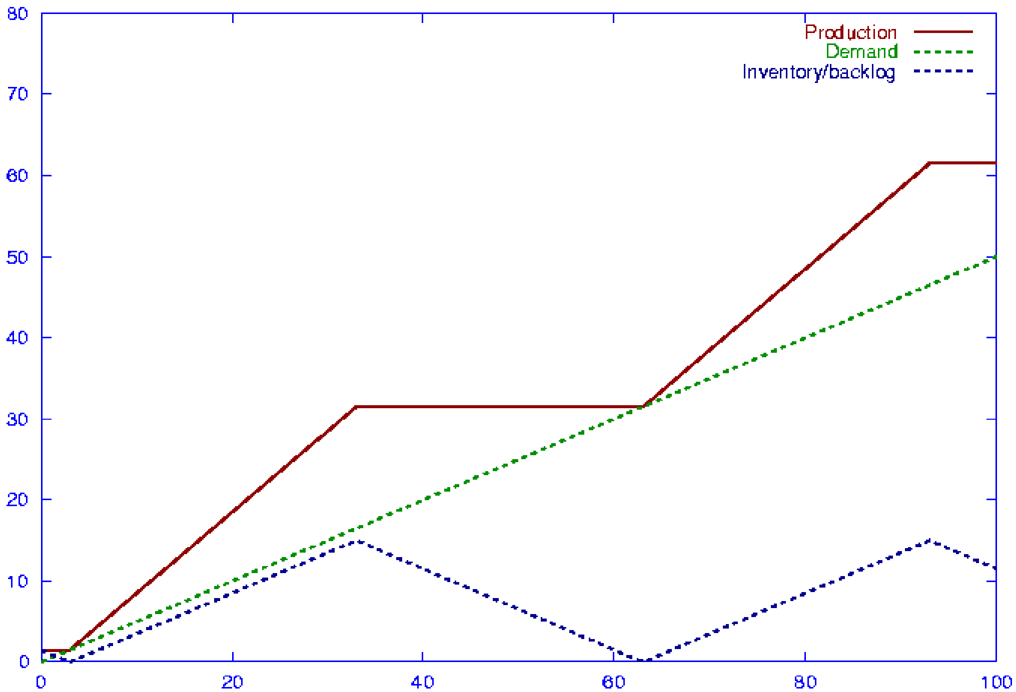
- Production period duration = $Q\tau = 10.$
- Idle period duration = 7.
- Total cycle duration = 20.
- Maximum inventory is $Q(1 - \tau d) = 5.$

Setups

Deterministic Example

Not frequent enough

$S = 3,\ Q = 30,\ \tau = 1,\ d = .5$



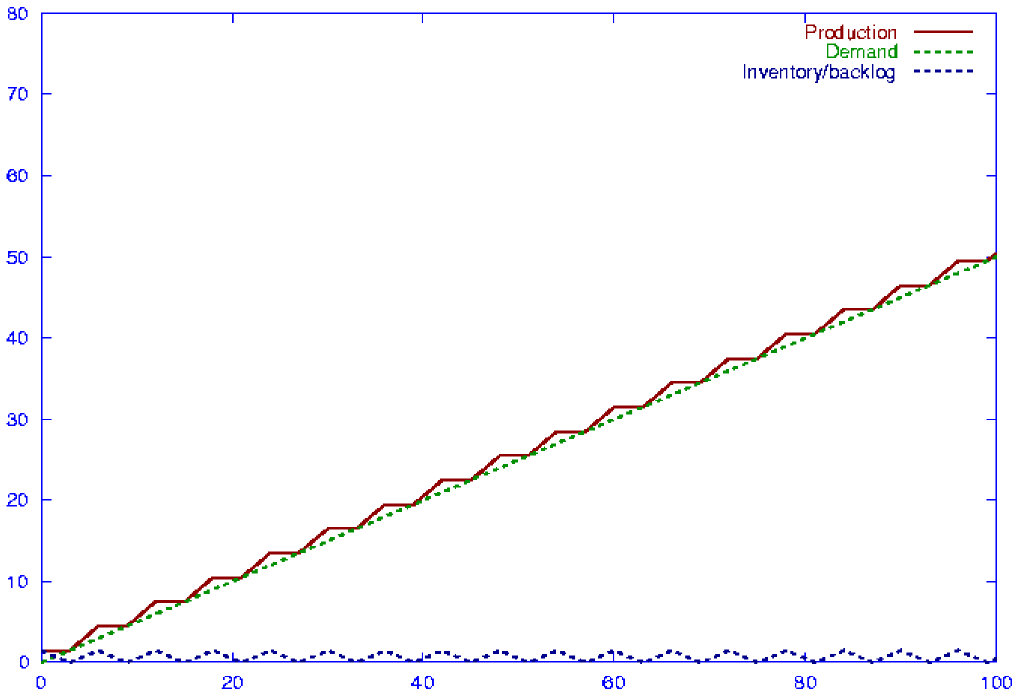
- *Large batches – big inventories.*
- Production period duration $= Q\tau = 30.$
- Idle period duration = 27.
- Total cycle duration = 60.
- Maximum inventory is $Q(1 - \tau d) = 15.$

Setups

Deterministic Example

Just right!

