

AND
RESIDENTIAL GROWTH
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The objective of this thesis is to quantify the general hypothesis of the existence of a relationship between the accessibility of an area and the potential rate and intensity of residential growth in that area; and further, to illustrate how this relationship, once quantified, can provide a basis for a residential land use model-- an estimating process of distributing total metropolitan growth to small areas within the metropolitan region.

Accessibility is defined in this thesis as the potential of opportunities for interaction and is a measure of the intensity of the possibility of interaction. The formula developed to measure accessibility is a variation of the gravitational principle and states that the accessibility to an activity is directly proportional to the size of the activity and inversely proportional to the distance to the location of the activity.

An empirical examination was conducted to determine the relationship between accessibility to employment, shopping, and social opportunities and the residential growth which occured in the Washington, D.C. metropolitan area during the seven year period from 1948 to 1955. Through statistical analyses the association between these variables was established and an estimating formula-- a residential land use model-- based on accessibility and the availability of vacant land, was developed. Tests of this model show that these two factors alone do not produce estimates of sufficient reliability for practical use ; however, methods of refining this basic model through the inclusion of other major variables; such as, land costs, zoning, and taxes, are outlined.

The models applicability to the metropolitan planning process is illustrated through the use of hypothetical examples. These illustrations show that the concept and approach developed in this thesis provide a potentially valuable tool for metropolitan planning purposes.

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## TABLR OF CONPENTS

ABSTRACT ..... $1 i$
ACKNOWLHDGMENTS . ..... iii
TABLE OF CONAHANS ..... iv
LIST OF TABLES AND FICURES ..... v
INTRODUCTION ..... 1
A MEASURBMENT OF ACCESSIBILITY ..... 2
Accessibility Defined ..... 4
Exponent of Distance. ..... 6
FMPIRICAL EXAMINATION \# WASHINGTON METROPOLITAN AREA. ..... 10
A RESIDEMIAL IAND USE MODRL ..... 21
Illustration of The Model ..... 22
CONCLUDING COMMENPS ..... 28
APPENDIX A
Determination of the Exponent of Distance ..... 31 in Gravity Models
APPENDIX B
Calculation of Accessibility. ..... 36
APPENDIX C :
Calculation of Residential Development Ratios ..... 41
SELECTHTD BIBLIOGRAPHY ..... 44
IIST OF TABLES AND FIGURES

## IIST OF TABLISS

## MABIE

1. Data For Accessibility Calculations . . . . . . . . . . . 11
2. Statistical Analysis of the Relationship. . . . . . . . . . 17

Botween Residential Development and the
Variūs Measures of Accessibility
Aml. Empirically Determined Exponents of Distance. . . . . . . . 32
B-1. Calculation of Residential Development Ratios . . . . . . . 42 and Summary of Accessibility Values

## LIST OF FIGURES

FIGURE

1. Pattern of Accessibility To Fmployment : . . . . . . . . . . 12 Washington Metropolitan Area, 1955
2. Pattern of Accessibility To Population : . . . . . . . . . 13 Wamhington Metropolitan Area, 1955
3. Pattern of Accessibility To Shopping : . . . . . . . . . . . 14 Washington Metropolitan Area, 1955
4. Accessibility and Residential Development . . . . . . . . . 16
5. Illustration of Land Use Model . . . . . . . . . . . . . 23
6. Successive Applications of the Model. . . . . . . . . . . . 26 Synthesize Growth

A-1. The Exponent of Distance In Gravity Models. . . . . . . . . 34
B-1. Sample Calculations of Accessibility. . . . . . . . . . . . 40

## ACCESSIBILITY AND RESIDENTIAL GROWTH

City planners, economists, and urban geographers have long emphasized the importance that accessibility has on the patterm of urban development. The general hypothesis, that the more accessible an area is to the various activities in a community the greater is its growth potential and probable intensity of development, has been expressed in one form or another for some time. One need only to glance at a master plan to observe the intuitive acceptance of this hypothesis by the planning profession.

Although there has been close to universal acceptance of this general proposition, there have been surprisingly few attempts to quantify this hypothetical relationship between accessibility and urban development. Therefore, this thesis is not an attempt to develop and propose a new theory concerning urban development; but rather, has as its purpose the narrower objective of measuring accessibility and examining the relationships which exist between it and the growth and structure of residential development.

Specifically, this study has four major objectives:
(1) To develop a definition and methodology for the measurement of accessibility.
(2) To determine empirically the relationships between residential growth and accessibility to employment, shopping, and social opportunities.
(3) To illustrate how accessibility can provide the planner with a clearer understanding of the dynamics of metropolitan growth and how the above relationships can provide a basis for a residential land use model--a process of distributing metro-
politan growth to small areas within the metropolitan region.
(4) To indicate, in light of the results of this study, avenues of potentially productive additional research.

## A MEASUREMENT OF ACCESSIBILITY

Various indices of accessibility have been developed and used to describe metropolitan structure. 1/ Distance to the center of the city, concentric circles or ischronal zones, and the ring sector measures are some of the simpler attempts to measure the effects of accessibility. The deficiencies of such indices as measurements of accessibility are too apparent to require elaboration.

More complex indices have been developed which attempt to measure the relative accessibility to a spatially dispersed activity rather than to a point in space. One such index is aggregate travel. Aggregate travel is defined as the accumulated distance that would be required to: concentrate that activity to which accessibility is being measured at any particular zone.

I/ Most of the work done at this time afford valuable insight into the structure of the metropolitan community but do not yield quantitative forecasts or insight into the process of growth; the problem towards which this paper is focused. This is because these studies reflect correlations at a specific time rather than trends through time. For an excellent study of this type see Don J. Bogue, The Structure of the Metropolitan Community: Ann Arbor, 1949.

For one attempt to quantify the process of growth see Hans Blumenfeld, 'The Total Wave of Metropolitan Expansion, "Journal of the AIP, Winter, 1954. Also see Creighton and Hamburg, "A Method of Land Use Forecasting Based Upon Historical Trends," (To be released) Journal of the AIP, Spring, 1959. All of these studies use distance to the CBD as an index of accessibility.

For example, aggregate travel to population from a zone would be the summation of the individual distances traveled if all persons were to get to that zone. The zone with the lowest number of "person miles "would be the most accessible to the population. This index could of course be refined to include aggregate travel to all work opportunities or shopping establishments.

The reciprosal of aggregate travel is in effect an index which indicates the proximity of any zone to the residential population of the city or to some specialized part of the city such as work places. The index is a relative measure of the ease of contact.

Another measurement of the relative accessibility of an area is the population divided by distance or " potential of population " concept developed by Stewari. 2/ In general terms the concept states that the possibility of having contact with a group of people is directly proportional to the size of the group ( activity level) and inversely proportional to the distance or separation from that group. 3/

2\% J. Q. Stewart, " Demographic Gravitation : Evidence and Application " Sociometry , 11: i-2 , Jan. 5, 1948, 31-58

3/ This concept is expressed in the equation :

$$
\mathrm{I}_{2}=\frac{\mathrm{P}_{2}}{\mathrm{D}_{1-2}}
$$

where:
$V_{2}$ equals the nocential of population at point 1 created by the pomulation at area 2.
$P_{2}$ equals the number of people at area 2 ; sometimes adjusted for varying capacities for interaction.
$D_{1-2}$ equals the distance of separation between point 1. and area 2; expressed in terms of miles, cost, or time.

