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High Directive Antenna with Virtual Aperture

Baile Zhang¹, Bae-Ian Wu^{*1}, and Hongsheng Chen^{1,2}

¹ Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA.

E-mail: biwu@mit.edu

² The Electromagnetics Academy at Zhejiang University, Zhejiang University, Hangzhou 310058, China.

Introduction

Transformational electromagnetics have recently attracted a lot of interests [1, 2]. It is possible to manipulate the electromagnetic waves to achieve interesting new effects. The previous idea of “perfect lens” has also been reinterpreted using coordinate transformation [3]. Recent studies showed that a passive object, like an object of PEC (perfect electric conductor), can be projected into an empty space at another position, and it can be used to conceal an empty entrance [4] and cloak an object exterior to the cloak [5]. Here we present a spherical shell structure that is able to form a virtual aperture in empty space by projecting a real aperture within the structure. The beam will be radiated as if it is from the virtual aperture. Reducing the dimensions of radiators is always of great interest in the antenna engineering community. It is well known that there is no mathematical upper limit to the level of directivity gain of an antenna of given size [6]. However, there still lacks a general design capable of producing arbitrarily large directivity. In our proposed model, any large directivity is possible to be physically achieved.

Theoretical Analysis

The proposed spherical shell structure is realized by taking a coordinate transformation in spherical coordinate system from virtual space $(\tilde{r}, \tilde{\theta}, \tilde{\phi})$ to physical space (r, θ, ϕ) as follows,

$$\tilde{r} = \frac{c}{a}r \ (r < a), \tilde{r} = -Ar + B \ (a < r < b), \tilde{r} = r \ (r > b) \quad (1)$$

where a and b are the inner and outer radii of the spherical shell, c is an arbitrary radius satisfying $c > b$, $A = \frac{c-b}{b-a}$ and $B = \frac{b(c-a)}{b-a}$. Using this transformation, the empty sphere of radius c is compressed into a smaller sphere of radius a , and the empty shell between $r = c$ and $r = b$ is mapped into another shell between $r = a$ and $r = b$. Under this transformation, the constitutive parameters of the material inside the core can be specified as $\mu_1 = \frac{c}{a}\mu_0$ and $\epsilon_1 = \frac{c}{a}\epsilon_0$. The material inside the shell has permittivity and permeability as the tensor forms: $\bar{\mu} = \mu_r \hat{r}\hat{r} + \mu_t(\hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi})$ and $\bar{\epsilon} = \epsilon_r \hat{r}\hat{r} + \epsilon_t(\hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi})$, where $\frac{\mu_r}{\mu_0} = \frac{\epsilon_r}{\epsilon_0} = \frac{(-Ar+B)^2}{r^2(-A)}(1-i\delta)$ and $\frac{\mu_t}{\mu_0} = \frac{\epsilon_t}{\epsilon_0} = -A(1-i\delta)$. Here δ corresponds to the loss tangent of all parameters.

We decompose the fields into TE and TM modes with respect to \hat{r} by introducing TE and TM scalar potentials, Φ_{TE} and Φ_{TM} . The expressions of Φ_{TE} and Φ_{TM} inside the shell can be obtained by applying the formula derived in [7], which gives

the expression $\Phi = k_c \tilde{r} b_n(k_c \tilde{r}) P_n^m(\cos \theta) e^{im\phi}$, where $k_c = \omega \sqrt{\mu_0 \epsilon_0} (1 - i\delta)$ and b_n is the spherical Bessel function of n th order.

Consider an electric dipole oriented in the direction of $\hat{\alpha}$ and located in (r', θ', ϕ') in the core. We can first expand the field from this dipole by assuming there is no reflection from the inner boundary as follows,

$$\Phi_{TM}^{inc} = \frac{I\ell}{k_1 \mu_1} \hat{\alpha} \cdot \nabla' \times \nabla' \times \hat{r}' \sum_{n=1}^{\infty} \sum_{m=-n}^n i \frac{2n+1}{4\pi n(n+1)} \frac{(n-|m|)!}{(n+|m|)!} \quad (2)$$

$$\cdot \psi_n(kr_{<}) \zeta_n(kr_{>}) P_n^{|m|}(\cos \theta') P_n^{|m|}(\cos \theta') e^{im(\phi-\phi')},$$

