

STATIC DEFORMATION OF FLUID-SATURATED ROCKS

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ABSTRACT

The static strain response of porous solids to combinations of confining stress and pore pressure is explained both theoretically and experimentally. The theoretical analysis is a synopsis of linear elasticity principles for porous media taken mainly from Biot (1941), Gassmann (1951), Biot and Willis (1956), and Geertsma (1957). From this analysis the conclusion is made that the "effective stress" of Terzaghi (1923, 1925), which is the difference between hydrostatic confining stress and pore pressure for strain properties, has no theoretical or experimental significance for the static strain response of intact rocks. The Terzaghi effective stress cannot account for the intrinsic bulk strain of minerals, a component of strain response important in consolidated sediments and rocks but not in muds and soils, for which the Terzaghi relation was originally intended. Effective stress "laws" for static deformation proposed by Nur and Byerlee (1971), Garg and Nur (1973), Robin (1973), and Carroll (1979) are shown only to be reformulations of linear elasticity relations. The effective stress so defined has no intrinsic physical meaning. Experimental bulk strain measurements on a suite of rocks as a function of hydrostatic confining stress and pore pressure are presented. Equilibrium strain at any combination of confining stress and pore pressure is predicted on the basis of 1) the zero pore pressure or drained jacketed stress-strain relation, and 2) the unjacketed stress-strain relation. Unjacketed strain measurements with a confining pressure fluid are emphasized as a means of directly measuring the intrinsic modulus of aggregate minerals in rocks. A technique is outlined for experimentally obtaining pore volume or porosity as a function of confining stress from finely digitized unjacketed and jacketed strain data by a straightforward application of linear elasticity principles incrementalized over small data steps. An argument is made, based on the linear elasticity analysis for strain response, that the differential hydrostatic stress, or what is commonly called effective stress, predicts many physical properties exclusive of bulk strain because of 1) the large intrinsic moduli of minerals, and 2) the definition of a stress as a force per unit area is maintained during deformation because of the small strains normally encountered in consolidated rocks and sediments.

INTRODUCTION

A remarkable variety of geological processes and geophysical properties as we know them would not exist on earth without water. Because rocks and sediments are porous in the crust, at least down to the brittle-ductile

transition, water may freely exist as a separate fluid phase at depth. Where lithosphere is subducted water may be carried to even greater depths. As a result of water, or more accurately, aqueous solutions in the crust there are phenomena as diverse as hydrothermal ore deposits, the lowering of solidus temperatures in basaltic magmas, diagenesis, substantial current flow through rock, and acoustic attenuation. Even hydrocarbons and organically-derived gas are possible only because of the water which sustained the original plant life.

The pressure in fluids at depth may be a significant factor in some processes. Pressure in the fluid phase can be generated by gravity, stress waves, tectonic stress, heat, and chemical or electrical potentials. A simple model for an ideal earth is a column of interconnected pore space filled with water in which pore pressure increases with depth proportional to density and the gravitational constant. There are obvious important exceptions, mostly because of the time scales involved. On one hand, some deep mines in crystalline rock are dry and pore pressure is insignificant. In developing sedimentary basins, on the other hand, rapid sedimentation and burial combined with heat and low permeability leads to overpressurized pore fluids. As a result, hydrocarbons are expelled from source rocks, fluids migrate over vast distances, and drilling holes is made interesting. On a shorter time scale, the instantaneous compressibility for an acoustic stress wave in rock may create local pressure gradients in the fluid phase because of pore shape. Another example of the importance of pore pressure is in the interpretation of earthquakes and overthrust faulting as complex reactions between regional stress fields and temporal and spatial variations in pore fluid pressure. Several instances have been documented of earthquakes being turned on and off by high-pressure fluid injection in wells.

Because of rock porosity, fluids to fill that porosity, and sources of fluid pressure, the overall mechanical response of rocks and sediments with fluids is determined by a two-component stress system. An overall frame or confining stress is transmitted through grain-to-grain contacts and may vary as a function of direction. The second stress component is pore pressure. Fortunately, fluid cannot support shear except over small distances. In low-frequency or static situations pore pressure is therefore hydrostatic.

A logical analysis of the mechanical response of rocks and sediments would treat confining stress and pore pressure as independent variables. There is, however, a simplifying functional relation generally known as "effective stress". Historically, Terzaghi (1923, 1925) showed in laboratory experiments that uniaxial consolidation and strength of fully-saturated soils is governed by the simple difference between normal stress and pore pressure. This difference became known as the "effective stress", and was identified as the controlling parameter in these types of deformation studies.

Over the years this simple observation has been extended and applied to cover the deformation of rocks and other porous materials besides soils. As other physical properties have been studied in the laboratory the term effective stress has been applied to measurements such as velocity, electrical resistivity, attenuation, and permeability. As a term, effective stress is often loosely used as a synonym for differential stress or pressure, again the simple difference between frame stress or confining pressure and pore pressure. In the literature effective stress sometimes innocuously turns up as a label on the

stress axis. At other times effective stress is embellished as a "law," a "principle," or as a "concept."

What effective stress really means for rocks with pore pressure is not clear. The term, however, is commonly used to interpret properties of rocks and sediments in situations where pore pressure and confining stress are variables. The principal concern here is whether there is any physics involved or if the Terzaghi relation is just a good approximation. In this paper the point is developed that the latter is true. This becomes very apparent when the property measured in the laboratory is static bulk strain.

The main purpose of this study is to investigate the equilibrium or static strain response of rocks to a combination of confining stress and pore pressure. This is the real key to understanding what effective stress means because changes in other physical properties, such as velocity and permeability, can be interpreted and modelled if elastic deformation is described. The first section of this paper is a review of the linear elastic material description of porous solids as initially developed by Biot (1941) for analysis of the three-dimensional consolidation of soils. The analysis is carried out in fair detail because, as will be shown in the second section, what has been proposed as an "effective stress law" for bulk strain (Nur and Byerlee, 1971; Garg and Nur, 1973; Carroll, 1979) is simply a reformulation of the linear elastic description. The "effective stress" so defined has no intrinsic physical meaning and actually obscures true interpretation of strain response. The third section uses some graphical examples to clarify the points.

Perhaps part of the confusion is due to the fragmentary and limited nature of previous experimental efforts aimed toward an understanding of the role of pore pressure and confining stress on static strain in rocks. There have only been a few experimental studies of static rock strain where both confining stress and pore pressure are variables (Van der Knapp, 1959; Nur and Byerlee, 1971).

A major effort of this paper is to present experimental static strain data for a suite of rocks including sandstones, granites, and limestone, where both pore pressure and confining stress are systematically varied. In the fourth, fifth, and sixth sections of this paper the experimental test and results are discussed. Inert nitrogen is used as the pore fluid and confining stress is hydrostatic pressure. Strain is measured with strain gages attached directly to rock samples. The experimental strain data agrees with the predictions of linear elasticity theory or the proposed effective stress laws for bulk strain when nonlinear behavior of the samples is incrementalized over many steps that are each nearly linear. This is possible through the power of the computer in collecting finely digitized experimental stress-strain data.

A second purpose for this study is to examine aspects of experimental strain data that may be useful in modelling other physical property measurements such as velocity, permeability, and resistivity. If the measurement of physical properties as a function of confining stress is a worthwhile effort, then the modelling of these properties with varying pore shapes must ultimately appeal to and agree with strain data. The sixth section of this paper points out how pore volume and porosity can be calculated quite accurately as a function of confining stress from jacketed and unjacketed

stress-strain measurements. The unjacketed strain test, which has been neglected in recent years, is emphasized as a direct measurement of the intrinsic bulk modulus.

LINEAR ELASTICITY OF FLUID-SATURATED POROUS MEDIA

Consider a homogeneous and isotropic porous solid. The following analysis is of a sample element large enough so that the porosity distribution doesn't interfere with the homogeneity assumption. For this sample element stress is defined as total force per unit area and strain is defined as relative displacement between points in the solid phase. Assume that the pores are empty. The strain tensor as a function of stress for this sample element is

$$\varepsilon_{ij} = \frac{1}{2G} (\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}) + \frac{1}{9K}(\sigma_{kk}\delta_{ij}), \quad (1)$$

where ε_{ij} is strain, σ_{ij} is deviatoric stress, σ_{kk} is hydrostatic stress, and δ_{ij} is the Kronecker's delta. The elastic bulk modulus is K and shear modulus is G . Strains are considered infinitesimal so that solid translation and rotational terms are not used. Repeated indices indicate summation. In this form the stress tensor is decomposed into two parts, the left term in parenthesis representing deviatoric stress and the right term representing pure hydrostatic stress. The deviatoric stress changes the shape and strain is a function of the shear modulus; the hydrostatic stress changes the volume and strain is a function of the bulk modulus.

Alternatively, the tensor expression can be expressed with the Poisson's ratio ν of the porous solid. The identity for an isotropic material is

$$K = \frac{2G(1+\nu)}{3(1-2\nu)},$$

and

$$\varepsilon_{ij} = \frac{1}{2G}(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}) + \frac{\nu}{2G(1+\nu)}(\sigma_{kk}\delta_{ij}). \quad (2)$$

Now consider the effect of a pore fluid at an equilibrium pressure of value p introduced throughout the pore space. No bulk shear strain will occur because fluid pressure is hydrostatic and the porous solid is isotropic. The result of pore pressure is a pure volumetric strain, equal in all directions. An additional term is added to the stress-strain relation so that the complete expression becomes

$$\varepsilon_{ij} = \frac{1}{2G}(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}) + \frac{1}{9K}(\sigma_{kk}\delta_{ij}) + \frac{1}{3H^2}p\delta_{ij} \quad (3)$$

Pore pressure p is positive and the sign of the additional term is positive so that bulk strain decreases with an increase in p . The convention used in this analysis is that compression is negative. The constant H was introduced by Biot

(1941) and is simply defined as the ratio between bulk volume strain and pore pressure change p with external stress constant. Biot (1941) and Biot and Willis (1957) suggested appropriate experimental tests for the determination of H . Bulk modulus K is measured with either the pores empty of fluid or else at constant pore pressure, i.e., a drained test.

As can be seen from the form of Eq. (3) the constant H is a modulus similar to the bulk and shear moduli. The constant H can, however, be defined in terms of bulk modulus K and solid matrix modulus K_s of the solid phase with the additional assumption that the solid phase is homogeneous and isotropic. This assumption was used by Gassmann (1951) and later Geertsma (1957). A short derivation follows.

For the sample element under consideration if there is only a hydrostatic confining pressure, such that $\sigma_{ij} = -\sigma\delta_{ij}$, and a pore pressure of value p , the expression for bulk strain ε from Eq. (3) becomes

$$\varepsilon = -\left[\frac{1}{K}\sigma - \frac{1}{H}p\right]. \quad (4)$$

The overall stress field can be decomposed into two stress states, individually analyzed, and the resulting strains summed by virtue of the overall linearity assumption. In Fig. 1 the decomposed stress states are schematically shown. Stress state (1) is an equal and uniform internal and external hydrostatic stress p . The normal stress over every boundary between pore and matrix is uniquely defined as p . An important point is that homogeneity requires that the porosity cannot split the sample, either because the sample element is too small or else because of a fracture. Since the solid phase is homogeneous and isotropic, with bulk modulus K_s , the result is a pure volumetric strain

$$\varepsilon_1 = -\frac{1}{K_s}p. \quad (5)$$

Stress state (2) is an external hydrostatic stress $\sigma - p$ with constant pore pressure and the resulting volumetric strain is

$$\varepsilon_2 = -\frac{1}{K}(\sigma - p) \quad (6)$$

Summation of the two volume strains gives

$$\varepsilon_1 + \varepsilon_2 = -\frac{1}{K}\sigma - \left[\frac{1}{K_s} - \frac{1}{K}\right]p. \quad (7)$$

Comparison with Eq. (4) reveals that

$$\frac{1}{H} = \frac{1}{K} - \frac{1}{K_g} \quad (8)$$

The stress-strain relation in Eq. (3) can therefore be written as

$$\varepsilon_{ij} = \frac{1}{2G} \left(\sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \right) + \frac{1}{9K} (\sigma_{kk} \delta_{ij}) + \frac{1}{3} \left(\frac{1}{K} - \frac{1}{K_g} \right) p \delta_{ij} \quad (9)$$

This is the overall stress-strain relation for a linear, isotropic, macro- and micro-homogeneous porous solid when pore fluid pressure is an additional state variable.

Solving Eq.(9) with respect to stresses gives the relation

$$\sigma_{ij} = \left(K - \frac{2}{3}G \right) \delta_{ij} \varepsilon_{kk} + 2G \varepsilon_{ij} - \zeta p \delta_{ij}, \quad (10)$$

where

$$\zeta = 1 - \frac{K}{K_g}. \quad (11)$$

This is perhaps the simplest expression and can be found in Biot (1941, 1962) and Rice (1980), with negative pore pressure. The first coefficient in parentheses can be identified with the Lamé parameter λ . The material constant ζ was called α by Biot (1941 and later) and by Nur and Byerlee (1971). If microhomogeneity and isotropy are assumed the equivalence in Eq. (11) is exactly correct. Otherwise, ζ is a material constant to be determined, and appropriate experimental tests are given by Biot (1941) and by Biot and Willis (1957). The quantity of fluid expelled during a jacketed compression test divided by bulk volumetric strain furnishes the value of ζ . This will be discussed in a later section.

To this point the assumption has been that pore pressure remains constant during external static deformation, as for a drained experiment or an open system. To complete the static linear description the fluid mass must be included with stress, strain, and pore pressure as fundamental state variables. Biot (1941) defined the response of fluid volume in the pores to stress and pore pressure as Θ , the increment of fluid volume per unit volume of porous solid. The expression for Θ is

$$\Theta = \frac{1}{3H_1} (\sigma_{kk}) + \frac{1}{R} p, \quad (12)$$

where H_1 and R are material constants. For constant confining stress the volume of fluid displaced into the pores of a unit volume is calculated with $\frac{1}{R}$.

Through the definition of a strain potential energy for the porous solid it can be shown that $H_1 = H$, the constant in Eq. (8). The argument used by Biot

(1941, 1973) is that $\sigma_{ij} d\epsilon_{ij} + p dv_p$, v_p being the pore fluid volume, is an exact differential, and there is a thermodynamic argument for why this is true (Biot, 1973). For the present purpose, however, the important result from his energy argument is the reciprocity between the response of pore volume v_p to hydrostatic confining stress σ and bulk volume response V_b to pore pressure p . This can be expressed as

$$\left(\frac{\partial V_b}{\partial p}\right)_\sigma = -\left(\frac{\partial v_p}{\partial \sigma}\right)_p \quad (13)$$

Love (1944) also gives several forms of the reciprocity theorem for elasticity. Gassmann (1951) and Geertsma (1957) also use reciprocity in their derivations.

Reciprocity can be used to show that $H_1 = H$. If the porosity is fully connected and fully saturated with pore fluid the increment of fluid content Θ in Eq. (12) can be identified with change in pore volume normalized to bulk volume. The equivalent to Eq. (12) is

$$\frac{1}{V_b} dv_p = \frac{1}{V_b} \left(\frac{\partial v_p}{\partial \sigma}\right)_p d\sigma + \frac{1}{V_b} \left(\frac{\partial v_p}{\partial p}\right)_\sigma dp \quad (14)$$

If the porous solid is completely immersed in fluid with pressure p bulk volumetric strain is given by $1/K_s$. If the external pressure p is removed such that only pore pressure p remains, the overall porous solid will expand by an amount inversely proportional to bulk modulus K . Summing these two stages gives the partial derivative of bulk volume to pore pressure with constant confining pressure

$$\frac{1}{V_b} \left(\frac{\partial V_b}{\partial p}\right)_\sigma = \left[\frac{1}{K} - \frac{1}{K_s}\right] \quad (15)$$

The same result can be seen in Eq. (7) if confining stress σ is set equal to zero. By reciprocity, Eq. (13), the partial derivative of pore volume to hydrostatic confining stress is given by

$$\frac{1}{V_b} \left(\frac{\partial v_p}{\partial \sigma}\right)_p = -\left[\frac{1}{K} - \frac{1}{K_s}\right] \quad (16)$$

With the matrix solid homogeneity and isotropy assumption the equivalence of Eq. (8) holds, and substitution of Eq. (16) into Eq. (14) gives

$$\frac{1}{V_b} dv_p = \frac{1}{H} d\sigma + \frac{1}{V_b} \left(\frac{\partial v_p}{\partial p}\right)_\sigma dp \quad (17)$$

Comparison with Eq. (12) indicates that $H_1 = H$ and that

$$\frac{1}{R} = \frac{1}{V_b} \left[\frac{\partial v_p}{\partial p} \right]_{\sigma} \quad (18)$$

and now it will be shown that R can be expressed as a function of porosity, bulk modulus, and solid matrix bulk modulus.

The change in pore volume with pore pressure can be equated to change in bulk volume V_b and solid matrix volume v_s by

$$\frac{1}{V_b} \left[\frac{\partial v_p}{\partial p} \right]_{\sigma} = \frac{1}{V_b} \left[\frac{\partial V_b}{\partial p} \right]_{\sigma} - \frac{1}{V_b} \left[\frac{\partial v_s}{\partial p} \right]_{\sigma} \quad (19)$$

Bulk volume and pore volume are related to solid matrix volume v_s through the porosity φ such that

$$\begin{aligned} V_b &= \frac{v_s}{1 - \varphi} \\ v_p &= \frac{\varphi}{1 - \varphi} v_s. \end{aligned} \quad (20)$$

Substitution of Eqs. (20) into the reciprocity theorem, Eq. (13), gives

$$\left[\frac{\partial v_s}{\partial p} \right]_{\sigma} = -\varphi \left[\frac{\partial v_s}{\partial \sigma} \right]_p \quad (21)$$

On the righthand side of Eq. (21) the differential for solid volume v_s is the difference between the differential for bulk volume, equal to $1/K$, and the differential for pore volume, Eq. (18). Therefore

$$\frac{1}{V_b} \left[\frac{\partial v_s}{\partial p} \right]_{\sigma} = \varphi \left[\frac{1}{K_s} \right] \quad (22)$$

and substitution of this with Eq. (15) into Eq. (19) gives

$$\frac{1}{V_b} \left[\frac{\partial v_p}{\partial p} \right]_{\sigma} = \frac{1}{K} - (1 + \varphi) \left[\frac{1}{K_s} \right] \quad (23)$$

Consequently, if the matrix solid is homogeneous and isotropic, the Biot constant R , referring to Eq. (18) and (23), can be identified as

$$\frac{1}{R} = \frac{1}{K} - (1 + \varphi) \left[\frac{1}{K_s} \right] \quad (24)$$

which is also given by Geertsma (1957). Therefore the increment of fluid volume Θ of Biot(1941) can be expressed as a change in pore volume

$$\frac{1}{V_b}(\Delta v_p) = \frac{1}{V_b} \left(\frac{1}{K} - \frac{1}{K_s} \right) \frac{\sigma_{kk}}{3} + \frac{1}{V_b} \left(\frac{1}{K} - \frac{1}{K_s} \right) p - \frac{\phi}{K_s} p, \quad (25)$$

or, equating pore volume with fluid volume v_f , a change in fluid volume

$$\Delta v_f = \left(\frac{1}{K} - \frac{1}{K_s} \right) \frac{\sigma_{kk}}{3} + \left(\frac{1}{K} - \frac{1}{K_s} \right) p - \frac{v_f}{K_s} p, \quad (26)$$

where v_f is the fluid volume in the unstressed state. Eq. (26) can be found in Rice and Cleary (1976, their Eq. (2)).

Following Rice and Cleary (1976), the variation in fluid mass Δm_f in response to pore pressure and confining stress may be expressed in a linear expansion of $m_f = \rho_f v_f$ to give

$$\Delta m_f = \Delta \rho_f v_f + \rho_f \Delta v_f, \quad (27)$$

where Δm_f is variation in fluid mass, $\Delta \rho_f$ is variation in fluid density, Δv_f is variation in fluid volume, and v_f and ρ_f are fluid volume and fluid density in the reference state. Using the identity of the fluid bulk modulus K_f ,

$$K_f = \frac{\rho_f p}{\Delta \rho_f}, \quad (28)$$

the substitution of Eq. (28) and Eq. (26) into Eq. (27) gives

$$\Delta m_f = \rho_f \frac{v_f}{K_f} p + \rho_f \left(\frac{1}{K} - \frac{1}{K_s} \right) \left(\frac{\sigma_{kk}}{3} - p \right) + \rho_f \frac{v_f}{K_s} p. \quad (29)$$

This is the equation for fluid mass variation with stress and pore pressure when the solid matrix modulus can be locally associated with K_s at every point in the solid. Together with Eq. (10) and Darcy's fluid flow equation a full set of constitutive relations are achieved for linear, isotropic, homogeneous, porous solids, with fully compressible constituents. Coupled with equilibrium conditions and compatibility conditions the full field equations, including diffusivity, can be generated (Rice and Cleary, 1976; Rice, 1980).

A crucial assumption in these derivations is that of solid matrix homogeneity and isotropy. This is also part of the reason why there are so many different symbols and material constants in various treatments. The assumptions can be distinguished by considering a macrohomogeneous and macroisotropic versus a microhomogeneous and microisotropic porous solid. Macrohomogeneity is fundamentally necessary for the analysis of stress and pressure on a sample element of the porous solid. Macroisotropy has been

assumed but is not necessary, and the extension to various symmetries was done by Biot (1956). The key issue is microhomogeneity and microisotropy. If the solid matrix is everywhere microhomogeneous and microisotropic, and the porosity is fully interconnected, then the solid matrix bulk modulus K_s can be used in all of the previous equations. Geologic materials, however, are far short of that ideal. Many different minerals exist in rocks and sediments and each mineral type is usually elastically anisotropic. A quick look at Birch's compilation in Clark (1966) shows that mineral elastic constants may vary by over 100%. The resolution of this fact to theory is by the definition of an equivalent homogeneous porous solid with equivalent solid matrix bulk modulus K_s' . This is a representative statistical average of all the different moduli of the individual minerals. It can be calculated as a Voigt-Reuss-Hill average (Brace, 1965) or else measured directly in an unjacketed stress-strain measurement. A rather noisy collection of K_s' values for some rocks can be found in Birch's compilation in Clark (1966, Table 7-13). The experimental section of this paper will present many more.

The use of K_s' in place of K_s in Eqs. (25), (26), and (29) is not, however, totally correct. The K_s in the last term of those equations can be traced to the variation in pore volume with constant differential pressure (Biot, 1973; Brown and Korringa, 1975). Brown and Korringa (1975) define the compressibility

$$\kappa_p = -\frac{1}{v_p} \left(\frac{\partial v_p}{\partial p} \right)_{p_d} \quad (30)$$

where v_p is pore volume, p is pore pressure, and p_d is differential pressure, $\sigma - p$. If the solid matrix is microhomogeneous and microisotropic the compressibility κ_p equals $1/K_s$, and the variation in pore volume v_p is equivalent to stress state (1) in Eq.(6). Biot(1973) also distinguished the compressibility C_m and Rice and Cleary (1976) distinguished the modulus $K_s'' (= 1/\kappa_p)$.

Using these two constants, K_s' and K_s'' , Eqs. (10), (11), and (29), the constitutive relations, are rewritten

$$\sigma_{ij} = \left(K - \frac{2}{3}G \right) \delta_{ij} \varepsilon_{kk} + 2G \varepsilon_{ij} - \zeta p \delta_{ij}, \quad (31)$$

$$\zeta = 1 - \frac{K}{K_s'}, \quad (32)$$

$$\Delta m_f = \rho_f \frac{v_f}{K_f} p + \rho_f \left[\frac{1}{K} - \frac{1}{K_s'} \right] \left[\frac{\sigma_{kk}}{3} + p \right] - \rho_f \frac{v_f}{K_s''} p. \quad (33)$$

The significance of K_s' and K_s'' is that they are material constants that, when experimentally measured, remove the homogeneity and isotropy requirement on the solid matrix. Equivalently, the H and R constants of Biot (1941) could be used. It may be noted that the analysis with K_s' and K_s'' and Biot's analysis using H and R are far more general, therefore, than the analysis of Gassmann (1951) and Geertsma (1957), both of whom specialize to the case of a homogeneous and isotropic matrix of modulus K_s . Biot almost certainly realized

this in 1941, but what must be remembered is that Biot's original application was soils. In a partially saturated, clay dominated soil the parameters such as porosity, matrix modulus, and pore fluid modulus are almost impossible to define and measure. By sticking to H and R Biot 1) defined measurable constants (H and R), 2) did not have to assume microhomogeneity and microisotropy, and 3) avoided any mention of porosity, so that the discussion and confusion with Terzaghi's "effective porosity" was avoided altogether.