As set forth by Stewart the concept of " potential measurements " may be thought of as a macroscopic measurement indicating the relative intensity of the possibility of interacting with a spatially dispersed activity.

Stewart's criginal fommation has been extended to measurements of " income potential ", "market potential ", and a variety of other indices describing the effect of an activity over space. More recently, this concept has formed the basis of the so-called " gravity models" being used for traffic estimation. The various applications of this concept have been well documented and need not be examined in detail at this time. 4/

In general, these investigations have shown that the intensity of a variety of phenomena; i.e., traffic flow, population density, price of a commodity, all vary directly with various potential measurements.

Accessibility Defined : Consistent with the potential concept, accessibility is defined as the potential of opportunities for interaction . This definition differs from the usual one in that it is a measure of the intensity of the possibility of interaction rather than just a measure of the ease of interaction.

In general terms, accessibility is a measurement of the spatial distri-

4/ For a summary discussion and extensive bibliography on the varicus applications of the potential and gravitational concepts of interaction: Gerald Carrothers, "An Historical Review of the Gravity and Potential Concepts of Human Interaction, " Journal of the AIP, Spring. 1956
bution of an activity ( opportunities for interaction ) adjusted for the ability and desire of people or firms to overcome spatial separation. More specifically, the formulation states that the accessibility at zone 1 to a particular type of activity at zone 2 (say employment) is directly proportional to the size of the activity in zone 2 ( number of opportunities for employment ) and inversely proportional to some function of the distance separating zones 1 and 2. Therefore, as nore opportunities for employment are offered at zone 2 , or as the distance separating zones 1 and 2 is decreased, the accessibility to employment at zone 1 will increase. The total accessibility to employment at zone l can be expressed as the summation of the accessibility to each of the individual employment areas. 5/

5/ More formally this concept can be expressed by the following formula:

$$
I_{A}=\frac{{ }^{S}{ }_{2}}{T_{1-2}^{X}}
$$

where:
$I_{2} A_{2}$ is a relative measure of the accessibility in zone 1 of an activity in zone 2.
$S_{2}$ equals the size of the activity in zone 2 ; i.e., the number of opportunities for interaction
$T_{1-2}$ equals the separation between zones 1 and 2 , expressed in terms of miles, time, or costs.
$x$ is an exponent describing the effect of the separation $T$ on the possibility of an interaction occurring.
Although the size of the activity $S$ and the separation $T$ are denoted by a single symbol; their actual calculation may take a more complex form.

If there are more than two zones involved the formula becomes:

$$
A_{1}=\frac{S_{1}}{T_{1-1}^{x}}+\frac{S_{2}}{T_{1-2}^{x}}+\cdots \cdot \frac{S_{n}}{T_{1-n}^{x}}
$$

There are of course many technical problems in the application of this concept. Most of these problems will be discussed briefly in the empirical examination section of this study; however, one aspect of the proposed accessibility model deserves special consideration and that is the meaning and function of distance in the denominator of the model.

Exponent of Distance: It is generally agreed and empirical examinations indicate, that an exponential function of distance should be used; that is, the measurement of distance separating the various areas should be raised to some power. It is the particular value that this exponent should assume that has been the source of most of the debate concerning gravity or potential models. Stewart contends that this exponent must be unity. He bases this contention on an intriguing but unconvincing analogy to Newton's Law of Gravity. 6/Of more interest are the results of the various studies which have empirically determined the value of this exponent. These examinations have resulted in exponents that range from 0.5 to 3.0 . This variation would at first glance seem to challenge the validity of the concept; however, re-examination of the concept of accessibility will make apparent the reasons for most of this variation.

Accessibility was defined as the potential of opportunities for interaction. Stated in another manner, accessibility may be thought of as a measure of the effective opportunities at any zone 1 created by the actual opportunities offered at any zone 2. The difference between these two quantities, actual opportunities minus effective opportunities

[^0]wuld be the opportunity reduction from area 2 to point 1 . If this reduction is expressed as a percentage of the actuel opportunities offered at area $\underline{2}$ and further divided by the distance separating area $\cong$ and point 1 the resultant would be the rate of opportunity reduction per unit distance.

Now, if a single exponent of distance is to be used in the accessibility model, it means that this rate of opportunity reduction must remain constant for all types of interactions. This in turn means that all types of opportunities nust be equally valued by the interacting party because the unit costs of travel, however measured, should remain relatively constant regardless of the destination or the purpose for which the trip is made. However, this latter statement does not appear to be reasonable. For example, it would not be expected that in choosing between two olternatives of work a person would be equally influenced by (place the same value on) differences of travel as he would in choosing between alternate places to buy aspirin or some other type of convenience good. The empirical examinations conducted within urban areas seem to support this expectation. These studies show that the exponent decrease in the following manner; schcol trips 2.0+, shopping trips 2.0, social trins 1.1 , and work trips 0.9. In short, the variation in the exponent above means that people are willing to travel farther to work than they are for any other trip purpose. 7/

I/ J. Douglas Carroll and H. W. Bevin, "Predicting Local Travel in Urban Areas," Papers and Proceedings of the Regional Science Associations, Vol. 3, 1957. See also Alan M. Voorhees, A General Theory

It is when the results of these intraurban studies are compared to the findings of interurban examinations that a variation in the value of the exponent appears, which is not explainable by reference to trip purpose. The analyses carried out on interurban travel, reveal exponents of between 2.5 and $3.0 .8 /$

One reason for this apparent variation in the results of the intra and interurban investigations, is that these studies did not include terminal time or terminal effect in the measurement of separation. In interurban travel, since most of the are relatively long, the effect of omitting 5 to 6 minutes of terminal time is probably negligible: In intraurban travel, however, where the median travel time is usually less than 20 minutes, a 5 to 6 minute terminal time would have considerable effect on the percentage increase on the measurement of separation.

To determine the effect of terminal time on the exponent of distance, the results of an analysis of travel patterns in Baltimore, Maryland, were examined. The details of this study, along with an explanation of the

7/ continued:
of Traffic Movement. Past Presidents Award Paper, Institute of Traffic Engineers.

These exponent values should be considered tentative. Additional research is being done by the Bureau of Public Roads to determine and statistically evaluate these exponents for the Washington D. C., area.

8/ J. Douglas Carroll, "Spacial Interaction and the Metropolitan Description," Papers and Proceedings of the Regional Science Association. Vol. I, 1955 .
procedure for determining exponents in gravity models are given in Appendix A.

Briefly, this examination showed that, if driving times alone were used as a measure of distance, the exponent of distance did not remain constant but increased as the time of separation increased. By incorporating 5 to 6 minutes of terminal time into this measurement of separation, the variation in the exponent for any particular type or purpose of trip was greatly reduced.

When the separation of areas was expressed in terms of driving time plus terminal time, the exponents for the various types of trips were found to be: shopping trips 3.00 ; social trips 2.35; work trips 2.20. These values are consistent with those determined by the various interurban studies and were incorporated into the formula for computing accessibility in the following empirical analysis.

## EMPIRICAL INVESTIGATION--WASHINGION MEIROPOLITTAN AREA

Data for the Washington, D. C., metropolitan area were used for empirical examinations of the relationships between residential development and accessibility. Origin and destination studies, conducted in 1948 and repeated in 1955, supplied the bulk of the information necessary to calculate the accessibility measurements and to determine the pattern of residential, commercial and industrial development over a period of seven years. 9/

Because a majority ( 80 percent) of all personal travel originating in residential areas is for work, shopping, and social purposes, this study has limited its investigation to the examination of the relationships between residential growth and accessibility to employment, shopping, and social opportunities.