$S = 3,\ Q = 3,\ \tau = 1,\ d = .5$



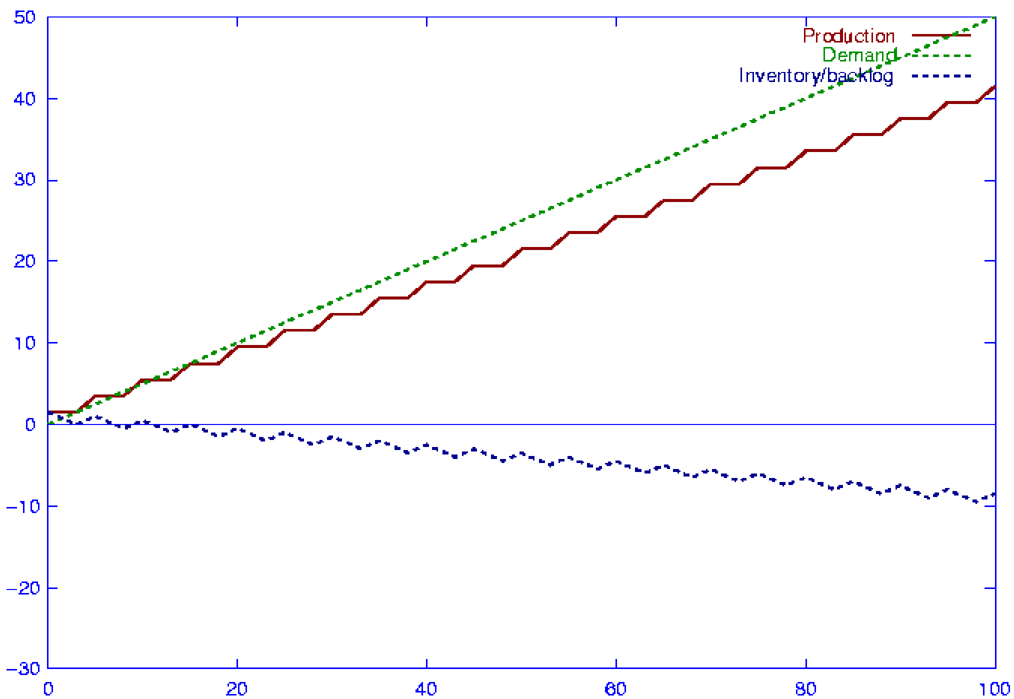
- *Small batches – small inventories.*
- Maximum inventory is $Q(1 - \tau d) = 1.5$.

Setups

Deterministic Example

Too frequent

$S = 3,\ Q = 2,\ \tau = 1,\ d = .5$



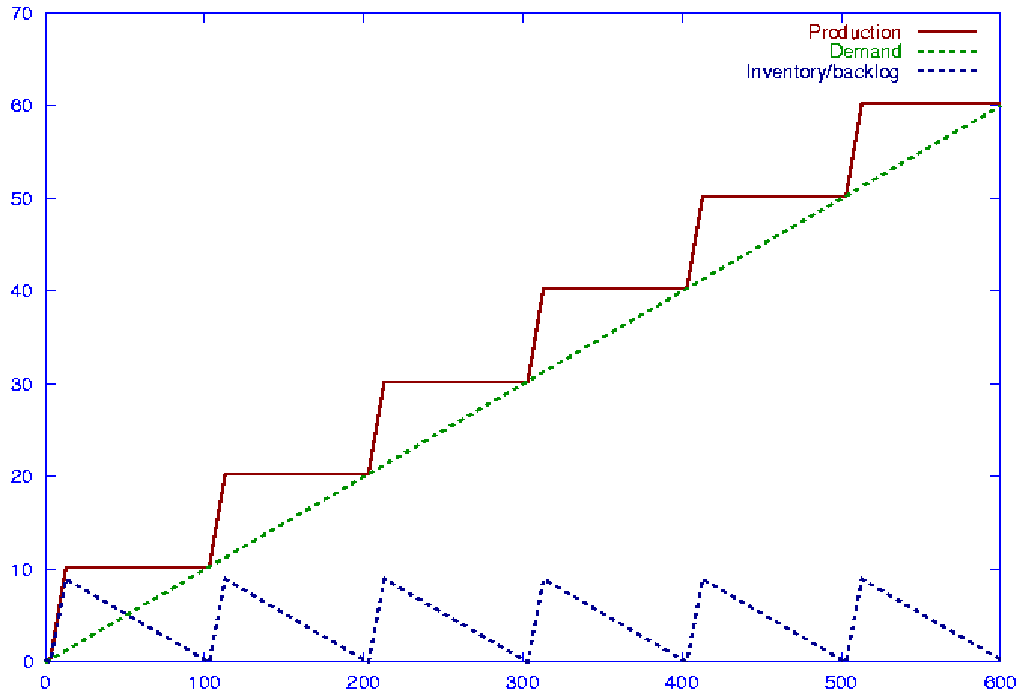
- *Batches too small – demand not met.*
- $Q(1 - d\tau) - dS = -0.5$
- Backlog grows.
- Too much capacity spent on setups.

Setups

Deterministic Example

Other parameters

$$S = 3,\ Q = 10,\ \tau = 1,\ d = .1$$

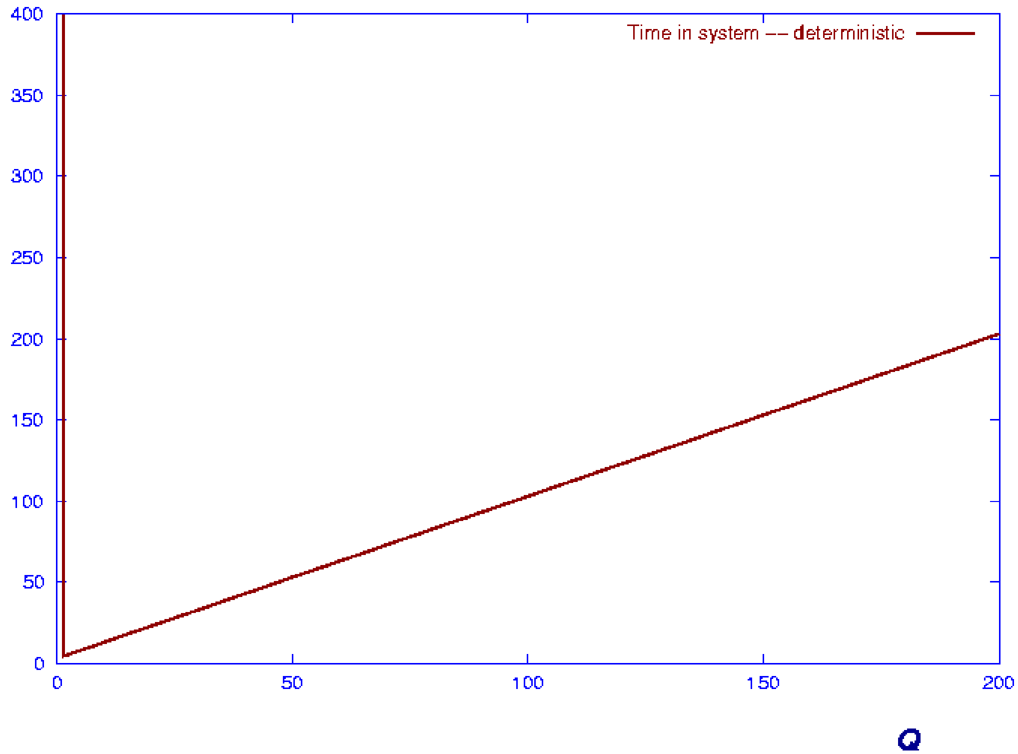


- Not always symmetrical.