$$\Phi_{TE}^{inc} = \frac{i\omega \mu_1 \epsilon_1 I\ell}{k_1} \hat{\alpha} \cdot \nabla' \times \hat{r}' \sum_{n=1}^{\infty} \sum_{m=-n}^n i \frac{2n+1}{4\pi n(n+1)} \frac{(n-|m|)!}{(n+|m|)!} \quad (3)$$

$$\cdot \psi_n(kr_{<}) \zeta_n(kr_{>}) P_n^{|m|}(\cos \theta') P_n^{|m|}(\cos \theta') e^{im(\phi-\phi')},$$

where $r_{<} = \min\{r, r'\}$ and $r_{>} = \max\{r, r'\}$. By denoting the coefficients of Φ_{TM}^{inc} and Φ_{TE}^{inc} as F_{mn}^{TM} and F_{mn}^{TE} when $r > r'$, we can express the reflected waves in the core as

$$\Phi_{TM}^{ref} = \sum_{n=1}^{\infty} \sum_{m=-n}^n F_{mn}^{TM} R_{mn}^{TM} \psi_n(k_1 r) P_n^{|m|}(\cos \theta) e^{im\phi},$$

$$\Phi_{TE}^{ref} = \sum_{n=1}^{\infty} \sum_{m=-n}^n F_{mn}^{TE} R_{mn}^{TE} \psi_n(k_1 r) P_n^{|m|}(\cos \theta) e^{im\phi}. \quad (4)$$

The potentials in the shell and outside can be subsequently expressed as

$$\Phi_{TM}^{shell} = \sum_{n=1}^{\infty} \sum_{m=-n}^n F_{mn}^{TM} (c_{mn}^{TM} \zeta_n(k_c \tilde{r}) + d_{mn}^{TM} \psi_n(k_c \tilde{r})) P_n^{|m|}(\cos \theta) e^{im\phi}, \quad (5)$$

$$\Phi_{TE}^{shell} = \sum_{n=1}^{\infty} \sum_{m=-n}^n F_{mn}^{TE} (c_{mn}^{TE} \zeta_n(k_c \tilde{r}) + d_{mn}^{TE} \psi_n(k_c \tilde{r})) P_n^{|m|}(\cos \theta) e^{im\phi}, \quad (6)$$

$$\Phi_{TM}^{out} = \sum_{n=1}^{\infty} \sum_{m=-n}^n F_{mn}^{TM} T_{mn}^{TM} \zeta_n(k_0 r) P_n^{|m|}(\cos \theta) e^{im\phi}, \quad (7)$$

$$\Phi_{TE}^{out} = \sum_{n=1}^{\infty} \sum_{m=-n}^n F_{mn}^{TE} T_{mn}^{TE} \zeta_n(k_0 r) P_n^{|m|}(\cos \theta) e^{im\phi}, \quad (8)$$

where T_{mn}^{TE} and T_{mn}^{TM} are the transmission coefficients.

By matching the boundary conditions at $r = a$ and $r = b$, we can solve the field distribution in all regions. It is seen that when $\delta = 0$, $R_{mn}^{TM} = R_{mn}^{TE} = 0$ and $T_{mn}^{TM} = T_{mn}^{TE} = a/c$. From the coordinate transformation point of view, the dipole $I\ell \hat{\alpha}$ at (r', θ', ϕ') now have a image $I\ell' \hat{\alpha}'$ at $(r'c/a, \theta', \phi')$. In the lossless case, an outgoing wave inside the core is perfectly transmitted to the outside without reflection. But the transmitted wave has an amplitude scaling of c/a . This is due to $E^{\lambda'} = \Lambda_{\lambda'}^{\lambda} E^{\lambda}$ with the Jacobian matrix $\Lambda_{\lambda'}^{\lambda} = c/a \bar{I}$ in this case. We can also interpret it from the transformation of the dipole itself. It is known that the