Table I gives the data used to calculate the various measurements of accessibility. Using the formulas developed in the preceding section, the accessibility to employment, shopping, and social opportunities were calculated for 70 areas in the Washington metropolitan area. A brief discussion of the components and calculations used to determine the measurements of accessibility can be found in Appendix B.

9/ The travel data used in the study were collected in 1948 and 1955 in two origin and destination surveys conducted by the Regional Highway Planning Committee for the Washington Metropolitan Area which was financed jointly by the highway departments of the District of Columbia, Maryland, and Virginia, and by the Bureau of Public Roads. Land use and economic data were obtained from the National Capital Planning Commission and the National Capital Planning Regional Planning Council.

TABLE I

Data for Accessibility Calculations

| Accessibility to | Units used to Express Activity Level of Size | Exponent of distance (minutes) * |
| :---: | :---: | :---: |
| Bmployment Opportunities | Number of Jobs | 2.20 |
| Shopping Opportunities | Annual Retail Sales | 3.00 |
| Social Opportunities | Population | 2.35 |

[^1]Since accessibility is a spatially continuous measurement, it can be mapped in much the same way as heights are depicted on a topographic map. 10/The map on Figure 1 shows the lines of equal accessibility to employment for the Washington area in 1955. Figures 2 and 3, are similar maps for accessibility to social and shopping opportunities.

10/That accessibility is in fact continuous is obvious. Every point in an urban area is accessible, in some manner, to every other point. However, in measuring accessibility, this continuity can only be approximated. This approximation becomes better as the metropolitan area is divided into increasingly smaller segments. The approximation does not materially effect the patterns shown on Figures 1 thru 3 . It does however, result in the loss of the smaller localized variations in the pattern of accessibility.


Figure 1 PATTERN OF ACCESSIBILITY TO EMPLOYMENT WASHINGTON METROPOLITAN AREA, 1955


Figure 2 PATTERN OF ACCESSIBILITY TO POPULATION


Figure 3 PATTERN OF ACCESSIBILITY TO SHOPPING

$$
\text { WASHINGTON METROPOLITAN AREA, } 1955
$$

To determine the relationship between residential development and accessibility, it is first necessary to account for the zonal variation in land available for residential development. This was done by distributing total metropolitan growth to the individual zones on the basis of vacant developable land; i.e., if zone A contained 10 percent of the vacant land in the metropolitan area, 10 percent of the expected growth was assigned to zone A. This proportion of the total metropolitan development assigned to each zone is termed the prorated development.

If there is a difference between this prorated development and the actual development, then some factor or factors other than the availability of land is affecting residential growth. One way of expressing this difference is to divide the actual development by the prorated development. This factor, called the development ratio; is a measure of actual growth per unit of prorated growth.

The residential development ratio was calculated for each of the areas shown in Figure 1. A discussion and summary of these calculations is given in Appendix C. Subsequently, these development ratios were examined to see whether or not they varied systematically with the various measures of accessibility. Observe the results in Figure 4. The relationship between the development ratios and distance to the center of the city is shown for comparative purposes.


Figure 4 ACCESSIBILITY AND RESIDENTIAL DEVELOPMENT

The data shown in Figure 4 substantiate the general hypothesis of the existance of a relationship between accessibility and urban growth, or at least residential growth. To quantify these relationships and to determine the actual degree of association between residential development and the various measures of accessibility, a standard regression analysis ( $\log \log$ ) was performed on this data. The results of these analyses are shown by the regression lines on Figure 4 and are summarized in Table 2.

## TABLE 2

Statistical Analysis of the Relationship between Residential Development Ratios and Various Measures of Accessibility

| INDEPENDENT VARIABIE ( $A_{1}$ ) | RESTIDENTIAL DEVELOPMENT RATIO (D1) |  |  | ESTEIMATING <br> EQUATION |
| :---: | :---: | :---: | :---: | :---: |
|  | Coefficient of Correlation | $\qquad$ | Standard Error |  |
| Accessibility To Employment | +0.9052 | . 82 | . 2690 | $\mathrm{D}_{1}=12.69 \mathrm{~A}_{1}^{3.04}$ |
| Accessibility To Population | +0.8917 | . 80 | . 2899 | $\mathrm{D}_{1}=0.93 \mathrm{~A}_{1}^{3.79}$ |
| Accessibility <br> To Shopping | +0.8563 | .74 | . 3274 | $\mathrm{D}_{1}=0.75 \mathrm{~A}_{1}^{2.31}$ |
| Driving Time To C.B.D. | -0.8534 | . 73 | -3293 | $D_{1}=\frac{719.0}{A_{1}} 2.23$ |

In addition to the regression analyses, a statistical " F " test was undertaken. This test showed that the relationships shown in Table 2 were significant and could not be the result of sampling variability and that the chance of their being random was less than 1 in 1,000 .

The sumary on Table 2 shows that of the variables tested, accessibility to employment is the best single indicator of residential growth. The correlation coefficient of 0.90 means that accessibility to employment alone explains $81 \%$ of the variation found in the residential development ratios. Considering the numerous factors which could influence the rate and intensity of land development, but which have not entered into these calculations, correlation coefficients as high as those shown in Table 2 are somewhat surprising.

One of the reasons for such high correlations is a result of the statistical procedures themselves and should be explained to avoid any misinterpretation of the results. In the type of regression analysis used here, the correlations are not developed on the actual numerical values of the variables themselves, but rather on the basis of the logarithms of these values. Therefore, the correlation coefficients and the resulting coefficients of determination are a measure of the association between the logarithms of the variables. Inasmuch as the variation between the logarithms of two numbers is numerically much less than the numerical variation between the two numbers, the correlation between the logarithms will be higher than a correlation between the actual variables.

To indicate the extent to which accessibility to employment can explain the actual variation in residential development, the equation shown in Table 2 can be used to estimate the distribution of the 1948--1955 residential growth(zonal increase in dwelling units). These estimates can then be compared to the actual growths recorded during this period
and the value of the relationship for estimating purposes may be assessed.

As previously described, the prorated development in any zone is determined on the basis of the proportion of the developable land in that zone. To estimate the actual development, the prorated development is multiplied by the development ratio, $D_{1}$, which as shown by the formula in Table 2, is equal to 12.9 times the accessibility to employment raised to the 3.04 power. 11/

A comparison of these estimated growths to the actual increases in dwelling units in each zone, showed that 40 percent of the estimates were within 30 percent of the actual figures, and that 70 percent were within 50 percent of the actual growths.

Although these results are quite promising and indicate that accessibility to employment and the availability of land are major factors in residential growth, they do not by themselves produce sufficiently precise results for estimating purposes. Other factors must be included if a usable estimating process is to be developed. The inclusion of the other measures of accessibility, by multiple correlation procedures, would undoubtedly improve the predictive ability of the model. Other factors such as zoning, land costs, and competition for land with other types of land use, must also be considered.

11/ This formula varies from the one previously reported in W. G. Hansen, "How Accessibility Shapes Land Use", Journal of the AIP, (To be released) Spring, 1959. The reason for this variation is that in the present case, the development ratio, $D$, was based on observed increases in dwelling units, while the previously reported $D$ was based on increases

The inclusion of some of these factors will be briefly discussed in the concluding section of this study. However, it is not the purpose of this paper to develop a usable residential land use model; but rather to outline its development and potential value.