Several other useful relations between bulk volume, pore volume, and porosity can be derived with the equations. For this purpose assume $K_s = K_s' = K_s''$. From Eq. (16) the differential of pore volume strain with hydrostatic confining stress σ was given by

$$\frac{1}{V_b} \left(\frac{\partial v_p}{\partial \sigma} \right)_p = - \left[\frac{1}{K} - \frac{1}{K_s} \right]. \quad (16)$$

The variation in porosity ϕ with hydrostatic confining stress σ can then be calculated from the expression

$$\left(\frac{\partial \phi}{\partial \sigma} \right)_p = \frac{\partial(v_p/V_b)}{\partial \sigma} = \frac{1}{V_b} \frac{\partial v_p}{\partial \sigma} - \frac{v_p}{V_b^2} \frac{\partial V_b}{\partial \sigma}. \quad (34)$$

The first term on the left is given by Eq. (16), and the second term on the right includes the definition of bulk modulus K and porosity ϕ . Eq. (34) can therefore be expressed as

$$\begin{aligned} \left(\frac{\partial \phi}{\partial \sigma} \right)_p &= - \left[\frac{1}{K} - \frac{1}{K_s} \right] + \frac{v_p}{V_b} \left(\frac{1}{K} \right) \\ &= - \frac{(1-\phi)}{K} + \frac{1}{K_s}. \end{aligned} \quad (35)$$

This means that porosity variation with external confining stress σ can be calculated from ϕ , K , and K_s ($=K_s'$). If porosity is very low and $K \ll K_s$, such as with crack-dominated solids at low pressure, the porosity can be ignored and

$$\left(\frac{\partial \phi}{\partial \sigma} \right)_p = - \left[\frac{1}{K} - \frac{1}{K_s} \right]. \quad (36)$$

This is the expression given by Walsh (1965). The expression in Eq. (35) is, however, more general.

EFFECTIVE STRESS

The modern idea of an effective stress in fluid-saturated porous media apparently originated with Karl Terzaghi, who is generally regarded as the

father of soil mechanics. For soils and muds the consideration of a pore fluid phase and possible pore pressure is obviously fundamental to the definition of materials and to the understanding of mechanical properties. From Terzaghi's experimental studies on the uniaxial consolidation and shear strength of fully-saturated soils (1923, 1925) he concluded that the controlling parameter for these properties was the simple difference between normal stress σ and pore fluid pressure p

$$\langle \sigma \rangle = \sigma - p. \quad (37)$$

In his relation σ is an external frame stress such as that applied to a sample in the laboratory by a deformation platen. It is positive in compression. Pore pressure p is kept constant during the application of normal stress, perhaps controlled through holes in the deformation platen and a low strain rate, and hence this is the definition of a drained test. The parameter $\langle \sigma \rangle$ became known as the effective stress and determined deformation and strength in these types of experiments. This was adequate for fully saturated soils as was experimentally shown by numerous subsequent studies. For partially saturated soils Bishop (1955) suggested a relation for effective stress of the form

$$\langle \sigma \rangle = \sigma - p_1 + \alpha(p_1 - p_2), \quad (38)$$

where p_1 is pore fluid gas pressure, p_2 is pore fluid water pressure, and α is a coefficient depending on the degree of saturation. In partially saturated soils gas and water pressure may be different due to surface tension and this relation attempts to account for this variation.

Apparently there was much disagreement over why and how Terzaghi's effective stress relation worked. The term "effective" to some minds references a micromechanical concept of how an external confining stress is distributed between grain contacts and pore fluid pressure, resulting in an intergranular "effective" stress between grains. For an isotropic aggregate the forces from fluid and solid across a given grain contact can be calculated quite simply (Skempton, 1960). If α is the area of solid particles per unit area of aggregate, the fluid forces on the plane sum to $(1 - \alpha)p$, where p is pore pressure. hence the intergranular stress σ_g is

$$\sigma_g = \sigma - (1 - \alpha)p, \quad (39)$$

where σ is the normal external stress. One minus the area of the solid over area of aggregate, however, is also a definition of porosity ϕ , and so

$$\sigma_g = \sigma - \phi p. \quad (40)$$

If intergranular stress σ_g is equated with effective stress $\langle \sigma \rangle$ the resulting expression is a function of porosity.

Terzaghi envisioned soils as individual plates and grains of solid material separated by springs that compressed during compaction. Terzaghi reasoned and argued that the simple differential pressure $\sigma - p$ could not determine deformation in the limit of zero porosity. The simple intergranular stress hypothesis above agrees with this. As the porosity goes to zero the effect of pore pressure goes to zero. In the other direction, however, soils and muds in his experiments that deform according to $\sigma - p$ did not have 100% porosity. Terzaghi suggested that because individual particles have surface roughness there is actually an "effective porosity" for intergranular stress that for soils was approximately equal to 1.

Biot (1935, 1941) established the physics of three dimensional consolidation. In his work he totally discarded effective stress and instead treated external stress and pore fluid pressure as independent variables. Stress was defined as total force per unit area of a representative homogeneous element and not separated between solid and fluid. The micromechanics of particle surfaces in contact were implicitly contained in the overall bulk and shear moduli and in the phenomenological constants H and R . Time effects in consolidation were included with Darcy's law. The linear elasticity of porous media presented in the previous section is the Biot formulation for consolidation. For muds and soils the bulk modulus is much greater than the intrinsic bulk modulus and the Biot stress-strain relation becomes equal to the Terzaghi relation.

In subsequent redevelopments of Biot's consolidation equations by Gassmann (1951) and Geertsma (1957) more readily defined physical parameters such as intrinsic bulk modulus (K_*), pore fluid modulus (K_f), and porosity (ϕ) were introduced in place of the original H and R constants of Biot. These too have been used in the previous section. It is significant to note, however, that effective stress was not referenced although the concept continued to be extensively used in the soil mechanics literature (Skempton, 1960; Süklje, 1969). Hubbert and Rubey (1959) argued for the Terzaghi relation as a general principle for stress in the crust.

When it became apparent that pore fluid injection could cause earthquakes (Healy *et al.*, 1968) or that pore pressure could be responsible for temporal velocity variations and aftershocks associated with earthquakes (Nur, 1971; Nur and Booker, 1972; Booker, 1974), pore pressure effects and effective stress were revived in a major way. For the most part effective stress was used as a synonym for differential pressure (Brace, 1972). In some cases, however, it gained additional status as the "law of effective stress" (Brace and Martin, 1968; Nur and Byerlee, 1971; Garg and Nur, 1973), the "effective stress concept" (Carroll, 1979), or as any of a number of "effective pressure laws" (Robin, 1973).

The mechanics of rock response to pore pressure and confining stress at small strains is well described by the linear elasticity of porous media in the first section. What is generally called effective stress or effective stress "laws" can be explained from the descriptions of the first section. Here it will be shown that the "effective stress law" for bulk strain (Nur and Byerlee, 1971; Garg and Nur, 1973; Carroll, 1979) and pore volume strain (Robin, 1973) are only reformulations of the linear elasticity analysis.

It is entirely possible to regroup terms in the stress-strain relationship (Eq. (10)) so that stress and pore pressure are combined as an "effective stress." This was done by Nur and Byerlee (1971), Garg and Nur (1973), and Carroll (1979). Nur and Byerlee (1971) actually started out with Eq. (10) and derived ζ (their α) in Eq. (11) along the same lines as Gassmann (1951) and Geertsma (1957). Eq. (10) from the first section can be regrouped as

$$\sigma_{ij} + \zeta p \delta_{ij} = (K - \frac{2}{3}G)\delta_{ij}\epsilon_{kk} + 2G\epsilon_{ij}$$

$$\zeta = 1 - \frac{K}{K_s} \quad (41)$$

The combination of stress and pore pressure on the left Nur and Byerlee (1971) termed "effective stress" $\langle \sigma_{ij} \rangle$ so that

$$\langle \sigma_{ij} \rangle = (K - \frac{2}{3}G)\delta_{ij}\epsilon_{kk} + 2G\epsilon_{ij}$$

$$\langle \sigma_{ij} \rangle = \sigma_{ij} + \zeta p \delta_{ij}. \quad (42)$$

Therefore $\langle \sigma_{ij} \rangle$ describes strain as a function of hydrostatic confining stress and pore pressure p as if there were no pore pressure. Rice and Cleary (1976) also use this definition in their development. Nur and Byerlee (1971) applied Eq. (10) to the specific case of a microhomogeneous and microisotropic linear solid and hence $\frac{1}{H} = \frac{1}{K} - \frac{1}{K_s}$.

The Nur and Byerlee (1971) and Garg and Nur (1973) definition of effective stress is an association of the confining stress and pore pressure in Eq. (10). There is no new physics introduced in doing so. Absolute strain could be predicted directly from Eq. (10) without any need for "effective stress." This is made clear in the section with graphical examples and in the experimental test.

Biot (1962a) referenced "effective stress" (σ') as the simple difference

$$\sigma'_{ij} = \sigma_{ij} + \delta_{ij}p. \quad (43)$$

the portion of the total stress in excess of local pore pressure p . With this definition, the stress-strain relation Eq. (10) becomes

$$\sigma'_{ij} - (1 - \zeta)\delta_{ij}p = (K - \frac{2}{3}G)\delta_{ij}\epsilon_{kk} + 2G\epsilon_{ij}. \quad (44)$$

obviously quite different from the Nur and Byerlee (1971) definition.

The Nur and Byerlee (1971) definition of effective stress is strictly valid only for an isotropic, linear, homogeneous, porous solid. Bulk isotropy is required because the bulk modulus K and intrinsic bulk modulus K_s are used in Eq. (11). Carroll (1979) analyzed the general anisotropic case although Biot

(1955, 1956) had extended his consolidation equations to general anisotropy some years earlier. The same derivation as that of Nur and Byerlee (1971) was followed except that the bulk modulus K was replaced by a tensor of elastic moduli M_{ijkl} and the intrinsic bulk modulus K_s was replaced by a tensor of elastic compliances C_{ijkl}^s . The effective stress for deformation of an anisotropic solid then becomes

$$\langle \sigma_{ij} \rangle = \sigma_{ij} + p(\delta_{ij} - M_{ijkl} C_{klmn}^s). \quad (45)$$

Comparing this expression with the Nur and Byerlee (1971) definition of effective stress,

$$\langle \sigma_{ij} \rangle = \sigma_{ij} + p\left(1 - \frac{K}{K_s}\right)\delta_{ij}, \quad (46)$$

anisotropy in either the solid matrix or overall porous solid means a different effective stress in different directions. Of course Eq. (10) could be formulated with tensor moduli and this was done by Biot (1955, 1956) although without the succinctness of tensor notations. The question of anisotropy will be further addressed and clarified in the graphical examples.

Linear elasticity equations in the first section describe both bulk strain and pore volume strain in response to pore pressure and confining stress. As Nur and Byerlee (1973) defined an "effective stress law" for bulk strain so too Robin (1973) defined an "effective stress law" for pore volume strain. From Eq. (16) of the first section the variation in pore volume Δv_p about some reference state at constant pore pressure normalized to pore volume is given by

$$\frac{\Delta v_p}{v_p} = -\frac{1}{\phi} \left(\frac{1}{K} - \frac{1}{K_s} \right) \sigma. \quad (47)$$

The variation in response to a combination of confining stress and pore pressure is derived from Eq. (25) of the first section and is given by

$$\frac{\Delta v_p}{v_p} = -\frac{1}{\phi} \left(\frac{1}{K} - \frac{1}{K_s} \right) (\sigma - p) - \frac{1}{K_s} p. \quad (48)$$

This equation can be expanded to

$$\frac{\Delta v_p}{v_p} = -\frac{1}{\phi} \left[\frac{K_s - K}{K_s K} \right] \left[\sigma - p + \frac{\phi K}{K_s - K} p \right]. \quad (49)$$

Comparison of this last equation with Eq. (48) gives an "effective stress" for pore volume strain

$$\langle \sigma \rangle = \sigma - \left[1 - \frac{\phi K}{(K_s - K)} \right] p. \quad (50)$$

This equation appears more substantial than the deformation that it is describing. Substitution of reasonable moduli and porosities for rocks shows that the coefficient in brackets is nearly 1, or that

$$\langle \sigma \rangle = \sigma - p. \quad (37)$$

If porosity rather than pore volume is described by an "effective stress law" the last equation is exact.

GRAPHICAL EXAMPLES

A few graphical examples with synthetic data will clarify most of the points and concepts made in the previous sections. Various plots shown in Fig. 2 are stress-strain relations for different idealized porous materials. On these plots the vertical axis is hydrostatic confining stress and the horizontal axis is strain.

In Fig. 2a the stress-strain relation for a perfectly linear, isotropic, homogeneous, porous solid is shown. Although the strain axis may be linear or volumetric strain, assume for this discussion that it is volumetric strain. The steeper of the two solid lines extending out from the origin, labelled U, is the stress-strain relation for the solid matrix material. If the porous solid is micro-homogeneous and micro-isotropic the inverse slope of this line is K_s , the solid matrix or intrinsic bulk modulus. If these attributes only apply on a macroscopic scale then the slope is K'_s , the solid matrix bulk modulus of an equivalent homogeneous material. The strain measured in an unjacketed stress-strain test, where confining pressure fluid freely infiltrates the pores, would follow line U.

The shallower solid line in Fig. 2a, labelled J, is the jacketed stress-strain relation with the pores empty or at least drained of pressure. The inverse slope is K , the jacketed bulk modulus, which includes features of pore closure in addition to solid matrix strain.

The upper solid line in Fig. 2a which is parallel to the jacketed stress-strain line represents the jacketed stress-strain relation for the porous solid with constant pore pressure of level p . Since pore pressure cannot be greater than confining pressure the origin for this line is at hydrostatic confining pressure p . At this point pore pressure equals confining pressure and the overall solid, as well as porosity and solid matrix, has experienced volumetric strain p/K_s . Assume that confining pressure σ combined with pore pressure p results in bulk volumetric strain ϵ . The stress field may be decomposed into two steps because of linearity. The first step is an equal internal and external pressure p that results in strain p/K_s . The second step is an external pressure $\sigma - p$ with constant pore pressure that results in a strain $(\sigma - p)/K$. Total strain is the sum of these two steps:

$$\frac{(\sigma - p)}{K} + \frac{p}{K_s} = \varepsilon, \quad (51)$$

or

$$\sigma - \left(1 - \frac{K}{K_s}\right)p = K\varepsilon \quad (52)$$

$$\zeta = 1 - \frac{K}{K_s}. \quad (11)$$

This is Eq. (10) of the first section for the case of hydrostatic confining pressure and a completely homogeneous and isotropic porous solid.

The "effective stress law" or "concept" of Nur and Byerlee (1971), Garg and Nur (1973), Robin (1973), and Carroll (1979), is to describe the strain at confining stress σ and pore pressure p in terms of the same strain at zero pore pressure. Their effective stress is simply the left term in the above equation. In Fig. 2a the dashed line down from the point σ, ε to the zero pore pressure relation line J and from there across to the stress axis graphically defines the effective stress or pressure. This is simply a matter of definition and the effective pressure so defined has no intrinsic physical meaning. Since K and K_s are both constants the coefficient $1 - K/K_s$ is constant with pressure.

The plot in Fig. 2b shows what happens when the pore space is structurally anisotropic. The solid matrix remains, on the average, isotropic, and so the single steeper solid line is solid matrix strain. The horizontal axis is linear strain, however, to show the anisotropy in 2 directions represented by the pair of shallower solid lines. For an anisotropic rock the effective stress will be different depending on direction. Linear strain at a confining stress σ and pore pressure p will, if related to the strain with zero pore pressure, correspond to two different "effective pressures" σ' and σ'' . This is indicated by the dashed lines in Fig. 2b. If the matrix solid is also anisotropic the lack of uniqueness in defining a single effective stress is further exacerbated because solid matrix strain will also vary with direction. In the experimental test problems with anisotropy in the samples were avoided by using bulk strain. Even if effective stress were calculated for linear strain, anisotropy is not a major problem in the samples studied. At low pressures modulus anisotropy is greatest, but since $K \ll K_s$, the coefficient ζ is not that sensitive to sizable differences in K .

The plot in Fig. 2c is more representative of a real rock. The horizontal axis is volumetric strain. The matrix solid stress-strain relation is nearly linear; any nonlinearity would be caused by occluded porosity or crystal defects. The jacketed stress-strain relation with zero pore pressure starts out nonlinear because of asperity closure effects. At high hydrostatic confining pressure those pore features responsible for most of the nonlinear strain have been removed. The inverse slope of the stress-strain relation at high pressure is the bulk modulus of a nearly linear porous solid, not of the solid matrix which is still given by the straight line.

The stress-strain relation at constant pore pressure p is the zero pore pressure curve with the origin relocated along the solid matrix stress-strain relation at hydrostatic confining pressure p . Bulk volumetric strain at confining pressure σ and pore pressure p is a sum of solid matrix strain p/K_s

and zero pore pressure strain at confining pressure $\sigma - p$. The bulk modulus K is a function of differential pressure $\sigma - p$. Consequently, the coefficient ζ is a function of differential pressure.

The effective stress σ' for hydrostatic confining pressure σ and pore pressure p is graphically calculated in Fig. 2c. It can be seen that the definition reduces the effect of pore pressure by coming down to a strain level higher on the zero pore pressure curve than that at $\sigma - p$. At higher pressures the slopes approach that of the solid matrix and the effect of pore pressure is further reduced. This simply reflects a decrease in the value of coefficient ζ . For rocks the solid matrix strain is an appreciable part of overall strain and the effective stress calculated is larger than differential pressure. For muds and soils bulk strain is much larger than solid matrix strain, ζ is nearly equal to 1, and the effective stress definition approximates differential stress. Essentially, the effective stress definition above for rocks obscures the solid matrix strain by reducing the effect of pore pressure. If the vertical axis is instead the solid matrix stress-strain relation in Fig. 2c, then differential pressure determines strain.

In Fig. 2d the idea of semilinearity (Biot, 1973) is graphically depicted for a nonlinear rock with finite strain. Semilinearity simply means that the solid matrix material behaves as a linear solid while the overall rock responds as a nonlinear solid. Semilinearity depends on strains being very small so that the definition of stress as a force per unit area is maintained before and after the deformation. In the plot the vertical axis is hydrostatic confining stress and the horizontal axis is volumetric strain. At constant pore pressure the bulk volumetric stress-strain relation is given by the curve labelled V_b . Because rocks in general behave as a semilinear material the bulk strain can be continuously broken down into pore volume strain and solid volume strain. Solid matrix volume strain is measured in anunjacketed stress-strain test and is the straight solid line labelled v_s . The difference between bulk volumetric strain and solid matrix volumetric strain is pore volume strain, the middle curve labelled v_p in Fig. 2d. All strains are normalized to overall bulk volume. With bulk and solid matrix volume strain measured, the decrease in pore volume with confining stress can be calculated quite accurately with finely incrementalized data points. Porosity decrease as a function of increasing confining stress can be calculated from the bulk volume decrease and pore volume decrease from a reference state.

EXPERIMENTAL TEST

Rocks, sediments, and soils fall far short of the ideal porous solids described in the elasticity analysis. Rocks are not homogeneous, rarely isotropic, almost never linear, and loathe to respond to infinitesimal strain. It should be of general interest, then, to investigate whether the equations fit real materials such as rocks. In the experiments to be described the bulk strain of a suite of rock samples was measured as a function of systematically cycled hydrostatic confining stress and internal gas pore pressure. For each sample, jacketed and unjacketed bulk moduli were calculated and the stress-strain measurements were compared with that predicted by the equations in the analysis. Additional measurements of jacketed and unjacketed strain were then made for a number of other rock samples.

The linear description in the first section originated with Biot (1941). Since that time there have apparently been only two relevant experimental stress-strain studies on rocks, where pore pressure was a variable in addition to confining stress. These are by Van der Knaap (1959) and by Nur and Byerlee (1971). Although he reports that experiments were run on a number of sandstones and limestones, Van der Knaap (1959) only gives data for a sample of Belait sandstone ($\phi=15\%$). He established experimentally the essential features of the analysis given in the first section and in the graphical examples. First, the stress-strain curve measured for this rock at a constant pore pressure of about 1 kbar was predicted from the zero pore pressure curve by using the intrinsic bulk modulus measured in an unjacketed stress-strain test. Without a jacket the confining pressure fluid fills the porosity of the sample and the resultant strain can be identified with an average modulus of the solid phase. The prediction corresponds to Fig. 2c in the graphical examples, where the zero pore pressure curve slides up the straight line of the intrinsic solid modulus to a point equal to pore pressure p .

The second feature of the analysis established experimentally by Van der Knaap (1959) is the division of bulk strain between pore volume strain and solid matrix volume strain. According to the linear elastic description the solid matrix volume strain should be inversely proportional to the intrinsic solid bulk modulus. For Belait sandstone Van der Knaap measured bulk strain as a function of increasing confining stress and, simultaneously, pore fluid expelled from the pore space at constant pore pressure. The difference between these two had to be the amount of solid matrix strain. This measured difference agreed with the predicted calculation based on the intrinsic solid bulk modulus.

Nur and Byerlee (1971) measured bulk strain on a sample of Weber sandstone ($\phi=9.5\%$, this paper) at various unspecified combinations of pore pressure and confining stress. Although their aim was to test an "effective stress law" for bulk strain, the prediction of strain with pore pressure with their "effective stress" indicates that the incrementalized linear elasticity description works. This is because the true source of their "effective stress" is the Biot linearized elasticity equations as shown in the first section. Garg and Nur (1973), however, working with the same data set on Weber sandstone, replot "effective stress" versus strain and show a serious negative discrepancy. They suggest that the data includes a time effect due to slow pore fluid diffusion, but why this did not show up in the earlier work is not clear. Nur and Byerlee (1971) do mention that up to 12 hours between data points was necessary for equilibration.

These two studies, then, are apparently the only ones to investigate the combined influence of pore pressure and confining stress on strain. Only one, by Van der Knaap (1959), appears reliable, and the data presented therein is only for a single sandstone. The conclusion is that not enough experiments on this fundamental issue have been made. In particular, experiments on rocks of extreme micro-inhomogeneity and micro-anisotropy, such as crystalline granites, have not been done.

Data can be found in the literature, however, that can be used in the equations. Fatt (1959) measured and calculated for Boise sandstone jacketed and unjacketed bulk modulus, porosity, and the coefficient ζ at several confining pressures. These values, in addition to shear modulus and fluid

compressibility, fix the properties necessary for predicting the static elastic response. A major emphasis in experimental studies has been the stress-strain response of jacketed samples that are dry or, if saturated, at constant pore pressure (Adams and Williamson, 1923; Zisman, 1933; Carpenter and Spencer, 1940; Hughes and Cook, 1953; Fatt, 1958; Brace, 1965; Simmons *et al.*, 1974). These kind of data provide the jacketed bulk modulus, and numbers have been compiled by Birch (Clark, 1966, Table 7-13). The important observation in these studies is the functional nonlinearity between stress and strain caused by local changes in contact area and asperity closure, especially at low stress. Because of nonlinearity the static moduli are a function of stress. Whether or not the static moduli are a function of differential stress, $\sigma - p$, is a question addressed in this experimental study.