Setups

Deterministic Example

Time in the system



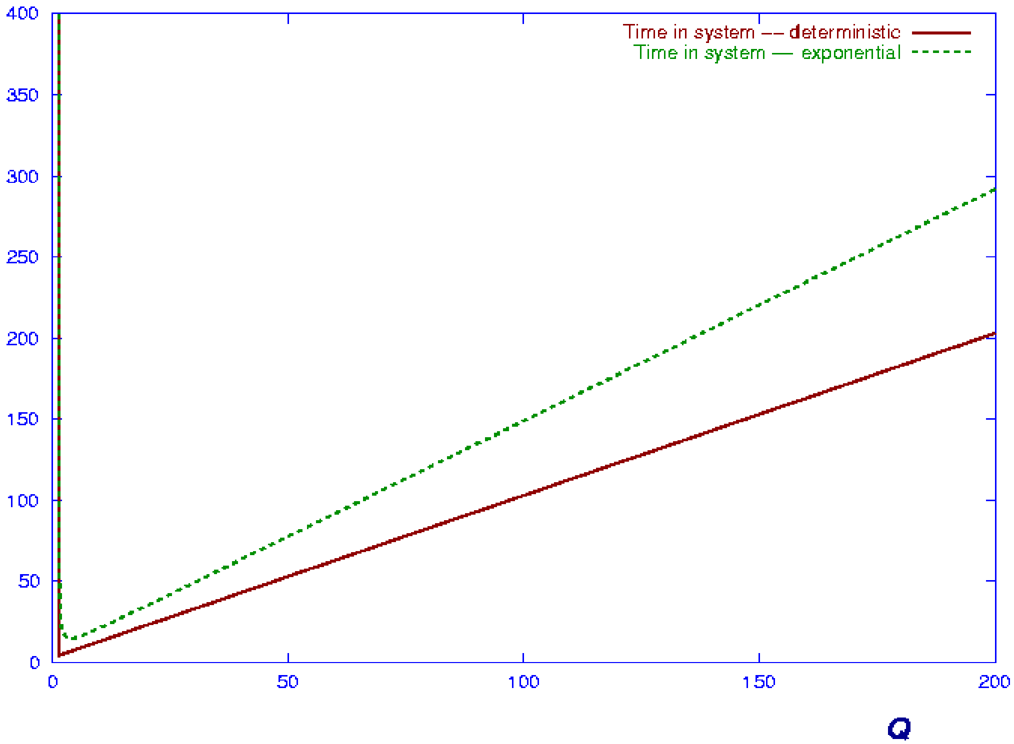
- Each batch spends $Q\tau + S$ time units in the system *if* $Q(1 - d\tau) - dS \geq 0.$
- Optimal batch size: $Q = dS/(1 - d\tau)$

Setups

- Batch sizes equal (Q); processing times random.
  - ⋆ Average time to process a batch is $Q\tau + S = 1/\mu$.
- Random arrival times (exponential inter-arrival times)
  - ⋆ Average time between arrivals of batches is $Q/d = 1/\lambda$.
- Infinite buffer for waiting batches

Setups

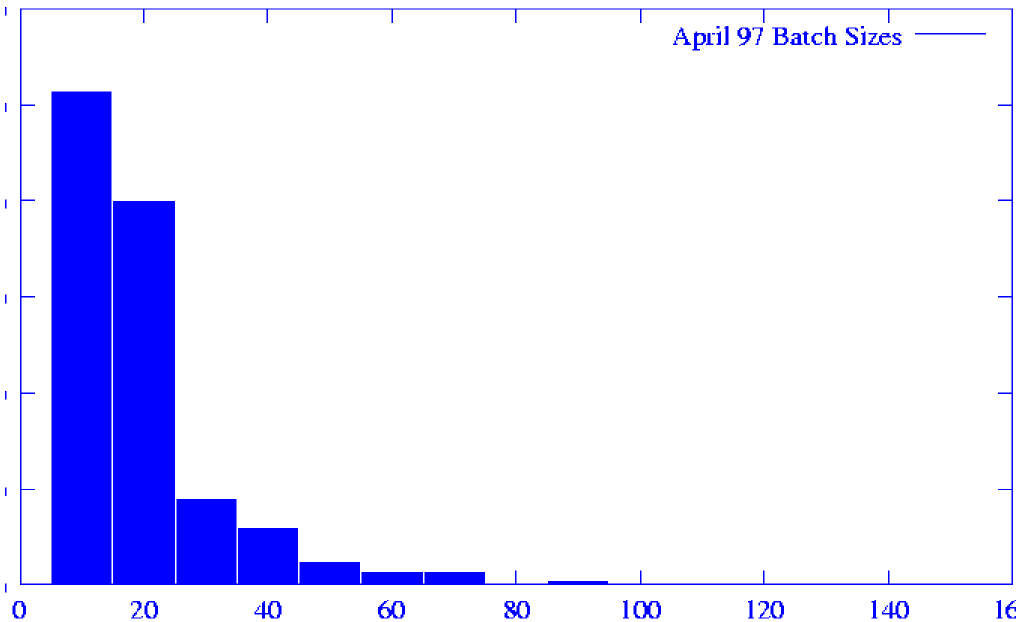
Stochastic Example



- Treat system as an $M/M/1$ queue in batches.
- Average delay for a batch is $1/(\mu - \lambda)$.
- *Variability increases delay* .