For these reasons, the development of the model in the following section, based solely on accessibility to employment, should not be interpreted as a proposal, but rather as an illustration.

11/ (cont) in population. Inasmuch as there is a fairly regular decrease in persons per dwelling unit as accessibility increases, the relationship between the variables produces a slightly higher exponent when $D$ is based on dwelling unit increase. The previous formula was:

$$
D_{1}=13.7 A_{1}^{2.7}
$$

A RESIDENIIAL IAND USE MODEL

Using the relationships determined in the preceding section, it becomes possible to develop a generalized estimating model based on accessibility and vacant or developable land. It was shown that the growth in the area could be determined by multiplying the prorated development by the development ratio. More formally:
(1) $\mathrm{G}_{1}=P \mathrm{Pd}_{1} \mathrm{D}_{1}$
where: $\quad G_{1}$ equals residential growth of zone 1 , in terms of dwelling units
$\mathrm{Pd}_{1}$ equals the probable development in zone 1
but:
(2) $P d_{1}=G_{t} \frac{O_{1}}{O_{t}}$
$G_{t}$ equals total metropolitan growth in terms of dwelling units
$O_{1}$ equals vacant land in zone 1
$O_{t} \quad$ equals total vacant land in metropolitan region.
therefore:
(3) $G_{1}=G_{t} \frac{O_{1}}{O_{t}} D_{1}$
also:
(4) $G_{t}=G_{1}+G_{2}+G_{3} \ldots \cdot G_{n}$
substituting (3) into (5)
(5) $G_{t}=G_{t} \frac{O_{1}}{O_{t}} D_{1}+G_{t} \frac{O_{2}}{O_{t}} D_{2} \cdots+G_{t} \frac{O_{n}}{O_{t}} D_{n}$
reducing to:
(6) $o_{t}=o_{1} D_{1}+o_{2} D_{2} \ldots \ldots o_{n} D_{n}$

Substituting into equation (3)
(7) $\frac{G_{1}}{G_{t}}=\frac{O_{1} D_{1}}{O_{1} D_{1}+O_{2}} D_{2} \ldots O_{n} D_{n}$

Substituting $\quad \mathrm{KAA}_{1}^{3.04}$ for $\mathrm{D}_{1}$ and cancelling K 's
(8)

$$
\frac{G_{1}}{G_{t}}=\frac{o_{1} A_{1}^{3.04}}{0_{1} A_{1}^{3.04}+0_{2} A_{2}^{3.04} \ldots o_{n} A_{n}^{3.04}}
$$

In effect this model or formula states that the proportion of the total metropolitan increase in dwelling units that can be expected in any given zone,is equal to the product of vacant land times the accessibility of that zone divided by the sumaation of the similar products for all zones. The following illustration is presented to clarify the mechanics of the model and to demonstrate its potential value to the city planning process.

Illustration of Model: Figure 5 is a diagram of a four-area hypothetical metropolitan region, showing the existing employment, vacant developable land, and the travel times between areas. Estimated growth for the entire metropolitan region, by some future point in time, is 2000 dwelling units and 1000 jobs. The distribution of residential growth will be determined for each of the following cases:

Case I. The travel times between areas are the same in 1965 as at present and the increase in employment takes place in zone 1.

Case II. By 1965 an express highway is built between zones $\underline{1}$ and $\underline{2}$ reducing the travel time from 26 minutes to 21 minutes. The increase in employment takes place in zone 1 .

Case III. The travel times between zones are the same in 1965 as at present and the increase in employment takes place in zone 3 .


Figure 5 ILLUSTRATION OF LAND USE MODEL

The calculations in Figure 5 demonstrate the potential value of this and similar land use models to the city planner. The model can assist the planner in assessing the probable effects of a given action; such as, the construction of an express highway (Case II) or a policy of decentralizing employment (Case III). Of particular importance is the fact that this determination of consequences need not be limited to some predetermined area of "influence", but can be assessed for all the areas within the metropolitan region. For example, comparing Case II to Case I in the preceding illustration, the fact that zones 3 and 4 will experience a decline in expected growth due to the construction of the expressway, riay frore fmportant than
 would be unable to make such assessments.

It should be pointed out that the reliability of this model, or for that matter, any estimating process which attempts to estimate thru time, but does not contain time as a variable, is extremely sensitive to the quantity of development being distributed. The model is only capable of accurately distributing fairly small increments of growth. The reasons for this limitation are quite apparent when it is remembered that the pattern of accessibility is constantly changing through time; and furthermore, that the accessibility and the availability of developable land at any point in time, is in part dependent upon the distribution of growth during the immediately preceding period.

This does not mean that the model cannot be used for long range forecasts; quite the contrary, When combined with knowledge concerning
the probable density of development in each area, successive applications of the model can integrate time or synthesize metropolitan growth up to any point in time.

The sensitivity of the model and this process of sythesizing growth are shown in Figure 6. These calculations are based on the same assumptions made for Case $I$ in the preceding illustration, the only difference being that the total growth will be distributed with two applications of the model. The growth of 2000 dwelling units and 1000 jobs is assumed to be linear; therefore 1000 dwelling units, based on an increase in employment of 500 jobs, will be distributed with each application of the model. It was assumed that all development would occur at a density of thirty dwelling units per acre.

Figure 6 illustrates quite clearly this sensitivity of the model. Two successive applications of the model produced zone estimates as much as 38 percent different then those estimated with a single application. This example also gives insight into one possible reason for the variation found in the empirical examination. It might be that the comparison between the actual and estimated growths would have been much improved if sufficient data had been available to use two or three applications of the model.

It is this aspect of the illustrated land use model--the ability to synthesize the process of metropolitan growth, which offers the greatest potential value to the planning process. It will allow the planner to assess the accumulated consequences, through time, of a given action taken at a given time. The incorporation of the fourth dimension, time, into the present planning


| 2 | $\frac{6000}{26^{2.2}}=4.60$ | 105 | 97.8 | 10,250 | $11.3 \%$ | 113 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | $\frac{6000}{26^{2.2}}=4.60$ | 105 | 195.6 | 20,500 | 22.7 | 227 |
| 4 | $\frac{6000}{16^{2.2}}=13.30$ | 2600 | 23.3 | $\frac{59,700}{90,450}$ | $\frac{66.0}{100.0 \%}$ | $\frac{660}{1000}$ |

SUMMARY: DISITRIBUITION OF TOTAL RESIDENYITAL GROWTH 2,000 D.U.

|  | USING ONE ITERATION (Figure 5 ) | USING | TWO ITERRATIONS (Above) | \% DIFFERENCE |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 130 | 179 | $(66+113)$ | +37.5\% |
| 3 | 260 | 359 | $(132+227)$ | +38.0\% |
| 4 | 1,610 | 1,462 | $(802+660)$ | - $9.2 \%$ |

Figure 6 SUCCESSIVE APPLICATIONS OF MODEL SYNHHESIZE RESIDENTIAL GROWTH
process would enable the planner to examine the path thru time by which a given objective or goal may be obtained. For example, it would be possible to determine the timing and sequence of constructing an expressway system which would best promote a desired distribution of metropolitan residential growth.

Thus far, this study has show the existence of a relationship between accessibility and residential growth and has developed an estimating process or model based on these relationships. The developed model was show to offer great potential benefits to the planning process. Before these benefits may in fact be realized, however, other factors affecting metropolitan development must be incorporated into the model. The final section of this thesis will briefly indicate how the effect of these other factors on the development of land, may be assessed and further, how this effect may be incorporated into the present model.