In addition to the jacketed bulk modulus K the unjacketed bulk modulus K_s is required. Experimental stress-strain measurement of an unjacketed sample, where confining pressure fluid penetrates the pores, is a direct measure of an average intrinsic modulus of the solid matrix. This is a measurement emphasized in this experimental study. Values for a number of rocks have been compiled by Birch (Clark, 1966, Table 7-13), but most of the data is quite old and there is a great deal of scatter in the measurements. The largest proportion of unjacketed moduli in Birch's compilation come from Zisman (1933). Instrumentation and techniques have seemingly improved since that time. Nur and Byerlee (1971) report a value of 0.454 Mbar for Westerly granite that as been cited (Rice and Cleary, 1976) but which is too low. Measurements by Knopoff (1954), Adams and Williamson (1923), and this study indicate that a more reasonable value is 0.56 Mbar. Values for a number of rocks are measured and tabulated in this study.

An alternative to direct measurement of intrinsic solid modulus is to calculate a Voigt-Reuss-Hill average from modal analysis and individual mineral moduli. This was done by Brace (1965). This calculation assumes that individual mineral moduli from other sources are appropriate and that microanisotropy is random.

Several points are clear in the previous experimental efforts with pore fluids and pore pressure. Nur and Byerlee (1971) had a serious problem with time-dependent effects because it took so long (~12 hours) for strain to equilibrate. Also clear from the experimental results of Mann and Fatt (1960) and Wyllie *et al.* (1958) is that liquids, particularly water, increased strain levels in the sandstones studied. Presumably, there is a chemical-physical interaction of pore fluid with matrix that increases elastic compliance. Todd (1973), however, reports agreement of static compressibilities in a dry and water-saturated sample of Fairfax diabase.

In order to accurately assess the elasticity relations and to avoid the deleterious effects of liquid pore fluids on strain, the decision was made early in this study to use nitrogen gas as the pore fluid. The three favorable aspects of gas are: 1) no interaction between pore fluid and rock matrix, 2) high compressibility, so that when pore volume is strained from confining stress the change in pore pressure is minimal, and 3) low viscosity, so that pore pressure equilibrates rapidly throughout the pore space and with the external nitrogen gas reservoir. Nitrogen gas was chosen because of readily available 6000 psi cylinders. Regulated pressures to that level were then possible without the

need of a pump.

The basic experimental method was to measure bulk strain on jacketed samples of rock subjected to systematic cycles of hydrostatic confining stress and pore fluid pressure. Rock strain was measured with strain gages attached directly to the rock surface, leads from which were brought out through the jacketing material. Hydrostatic confining stress to 1 kbar was generated on the samples in a pressure vessel. A small high-pressure fitting penetrated the sample jacket and associated tubing brought nitrogen gas pore pressure up to 350 bars from an external gas reservoir.

For the measurement of unjacketed bulk modulus jackets were removed from the samples and kerosene confining pressure fluid penetrated the pores. Strain was measured to 1 kbar pressure.

With stress-strain data from jacketed measurements at zero pore pressure and unjacketed strain measurements, bulk modulus and intrinsic bulk modulus could be calculated. The theory of the first section could then be used to predict and compare with strain measurements at various pore pressures. In the following paragraphs the experimental test is broken down into discussions of rock samples, instrumentation, procedure, and data reduction.

The rocks studied included sandstones, granites, and limestone, and a list of samples is given in Table 1. The first seven samples in the list were fully investigated with various pore pressure runs, and jacketed and unjacketed measurements. The remaining rocks in the list were measured jacketed and then unjacketed.

Rock samples were of two types. Large 3" diameter by 2" long finely finished cores were prepared for the jacketed measurements with pore pressure. Small blocks, usually about 1" square by 1/2" thick were prepared for the unjacketed measurements so as to preserve the larger cores from kerosene contamination. The small blocks were first measured jacketed and then unjacketed. Therefore, multiple data sets could be compared from different samples of the same rock.

Rock samples were cored from various larger blocks of rock using a water-cooled diamond core drill. Samples were finished to a nominal 2" long by 3" diameter size with diamond-wheel cylindrical and surface finish grinders. Small block samples were cut to dimension with a diamond saw. Samples were flushed with water and acetone and then saturated in acetone by pulling a vacuum on the sample while submerged in an acetone-filled beaker. Samples were kept saturated for at least 24 hours and then vacuum dried (20 micron) at 60 degrees C for at least 24 hours.

Strain gages were applied directly to the finished surfaces of the samples. For the core samples three strain gages (BLH FAE-50-S6E) were attached in three mutually perpendicular directions on the cylindrical surface, one axially and two circumferentially. For the block samples normally two gages, sometimes three, were attached in mutually perpendicular directions. If bedding or a rift plane were present gages were oriented according to the planar feature. The general area on the sample where a gage was to be attached was always sanded with 400 grit sandpaper. For all of the samples

with appreciable porosity the rock surface was first filled with a viscous epoxy (BLH EPY 150) to prevent gage collapse into the pores under pressure. This epoxy, when dry, was sanded until the minerals were uniformly exposed, finishing with 400 grit paper. This procedure was found to be much easier when the epoxy was colored with a red dye. After surface preparation gages were applied with epoxy (EPY 150) and clamped to the samples with teflon sheets, sections of 3 inch thin wall brass tubing, and hose clamps.

The jacketed sample arrangement within the pressure vessel apparatus is shown diagrammatically in Fig. 3. Pore pressure in the sample is controlled through a stainless steel fitting that penetrates the jacket. High pressure tubing from the fitting to outside of the vessel goes to either a vacuum pump or a regulated nitrogen gas tank. The sample jacket contains two layers. First is a polyurethane sheet that is slit around the gages for lead attachment. After clean #22 copper wire is soldered to the gages the entire assembly of fitting, sample, and jacket is potted with an air-drying liquid polyurethane compound (Flexane, Devcon Corp.). The polyurethane sheet prevents the compound from entering the pores of the sample. Stress-strain tests on jacketed and unjacketed nonporous aluminum and lucite indicated no interference from the jacket on strain measurements.

A 3-wire lead system was used between the sample strain gages and the bridge completion network. Each gage had an individual bridge conditioning module (Analog Devices 2B30K) with one half of the bridge completed by shared 120 ohm resistors (Vishay 0.01%). A common shunt resistor on one of the shared resistors allowed for simultaneous calibration of all of the strain gages. The bridge conditioning modules output connects to the HP 3497A data acquisition/control unit and from there to the HP 1000 computer.

The pressure vessel apparatus in Fig. 3 is capable of subjecting samples up to 15 inches long and almost 3.5 inches in diameter to hydrostatic confining pressures up to 4 kbar. In these experiments, however, the samples were much smaller and pressures never exceeded 1.2 kbar. The confining pressure fluid is kerosene pumped up to pressure by an air-driven Haskell pump. Air to the pump and air to an air-operated high pressure relief valve is controlled by individual solenoid valves. These solenoid valves are in turn controlled by relays on a board in the HP 3497A data acquisition/control unit. Confining pressure and pore pressure are monitored by pressure transducers whose signals are conditioned with Analog Devices 2B31K modules and then fed into the HP 3497A data acquisition/control unit. This device is in turn controlled by the HP 1000 computer. Software developed on the computer controlled the cycling of confining fluid pressure and the collection of data from the strain gages and pressure transducers.

The procedure for a typical stress-strain measurement with pore pressure is now described. The jacketed sample arrangement with leads attached was located within the pressure vessel and a 20 bar confining pressure was applied. Allowing the sample to stay at zero pressure for lengths of time without this pressure made it susceptible to leaks. A vacuum (20 micron) was pulled on the sample for no less than 24 hours. Several slow cycles of confining pressure were then automatically applied up to the maximum desired confining pressure in order to get a repeatable stress-strain curve. This was determined by the collected data and was usually achieved after 3 cycles. For the sandstones it

was found that strain, after initially decreasing at a given pressure over the first few cycles, increased with additional cycles. The maximum confining pressure for the sandstones was 1000 bars, for the granites 1200 bars, and for the limestone 750 bars. Hysteresis and repeatability of the vacuum stress-strain curves was usually less than 10 microstrain (10×10^{-6} inch/inch of sample).

After cycling to obtain a repeatable vacuum stress-strain curve the measurements with pore pressure could begin. For most of the rock samples confining pressure was cycled with constant nitrogen gas pore pressures of 50, 100, 150, 200, 250, and 300 bars. The procedure was to raise confining pressure to a value about 5 bars above the level of pore pressure which was subsequently introduced. Strain levels immediately dropped and equilibrium was assumed when strain readings leveled off at a stable value. For the sandstones and limestone this took about 15 minutes and for the granites about 30 minutes. After the system equilibrated, the confining pressure was cycled up with strain and pressure measurements recorded every 10 bars at a stress rate of about 5 bars/minute. Pore pressure was maintained at a constant level by periodic adjustments with the regulator or released via a valve. After reaching the desired maximum, confining pressure was reduced with measurements recorded every 25 bars. Pore pressure was then slowly released, confining pressure was reduced to 20 bars, and the overall system was allowed to equilibrate for 6 to 24 hours before the next cycle. For most of the samples multiple cycles were run with the same pore pressure.

Initially, theunjacketed bulk modulus, or the intrinsic bulk modulus, was calculated from the different pore pressure strain measurements at constant differential pressure. If differential pressure is kept constant as the absolute magnitudes vary, the strain undergone by the sample is a measure of the intrinsic bulk modulus K_s' . The data for this calculation were limited by the number of runs with different pore pressures and the results were not precise. To accurately fix the solid matrix modulus K_s' unjacketed stress-strain measurements were made on smaller "block" samples. Two or three gages attached to these block samples were oriented in directions similar to the larger cores. The first step for these samples was a set of jacketed stress-strain cycles. The samples were fully encapsulated and suspended in the pressure vessel without the pore pressure fitting. These measurements were compared with the measurements on the larger cores for repeatability. The jacket was then snipped off, the sample saturated with kerosene, again placed in the pressure vessel, and unjacketed cycles of stress-strain were measured to 1 kbar. With the system automated these cycles were slowed down to ensure that pore pressure equalled confining pressure. Typically one cycle took about 10 hours. Permeability, however, does not change with pressure in this measurement.

One drawback of the strain gage technique is that once attached, the gage cannot be calibrated except electronically by the gage factor assigned by the manufacturer ($G.F. = \frac{\Delta R/R}{\Delta L/L} \approx 2.00$). From the strain gage to the computer the system is exceedingly accurate and calibration is based on the gage factor. Ambient temperature and electronic fluctuation result in at most a ± 5 microstrain drift over a 10 hour period. The real question is how well the resistance change of the gage reflects true bulk strain in the rock. Points to consider in answering this question include: 1) filling the pores with epoxy

probably stiffens the rock directly below the gage, 2) the epoxy layer between gage and rock surface absorbs some of the true strain, 3) repeatability of the measured stress-strain relation at a particular point on the rock between different gages, and 4) repeatability of the measured stress-strain relation in the same orientation for different samples of the same rock.

During the course of this study approximately 165 strain gages were attached to 35 rocks and standards. Perhaps the best estimate of error is gained in that process. Point 1 in the list above is perhaps not that serious. Jacketed data on some of the sandstones in this study compares favorably with that of Fatt (1958) and Mann and Fatt (1960). They measured some of the same rocks with an external cantilever-type device. For the unjacketed measurements the calculated bulk modulus for some of the clean sandstones is nearly that of quartz, so the epoxy in the pore space does not seriously interfere with that measurement. Point 2 above was studied by Brace (1964) and is termed the "pressure effect." All of the data in this study have been corrected for pressure effect by adding 0.06 microstrain per bar of confining pressure.

Points 3 and 4 in the list above are addressed from the standpoint of having made so many measurements. For almost all of the rocks at least 2 samples were measured jacketed. For Westerly granite, Chelmsford granite, and Berea sandstone 4 samples were measured. About 10% of the gages applied resulted in questionable data. This was indicated by 1) strongly negative hysteresis strain in jacketed tests or 2) failure to agree with other gages in the same orientation. For the few jacketed tests where good gages were replaced the subsequent measured strains were repeatable to about $\pm 1\%$. For the different samples of Westerly and Chelmsford the jacketed absolute linear strains at 1 kbar compared to within ± 40 microstrain at levels of 1000-2500 microstrain.

In the unjacketed tests the intrinsic solid strain was amazingly reproducible. For samples of Westerly granite, Bedford limestone, and Berea sandstone linear strain was repeatable to within 2%.

Jacketed and unjacketed linear and bulk moduli were calculated on the computer from the digitized stress-strain data. The analysis program was that used by Cheng and Johnston (1981). A Chebyshev polynomial, usually 6th to 8th order, was fit to the data and differentiated for modulus. The first few strain points at low pressure were sometimes filtered and fit separately with a low-order polynomial to make the modulus monotonically increase with pressure. For the calculation of bulk modulus strain data from three directions were first summed.

During the course of measuring rocks three different synthetic samples were measured. These were titanium (Ti-6Al-6Vn-2Sn), fused silica (GE125), and aluminum (T2024). The titanium and aluminum were not heat treated. Bulk moduli calculated from the stress-strain data were 1.10 Mb for the titanium, 0.385 Mb for the quartz, and 0.735 Mb for the aluminum. The value for aluminum is similar to the 0.744 Mbar value observed by Bridgman (1923, p. 166) and the 0.752 Mbar value observed by Brace (1965).

EXPERIMENTAL RESULTS

Volume microstrain versus pressure plots for the seven rocks systematically studied with pore pressure are shown in Figs 4-10. There is a dichotomous interpretation for the data and the two interpretations define the axes differently as will be discussed. The solid black curve in each plot, however, is the measured zero pore pressure (vacuum) volumetric strain versus hydrostatic confining pressure. Volumetric strain is the sum of strains from three mutually perpendicular gages on each rock sample. The squarish symbols in these plots are the strain measurements with different pore pressures, i.e., 50, 100, etc., bars. There is no need to distinguish absolute values of pore pressure in the symbols because there are no systematic trends beyond the interpretations given. The immediate conclusion from the profusion of data points on top of the solid black line is that bulk strain in the tests with pore pressure is understood and very predictable.

The first interpretation of the data in Figs. 4-10 is with the "effective stress law" for bulk strain of Nur and Byerlee (1971), Garg and Nur (1973), and Carroll (1979). For this interpretation the vertical axis is effective stress or effective pressure and is defined by

$$\langle \sigma \rangle = \sigma - \zeta p \quad (53)$$

$$\zeta = 1 - \frac{K}{K_s'} \quad (54)$$

where $\langle \sigma \rangle$ is effective stress, σ is hydrostatic confining stress, p is pore pressure, K is bulk modulus, and K_s' is solid matrix bulk modulus. The horizontal axis is volumetric microstrain and the origin is at zero pressure. From the zero pore pressure stress-strain data and unjacketed stress-strain data shown in Figs. 19-25 the two moduli and the coefficient ζ were calculated as a function of pressure. For each of the rocks these values are plotted in Figs. 11-17. For each data point of volume microstrain measured with pore pressure and confining pressure the effective pressure was calculated with the above equation. In calculating effective stress the moduli at a pore and confining pressure were taken as a function of the simple difference between pore and confining pressure. The square symbols in Figs. 4-10 represent calculated effective pressure versus volume microstrain. Agreement is excellent, regardless of rock-type. This means that bulk strain at any combination of pore and confining pressure is predictable from the zero pore pressure data and unjacketed data. Strain in the Weber sandstone with pore pressure that troubled Nur and Byerlee (1971) and Garg and Nur (1973) is predicted exactly in Fig. 4 for all pore pressures. Strain in the granites and in the limestone is also predicted exactly with the definition of effective stress.

The coefficient ζ decreases with increasing confining pressure according to the plots in Figs. 11-17. Since K_s' is nearly constant with pressure, ζ decreases with increasing pressure in a manner that reflects the increase in K . In Fig. 18 the value of ζ between zero and 1 kbar pressure is summarized for the seven rocks studied in detail. The largest value of ζ is at low confining pressure since bulk modulus K is lowest relative to solid matrix bulk modulus K_s' . Of the three sandstones studied Navajo had the stiffest response, and the value of ζ never

reaches above 0.75. For the other two sandstones, Berea and Weber, ζ almost reaches 1 at low pressure. At high pressure ζ for these sandstones is around 0.5, Bedford limestone has very little nonlinear strain, few crack like pore structures, and ζ decreases very little with pressure. The greatest range of ζ is for the three granites. At low pressure, Chelmsford granite ζ is nearly 1; at high pressure it decreases to about 0.25.

In these data the effective stress law for bulk strain of Nur and Byerlee (1971), Garg and Nur (1973), and Carroll (1979) has been shown experimentally correct for a large variety of rocks. The calculated effective stress, however, has no physical significance beyond predicting strain in rocks with pore pressure. As shown in the second section effective stress is simply a reformulation of Biot's linearized elasticity equation. There is a far easier means of predicting the strain without calculating moduli or effective stress. This is a second interpretation of the data in Figs. 4-10. If Φ_j is the drained and jacketed stress-strain response of the porous solid and Φ_u is the unjacketed stress-strain response, the strain ε at any combination of pore pressure p and hydrostatic confining stress σ is

$$\varepsilon = \Phi_j(\sigma - p) + \Phi_u(p). \quad (55)$$

Total strain is simply a sum of the drained and jacketed response at the differential pressure plus the unjacketed response at the pore pressure. Conceptually the overall stress field has been decomposed into an equal internal and external pressure p and an external pressure $\sigma - p$. For this interpretation the vertical axis in Figs. 4-10 is now differential pressure and the horizontal axis is the drained, jacketed response to differential pressure. The origin of the plots is $\Phi_u(p)$, which varies as a function of pore pressure, and in this interpretation is being subtracted out. There is no need to replot the data in Figs. 4-10 because agreement is still excellent.

The main result is that bulk strain in response to pore and confining pressure is understood and predictable from the drained, jacketed and unjacketed stress-strain relation. For this purpose the "effective stress" of Nur and Byerlee (1971), Garg and Nur (1973), and Carroll (1979) is a rather redundant calculation with no intrinsic physical meaning. The data necessary to calculate effective stress can be used directly to predict strain with pore pressure.

A more fundamental aspect of the data is the duplication of the drained, constant pore pressure stress-strain function regardless of pore pressure. Particularly for the granites studied there is severe microinhomogeneity and microanisotropy of mineral grains. The linear compressibility of quartz along the c axis is 0.718 Mb^{-1} ; perpendicular to the c axis it is 0.995 Mb^{-1} (Birch, Table 7-12, in Clark, 1966). The linear compressibility of orthoclase is much more anisotropic: along the a axis 1.013 Mb^{-1} ; along the c axis 0.468 Mb^{-1} . Microcline is a solid solution of orthoclase and albite and is a common phase in the granites studied. With these different mineral grains adjacent, cracks develop due to mechanical anisotropy as well as thermal expansion anisotropy during cooling (Nur, 1969). In the application of pore pressure and confining pressure in the laboratory the process is conceptualized as two stages. First an equal internal and external pressure at the value of the pore pressure, then an

external pressure equal to the differential pressure. The application of an equal internal and external pressure necessarily causes shear stresses and grain boundary reorganization where minerals with different compressibilities meet. For example, where the c axis of orthoclase lies parallel to the c axis of quartz the application of 1 kbar pore and confining pressure during an unjacketed test will result in 468 microstrain in orthoclase and 995 microstrain in the quartz. If a small crack separates the two minerals surface topographies will shift relative to each other. When confining pressure is then applied at constant pore pressure of 1 kbar the nonlinear aspect of strain as the crack closes may be different than when pore pressure is zero. Whether there is more strain or less strain is not predictable as that is a function of an exact description of topography. The order in which stresses are introduced will also be a factor. In the data collected on granites with pore pressure no measurable consistent trend in bulk strain was detected with increasing pore pressure. Therefore the conclusion is that microanisotropy and microinhomogeneity do not seriously affect the nonlinear constant pore pressure strain response after an initial pore and confining pressure are placed on the rock.

ζ AND POROSITY

The material constant ζ introduced in the stress-strain analysis of linear porous solids is quite useful for visualizing the experimentally measured strain response of rocks. In this discussion the symbol ζ is used, as in recent derivations (Rice and Cleary, 1976; Garg and Nur, 1973; Rice, 1980), although in Biot's work (1941, 1961) and in Nur and Byerlee's formulation (1971) the symbol α is used. Biot (1941) and Biot and Willis (1957) pointed out that ζ (their α) is the ratio between pore volume strain and bulk volume strain in a drained, jacketed test, and that the lower limit on ζ is the porosity ϕ and the upper limit is 1.

This identity and the bounds can be easily derived for a linear porous solid with a homogeneous and isotropic solid matrix. For such a material ζ can be associated with bulk modulus K and intrinsic bulk modulus K_s in the following relation:

$$\zeta = 1 - \frac{K}{K_s}. \quad (11)$$

Bulk strain for the solid in response to hydrostatic confining pressure σ at constant pore pressure is given by

$$\frac{1}{V_b} \left(\frac{\partial V_b}{\partial \sigma} \right)_p = \frac{1}{K}. \quad (56)$$

Pore volume strain for the solid is given by

$$\frac{1}{V_b} \left(\frac{\partial v_p}{\partial \sigma} \right)_p = \frac{1}{K} - \frac{1}{K_s}. \quad (16)$$

The ratio of these two expressions gives the stated result,

$$\left(\frac{\partial v_p}{\partial V_b} \right)_p = 1 - \frac{K}{K_s} \quad (57)$$

which is ζ .

The numerical value of ζ is bounded by $\varphi \leq \zeta \leq 1$. First, as can be seen from the expression for ζ , the value must be positive but cannot be greater than 1 since $K \leq K_s$ is always true. The lower limit may be obtained from Eq. (35) which describes the porosity response to confining pressure

$$\left(\frac{\partial \varphi}{\partial \sigma} \right)_p = \frac{(1 - \varphi)}{K} - \frac{1}{K_s} \quad (35)$$

Multiplying through by K gives

$$K \left(\frac{\partial \varphi}{\partial \sigma} \right)_p = -\varphi + \left[1 - \frac{K}{K_s} \right] \quad (58)$$

The expression in parentheses is ζ . The left side of the equation cannot be negative with increasing σ . Therefore $\zeta \geq \varphi$.

The significance of the coefficient ζ is that it represents the ratio of pore volume to bulk volume strain in a jacketed, drained test. For laboratory studies of rock properties measured as a function of confining stress the source of information on physical dimension is given by rock strain. The bulk strain of rocks under pressure, an easily measured quantity, is a combination of solid matrix strain in addition to pore volume strain. Pore volume strain is the desired quantity, and the coefficient ζ bridges bulk strain data to pore volume strain. The numerical value is a direct indication of pore volume response to pressure with one assumption. This interpretation of ζ is only strictly true for fully homogeneous and isotropic solids. The experimental evidence of Van der Knapp (1959) indicates that, at least for the sandstone studied, this assumption is not too severe.

Therefore, in Fig. 18 or in any of the plots in Figs. 11-17, the value of ζ can be interpreted in a manner quite different than as the pore pressure coefficient in the effective stress law for bulk strain. For the Berea and Weber sandstones and Chelmsford granite the value of ζ approaches 1 at low pressure, indicating that almost all of the measured bulk strain is pore volume strain. For soils and muds ζ is also 1. For granites at high pressure the greatest portion of bulk strain is the deformation of the matrix minerals, so that the value of ζ is low. For Westerly granite at 1 kbar confining pressure ζ is approaching a fairly constant value of about 0.22, indicating that 22% of external bulk strain is being absorbed by the pore space. The lower limit of ζ , the porosity, is never approached in these samples over the pressure range to 1 kbar. A rock with negligible nonlinear strain, as for the sample of Bedford limestone, has a nearly constant, small range of ζ values. Below the pressure at which pores collapse

and the stress-strain relation is severely altered, ζ has a value of about 0.6, meaning that about 60% of the bulk strain measured is a pore volume strain.