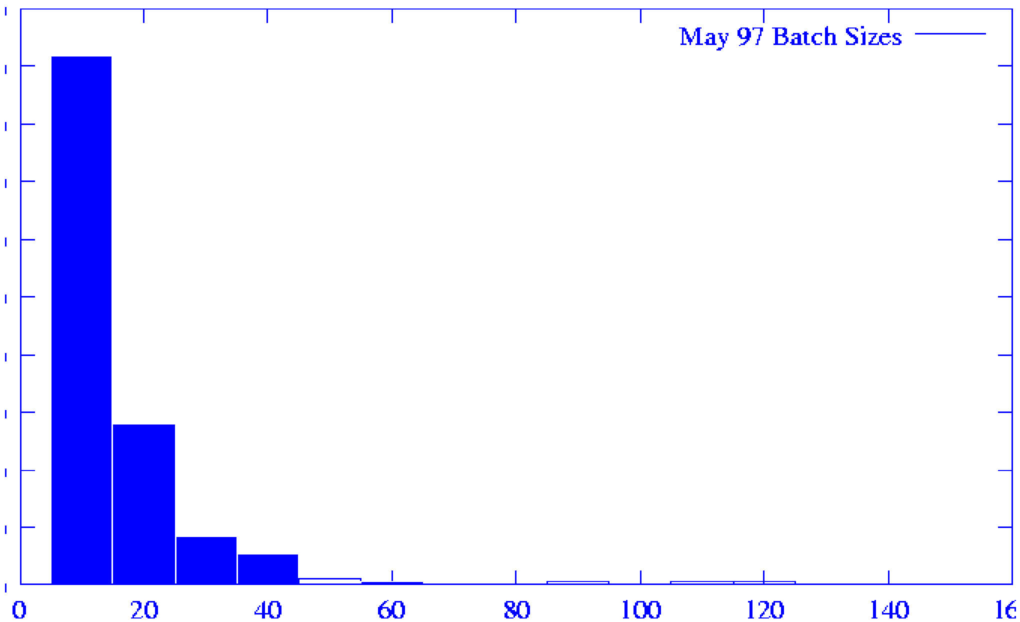
Setups

Batch size data from a factory



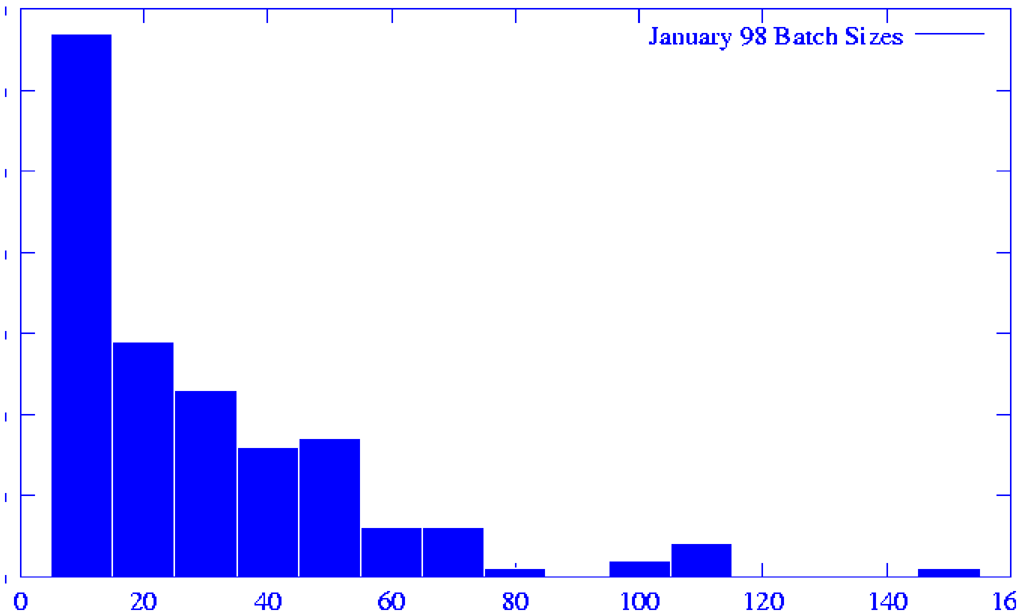
Setups

Batch size data from a factory



Setups

Batch size data from a factory



Avg Lot Size=25
Std Dev=27

Setups

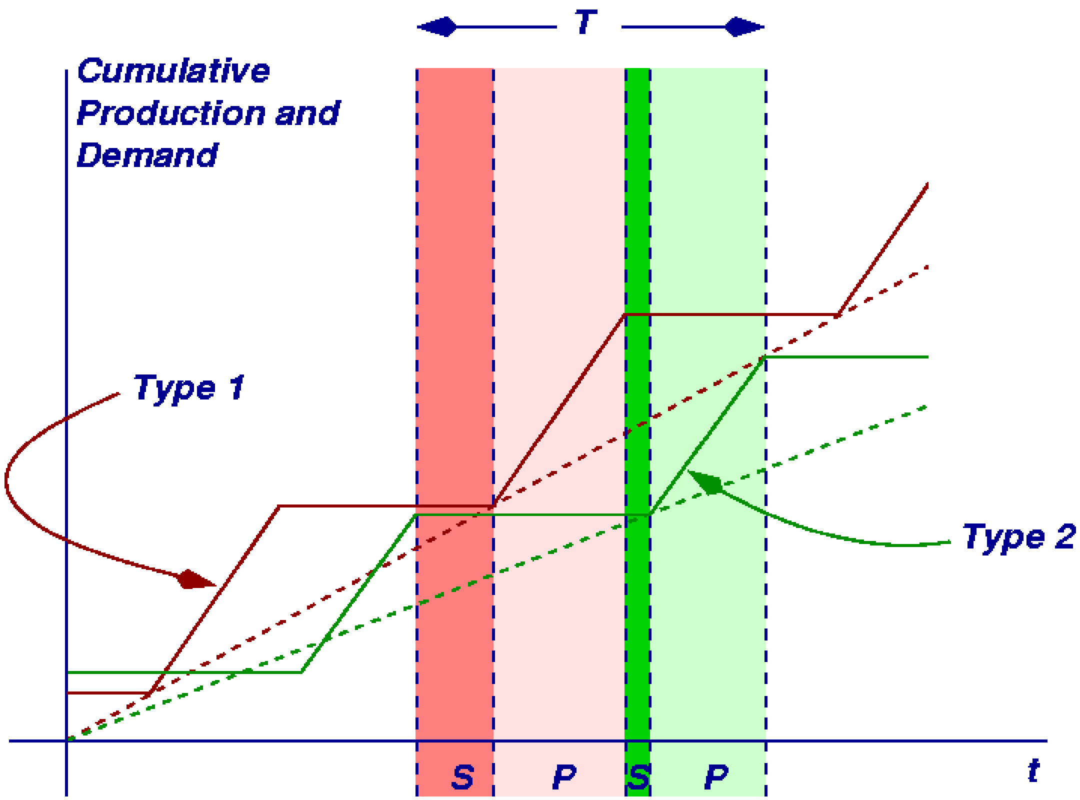
Two Part Types

- Assumptions:
  - ⋆ Cycle is *produce Type 1, setup for Type 2, produce Type 2, setup for Type 1* .
  - ⋆ Unit production times: τ_1, τ_2.
  - ⋆ Setup times: S_1, S_2.
  - ⋆ Batch sizes: Q_1, Q_2.
  - ⋆ Demand rates: d_1, d_2.
  - ⋆ No idleness.

Setups

Two Part Types

Cycle:



Setups

Let T be the length of a cycle. Then

$$S_1 + \tau_1 Q_1 + S_2 + \tau_2 Q_2 = T$$

To satisfy demand,

$$Q_1 = d_1 T; \quad Q_2 = d_2 T$$

This implies

$$T = \frac{S_1 + S_2}{1 - (\tau_1 d_1 + \tau_2 d_2)}$$

Setups

- $\tau_i d_i$ is the fraction of time that is devoted to producing part i.
- $1 - (\tau_1 d_1 + \tau_2 d_2)$ is the fraction of time that is *not* devoted to production.
- We must therefore have $\tau_1 d_1 + \tau_2 d_2 \leq 1$. This is a *feasibility condition* .

Setups

- New issue: *Setup sequence* .
  - ⋆ In what order should we prduce batches of different part types?
- S_{ij} is the setup time (or setup cost) for changing from Type i production to Type i production.
- *Problem:*
  - ⋆ Select the setup sequence $\{i_1, i_2, ..., i_n\}$ to minimize $S_{i_1i_2} + S_{i_2i_3} + ... + S_{i_{n-1}i_n}$.