## CONCLUDING COMMENTIS

The immediate value of the relationships and resulting model described in this study is that it makes possible its own improvement or reliability. The use of this model will make it possible to isolate and examine empirically, the effect of other factors on land development; such as, income, zoning, taxes and land costs. The assessment of the effect of such factors can be accomplished by empirically studying the relationship between them and the differences between observed growth and growth estimated on the basis of accessibility alone.

The results of such studies would provide the planner with a clearer understanding of the metropolitan community. It would also enable him to determine the effectiveness of the various types of land controls at the metropolitan scale. In addition, the results of these studies could be incorporated into the present model, thereby, increasing its reliability for estimating purposes.

The effect on residential growth of factors, other than accessibility, can be incorporated into the model in three ways: as modifying factors im the calculation of accessibility; as adjustments in the determination of prorated development; and as restraints in the output of the model itself.

Modification of the accessibility calculations can be accomplished by changing the measurement of opportunities from a single quantity, such as people or jobs as was used in this study, to a complex index. For example, population could be modified by some measurement of opportunities for social interaction. Opportunities for employment could probably be better measured by some estimate of the actual opportunities offered during the time period
studied rather than using total employment.

Another method of refining the accessibility index would be to make calculations for more precise types of accessibility. For example, it would be possible to determine the accessibility to employment opportunities for each of the major types of industries or occupations. One such refinement that should be made, but which was not possible in this study because of insufficient data, is to separate accessibility to shopping into the two major categories of shopping goods and convenience goods.

The determination of the prorated development expected in each area offers many opportunities to include the effect of other factors in the estimating process. Instead of determining prorated development on the basis of area of vacant land, these calculations could be based on the ultimate capacity of each zone in terms of dwelling units. 12/These capacity figures could be based on a combination of many factors i.e., existing density of development, present or planned zoning, subdivision regulations, and taxing policies. These capacity calculations should of course be based on factors which empirical examinations have shown have an effect on residential growth.

12/ Zonal capacity was one of the variables used to estimate future residential growth in Chicago Transportation Study. For an explanation of how these capacities were determined see: Klopprodt, "Allocating Population to Small Areas," Chicago Area Transportation Study pamphlet June, 1957.

Referring to the calculations in Figure 5 in the preceding section it would be possible to place restraints on the output of the model. By not allowing the amount of growth in a particular zone to exceed a predetermined amount, any growth which would normally be distributed to the restrained zone in excess of this amount would automatically be redistributed to the other zones. The effect of zoning might be incorporated into the model in this manner .

In conclusion, this study has developed a residential land use model-a process of distributing total metropolitan growth to small areas within the metropolitan region. The model was shown to have great potential value to the planning of metropolitan communities. Although considerable additional research and refinement are required before the model will be sufficiently precise for actual application, the model is sufficiently flexible to incorporate factors other than accessibility which affect residential growth.

The additional refinement of this estimating process, although feasible, would require considerable time and effort. It is the opinion of the writer that the benefits accruing from such additional study as may be required would more than justify the expenditure.

## APPENDIX A

## DETERMLNATION OF THR EXPONENT OF DISTANCE

IN GRAVIIY MODEIS

Before it is possible to empirically determine the effect of distance or spacial friction on travel, it is necessary to correct or account for the variation in the size of the zones (in terms of trip generation). This is done by assuming that it is equally easy to travel between all zones; i.e., all zonal pairs have equal time, cost and distance frictions. If this assumption were valid each zone would draw or attract only its proportion of the trips generated by any of the other zones. For example, assume that zone 1 attracts a total of 1,000 trips and that there are 10,000 trips made in the entire metropolitan area. Then if all interzonal travel frictions are assumed to be equal, zone 1 could be expected to attract 1,000 divided by 10,000 or $10 \%$ of the trips produced by each of the other zones. These resultant interzonal volumes, called the probability interchange are the volumes that could be expected if distance of separation had no effect on the volume of interchange between zones.

To the extent that the probability interchange differs from the actual or observed interchange some factor (s) other than zone size are effecting travel volumes. One way of expressing this difference is to divide the actual and probability interchange. This ratio will be termed the interchange ratio, and is a measure of the actual trips per probable trips.

By comparing the resulting interchange ratio between two zones, to the distance of separation between those zones, it is possible to assess the effect of distance, measured in terms of miles, time, or cost; on the interchange volume.

Several studies, conducted in various cities have been made along the lines outlined above. These studies have shown that the interchange ratio varies systematically with the reciprocal of distance to some power. A summary of the results of these studies is shown in Table A-l.

TABLE A-1
Empirically Determined Exponent of Distance

| Purpose of Trip |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Work | Social | Shopping | All Purpose |
| Detroit * | 1.20 | 1.0 | 2.60 | 1.63 |
| Fort Wayne ** | .805 | - | - | - |
| Oklahoma ** | .745 | - | - | - |
| South Bend ** | .703 | - | - | - |
| Wichita ** | .680 | - | 2.00 | 1.60 |
| Baltimore *** | .900 | 1.0 |  | - |

Sources:

* Carroll and Bevis, "Predicting Local Travel," Papers and Proceedings of the Regional Science Association, Vol. 3, 1957.
** Alan M. Voorhees, "Forecasting Peak Hours of Travel", Highway Research Board Bulletin. 203, 1958.
*** Unpublished data Baltimore Transportation Study.

Although there is considerable variation in these developed exponents there is a certain consistency in the variation of exponent for different trip purposes. The effect of distance seems to become less restrictive i.e., the exponent decreases, as purpose of trip becomes more important. 13/

Effect of Terminal Time To determine the effect that the inclusion of terminal time into measurement of separation would have on the exponent of distance, an analysis was made of travel patterns in Baltimore, Maryland.

Figure A-1 shows the interchange ratio, actual trips divided by probable trips, plotted on log-log-co-ordinates. (A straight line on this type of paper has an equation of the form $Y=K X^{2}$ where $a$, the exponent, is equal to the slope of the line.. As the slope of the line becomes steeper, the exponent increases. A 45 degree line, has an exponent of $\pm 1)$

The Baltimore data are plotted twice, once using driving time alone as a measure of separation and again, using driving plus 6 minutes of terminal time as the measure of separation. The Detroit data is presented to show the similarity between the two cities and is plotted using driving time alone as the measure of separation. Observe in Figure A-1 that when the Baltimore data were plotted using driving time alone the slope of the line does not remain constant but increases as driving time increases. However, when separation is expressed in terms of driving time plus terminal time, this variation in slope (and exponent) is greatly decreased. The resultant pattern of solid points can be approximated with a straight line.

13/ continued purpose see, Carrol and Beuis, "Predicting Local Travel in Urban Areas," Papers on Proceedings of the Regional Science Association, Vol. 3, 1957.

$\ldots$ BALTIMORE DATA (DRIVING TIME ONLY) • - (DRIVING TIME + TERMINAL TIME) ———— DETROIT (DRIVING TIME ONLY)

Figure A-I THE EXPONENT OF DISTANCE IN GRAVITY MODELS

Also to be noted on this figure is the decreasing effect that the inclusion of terminal time has on the slope of the line as total separation increases. There would be practically no difference between the slopes of lines dram through either the solid points or the open points when the distance of separation becomes greater than 50 minutes. For this reason it may be concluded that the results of the empirical examinations of interurban travel, where most of the trips would be greater than 40 minutes, are not in appreciable error because of the ommission of terminal time in the measurement of separation. Such studies of interurban travel have resulted in exponents of 2.5 to 3.0 .