The numerical value of ζ calculated as a function of confining pressure can be used to calculate pore volume strain and hence porosity as a function of pressure from bulk strain. A more direct technique based on the data would use the digitized jacketed and unjacketed stress-strain relations. The initial porosity must be known. Starting at zero pressure with a unit bulk volume and pore volume from the initial porosity, the bulk stress-strain relation is stepped through incrementally. Solid volume strain in the unjacketed data is subtracted from bulk volume strain in the jacketed data over each step to arrive at pore volume strain. Essentially this is an incrementalized application of Eq. (16) over steps small enough that the bulk strain appears linear. New bulk and pore volumes are calculated at each step from the bulk and pore volume strains. At each step porosity is calculated from current pore and bulk volumes. In this manner a porosity or pore volume versus pressure relation can be constructed based only on results from linear elasticity and assuming that microinhomogeneity and microanisotropy do not severely distort volume strain. This is a point amenable to further experimental test as was done by Van der Knapp (1959) for the Belait sandstone.

For the seven rocks studied with pore pressure porosity versus confining stress has been calculated with this procedure. The results are plotted in Figs. 26-32. The scales on all of the plots are the same for easy comparison. Initial porosities for these samples were calculated from dry and saturated weighings of the large cores and are recorded in Table 1. Accuracy of the initial porosity is not that critical for calculating the porosity decrease with confining pressure. The curves in Figs. 26-32 can therefore be shifted up and down the porosity axis, particularly the low porosity granites. Of the three sandstones Berea shows the most rapid decrease of porosity with confining pressure. This correlates with the low bulk modulus measured for this rock. The Navajo sandstone, a relatively stiff sandstone because of the extensive silica cementation, only decreases a little more than 0.2% in porosity at 1 kbar. The Weber sandstone porosity decreases dramatically over the first few hundred bars of confining pressure. Bedford limestone, almost a linear porous solid, shows a very small decrease in porosity to 500 bars. The three granites represent an interesting range with the Chelmsford porosity decreasing the greatest amount, from 1.1% down to 0.75% at 1 kbar. After about 500 bars the rate of porosity decrease with pressure for the Chelmsford and Barre are nearly the same. Westerly granite is the stiffest of the three granites. Porosity decreases by only 0.1% between 0 and 1 kbar.

GENERAL DISCUSSION

The theoretical development of linear elasticity and the experimental test of bulk strain clearly indicate that effective stress is an unnecessary definition in the description of bulk deformation of intact rocks where pore pressure and confining stress are variables. With the drained, jacketed stress-strain relation and the unjacketed stress-strain relation measured, deformation behavior at any combination of confining stress and pore pressure is completely described. To calculate the "effective stress law" for bulk strain of Nur and Byerlee (1971), Garg and Nur (1973), and Carroll (1979) is clearly not necessary to predict

strain and the "effective stress" so calculated is without physical significance.

The observation remains, however, that differential stress, or pressure, adequately describes various physical properties of rocks measured in the laboratory with pore pressure in addition to confining stress. These properties include acoustic velocities, both P- and S-wave (Gardner *et al.*, 1965; Coyner, 1984), fluid permeability (Knutson and Bohor, 1963; Brace *et al.*, 1968; Coyner, 1984), pore phase electrical resistivity (Coyner, unpublished results), and fracture strength (Robinson, 1959; Handin *et al.*, 1963; Brace and Martin, 1968). These observations, that pore pressure is equally "effective" as confining stress in laboratory measurements, have been associated with Terzaghi's effective stress relation (Brace, 1972). Terzaghi, however, introduced the concept to describe deformation and failure in muds and soils. The question to be answered is why these other physical properties should be described by the same relationship, especially since bulk strain is clearly not.

In this paper the linear static theory of Biot for fluid-saturated porous solids has been developed and applied to the bulk and pore volume deformation of rocks. Even though linearity and infinitesimal strain are fundamental assumptions, aspects of the analysis can be applied to the physical properties listed above. This is in spite of the fact that these properties may vary in a highly nonlinear fashion with stress. The purpose here is not to model this nonlinearity with pore closure models, but to explain the strain response to a combination of stress and pore pressure which would lead to the Terzaghi relation.

The linear analysis of the first section allows for the decomposition of stresses given in Eqs.(5) and (6) and shown in Fig. 1. From the viewpoint of this decomposition the effect of a pore pressure p and a confining stress σ on other physical properties can be addressed. The argument is similar for all of the properties listed above exclusive of bulk strain. Recall that the two-step decomposition is an equal and uniform internal and external stress equal to pore pressure p and an external stress of value $\sigma - p$. This decomposition, in fact, mimics a possible laboratory procedure in testing for a pore pressure effect.

The first step in the analysis ideally reduces every linear dimension by an amount inversely proportional to the intrinsic bulk modulus of the solid phase. For rocks, which are composed of various anisotropic minerals and perhaps a fraction of occluded porosity, this statement is only an approximation. This statement is exactly true for a microhomogeneous and microisotropic porous solid with fully interconnected porosity. Given this assumption, there is an equal fractional decrease in solid and pore phases. Therefore, the porosity, the aspect ratios of all the cracks and pores, and the relative orientation of all cracks and pores, all remain constant. For silicate and salt-like crystalline structures the intrinsic modulus is quite large, about 0.37 Mb for quartz and larger for other silicates and ionic solids. This means that an equal external and internal pressure of 1 kbar reduces linear dimensions by approximately 0.1%. A crack 10.0 microns in diameter becomes 9.99 microns in diameter.

The effect on physical properties of this first step is negligible. Since porosity remains constant electrical resistivity does not change if Archie's Law holds. Fluid permeability, even though quite sensitive to 3-dimensional pore

shape, should not decrease measurably when pore dimensions decrease by 0.1%. Acoustic velocity does not change because aspect ratios are constant and the bulk density increase is negligible. Fracture strength does not change because crack and pore orientation has not changed and the amount of strain is so small that the stress definition as a force per unit area changes negligibly.

The second step in the analysis, the application of external pressure $\sigma - p$, is what causes the asperities to close and pore surfaces to come into contact. The differential stress, $\sigma - p$, is then seen as the "effective" stress because it determines the deformation of pores that leads to a change in the physical properties. Because of the inherent stiffness of minerals that make up rocks and sediments and the hydrostatic pressure of pore fluid the differential stress or the "effective stress" describes the response of physical properties. The Terzaghi relation has the status of a very good approximation.

There is a general usefulness for measuring the static strain of rocks. A laboratory technique for investigating rock properties has been the measurement of properties such as velocity, attenuation, resistivity, and permeability, as the rock sample is subjected to confining stress. As the cracks and pores close the properties may change dramatically. The reason these tests are useful is that the mineralogy of the phases remains constant, so that variable is removed. Properties are measured and can be modelled with cracks or ellipsoids of various shape that shrink and change in number with pressure. An appeal to what is actually happening to the rock under stress must go to strain measurements because they provide dimensional information. In the previous section it was shown how subtraction ofunjacketed stress-strain data from jacketed stress-strain data gives pore volume deformation as a function of confining stress. This result should be useful in modelling the physical properties of these rocks.

The other point to emphasize here is the unjacketed stress-strain test. This test gives a direct measurement of solid matrix modulus averaged over many elastically anisotropic composite grains in various orientations. The measurement is useful for two reasons. One, as was previously discussed, pore volume strain as a function of confining stress can be calculated. Two, the intrinsic solid matrix modulus is necessary in most velocity modelling as well as in deformation modelling. For rocks, if there is little occluded porosity or crystal defects, the static calculation of intrinsic bulk matrix modulus from unjacketed stress-strain data can be directly used in velocity modelling. The static and dynamic modulus should be nearly the same for the solid phase. For the overall bulk rock dynamic modulus can be much larger than static modulus (Cheng and Johnston, 1981).

CONCLUSIONS

- (1) The static bulk strain of rocks in response to a combination of hydrostatic confining stress and pore pressure is predictable and understood by the linear elastic description of porous media. Strain at any combination of hydrostatic confining stress and pore pressure can be predicted from 1) the drained stress-strain response of the porous solid, and 2) the intrinsic bulk modulus K_s measured in an unjacketed stress-strain test.

- (2) The lack of microhomogeneity and microisotropy does not measurably influence the drained bulk stress-strain relation with an initial pore pressure. Experimental observations show the zero pore pressure stress-strain response is followed in a rock with pore pressure p , the only difference being a constant shift in pressure and strain p/K_s' due to the intrinsic modulus K_s' of the rock. Silicate minerals are so stiff that grain boundary shear due to pore pressure has a minimal effect on the subsequent nonlinear strain response to confining stress.
- (3) The differential stress $\sigma - p$ will generally determine those physical properties exclusive of bulk strain which are strongly influenced by the presence of cracks and pores, i.e., velocity, attenuation, permeability, resistivity, and fracture strength. The reason is because of the large intrinsic modulus of crystalline minerals and the small bulk strains of rock in response to stress.
- (4) The "effective stress law" for strain proposed by Nur and Byerlee (1971), Garg and Nur (1973), Robin (1973), and Carroll (1979) is simply a reformulation of the Biot linearized description of fluid-saturated porous media. Effective stress so defined for intact materials entails no fundamental description of micromechanic distribution of stress between grains in intact rock. To avoid confusion, effective stress should only be used as a synonym for differential stress, the difference between confining stress and pore pressure.
- (5) The coefficient ζ was initially introduced by Biot (1941, his α) and is a fairly useful material constant. Experimentally determined, ζ equals the ratio of change in pore volume to change in bulk volume at constant pore pressure. This coefficient, in addition to bulk modulus and shear modulus, complete the constitutive equation for drained response. The coefficient ζ can be calculated from the bulk modulus K and the matrix solid bulk modulus K_s by the relation

$$\zeta = 1 - \frac{K}{K_s}$$

The moduli can be incrementalized with stress to account for nonlinearity in stress-strain. The value of ζ is theoretically $0 \leq \zeta \leq 1$ for a solid of porosity ϕ .

- (6) Assuming that semilinearity (Biot, 1973) is generally true for rocks, the porosity or pore volume change with confining stress can be calculated from the initial porosity and carefully measured jacketed and unjacketed stress-strain relations. Porosity versus hydrostatic confining stress has been calculated for the seven rocks studied with pore pressure. The results show that porosity change with confining stress to 1 kbar is largest for the sandstones in the order Berea > Weber > Navajo. For the Bedford limestone porosity change is very small. Of the three granites studied the porosity decrease is greatest for Chelmsford, less for Barre, and least for Westerly.
- (7) A large volume of jacketed and unjacketed static stress-strain data for a number of rocks has been presented. Bulk moduli and intrinsic bulk moduli have been calculated for these rocks as a function of confining

stress. The intrinsic bulk modulus K_0' fromunjacketed stress-strain tests has been emphasized as a simply measured parameter, the value of which is quite useful in modelling studies. The moduli substantially add to the available data on static elastic response of rocks.

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Table 1: Rock Samples in this Study

Rock	Density (g / cm^3)	Grain Size (mm)	Porosity (%)
Berea Sandstone	2.197	0.1	17.8
Navajo Sandstone	2.316	0.15	11.8
Weber Sandstone	2.392	<0.05	9.5
Bedford Limestone	2.360	0.75	11.9
Westerly Granite	2.639	0.75	0.8
Chelmsford Granite	2.606	1.5	1.1
Barre Granite	2.635	3	0.7

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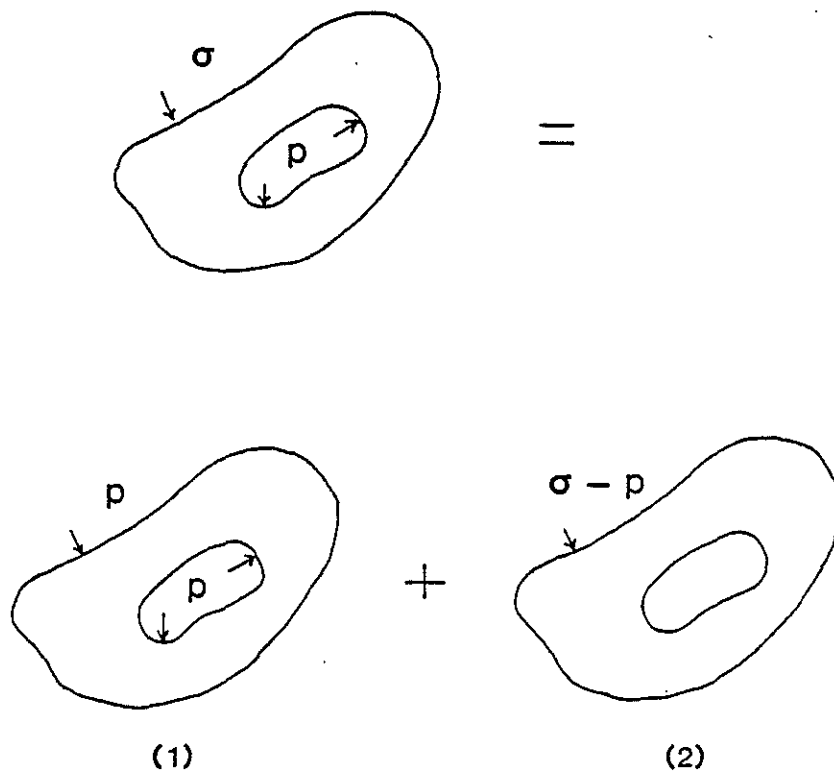


Figure 1. Stress field decomposition for porous solid (rock) with hydrostatic confining stress σ and pore pressure p . Stress state (1) is an equal internal and external stress p ; stress state (2) is an external stress $\sigma - p$.

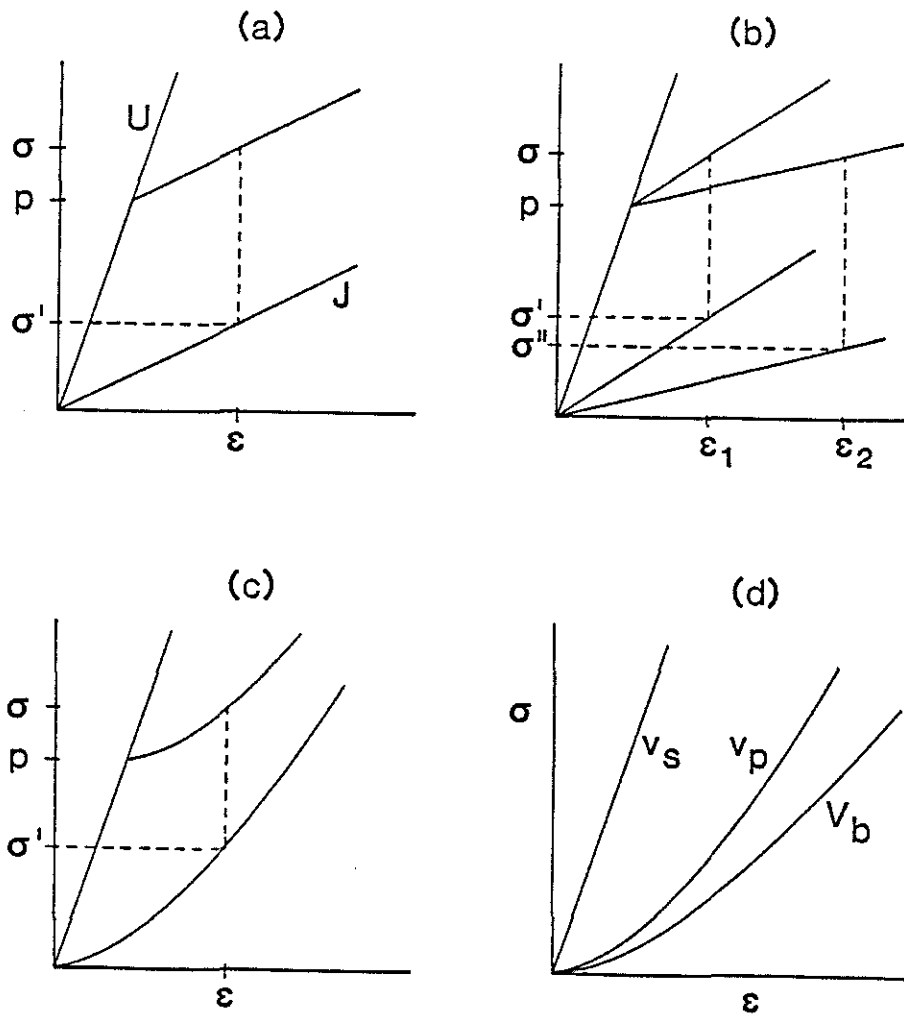


Figure 2. Static stress-strain relations with synthetic data. In (a), (b), and (c), the upper parallel relation is with pore pressure p . (a) linear, isotropic, porous solid. Jacketed strain with zero pore pressure is line J and unjacketed strain is line U; effective stress σ' defined at stress σ and pore pressure p . (b) linear, anisotropic, porous solid. Note nonuniqueness of effective stress definition for two directions. (c) nonlinear solid (rock). Note how effective stress σ' for strain ϵ obscures solid matrix strain due to pore pressure p by making pore pressure appear less "effective." (d) bulk strain as a sum of pore volume strain and solid matrix strain based on semilinear behavior. Pore volume strain and hence porosity as a function of confining stress can be calculated from bulk strain (jacketed data) and solid matrix strain (unjacketed data).

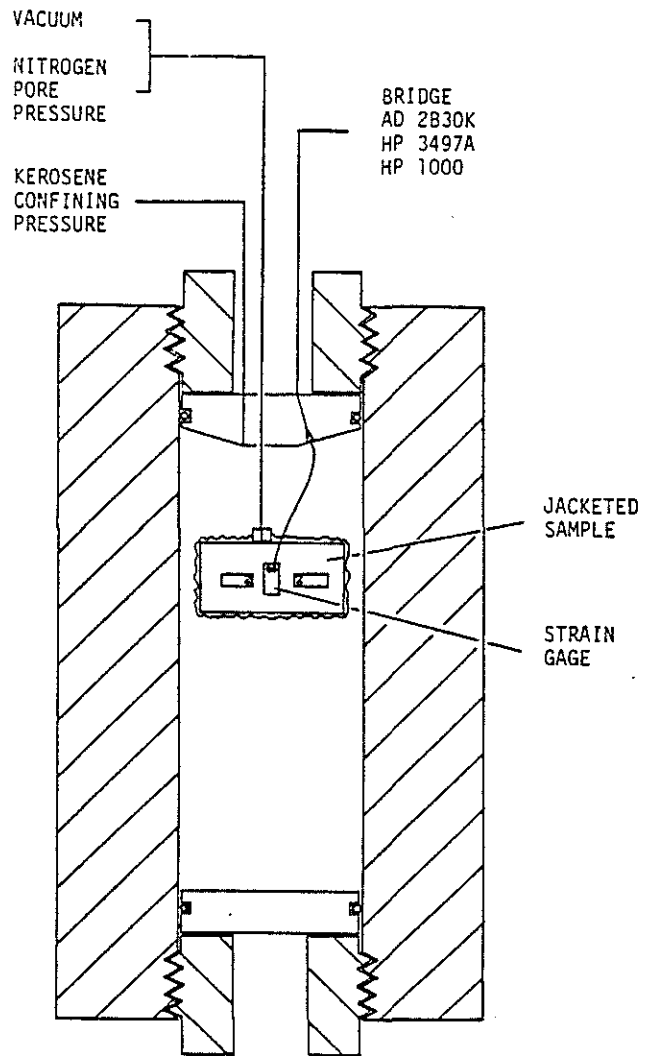


Figure 3. Schematic diagram of experimental apparatus for strain measurements. Sample with strain gages is shown loaded inside of the pressure vessel.

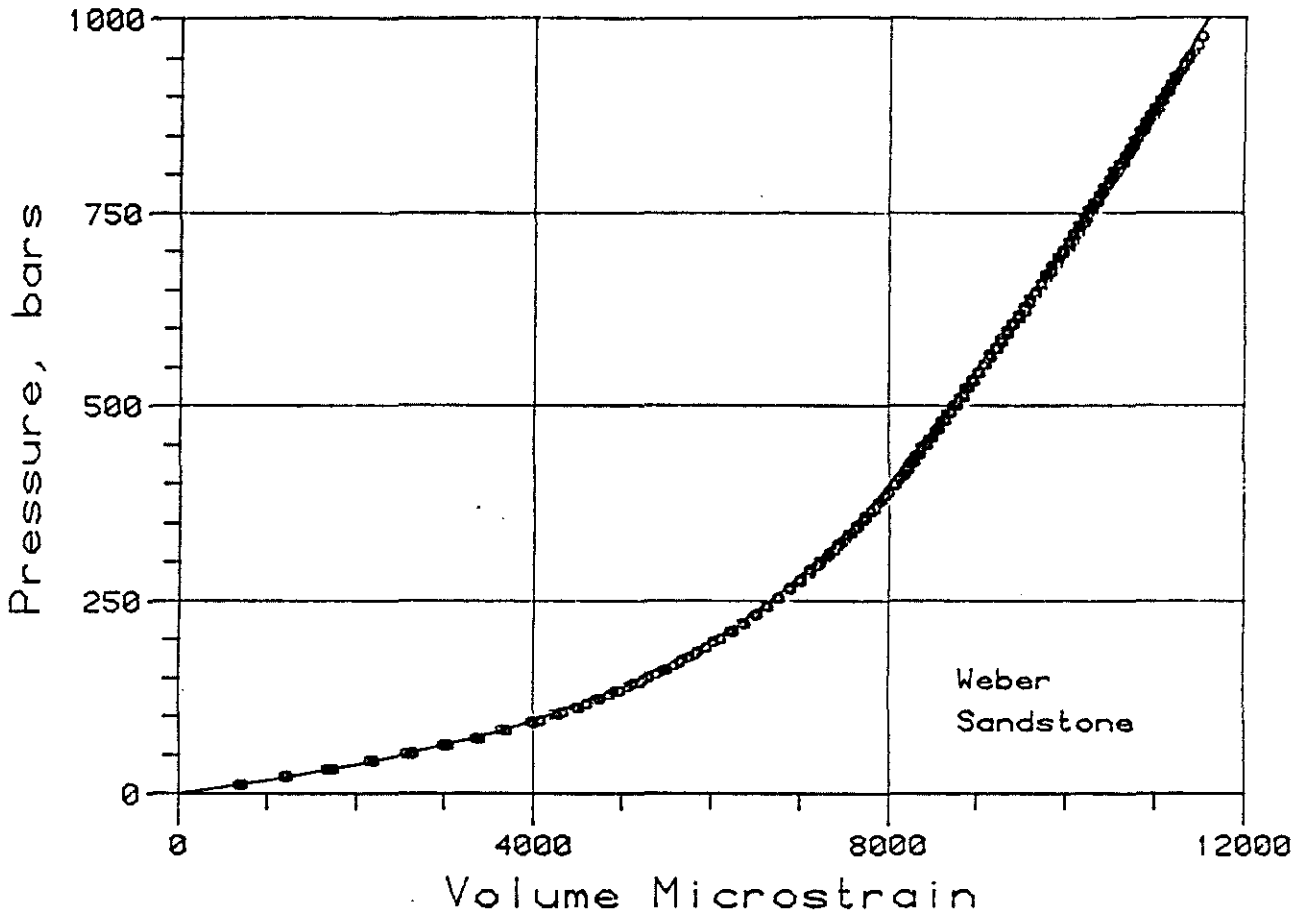


Figure 4. Volumetric strain versus confining pressure for Weber sandstone. Solid line is vacuum dry stress-strain relation. Squarish symbols are volumetric strains measured at various pore pressures and are predicted by 1) displacing the zero pore pressure curve along the solid matrix stress-strain relation or 2) calculating the 'effective stress.' In the first case the pressure axis is differential pressure and the strain axis origin is unjacketed sample strain at each pore pressure; in the second case the pressure axis is effective pressure and the strain axis is absolute strain from zero confining pressure.