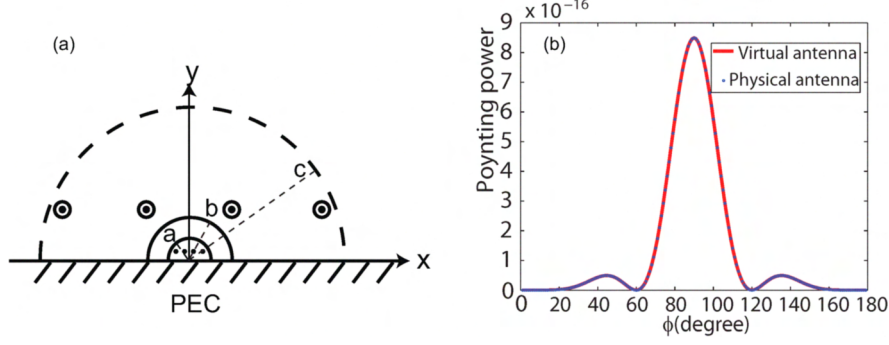


Figure 1: (a) Geometry of a four-dipole antenna array on top of a PEC plane. (b) Comparison of radiation patterns of the physical four-dipole array and the virtual four-dipole array.

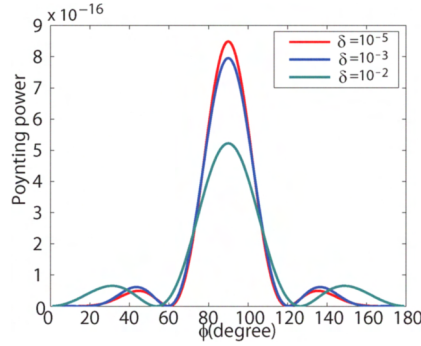


Figure 2: Degradation of directivity with increase of the loss tangent of the constitutive parameters inside the shell. All other parameters follow Fig. 1

current density transforms according to $j^{\lambda'} = |\Lambda_\lambda^{\lambda'}|^{-1} \Lambda_\lambda^{\lambda'} j^\lambda$ and the small volume element transforms according to $dV' = |\Lambda_\lambda^{\lambda'}| dV$. Therefore, an electric dipole $I\ell$ will transform as $I\ell^{\lambda'} = \Lambda_\lambda^{\lambda'} I\ell^\lambda$. In this case, it means $I\ell' = c/a I\ell$. Due to the concern of convergence of the field expansions, the loss tangent δ can not be set to zero. So, a perfect antenna projection is not possible. However, as long as the loss is sufficiently small, we can still construct an acceptable virtual antenna.

Four-dipole antenna array on top of a PEC plane

We consider a four-dipole antenna array on a PEC plane, as shown in Fig. 1(a). The locations of the four dipoles are $(-0.75\lambda_0, 0.25\lambda_0, 0)$, $(-0.25\lambda_0, 0.25\lambda_0, 0)$, $(0.25\lambda_0, 0.25\lambda_0, 0)$ and $(0.75\lambda_0, 0.25\lambda_0, 0)$, each with amplitude $1/\omega$. To construct an equivalent antenna which can project an equivalent virtual aperture same as the original four-dipole antenna but with smaller dimension, we use the spherical shell structure discussed in last section. Due to the ground plane, we only need to consider a semi-spherical structure. We set $c = 0.8\lambda_0$, $b = 0.25\lambda_0$ and $a = 0.15\lambda_0$. The dipoles we put inside the core are now located in $(-0.14\lambda_0, 0.047\lambda_0, 0)$, $(-0.047\lambda_0, 0.047\lambda_0, 0)$, $(0.047\lambda_0, 0.047\lambda_0, 0)$ and $(0.14\lambda_0, 0.047\lambda_0, 0)$, and their amplitudes are $0.1875/\omega$. The loss tangent δ of the constitutive parameters of the shell is set to be 10^{-5} .

In Fig. 1(b) we plot the far field radiation pattern in the xy plane. The radiation pattern from the virtual antenna formed by a smaller shell-core structure is equivalent to that from a physical antenna with a larger dimension. Since c can be arbitrarily large, using this structure with a small dimension we are able to design an arbitrarily large directivity. As mentioned before, the loss tangent δ can not be set as exactly zero as loss is unavoidable in practice. Figure 2 shows that with the increase of loss, the directivity decreases and the beam-width expands.

Conclusion

Here we propose a spherical shell-core structure which is able to achieve arbitrarily large directivity. The structure is obtained from coordinate transformation. A small antenna can be projected to free space with a large dimension, which subsequently leads to a large directivity.

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