Circuits

The Circuit Abstraction

Circuits represent systems as connections of component
• through which currents (through variables) flow and
• across which voltages (across variables) develop.

The Circuit Abstraction

Circuits are important for two very different reasons:
• as physical systems
  – power (from generators and transformers to power lines)
  – electronics (from cell phones to computers)
• as models of complex systems
  – neurons
  – brain
  – cardiovascular system
  – hearing

The Circuit Abstraction

Circuits are the basis of our enormously successful semiconductor industry.

What is a Circuit?

Circuits are connections of components
• through which currents (through variables) flow and
• across which voltages (across variables) develop.

Figure by MIT OpenCourseWare.
Rules Governing Flow

**Rule 1:** Currents flow in loops.

*Example: flow of electrical current through a flashlight*  

When the switch is closed, electrical current flows through the loop. The same amount of current flows into the bulb (top path) and out of the bulb (bottom path).

\[ i_1 = i_2 + i_3. \]

The dot represents a “node” which represents a connection of two or more segments.

**Nodes**

Nodes are represented in circuit diagrams by lines that connect circuit components.

The following circuit has three components, each represented with a box.

There are two nodes, each indicated by a dot. The net current into or out of each of these nodes is zero. Therefore \( i_1 + i_2 + i_3 = 0. \)

What is a Circuit?

Circuits are connections of components

- through which currents (through variables) flow and
- across which voltages (across variables) develop.

**Rules Governing Voltages**

Voltages accumulate in loops.

*Example: the series combination of two 1.5 V batteries supplies 3 V.*

\[ 1.5 \text{ V} + 1.5 \text{ V} = 3 \text{ V}. \]

**Kirchoff’s Voltage Law:** the sum of the voltages around a closed loop is zero.
Alternative Representation: Node Voltages

Node voltages represent the voltage between each node in a circuit and an arbitrarily selected ground.

Node voltages and component voltages are different but equivalent representations of voltage.
- **Component voltages** represent the voltages across components.
- **Node voltages** represent the voltages in a circuit.

Rules Governing Components

Each component is represented by a relationship between the voltage across the component to the current through the component.

Examples:

\[ v = iR \quad \text{(regardless of } i) \]
\[ v = V_0 \quad \text{(regardless of } v) \]

Node-Voltage-and-Component-Current (NVCC) Method

Combining KCL, node voltages, and component equations leads to the NVCC method for solving circuits:
- Assign **node voltage variables** to every node except ground (whose voltage is arbitrarily taken as zero).
- Assign **component current variables** to every component in the circuit.
- Write one **constitutive relation** for each component in terms of the component current variable and the component voltage, which is the difference between the node voltages at its terminals.
- Express **KCL** at each node except ground in terms of the component currents.
- **Solve** the resulting equations.

Analyzing Simple Circuits

Analyzing simple circuits is straightforward.

The voltage source determines the voltage across the resistor, \( v = 1V \), so the current through the resistor is \( i = \frac{v}{R} = \frac{1}{1} = 1 \text{A} \).

The current source determines the current through the resistor, \( i = 1 \text{A} \), so the voltage across the resistor is \( v = iR = 1 \times 1 = 1V \).

Check Yourself

What is the current through the resistor below?

1. 1A
2. 2A
3. 0A
4. cannot determine
5. none of the above

Common Patterns

There are a number of **common patterns** that facilitate design and analysis:
- series resistances
- parallel resistances
- voltage dividers
- current dividers
Series Combinations
The series combination of two resistors is equivalent to a single resistor whose resistance is the sum of the two original resistances.  

\[ v = R_1i + R_2i \quad \text{and} \quad v = R_si \]

\[ R_s = R_1 + R_2 \]

The resistance of a series combination is always larger than either of the original resistances.

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Parallel Combinations
The parallel combination of two resistors is equivalent to a single resistor whose conductance (1/resistance) is the sum of the two original conductances.  

\[ i = \frac{V}{R_1} \quad \text{and} \quad \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \]

\[ R_p = \frac{R_1R_2}{R_1 + R_2} \equiv R_1||R_2 \]

The resistance of a parallel combination is always smaller than either of the original resistances.

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Check Yourself
What is the equivalent resistance of the following circuit.

```
1 2 3 4 5
1 2 3
V
```

1. 1 2. 2 3. 0.5 4. 3 5. 5

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Voltage Divider
Resistors in series act as voltage dividers.  

\[ I = \frac{V}{R_1 + R_2} \]

\[ V_1 = R_1I = \frac{R_1}{R_1 + R_2}V \]

\[ V_2 = R_2I = \frac{R_2}{R_1 + R_2}V \]

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Current Divider
Resistors in parallel act as current dividers.  

\[ V = (R_1||R_2)I \]

\[ I_1 = \frac{V}{R_1} = \frac{R_1||R_2}{R_1}I = \frac{R_1R_2}{R_1 + R_2}I = \frac{R_2}{R_1 + R_2}I \]

\[ I_2 = \frac{V}{R_2} = \frac{R_1||R_2}{R_2}I = \frac{R_1R_2}{R_1 + R_2}I = \frac{R_1}{R_1 + R_2}I \]

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Check Yourself
Find \( V_o \).

\[ +12V \]

\[ +8\Omega \quad +12\Omega \quad 6\Omega \quad +V_o \]

\[ - \]
Loading

Adding (or changing the value of) a component alters all of the voltages and currents in a circuit (except in degenerate cases).

Consider identical light bulbs connected in series across a battery.

Because the same current passes through both light bulbs, they are equally bright.

Check Yourself

What happens if we add third light bulb?

Closing the switch will make
1. bulb 1 brighter 2. bulb 2 dimmer
3. 1. and 2. 4. bulbs 1, 2, & 3 equally bright
5. none of the above

Loading

Loading did not occur in LTI systems.

Example: adding \( H_2 \) has no effect on \( Y \)

\[
Y = H_1X \text{ regardless of } H_2.
\]

Q: What’s different about a circuit?

A: A new component generally alters the currents at the nodes to which it connects.

Buffering

Effects of loading can be diminished or eliminated with a buffer.

An “ideal” buffer is an amplifier that
- senses the voltage at its input without drawing any current, and
- sets its output voltage equal to the measured input voltage.

We will discuss how to use op-amps to make buffers in next lecture.

Summary

Circuits represent systems as connections of components
- through which currents (through variables) flow and
- across which voltages (across variables) develop.

There are a number of common patterns that facilitate design and analysis:
- series resistances
- parallel resistances
- voltage dividers
- current dividers

Buffers eliminate loading and thereby simplify design and analysis.