Setups

Multiple Part Types

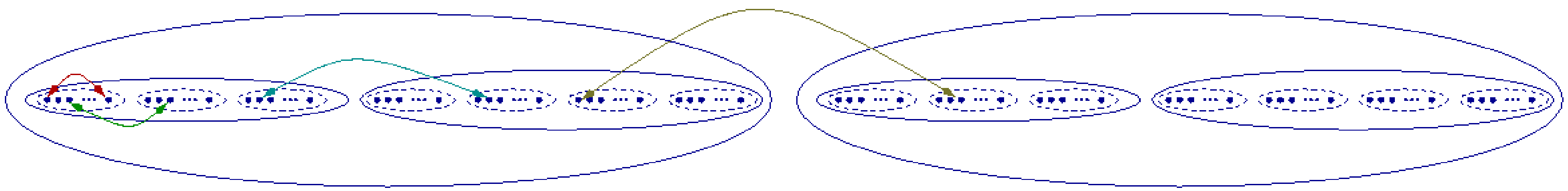
Cases

- Sequence-independent setups: $S_{ij} = S_j$. Sequence does not matter.
- Sequence-dependent setups: traveling salesman problem.

Setups

- Paint shop: i indicates paint color number.
- S_{ij} is the time or cost of changing from Color i to Color j.
- If $i > j$, i is darker than j and $S_{ij} > S_{ji}$.

Setups



- Hierarchical setups.
- Operations have several attributes.
- Setup changes between some attributes can be done quickly and easily.
- Setup changes between others are lengthy and expensive.

Setups

- *Wagner-Whitin (1958)* problem
- Assumptions:
  - ⋆ Discrete time periods (weeks, months, etc.); $t = 1, 2, ..., T$.
  - ⋆ Known, but non-constant demand D_1, D_2, ..., D_T.
  - ⋆ Production, setup, and holding cost.
  - ⋆ Infinite capacity.

Setups

Other notation

- c_t = production cost (dollars per unit) in period t
- A_t = setup or order cost (dollars) in period t
- h_t = holding cost; cost to hold one item in inventory from period t to period $t+1$
- I_t = inventory at the end of period t — the state variable
- Q_t = lot size in period t — the decision variable

Setups

minimize $\sum_{t=1}^{T}(A_t\delta(Q_t) + c_tQ_t + h_tI_t)$

(where $\delta(Q) = 1$ if $Q \geq 0$; $\delta(Q) = 0$ if $Q = 0$)