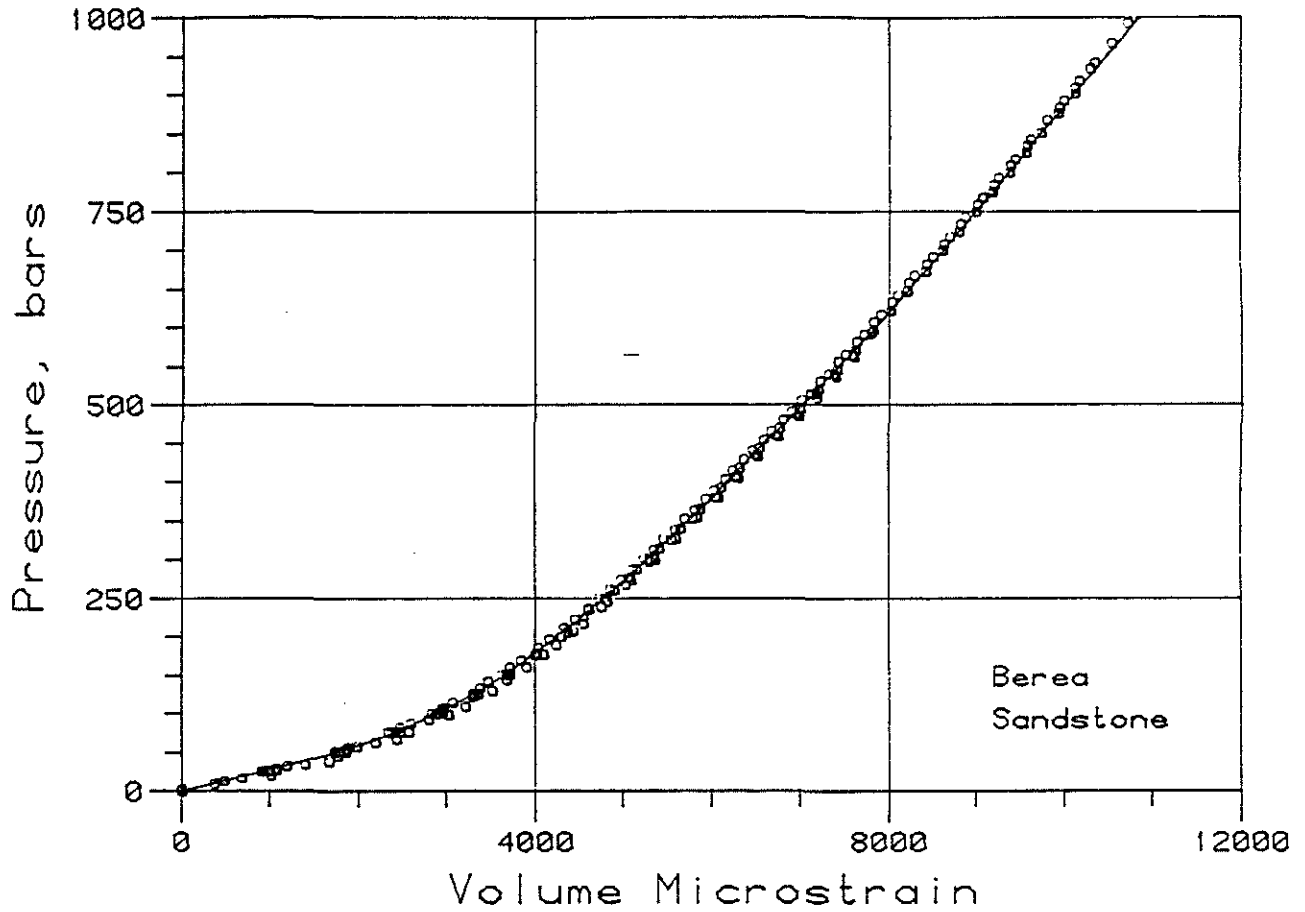


Figure 5. Volumetric strain versus confining pressure for Berea sandstone. Solid line is vacuum dry stress-strain relation. Square symbols are volumetric strains measured at various pore pressures.

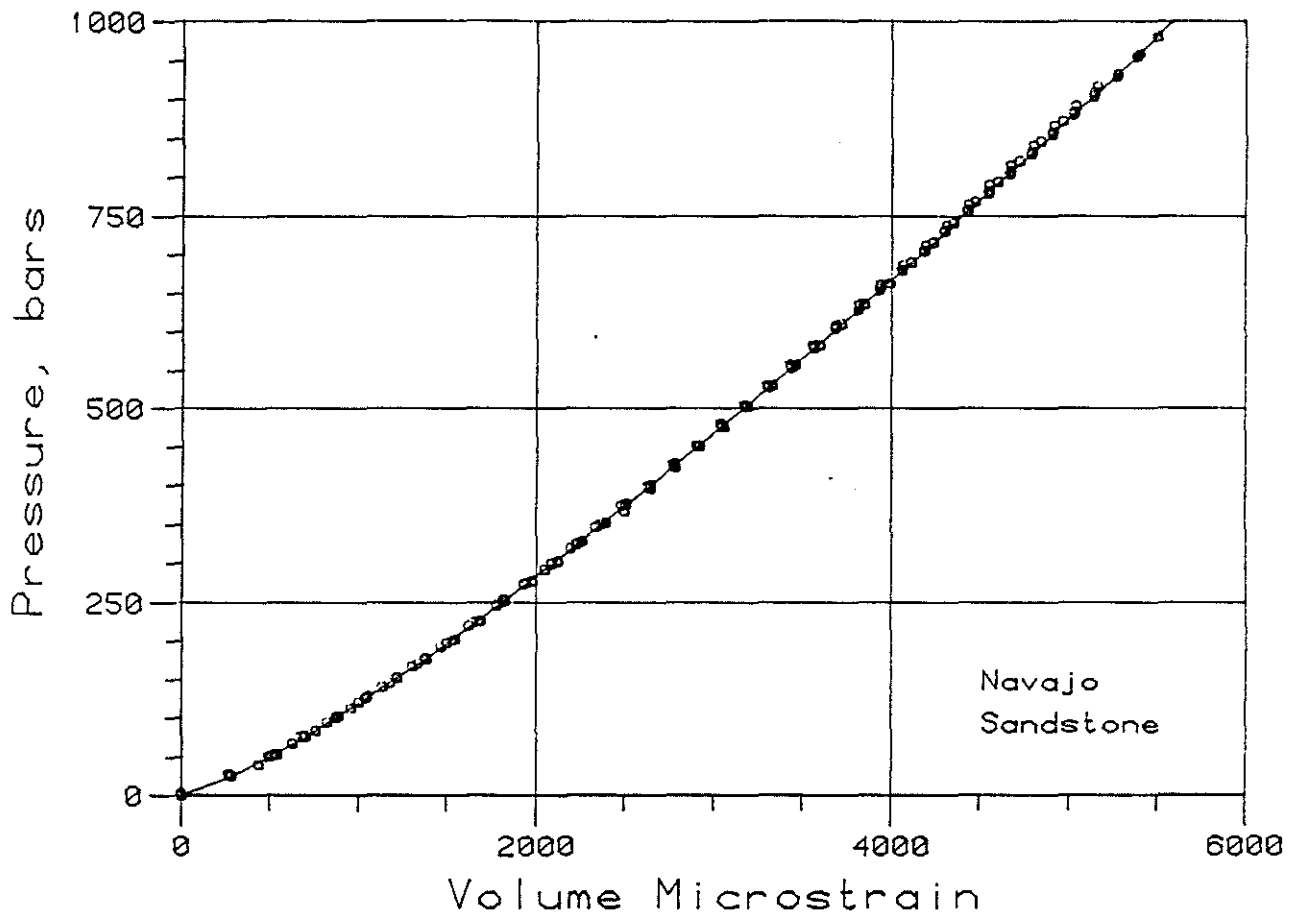


Figure 6. Volumetric strain versus confining pressure for Navajo sandstone. Solid line is vacuum dry stress-strain relation. Square symbols are volumetric strains measured at various pore pressures.

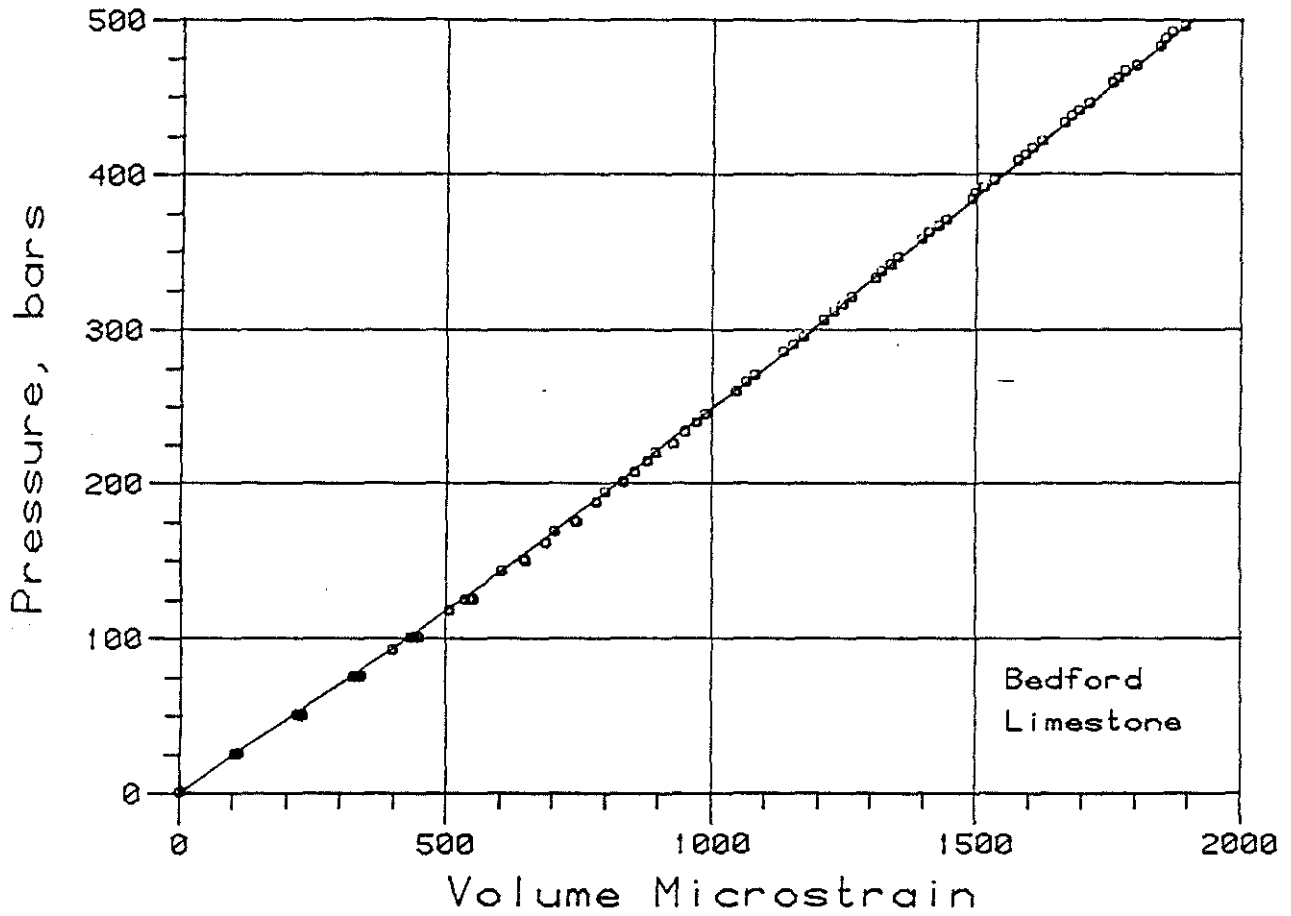


Figure 7. Volumetric strain versus confining pressure for Bedford limestone. Solid line is vacuum dry stress-strain relation. Square symbols are volumetric strains measured at various pore pressures.

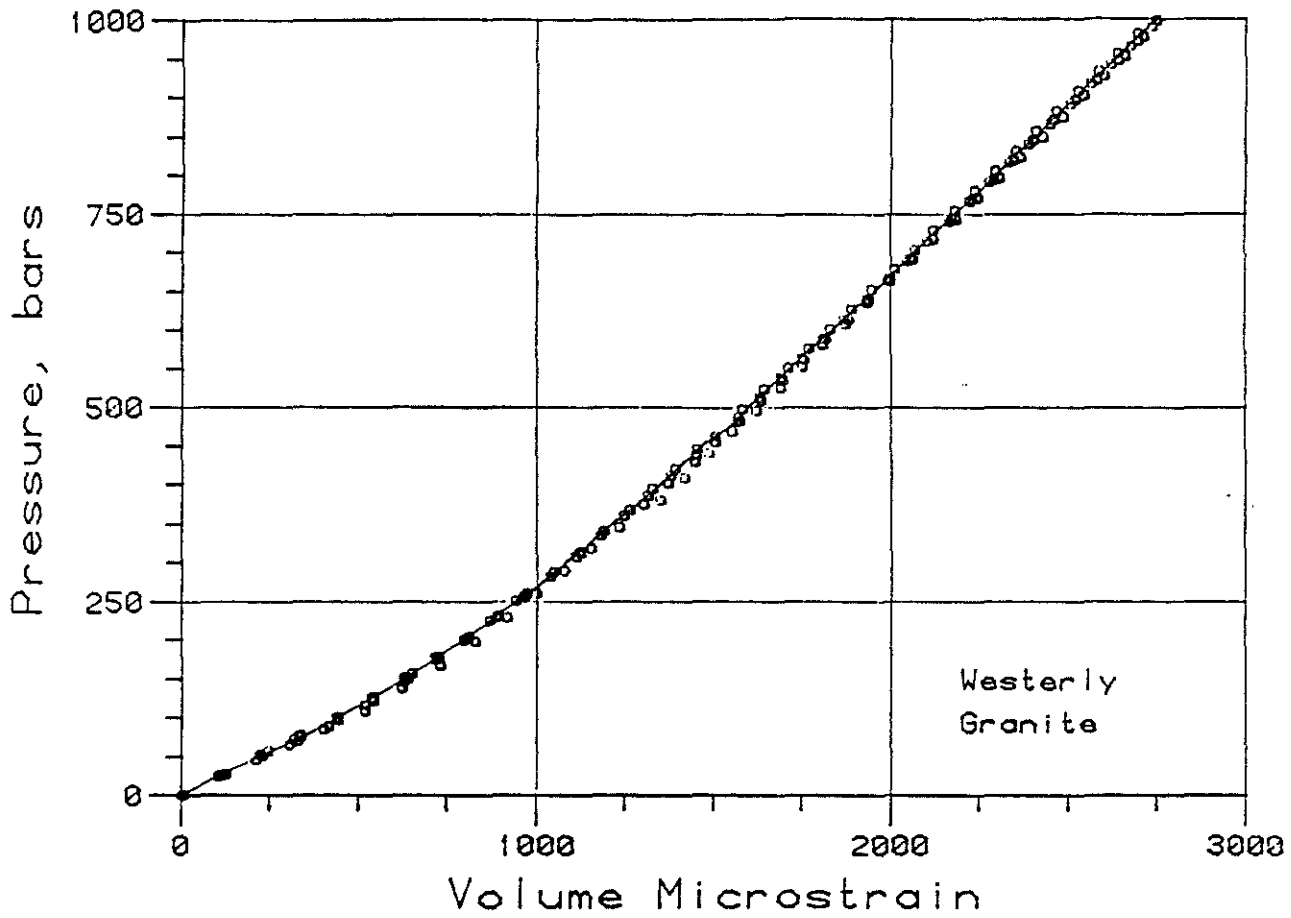


Figure 8. Volumetric strain versus confining pressure for Westerly granite. Solid line is vacuum dry stress-strain relation. Square symbols are volumetric strains measured at various pore pressures.

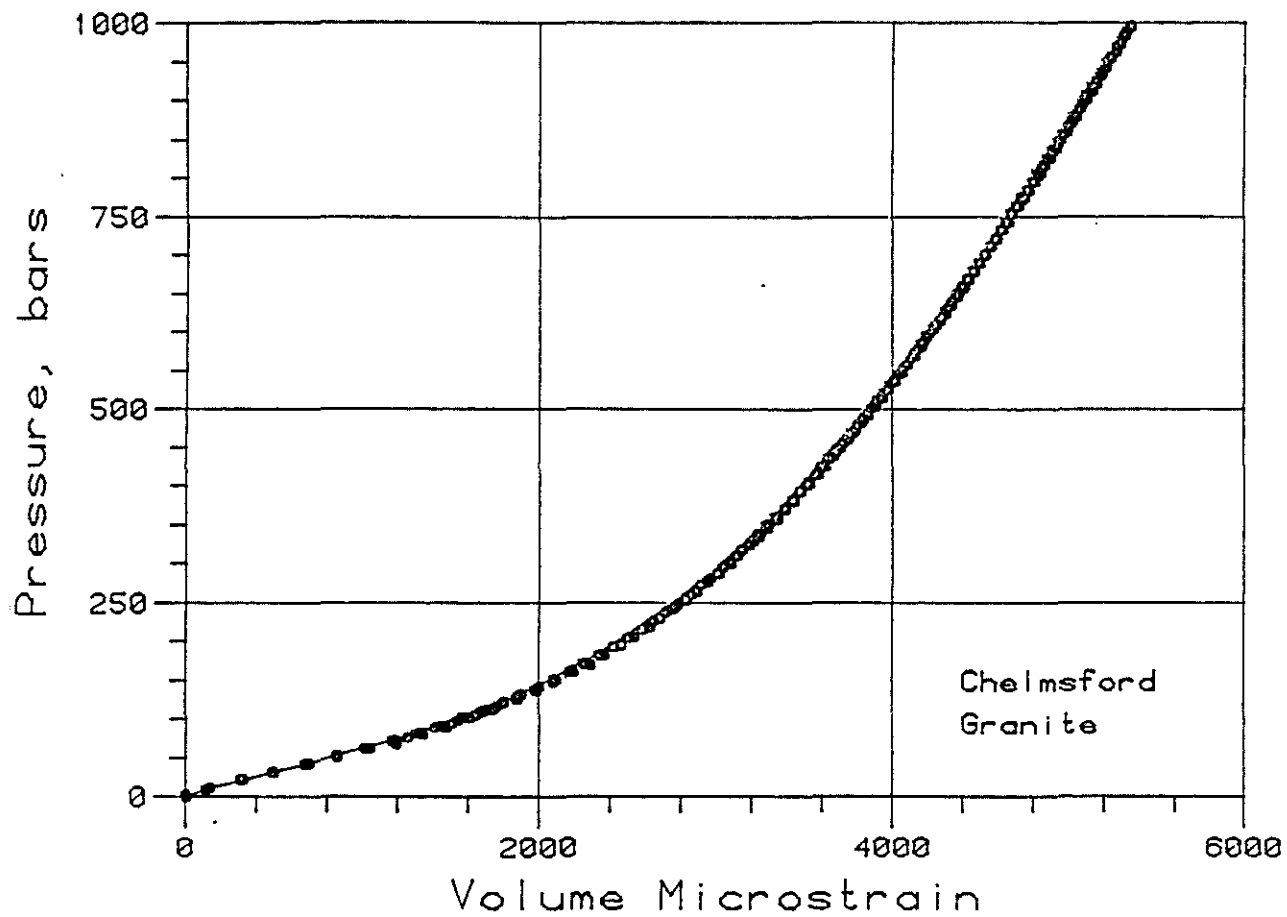


Figure 9. Volumetric strain versus confining pressure for Chelmsford granite. Solid line is vacuum dry stress-strain relation. Square symbols are volumetric strains measured at various pore pressures.

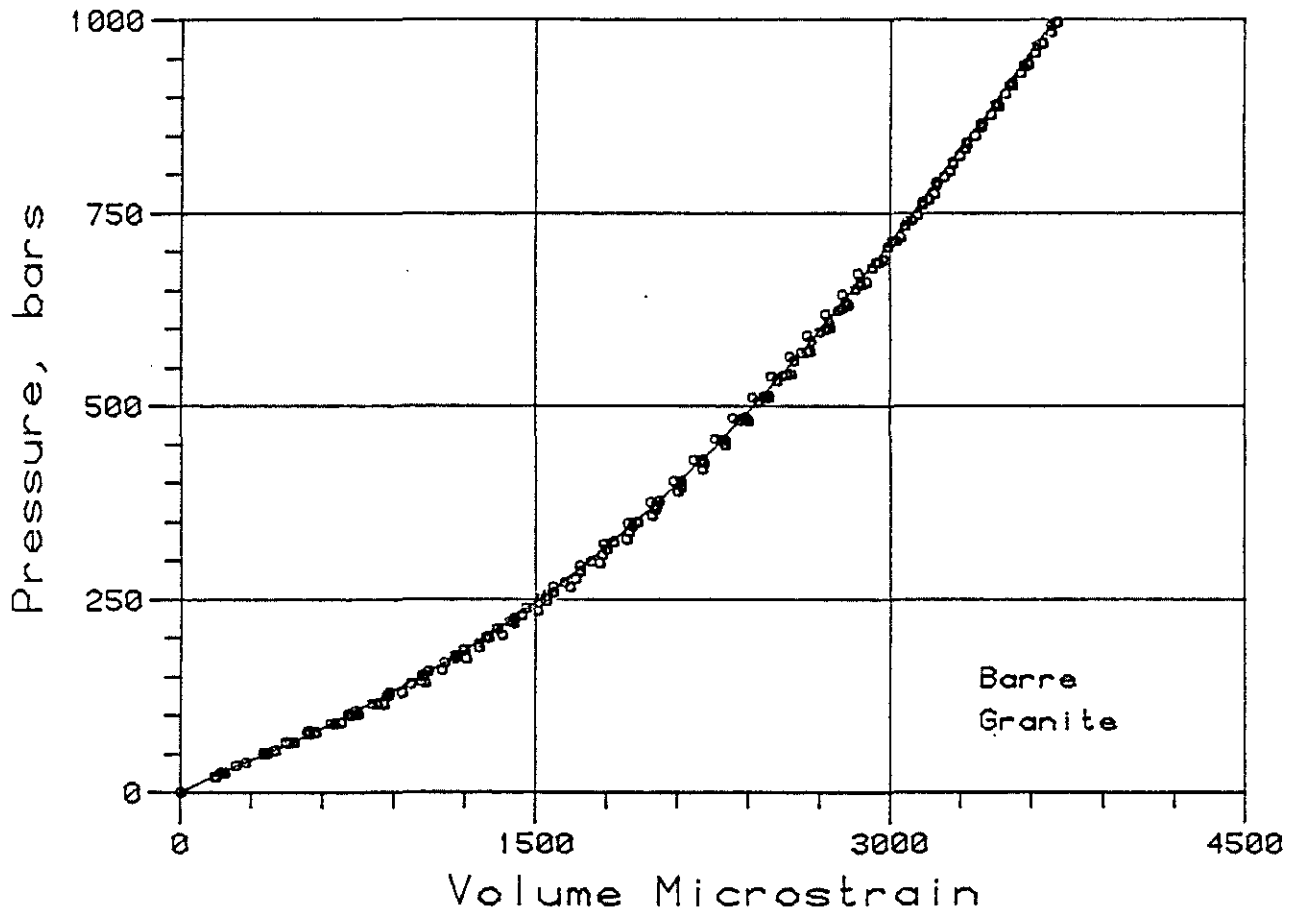


Figure 10. Volumetric strain versus confining pressure for Barre granite. Solid line is vacuum dry stress-strain relation. Square symbols are volumetric strains measured at various pore pressures.