The slopes of the free hand lines approximating the relationship between the interchange ratio and separation, measured in terms of minutes of driving time plus terminal lines, result in the following exponents: Work trips 2.20, Social trips 2.35, and Shopping trips 3.00. These exponents are in general agreement with those resulting from the interurban examinations and were utilized to calculate accessibility in the present study.

Implicit in the use of these exponents, is the assumption that the findings of the Baltimore study are applicable to the Washington area. While not strictly valid, this assumption should not produce an appreciable error in the accessibility calculations. First, because the exponent for any particular purpose of trip does not show great variation between cities (See table A-1); and second, because the relative measures of accessibility are not particularly sensitive to small variations in the exponent of distance.

## APPENDIX B

## CALCULAITION OF ACCESSIBILITY

Prior to applying the accessibility formulas developed in the body of this thesis it is necessary to consider three important aspects of accessibility. These elements can best be expressed in the form of three questions. Accessible to what? Accessible by what means of travel? Accessible at what point in time? In the present examination these questions were for the most part answered or partially answered by the limitations of the available data; however, to assist in possible future empirical tests and to assess the consequences of data limitations in the present analysis it is desirable to discuss briefly each of these aspects of accessibility.

Accessible to what? First it should be explained that this question does not imply a pre-evaluation of the relative importance of accessibility to different types of activities; but rather that it is necessary to determine which of the limitless types of accessibility are going to be calculated and compared to the pattern of growth. Also that it is necessary to determine the units which are to be used to express the size of the activity to which accessibility is being calculated.

The method used in the present examination to determine the types of accessibility to be calculated, was to observe the types of activities with which people had repeated contact. Using the results of various origindestination surveys it was found that on the average 80 percent of all the trips made in an urban area had either their origin or destina-
tion at home; 40 percent had their origin or destination at work; 20 percent at shopping or commercial recreation areas; 20 percent at residential areas for social purposes; and the remaining 40 percent at a variety of places for purposes such as medical, dental, change travel mode, etc. (Note: each trip has both an origin and destination hence the 200 percent.) With the mejority of trips, other than "home", concentrated in the work, social, and shopping catagories these were the types of accessibility which were calculated. It could be argued that frequency of contact is but one aspect of indicating the importance of accessibility; however, lacking any other criteria, the determination was based solely on this factor. Other types of accessibility should be evaluated in light of residential growth in future examinations.

Once the types of activities to which accessibility is to be calculated have been determined, the units to measure the relative size of the activity areas must be developed. In the present analysis, total employment ( number of jobs ) was used to indicate the number of employment opportunities in an area. Total population (number of people) was used to indicate the number of opportunities for social interaction offered by an area. Annual retail sales ( dollars) were used to indicate the activity level or opportunities for shopping interaction offered by an area. It is possible that these measures could be modified so that they would indicate better the actual opportunities for interaction offered by each zone or area. For example, instead of total employment, the average annual number of job openings would probably be a better measure of employment opportunities. Opportumities for social
interaction could be measured by some combined index of number of people, average income, and social status. Numerous other modifying factors could be developed; however, only through empirical analysis will the desirability of their inclusion into the accessibility calculations be assessed.

It should be remembered that absolute figures are unimportant in expressing the size or activity level of an area. The model only considers the size of one area relative to the size of other areas, whether this relationship is expressed as 1 to 2 or 1,000 to 2,000 makes no difference.

Accessible by what means of travel ? It will be noted that in the formula for calculating accessibility the distance between any given pair of zones can only assume one value at any particular point in time. However, the actual distance between any pair of zones can have a variety of values depending upon the mode of travel used. This weakness is present in all locational analysis proceedures and as yet no adequate solution has been offered. One method which has been used is to express interzone distance in terms of average costs or average time; however, this is at best an approximation, an in addition, before these average values can be computed it is necessary to know what proportion of each interzone movement utilizes each mode of travel, which in some cases is the intended output of the model.

In this analysis distance is expressed in terms of off-peak driving time plus 5 to 8 minutes of terminal time. No attempt was made to adjust for mass transit. In this particular case this omission does not intwoduce
an appreciable error into the accessibility calculations because the Washington, D.C. area has only surface transit (streetcars and buses) and the interzone travel time by transit does not vary from that by car. If a similar examination were going to be made for a city such as Boston, some method would have to be devised to account for transit or the relative accessibility of the central area would be much too low.

Accessible at what point in time ? One of the major problems encountered in this study was to determine which pattern of accessibility should be compared to the observed distribution of residential growth. Theoretically, the accessibility which existed at the exact time the land development occured should have been used; however, the process of growth is continuous and the pattern of accessibility is constantly changing. In Washington only the total growth over a seven year period was available. Data was not available to break this into smaller increments. Also, the only travel time data available was for the end point of this seven year interval. For these reasons the growth which occured from 1948 to 1955 was compared to the pattern of accessibility in 1955. It is now felt that this proceedure possibly introduced an appreciable unknown into the analysis and attempts are now being made to estimate travel times for 1948 and 1952 and also to divide the total growth into two increments.

When these data become available the residential growth which occured between 1948 and 1952 will be compared to the average accessibilty during this period. The remaining residential growth will be compared to the 1952-1955 average accessibility values. It is hoped that this proceedure will improve the general findings of this thesis.

| ACCES | IBIITITY CAL | UATIONS FOR | ZONE $\quad 37$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| to | TOTAL | TOTAL | RETAIL | TRAVEL ${ }^{*}$ | FRIC | ION | mors | ACCESSIB | ITTY TO | *** |
| ZONE | POPULATION | EMPLOYMENT | $\begin{aligned} & \text { SAIES } \\ & (000) \$ \end{aligned}$ | $\begin{aligned} & \text { TTME } \\ & (\min .) \end{aligned}$ | $\mathrm{F}_{1}$ | $\mathrm{F}_{2}$ | $\mathrm{F}_{3}$ | $\begin{gathered} \text { POPULATION } \\ A_{1} \\ \hline \end{gathered}$ | $\begin{gathered} \text { EMPLOYMENT } \\ \mathrm{A}_{2} \end{gathered}$ | $\begin{gathered} \text { SALES } \\ \mathrm{A}_{3} \\ \hline \end{gathered}$ |
| 01 | 12,218 | 45,355 | - | 24 | . 39 | . 41 | . 30 | 4,770 | 18,550 | - |
| 02 | - | 25,696 | - | 27 | . 30 | . 32 | . 21 | - | 8,230 | - |
| 03 | 11,999 | 35,482 | 5,684 | 23 | , 43 | ,45 | . 35 | 5,150 | 16,000 | 1,990 |
| 04 | 27,360 | 11,257 | 5,684 | 27 | . 53 | . 55 | . 45 | 14,500 | 6,200 | 2,550 |
| - | - | - | - | - | - |  |  |  |  | - |
| $\dot{8}$ | 13,764 | 1,602 | 400 | 34 | .17 | . 19 | . 11 | 2,330 | 300 | - 40 |
|  |  |  |  |  |  |  |  |  |  |  |

** Inverse of travel time $: F_{1}=\frac{1}{t t^{2.35}} \quad F_{2}=\frac{1}{t t^{2.20}} \quad F_{3}=\frac{1}{t t^{3.00}}$
*** $A_{1}=F_{1}$ times population; $A_{2}=F_{2}$ times employment; $A_{3}=F_{3}$ times Retail sales

Figure B-1 SAMPLE CALCULATIONS OF ACCESSIBIITTY

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APPENDIX C GALCULATION OF RESIDENFIAI DEVELOPMENT RAIIOS
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The residential development ratio is defined as the ratio of actual growth to the prorated growth, where the prorated growth of any area 1 is equal to the total metropolitan growth divided by the total vacant land in the metropolitan area and multiplied by the vacant land in area $i$. In other words, the proportion of the total prorated growth in area $i$ is equal to area i's proportion of the total vacant land in the metropolitan region.