subject to

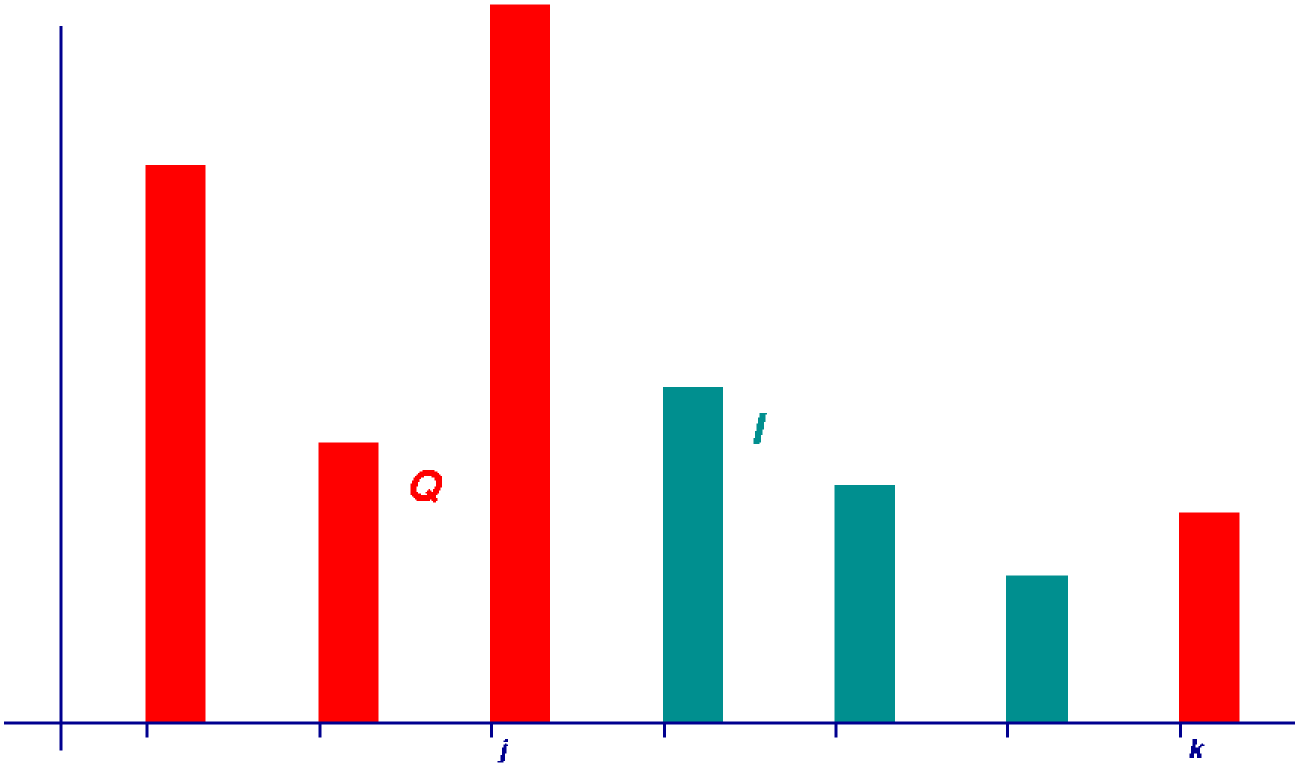
- $I_{t+1} = I_t + Q_t - D_t$
- $I_t \geq 0$

Setups

Dynamic Lot Sizing

Wagner-Whitin Property

Characteristic of Solution:



Setups

Characteristic of Solution:

- *Either* $I_t = 0$ *or* $Q_{t+1} = 0$. *That is, produce only when inventory is zero.* Or,
  - ⋆ If we assume $I_j = 0$ and $I_k = 0$ $(k > j)$ and $I_t > 0, t = j + 1, ..., k$,
  - ⋆ then $Q_j > 0$, $Q_k > 0$, and $Q_t = 0, t = j + 1, ..., k$.

Setups

Then

- $I_{j+1} = Q_j - D_j$,
- $I_{j+2} = Q_j - D_j - D_{j+1}$, ...
- $I_k = 0 = Q_j - D_j - D_{j+1} - ... - D_k$

Or, $Q_j = D_j + D_{j+1} + ... + D_k$

which means *produce enough to exactly satisfy demands for some number of periods, starting now.*

- This is not enough to determine the solution, but it means that the search for the optimal is limited.
- It also gives a qualitative insight.

Setups

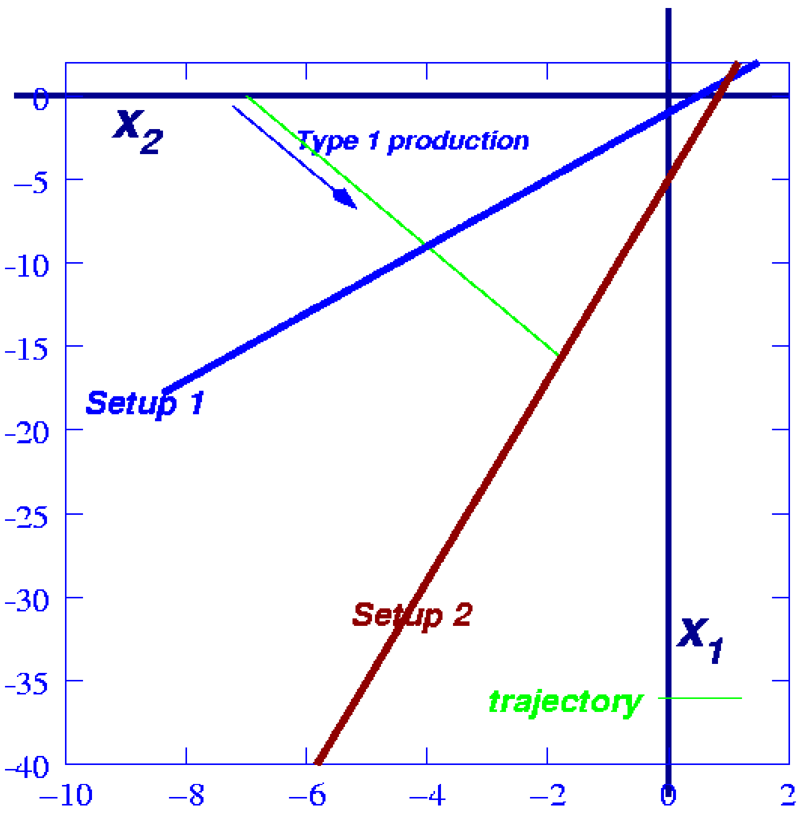
- *Problem:* How to decide on batch sizes (ie, setup change times) in response to events.
- *Issue:* Same as before.
  - ⋆ Changing too often causes capacity loss; changing too infrequently leads to excess inventory and lead time.