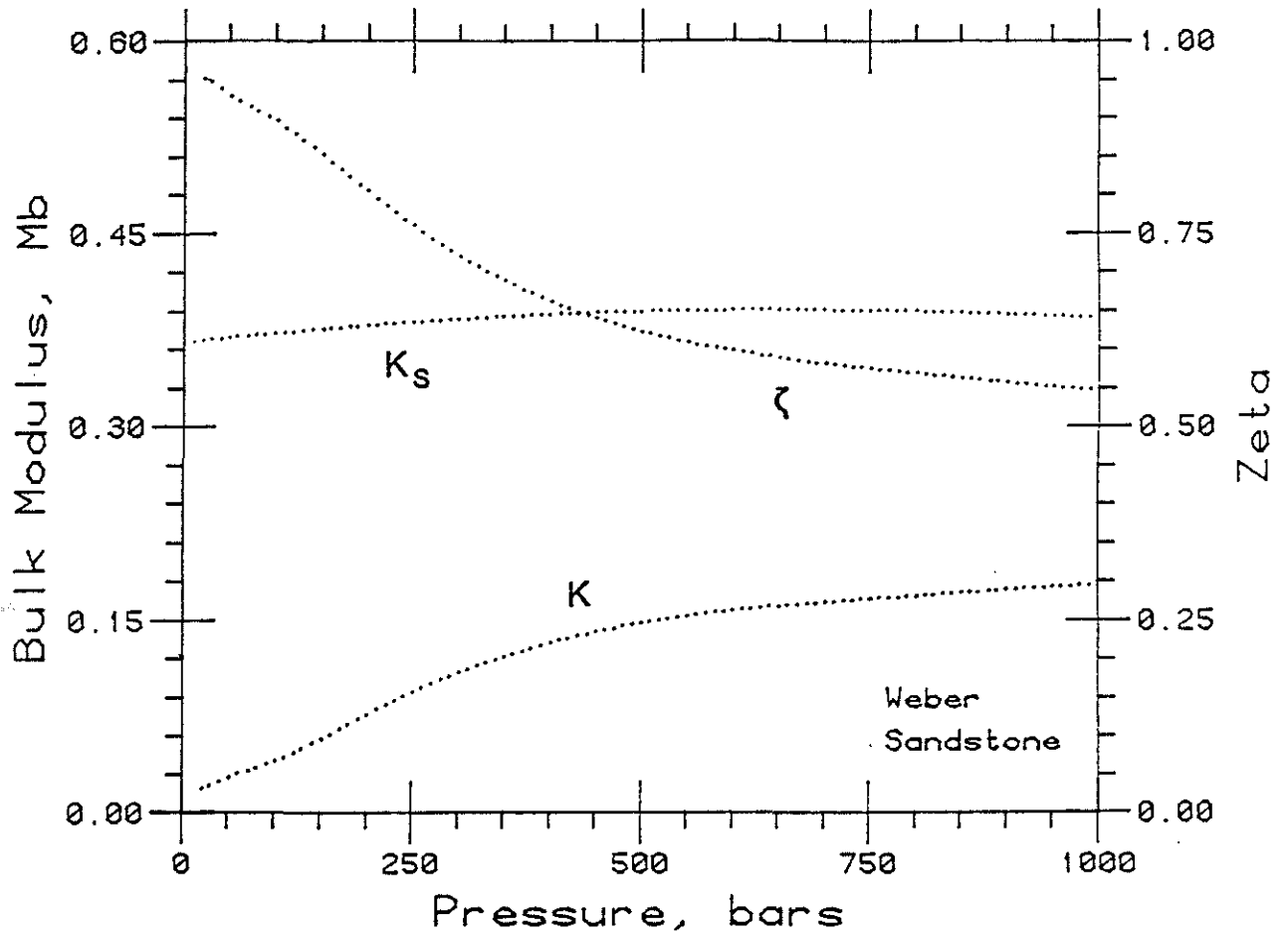


Figure 11. Jacketed bulk modulus K ,unjacketed bulk modulus K_s , and ζ as a function of hydrostatic confining pressure for Weber sandstone.

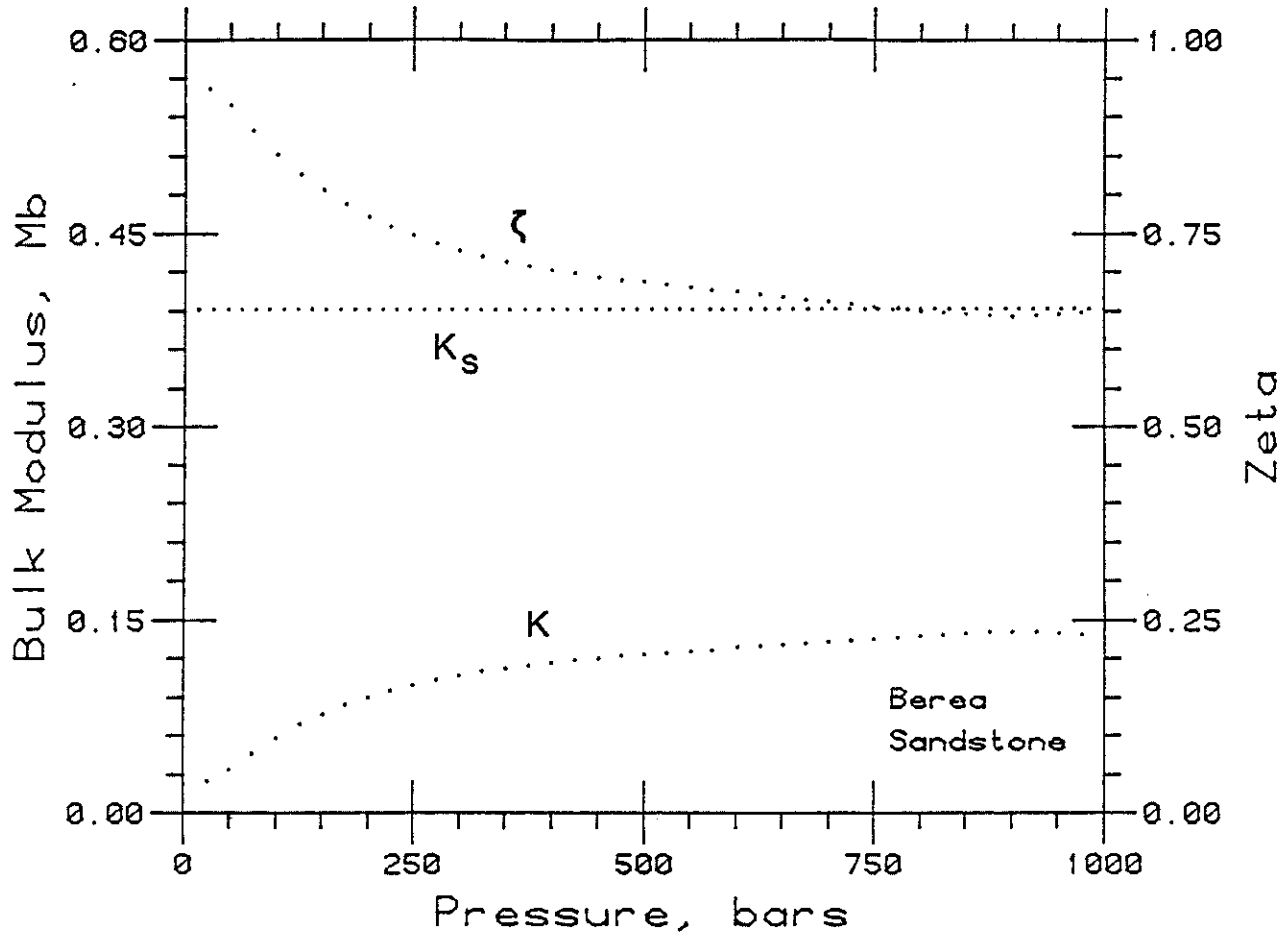


Figure 12. Jacketed bulk modulus K ,unjacketed bulk modulus K_s , and ζ as a function of hydrostatic confining pressure for Berea sandstone.

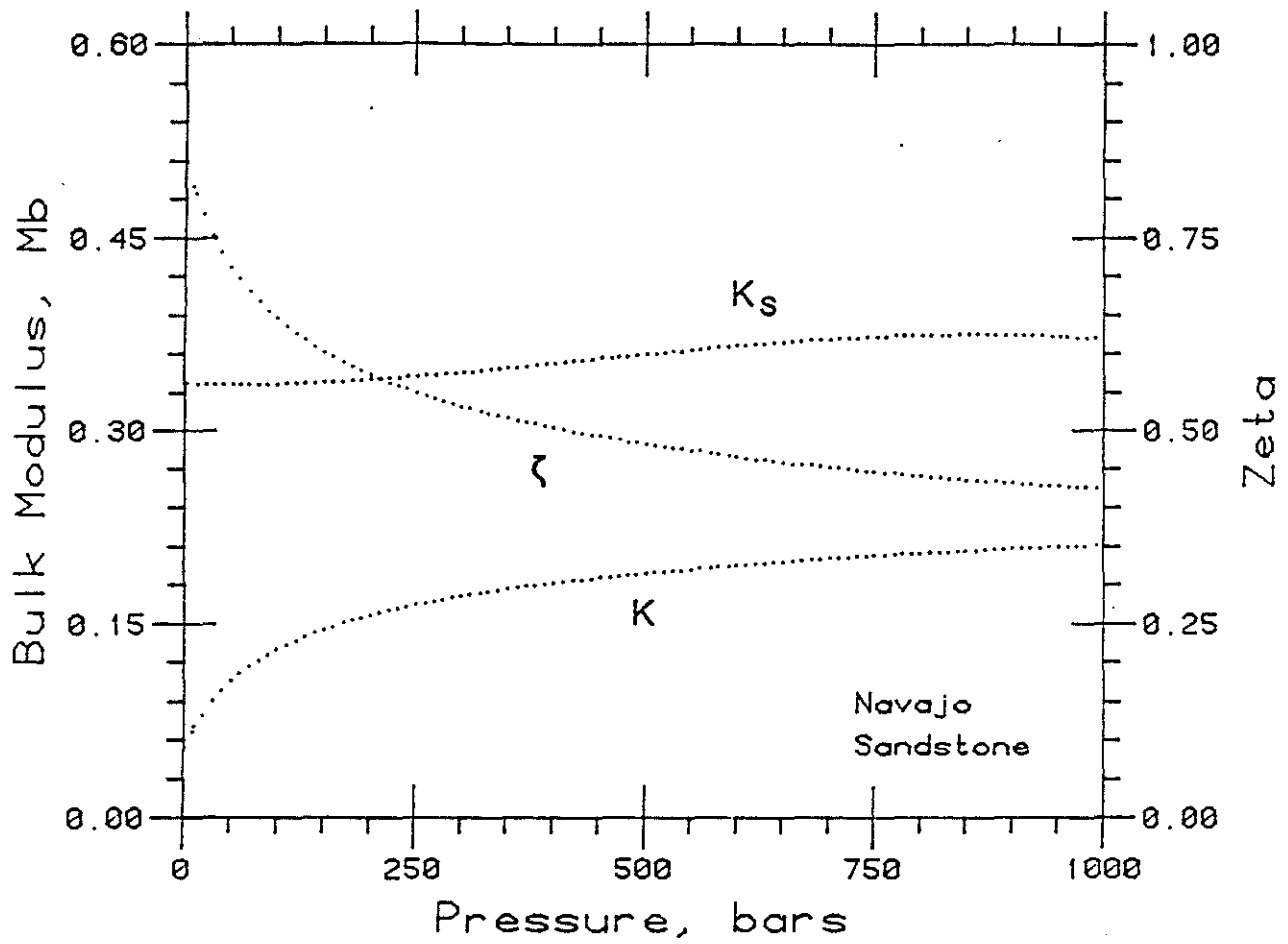


Figure 13. Jacketed bulk modulus K ,unjacketed bulk modulus K_s , and ζ as a function of hydrostatic confining pressure for Navajo sandstone.

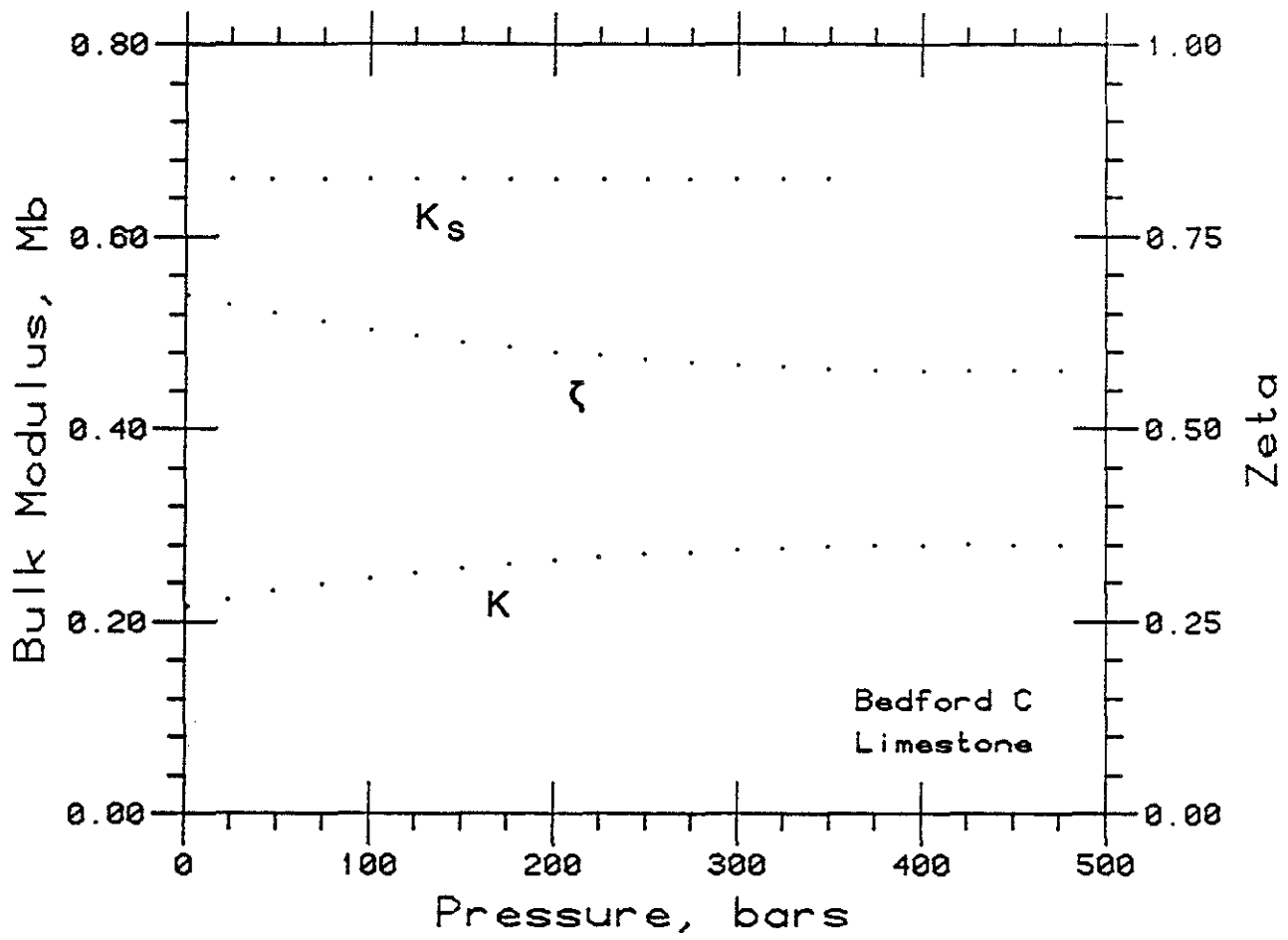


Figure 14. Jacketed bulk modulus K ,unjacketed bulk modulus K_s , and ζ as a function of hydrostatic confining pressure for Bedford limestone.

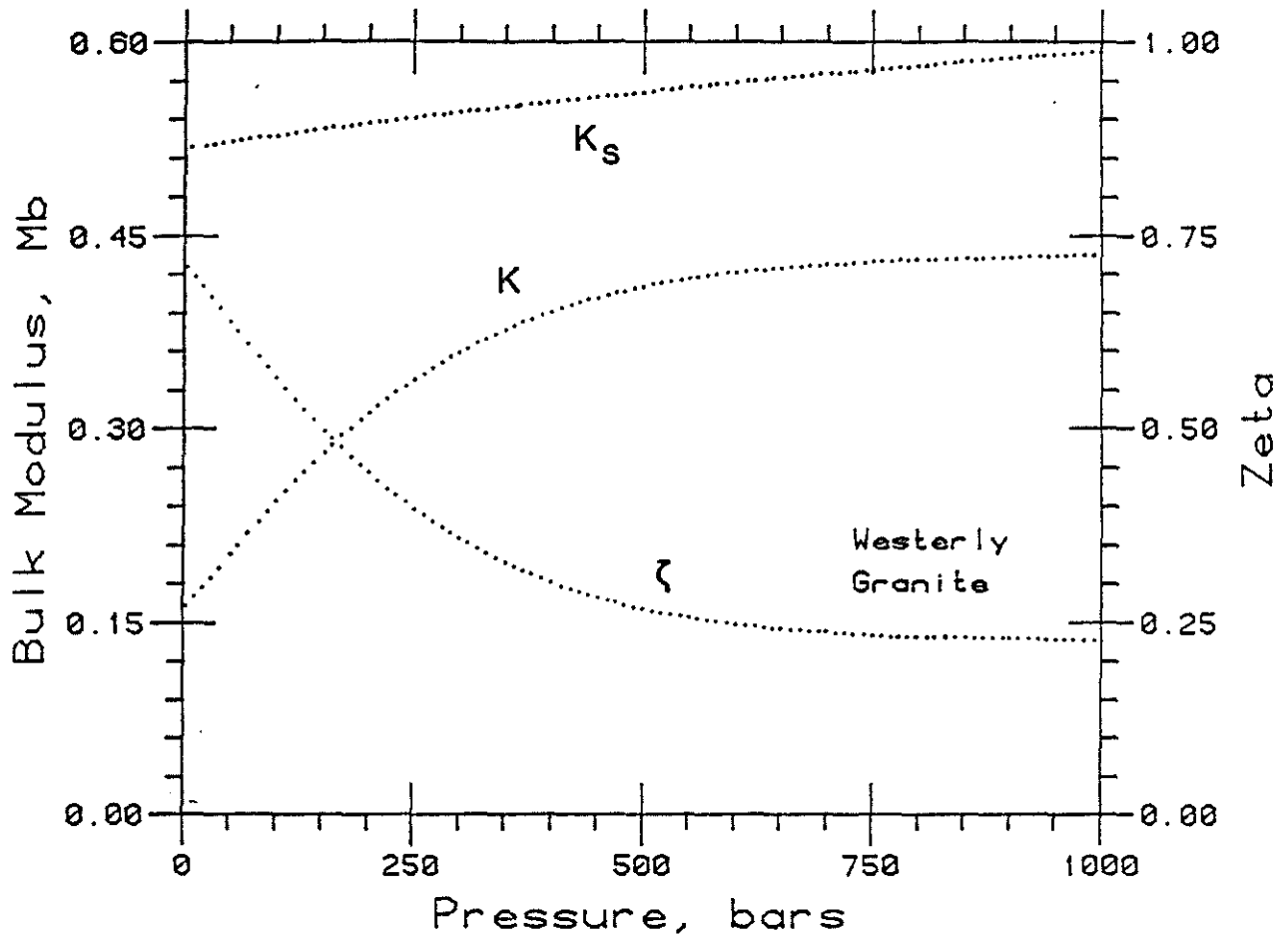


Figure 15. Jacketed bulk modulus K ,unjacketed bulk modulus K_s , and ζ as a function of hydrostatic confining pressure for Westerly granite.

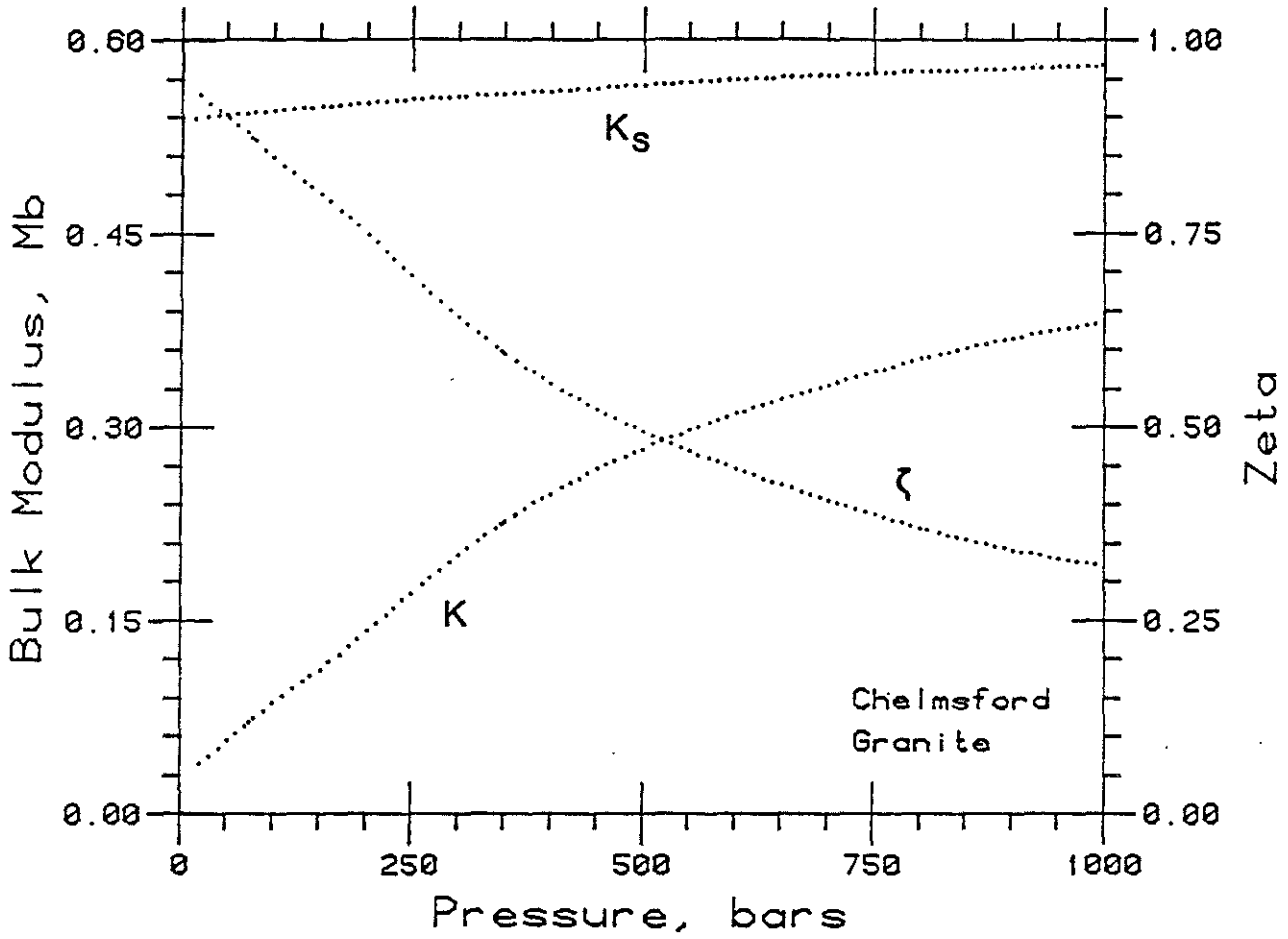


Figure 16. Jacketed bulk modulus K ,unjacketed bulk modulus K_s , and ζ as a function of hydrostatic confining pressure for Chelmsford granite.