In Washington, the amount of vacant land in each zone was not directly available; therefore, to estimate this item the following proceedure was used. First, the amount of vacant land, residential land, and total number of dwelling units in each zone was known. for 1955 , The change in the number of dwelling units in each zone between 1948 and 1955 was known. The following equation was used to estimate the vacant land in each zone in 1948.

1948 Vacant Land $=1955$ Vacant Land +1955 Residential Land $\times$ 1955-48 D.U. 1955 D.U.

The results of these calculations were adjusted in specific cases where it was known that the density of development which occured between 1948 and 1955 was significantly different than that which occured prior to 1948. These adjustments were for the most part quite minor. Table C - 1 gives a summary of these calculations along with the 1955 accessibility index for each zone.

Table ，G－1 CALCULATION OF RESIDENTIAL DEVELOPMENT RATIO \＆SUMMARY OF ACCESSIBILITY VALUES

| ${ }_{\text {zas }}$ | dmaxrso unis | daxziom unim | dmalmo wits | reatamite Livo | vowt Lind ${ }^{1 /}$ | viewir wid | vacut lasp | HRowas meveip | AOTVIS penticer． | vestratil |  |  |  | Thant Tus <br> TO C．B．D． | nestramina <br> vessert 1955 <br> （D，V，／Aasial |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 198 | 1955－1948 | （ 4 CRES3） 1995 | 1955－1948 | 195 | 1988 （tart） | noil | now | 20 | उगकwer | TOULTOT |  |  |  |
| 1 | 130 | 65 | 65 | 2 | 1 | $\checkmark$ | 1 | 2 | 43 | $2=00$ | 100 | 173 | 264 | 11 | 650 |
| 2 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | － | 110 | 160 | 280 | 11 | － |
| 3 | 5562 | 7315 | 2247 | $=76$ | 41 | 3 | 44 | 90 | 1500 | 1690 | 105 | 192 | 311 | 11 | 540 |
| 4 | 3040 | 2032 | － | 56 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | 5 | $\checkmark$ | 117 | 203 | 346 | 11 | 540 |
| 5 | 564 | 566 | － | 1 | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | 121 | 177 | 316 | 11 | 5640 |
| 6 | 5202 | $61=9$ | 1073 | 72 | 8 | $\checkmark$ | 0 | 16 | 716 | $4480^{\circ}$ | 114 | 197 | 350 | 11 | 1280 |
| 7 | 7377 | 5282 | 2085 | 32. | $\stackrel{ }{ }$ | $\checkmark$ | 9 | 19 | 1390 | 7300 | 114 | 164 | 326 | 11 | 2 300 |
| 3 | 4367 | 3778 | 609 | 50 | 7 | $\checkmark$ | 7 | 14 | 406 | 2800 | 107 | 187 | 329 | 12 | 880 |
| 9 | 2594 | 1991 | 603 | 47 | 11 | 7 | 18 | 37 | 402 | 1080 | 114 | 215 | 354 | 13 | 550 |
| 11 | 11037 | 6367 | 2670 | 401 | 97 | 20 | 117 | 240 | 1780 | 740 | 86 | 162 | 282 | 16 | 260 |
| 12 | 2212 | 1507 | 705 | $5: 2$ | 166 | $\checkmark$ | 166 | 341 | 470 | 140 | 67 | 134 | 236 | 17 | 42 |
| 13 | 1802 | 1020 | 762 | 474 | 205 | 77 | 202 | 580 | 521 | 20 | 49 | 104 | 115 | 21 | 30 |
| 16 | 3207 | 1832 | 1375 | 785 | 337 | 633 | 270 | 2000 | 216 | 46 | 39 | 95 | 153 | 26 | 41 |
| 17 | 1058 | 479 | 579 | 498 | 273 | $1: 27$ | 1400 | 2800 | 386 | 14 | 27 | 57 | 61 | 29 | 21 |
| 22 | 654 | 6375 | 2166 | 324 | 100 | 20 | 120 | 250 | 1445 | 580 | 75 | 142 | 263 | 17 | $2=0$ |
| 23 | 6575 | 5193 | 1377 | 562 | 120 | 22 | 140 | 290 | 216 | 320 | 61 | 138 | 236 | 20 | 117 |
| 24 | 3669 | 2095 | 274 | 468 | 117 | 41 | 150 | 330 | 650 | 200 | 50 | 126 | 204 | 21 | 83 |
| 25 | $4=20$ | $3: 20$ | 294 | 522 | 126 | 42 | 168 | 350 | 662 | 190 | 47 | 125 | 169 | 25 | 79 |
| 26 | 6189 | $25 \leq 3$ | 2646 | 236 | 320 | 312 | 710 | 1460 | 1760 | 120 | 45 | 110 | 150 | 26 | 66 |
| 27 | 3521 | 2442 | 1079 | 061 | 264 | 326 | 590 | 1210 | 672 | 55 | 43 | 115 | 164 | 28 | 41 |
| $=0$ | 3727 | 2725 | 1002 | 218 | 246 | 422 | 66 e | 1370 | 667 | 49 | 33 | 79 | 91 | 29 | 41 |
| $=9$ | 6754 | 2066 | 4688 | 1438 | 1000 | 570 | 1270 | 4050 | 2120 | 77 | 30 | 8 \％ | 76 | 37 | 47 |
| 31 | 6510 | $4{ }_{4} 32$ | 1878 | 110 | 52 | 6 | so | 76 | 1255 | 1650 | $=05$ | 198 | 340 | 14 | ¢90 |
| 32 | 4－723 | 37402 | 6321 | CO2 | 87 | 6 | 23 | 291 | 4220 | 2210 | 94 | 214 | 322 | 14 | 730 |
| 33 | 16536 | 14039 | 3687 | 442 | 23 | 33 | ： 26 | 259 | 二 000 | 1000 | 76 | 129 | 277 | 17 | 420 |
| 34 | 12395 | 12092 | 303 | 256 | 15 | 16 | 31 | 64 | 204 | 二20 | 63 | 178 | 255 | 20 | 206 |
| 35 | 11400 | 6040 | 2352 | 005 | 253 | 08 | 342 | 704 | 2240 | 320 | 54 | 141 | 231 | 21 | 132 |
| 36 | 7332 | 4283 | 2349 | 1290 | 375 | 386 | 763 | 1560 | 1570 | 200 | 45 | 128 | 200 | 25 |  |
| 27 | 13744 | 4337 | 5407 | 2344 | $8=0$ | 709 | $16=9$ | 2350 | 0260 | $\pm$ ¢ | 44 | 121 | 200 | 24 | 100 |
| 30 | 7330 | 2561 | 2760 | 1254 | 595 | 743 | 1320 | 2740 | 2520 | 92 | 33 | 92 | 93 | 31 |  |
| 41 | $8=64$ | $7 \pm 56$ | 1008 | 181 | 22 | 5 | 27 | E 5 | 667 | 1210 | 97 | 212 | 282 | 14 |  |
| 42 | 3404 | $32=1$ | 263 | 297 | 2 2 | 25 | 47 | 87 | $=76$ | 二eo | 65 | 161 | 157 | 20 |  |
| 43 | 8663 | －0 43 | 2840 | © © 6 | 267 | $=24$ | 391 | 605 | 2560 | $3=0$ | $=6$ ． | $1: 1$ | 120 | 12 |  |
| 44 | 4530 | 2000 | 332 | ass | 11 | 10 | 21 | 43 | 221 | 510 | 20 | 151 | 312 | 14 |  |
| 45 | 0664 | 1265 | 1459 | 065 | 548 | $2=0$ | 1470 | ：0 $=0$ | 2670 | 120 | 40 | 101 | ： 04 | 26 | － 00 |
| 46 | 9272 | $70 \leq 3$ | 二2s） | 586 | 242 | 300 | 542 | 1210 | 1425 | 155 | 47 | 117 | －53 | 20 |  |
| 47 | 2094 | 2016 | 1878 | 834 | $2=0$ | 1047 | 1303 | － 6 © 0 | $12=5$ | 47 | 35 | $=09$ | 103 | 23 | 73 |