Setups

Model:

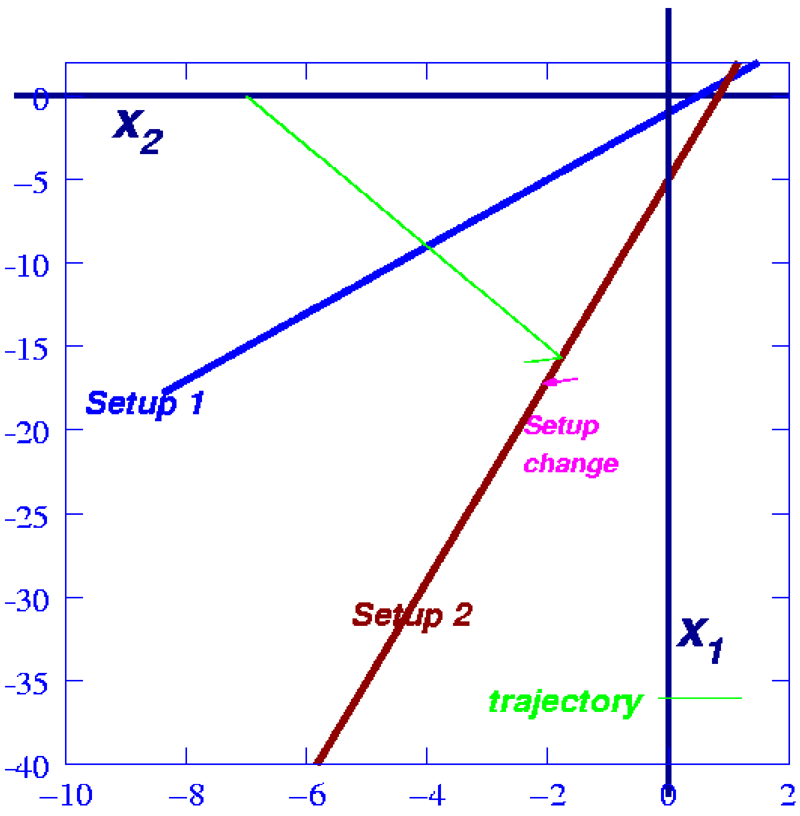
- d_i = demand rate of Type i
- $\mu_i = 1/\tau_i$ = maximum production rate of Type i
- S = setup time
- $u_i(t)$ = production rate of Type i at time t
- $x_i(t)$ = surplus (inventory or backlog) of Type i
- $\dfrac{dx_i}{dt} = u_i(t) - d_i, i = 1, 2$

Setups



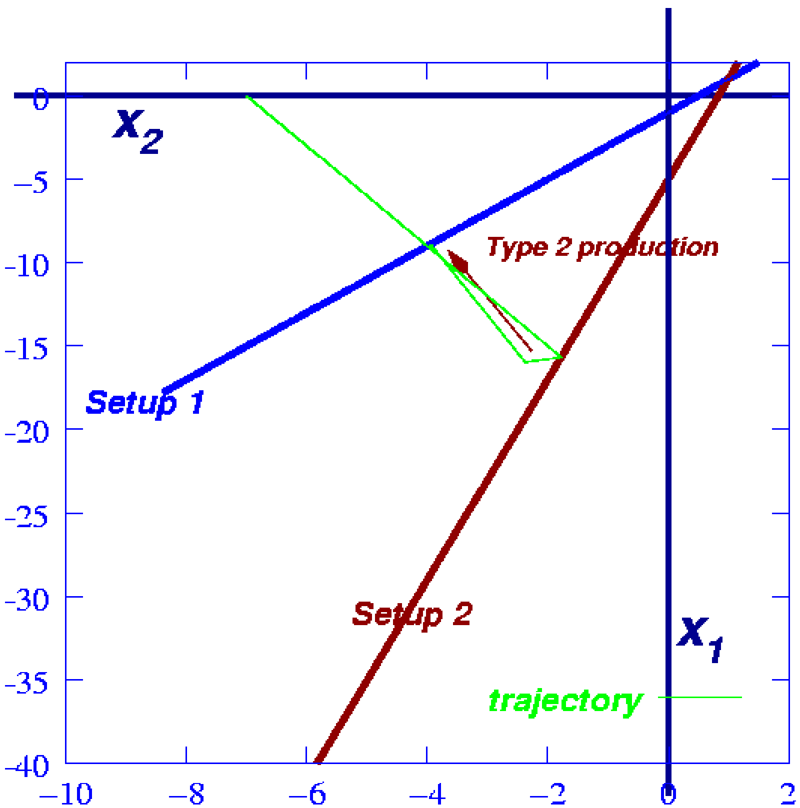
- Draw two lines, labeled *Setup 1* and *Setup 2*.
- Keep the system in setup i until $x(t)$ hits the *Setup* j line.
- Change to setup j.
- Etc.

Setups


0
-5
-10
-15
-20
-25
-30
-35
-40
-10
-8
-6
-4
-2
0
2
x_2
x_1
Setup 1
Setup 2
Setup change
trajectory