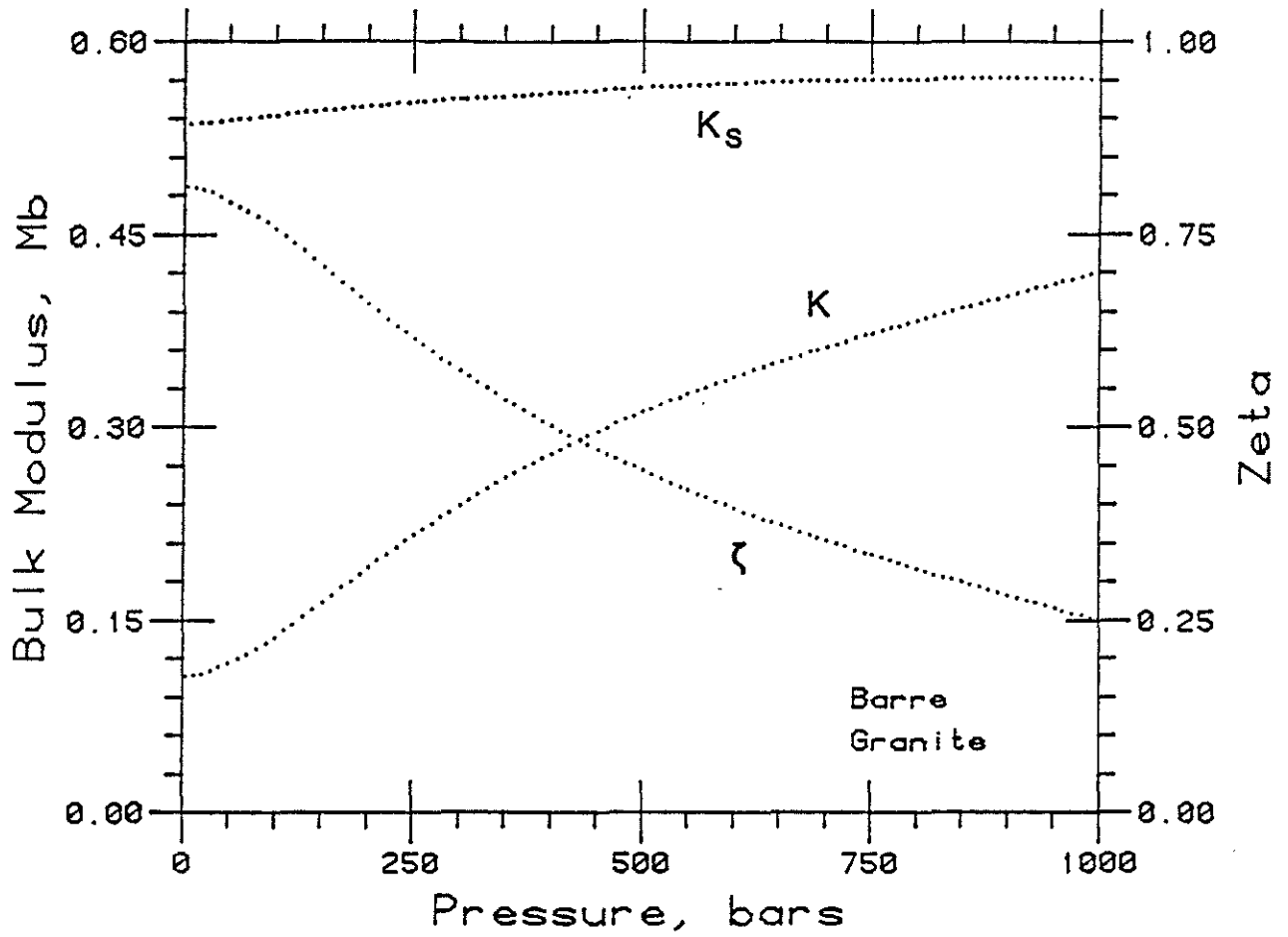


Figure 17. Jacketed bulk modulus K ,unjacketed bulk modulus K_s , and ζ as a function of hydrostatic confining pressure for Barre granite.

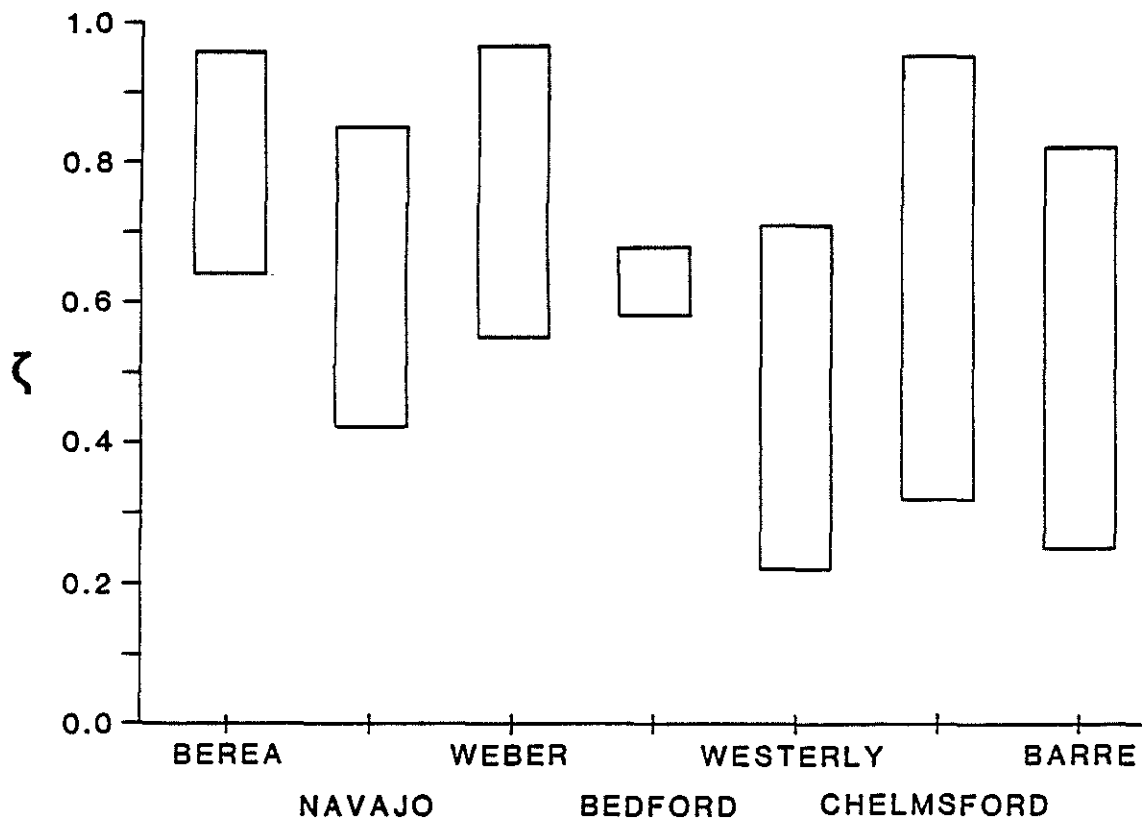


Figure 18. Summary plot of range of ζ values between 0 - 1 kbar confining pressure for seven rocks.

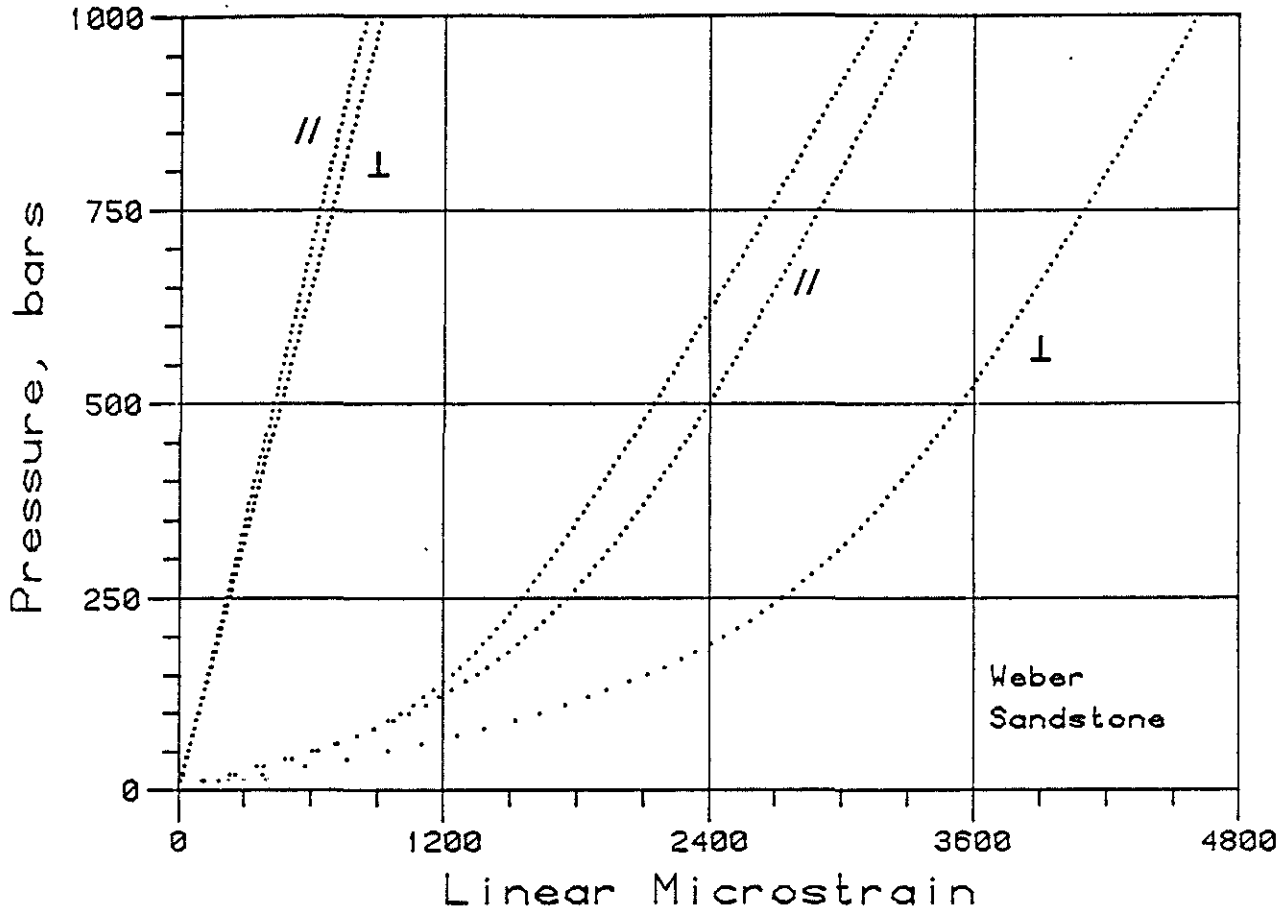


Figure 19. Linear strain versus confining pressure for Weber sandstone. The highly nonlinear curves to the right are jacketed strain; the nearly linear lines to the left areunjacketed strain. Symbols correspond to directions oriented parallel and perpendicular to bedding.

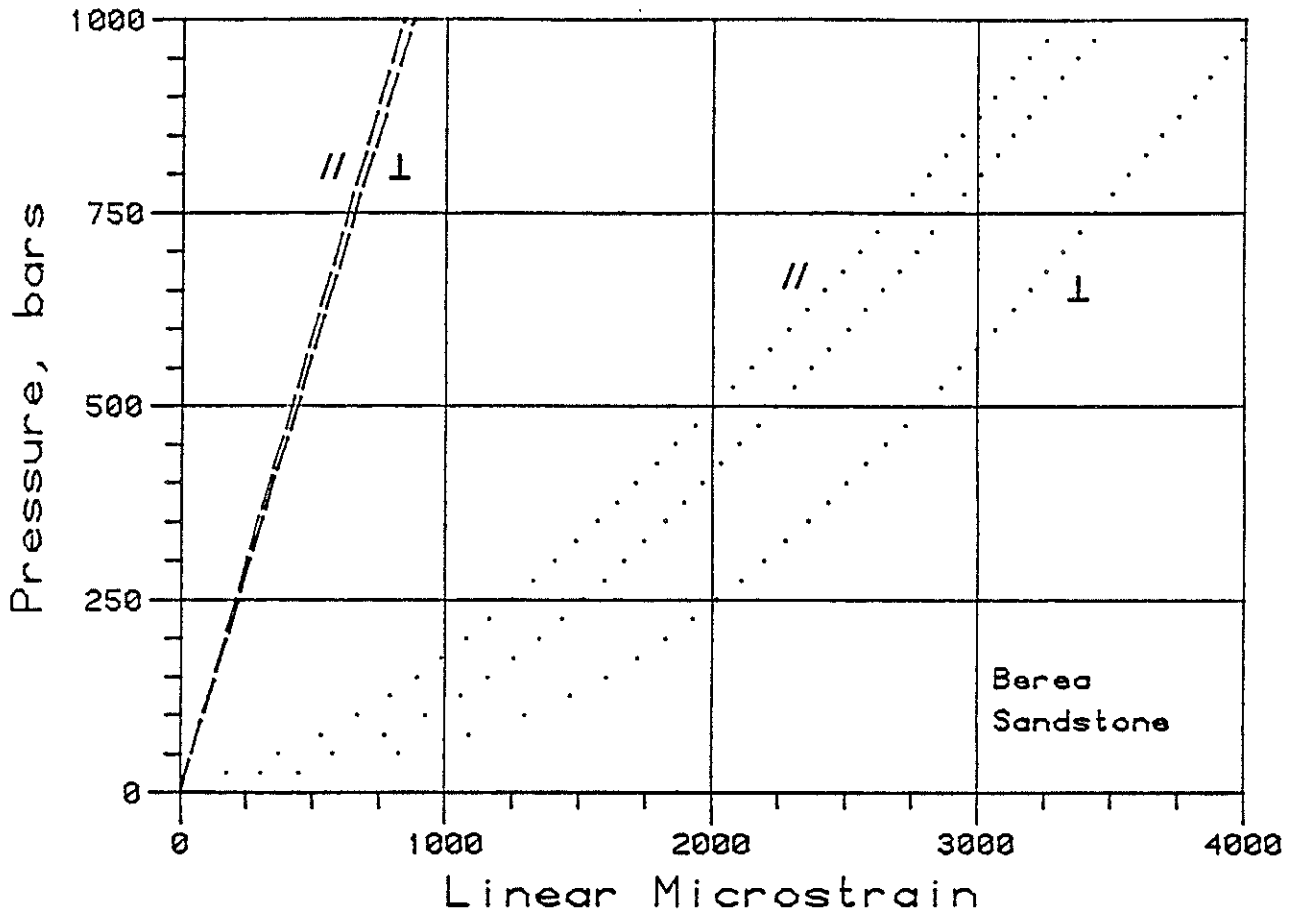


Figure 20. Unjacketed and jacketed linear stress-strain measurements for Berea sandstone. Symbols correspond to directions oriented parallel and perpendicular to bedding.

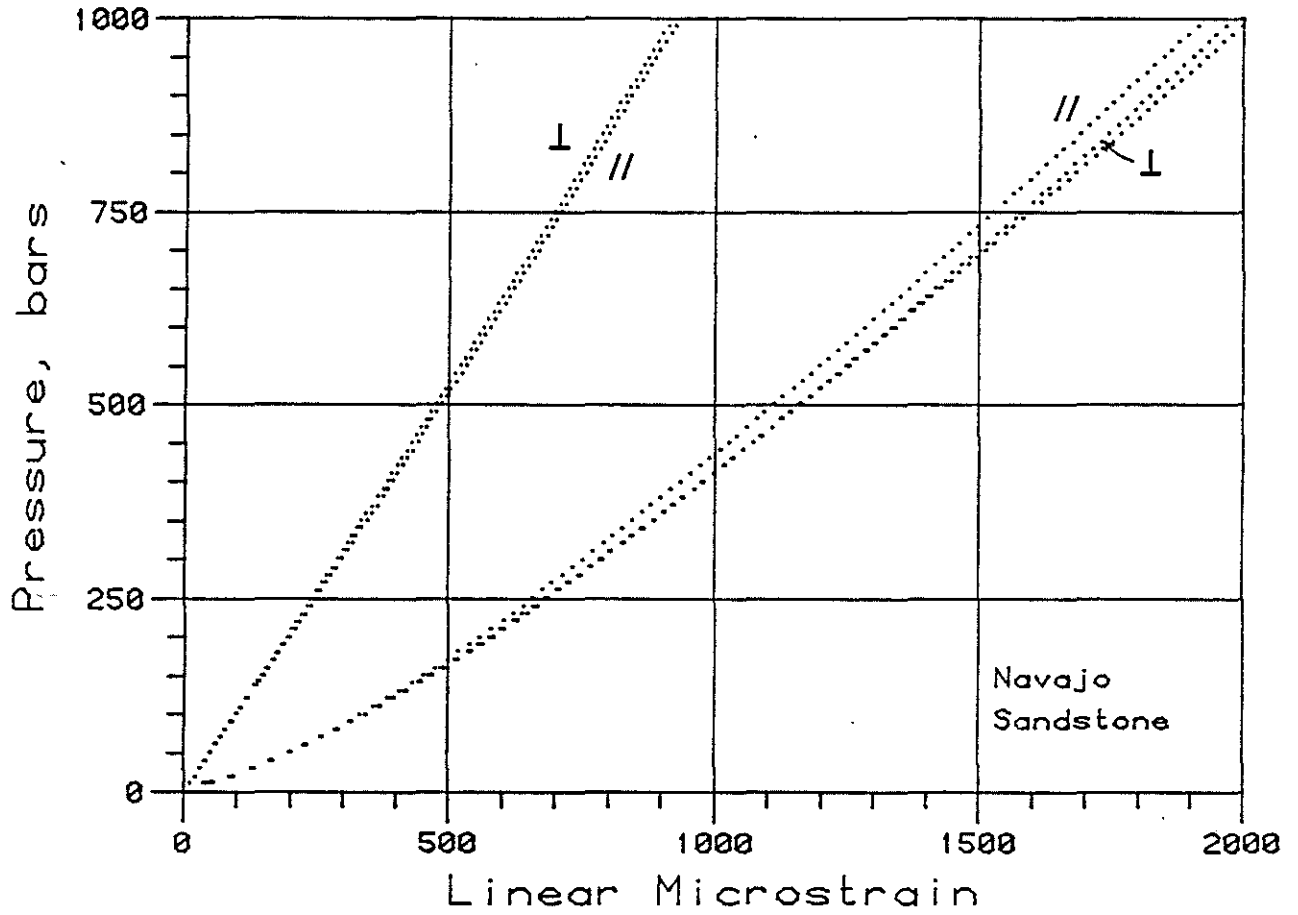


Figure 21. Unjacketed and jacketed linear stress-strain measurements for Navajo sandstone. Symbols correspond to directions oriented parallel and perpendicular to bedding.

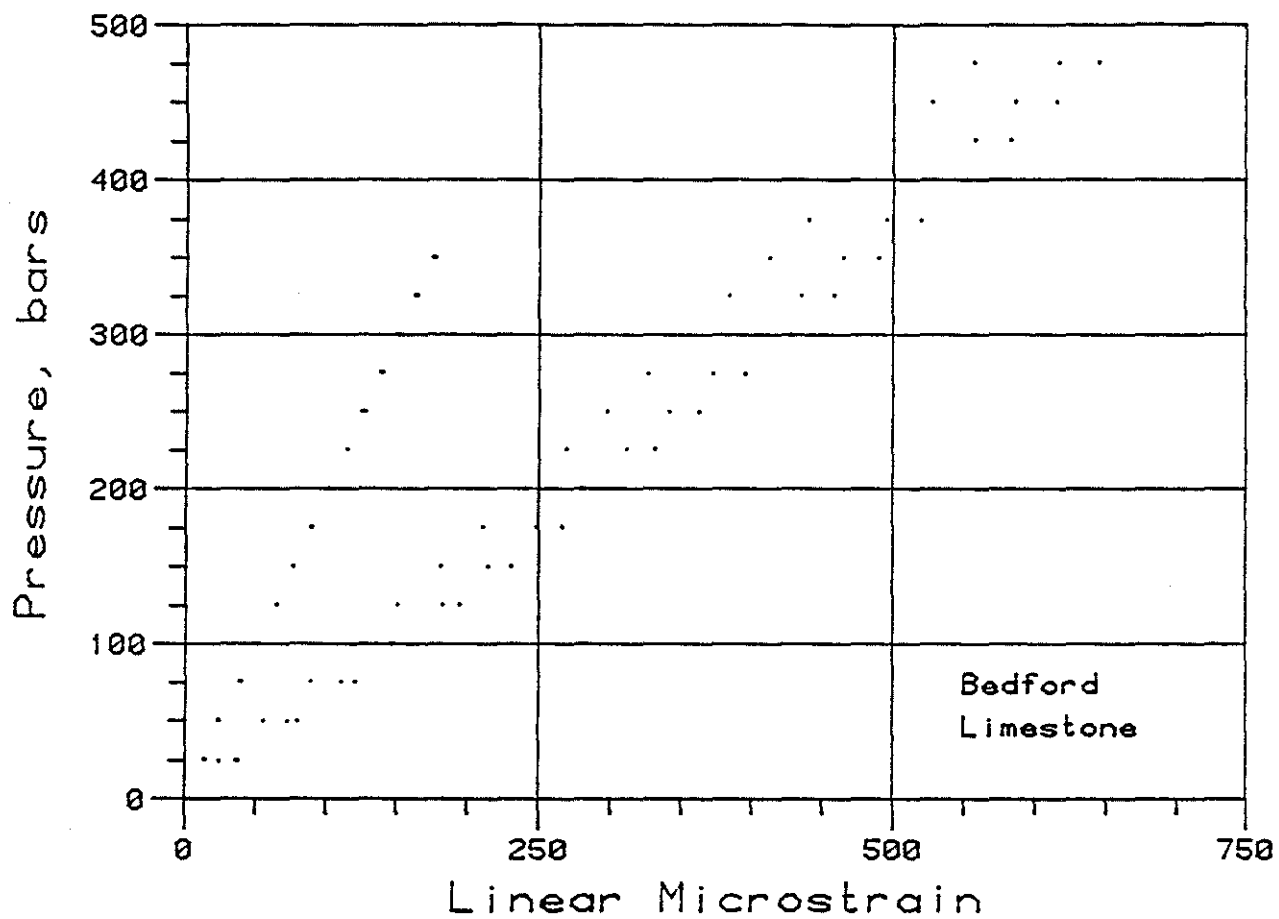


Figure 22. Unjacketed and jacketed linear stress-strain measurements for Bedford limestone.

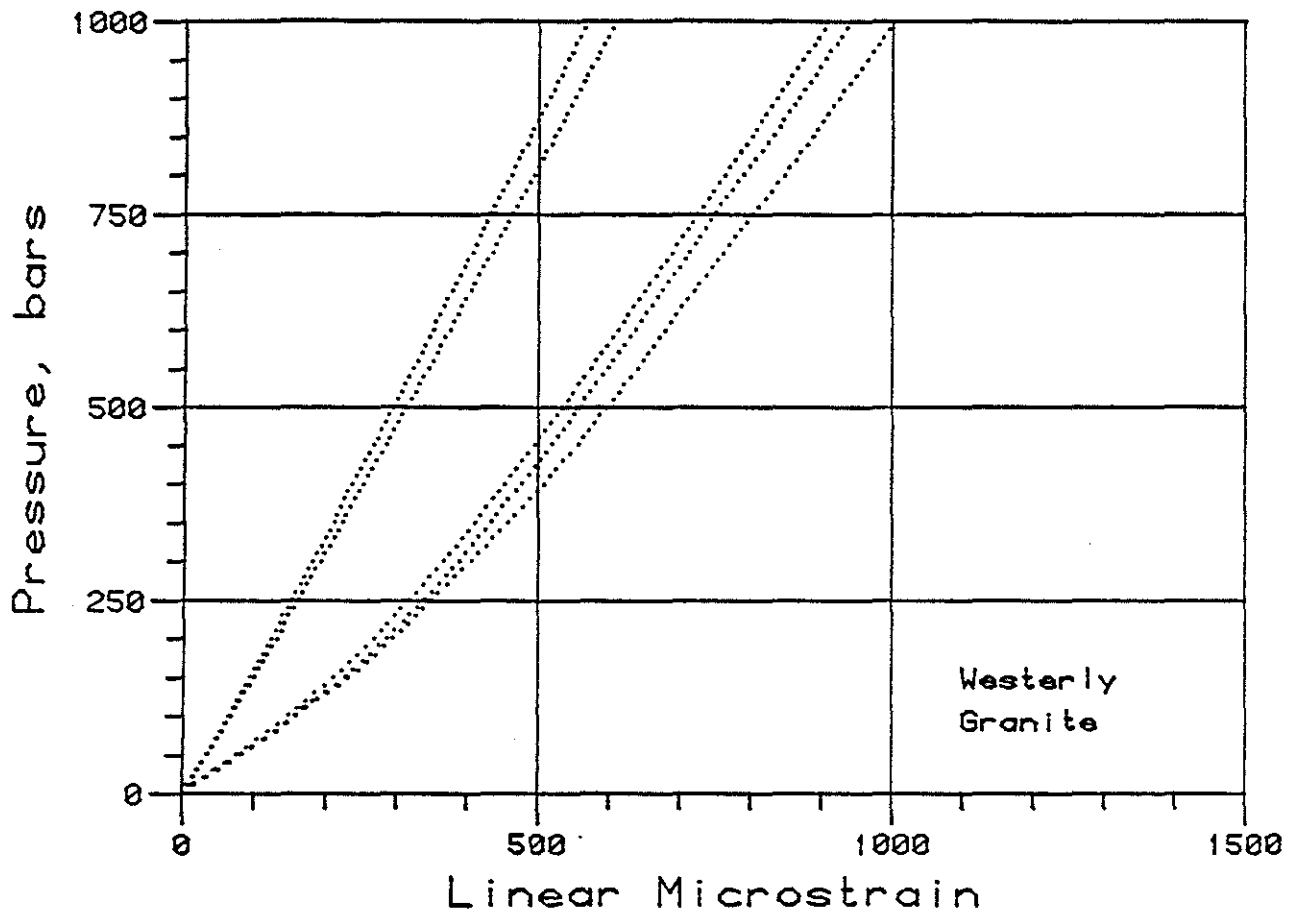


Figure 23. Unjacketed and jacketed linear stress-strain measurements for Westerly granite.