Table C-I Con't.

| $\underset{\text { moresk }}{\text { zars }}$ | Deaturno unrm <br> 1955 | D 1948 | $\begin{gathered} \text { Dmazamo wixis } \\ 1955=1948 \end{gathered}$ | Resimemal Land <br> ( Acress ) 1995 | $\begin{aligned} & \text { Mown Lurd }{ }^{\frac{3}{4}} \\ & \text { l9ss - } 1948 \end{aligned}$ | $\begin{aligned} & \text { manr wid } \\ & \text { 195s } \end{aligned}$ | vachet Lawd 1948 ( EsT.) | PRCBARTE IEvELOP, <br> \& © TOTAL | aOmal mevilop. <br> \& or rotal | mastiantiac | nooseersmitr minax |  |  |  | restramith <br> pmsity 1955 <br> (D.V. / AOME |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  | neroneer | Portarion | Sommo |  |  |
| 48 | 5300 | 1857 | 3443 | 819 | 533 | 1270 | 1803 | 3700 | 2300 | 62 | 30 | 90 | 80 | 27 |  |
| 49 | 2667 | 1643 | 1024 | 473 | 181 | 416 | 597 | 1225 | 684 | 56 | 28 | 79 | 76 |  | $65$ |
| 51 | 13510 | 11030 | 2480 | 281 | 52 | V | 52 | 107 | 1655 | 1550 | 94 | 198 | 234 | 15 |  |
| 52 | 11392 | 8326 | 3073 | 714 | 192 | 146 | 338 | 695 | 2050 | 300 | 52 | 151 | 145 |  |  |
| 53 | 7677 | 4888 | 2789 | 461 | 168 | 105 | 273 | 562 | 1860 | 330 | 52 | 147 | 133 | 22 | 165 |
| 56 | 5102 | 1472 | 3630 | 671 | 478 | 2030 | 2508 | 5160 | 2420 | 47 | 34 | 92 | 82 | 26 | $76$ |
| 57 | 5204 | 1984 | 3220 | 707 | 436 | 3258 | 3694 | 7580 | 2150 | 28 | 29 | 62 | 64 | 29 | 74 |
| 61 | 7155 | 6596 | 559 | 193 | 15 | $\checkmark$ | 15 | 31 | 374 | 1210 | 87 | 146 | 185 | 15 |  |
| 62 | 12397 | 9830 | 2567 | 297 | 61 | $\checkmark$ | 61 | 125 | 1715 | 1370 | 90 | 180 | 22.6 | 14 | 420 |
| 63 | 12494 | 7601 | 4893 | 573 | 224 | 143 | 367 | 754 | 3260 | 430 | 55 | 115 | 115 | 20 |  |
| 64 | 19715 | 11236 | 8479 | 1097 | 471 | 302 | 773 | 1590 | 5660 | 360 | 67 | 161 | 173 | 18 | 180 |
| 66 | 5218 | 2309 | 2909 | 658 | 367 | 2252 | 2619 | 5380 | 1940 | 36 | 34 | S 6 | 73 |  | eo |
| 67 | 3912 | 437 | 3475 | 428 | 380 | 1675 | 2055 | 4220 | 2310 | 55 | 43 | 26 | 84 | 22 | 91 |
| 71 | 111 | 111 | $\checkmark$ | 10 | $\checkmark$ | $\checkmark$ | v | ~ | - | $\checkmark$ | 92 | 130 | 249 | 17 | 110 |
| 72 | 2972 | 3640 | 668 - | 409 | $\checkmark$ | 295 | 295 | 605 | 446 - | $\checkmark$ | 87 | 141 | $2=0$ | 17 | 73 |
| 73 | 3874 | 3401 | 473 | 371 | 45 | 126 | 171 | 352 | 315 | 90 | 83 | 134 | 210 | 17 |  |
| 74 | 11004 | 7703 | 3301 | 1208 | 361 | 415 | 776 | 1590 | 2200 | 130 | 52 | 98 | 110 | 21 | 21 |
| 75 | 5615 | 411 | 5204 | 1097 | 1020 | 2551 | 3571 | 7350 | 3470 | 47 | 45 | 77 | 77 | 23 |  |
| 77 | 8 579 | 7021 | 1558 | 350 | 63 | 145 | 200 | 420 | 1040 | 240 | 63 | 109 | 176 | 18 |  |
| 78 | 13551 | 9615 | 3936 | 1789 | 520 | 287 | 807 | 1660 | 2630 | 160 | 56 | 100 | 145 | 22 |  |
| 81 | 12881 | 9292 | 3569 | 245 | 264 | 123 | 387 | 795 | 2390 | 300 | 8 3 | 134 | 233 | 18 | 136 |
| 62 | 6524 | 4985 | 1539 | 204 | 213 | 313 | 526 | 1080 | 1030 | 95 | 53 | 107 | 155 | 21 |  |
| 63 | 4533 | 2922 | 1611 | 1280 | 455 | 730 | 1185 | 2440 | 1075 | 44 | 51 | 105 | 132 | 20 | 35 |
| 84 | 6165 | 2482 | 5683 | 1267 | 755 | 261 | 1016 | 2090 | 2450 | 120 | 47 | 23 | 103 | 24 |  |
| 06 | 6390 | 2418 | 3972 | 1432 | 891 | 1373 | 2264 | 4660 | 2650 | 57 | 30 | 70 | 66 | 29 |  |
| 87 | 4002 | 362 | 3640 | 1175 | 1070 | 1198 | 2268 | 4660 | 2430 | 52 | 33 | 73 | 64 | 28 |  |
| 88 | 1881 | 300 | 1501 | 1899 | 1510 | 2854 | 4364 | 8970 | 1000 | 11 | 24 | 55 | 48 | 29 | 10 |
| mones | 465779 | 315839 | 149940 | 40502 | 17989 | 30719 | 48708 | 100102 | . 9252 |  |  |  |  |  |  |

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[^1]:    * Distance is expressed in minutes of offpeak driving time plus 5 to 8 minutes of terminal time.