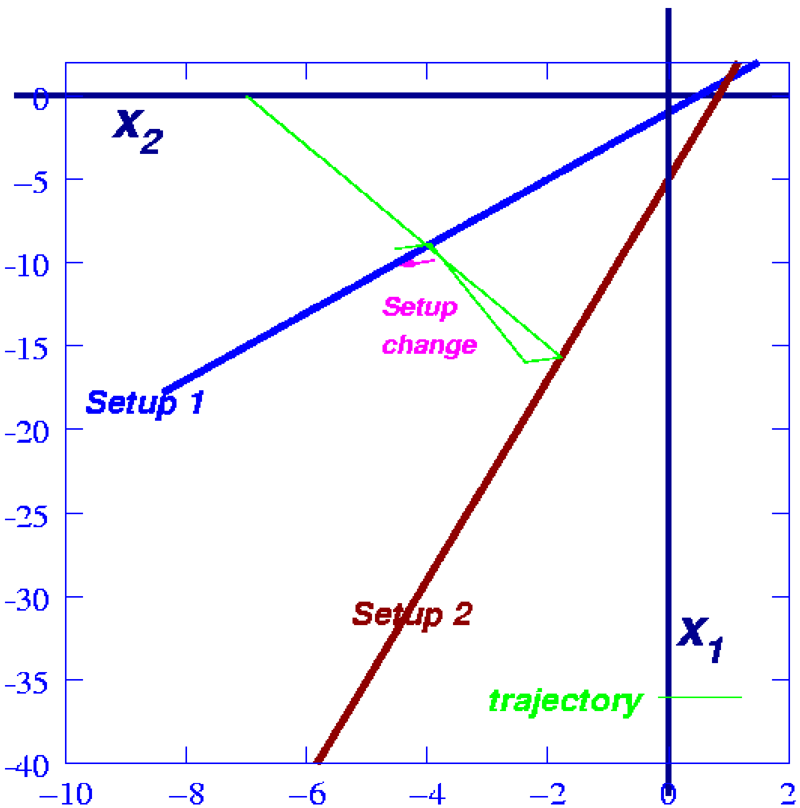
Setups

Heuristic: Corridor Policy



Setups

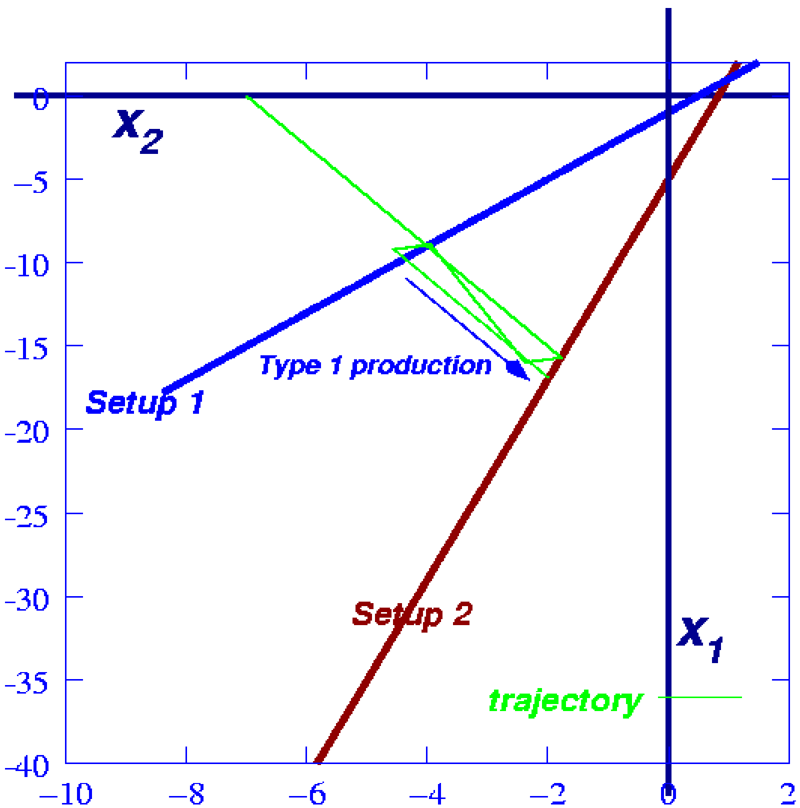
Heuristic: Corridor Policy



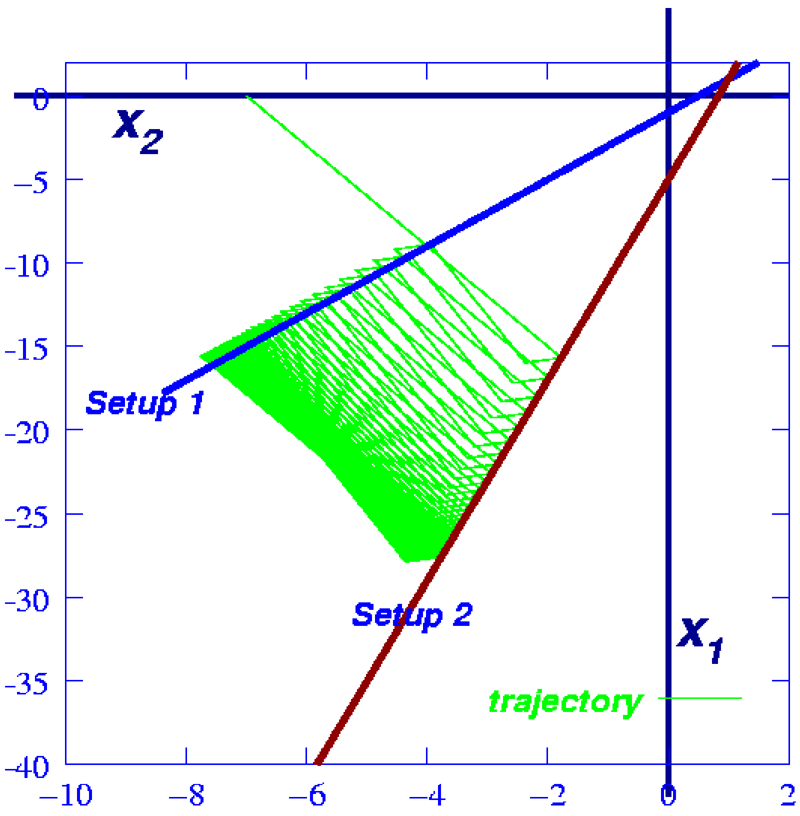
Setups

Real-Time Scheduling

Heuristic: Corridor Policy



Setups


0
-5
-10
-15
-20
-25
-30
-35
-40
-10
-8
-6
-4
-2
0
2
x_2
x_1
Setup 1
Setup 2
trajectory

Setups

- In this version, batch size is a function of time.
- Also possible to pick parallel boundaries, with an upper limit. Then batch size is constant until upper limit reached.

Setups

Two possibilities (for two part types):

- Converges to limit cycle — only if demand is within capacity, ie if $\sum_i \tau_i d_i < 1$.
- Diverges — if
  - ⋆ demand is not within capacity, or
  - ⋆ corridor boundaries are poorly chosen.

Setups

Three possibilities (for more than two part types):

- Limit cycle — only if demand is within capacity,
- Divergence — if
  - ⋆ demand is not within capacity, or
  - ⋆ corridor boundaries are poorly chosen.
- *Chaos* if demand is within capacity, and corridor boundaries chosen ... not well?