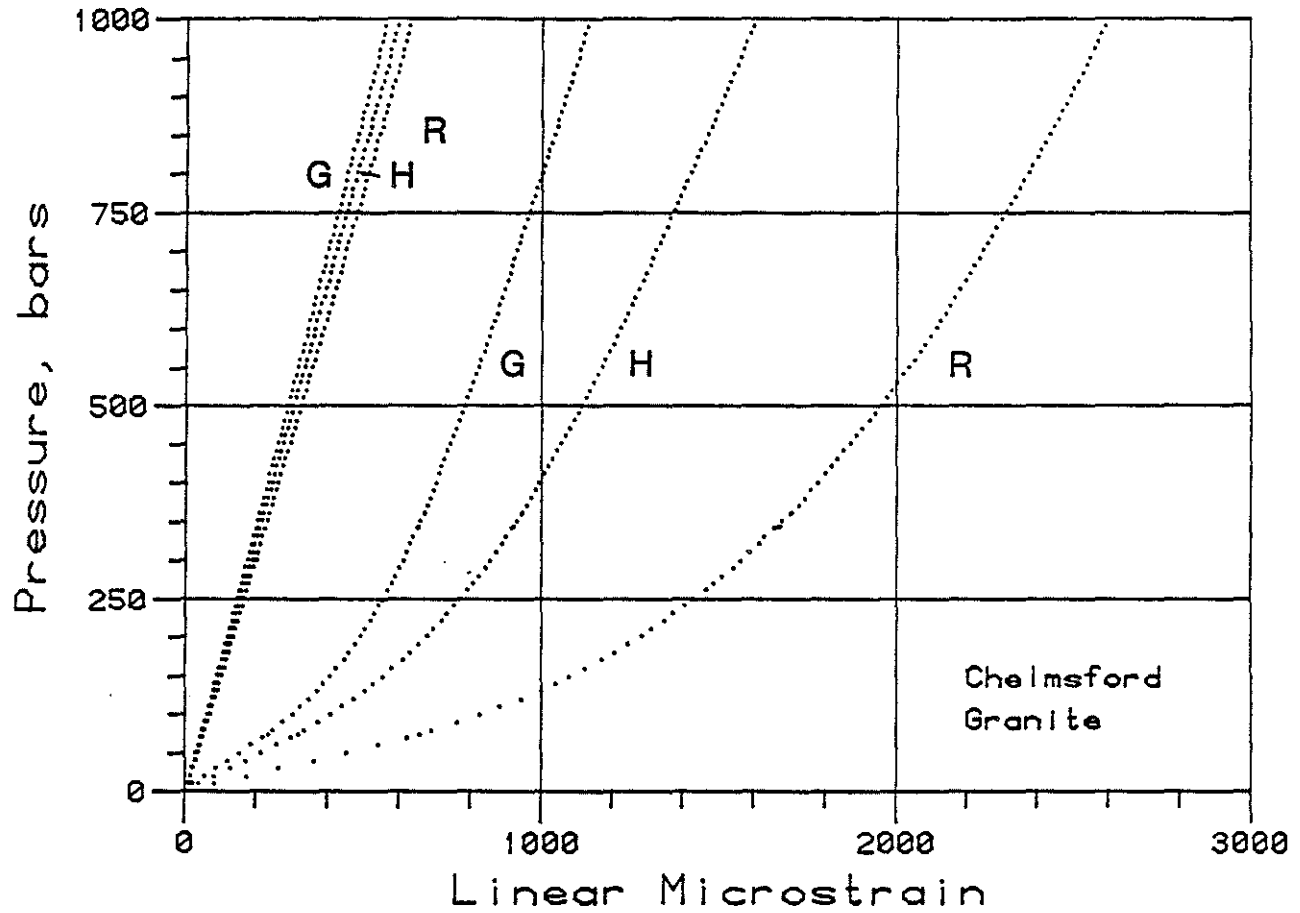


Figure 24. Unjacketed and jacketed linear stress-strain measurements for Chelmsford granite. Curves labelled G, H, and R correspond to orientations perpendicular to grain, headgrain, and rift directions in the quarry.

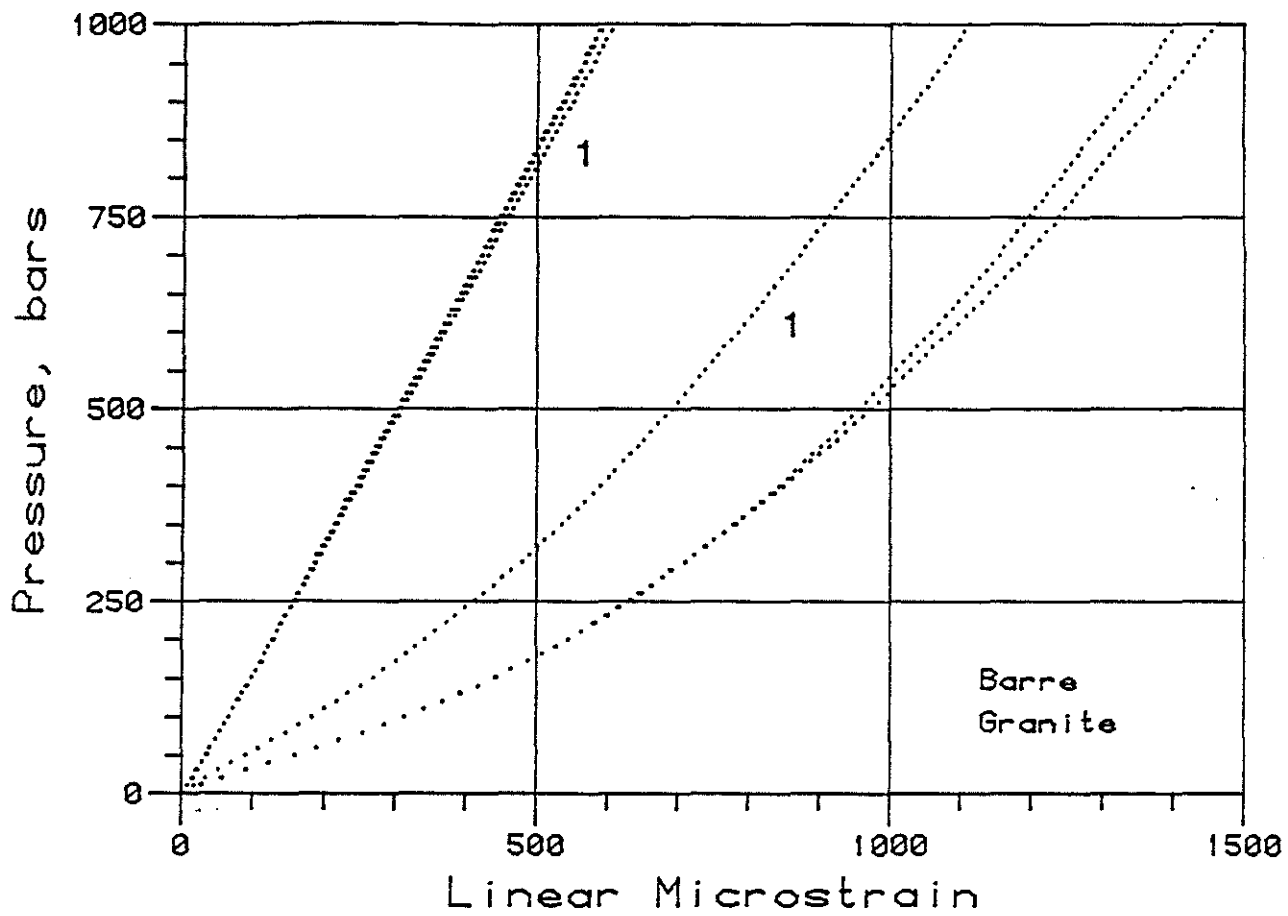


Figure 25. Unjacketed and jacketed linear stress-strain measurements for Barre granite. Curves labelled 1 correspond to same orientation.

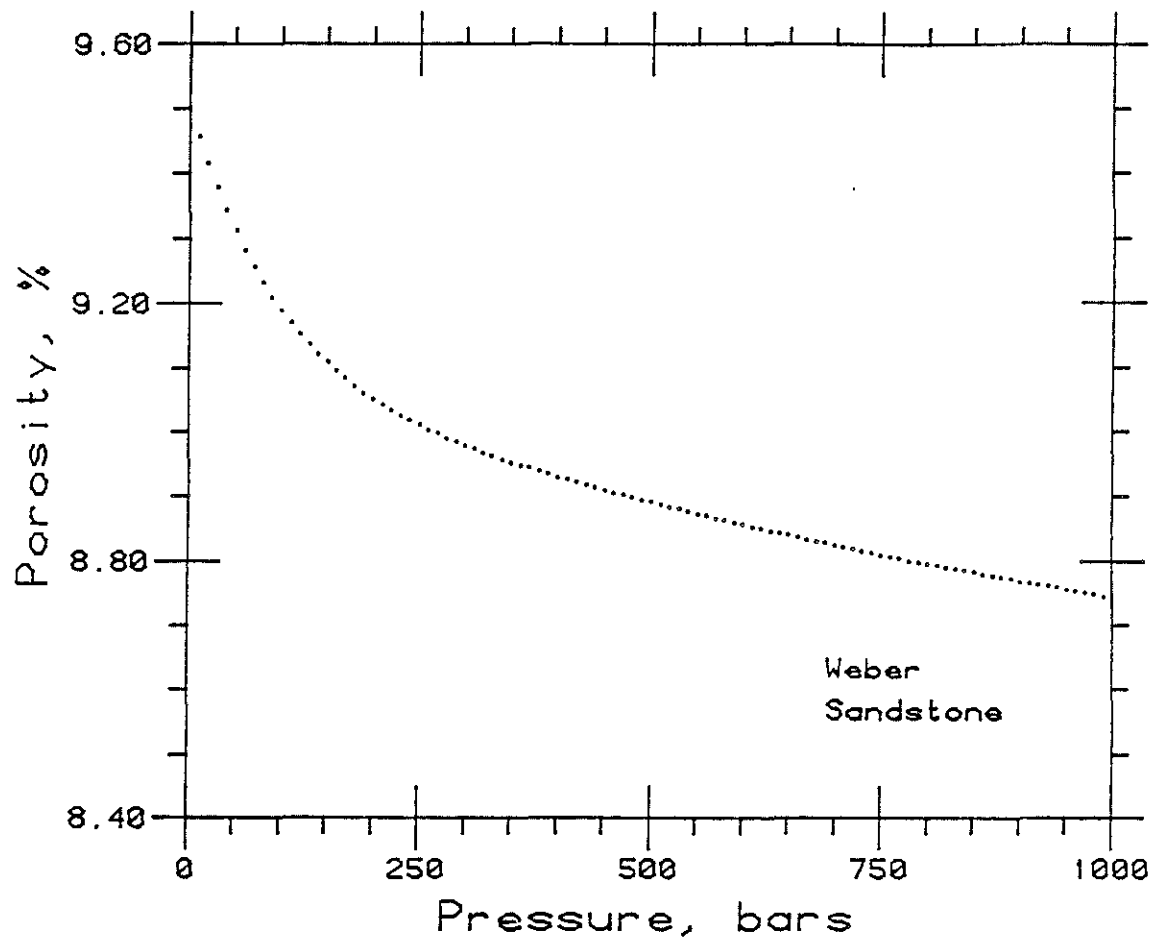


Figure 26. Porosity versus hydrostatic confining pressure for Weber sandstone calculated fromunjacketed and jacketed stress-strain measurements.

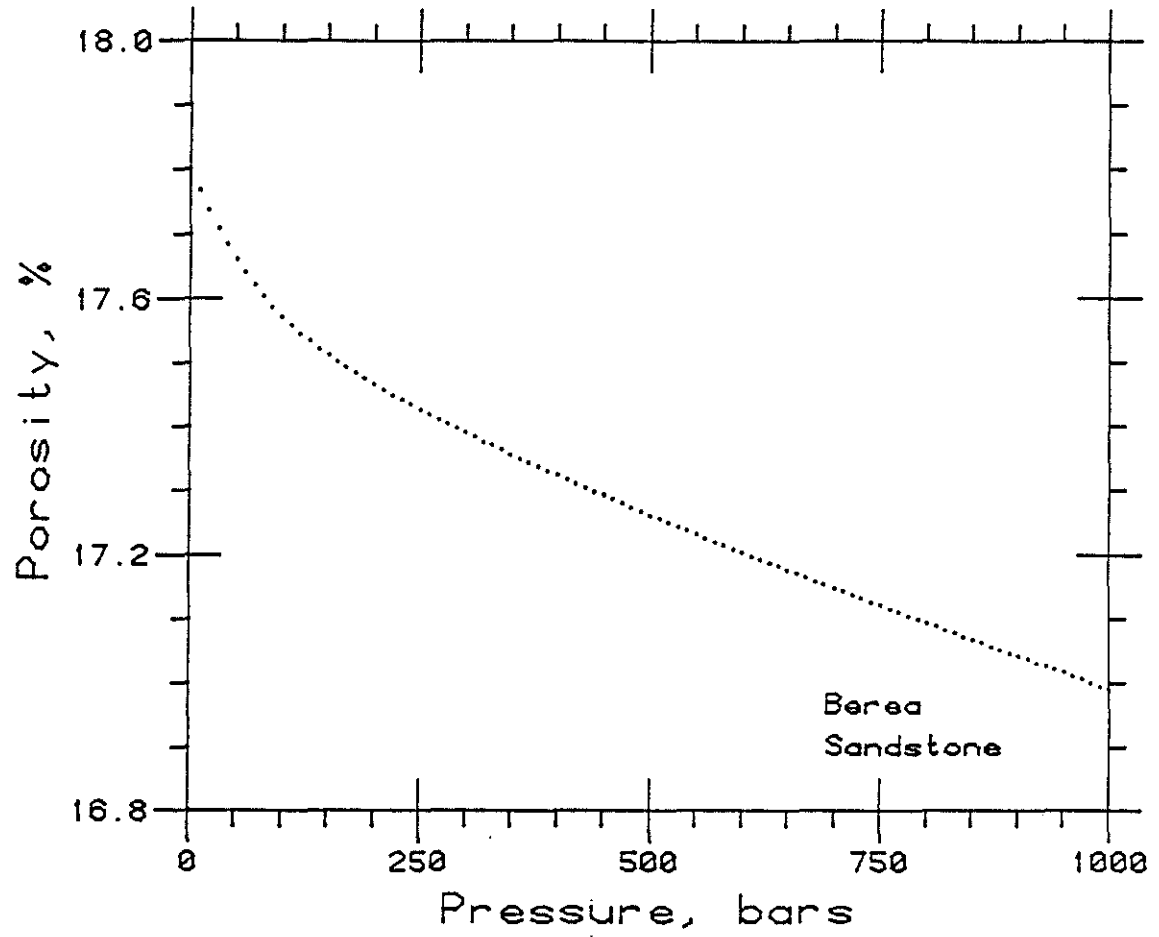


Figure 27. Porosity versus hydrostatic confining pressure for Berea sandstone calculated fromunjacketed and jacketed stress-strain measurements.

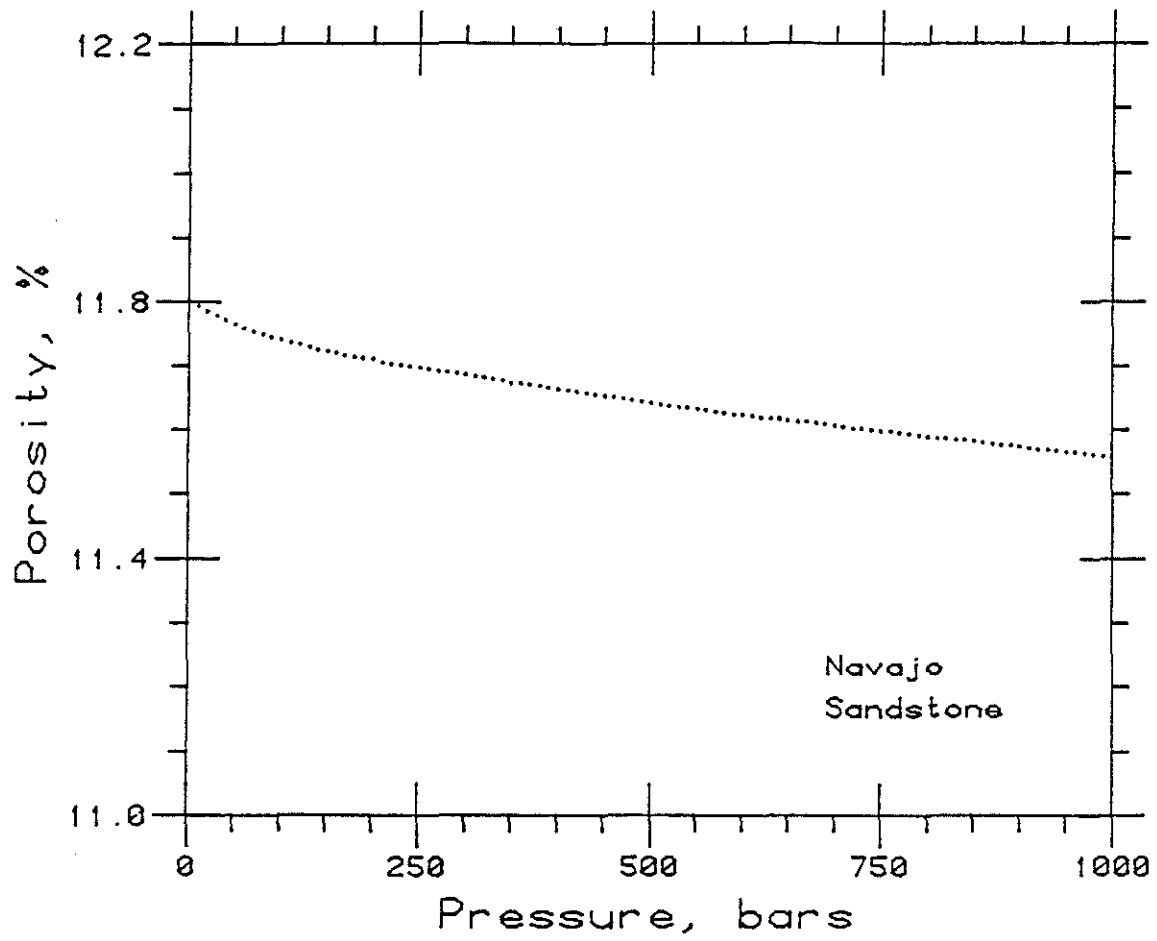


Figure 28. Porosity versus hydrostatic confining pressure for Navajo sandstone calculated fromunjacketed and jacketed stress-strain measurements.

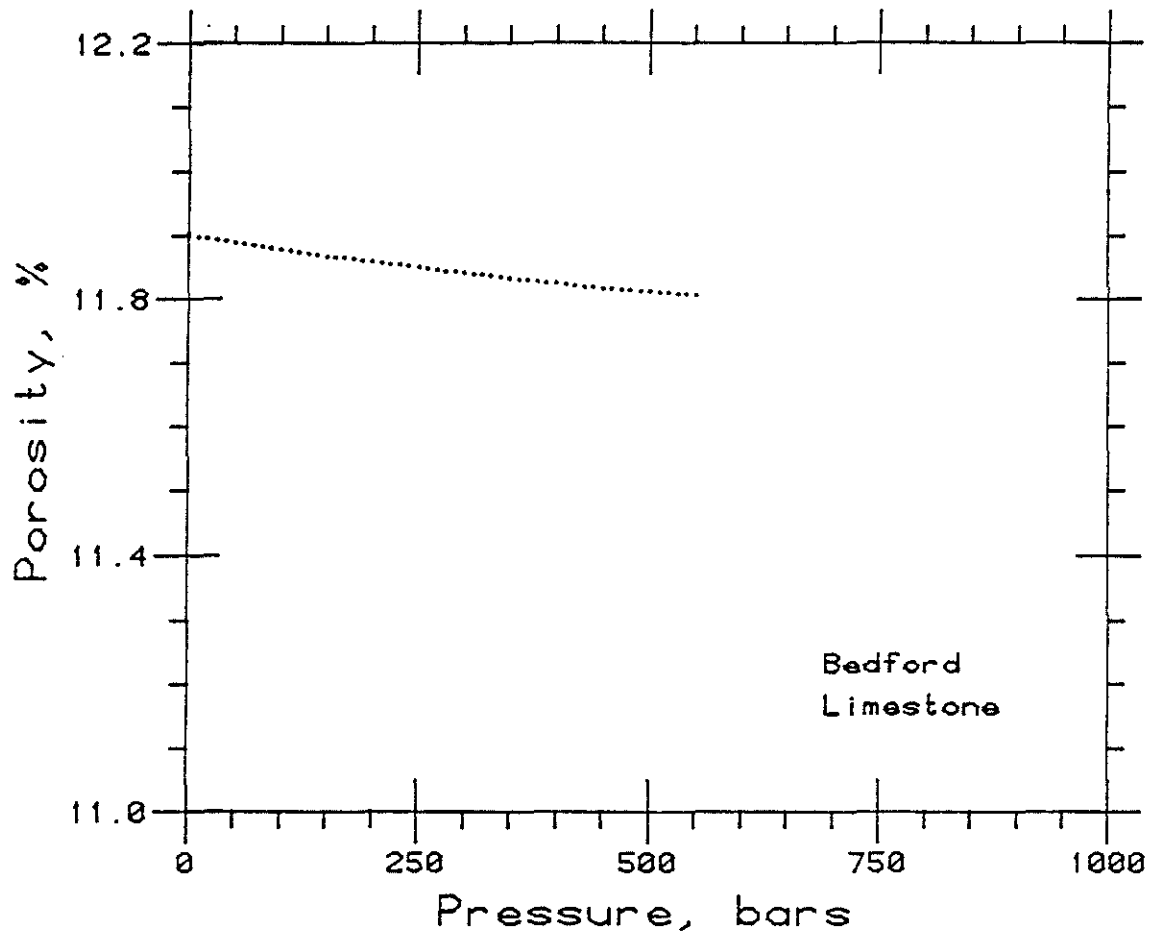


Figure 29. Porosity versus hydrostatic confining pressure for Bedford limestone calculated fromunjacketed and jacketed stress-strain measurements.

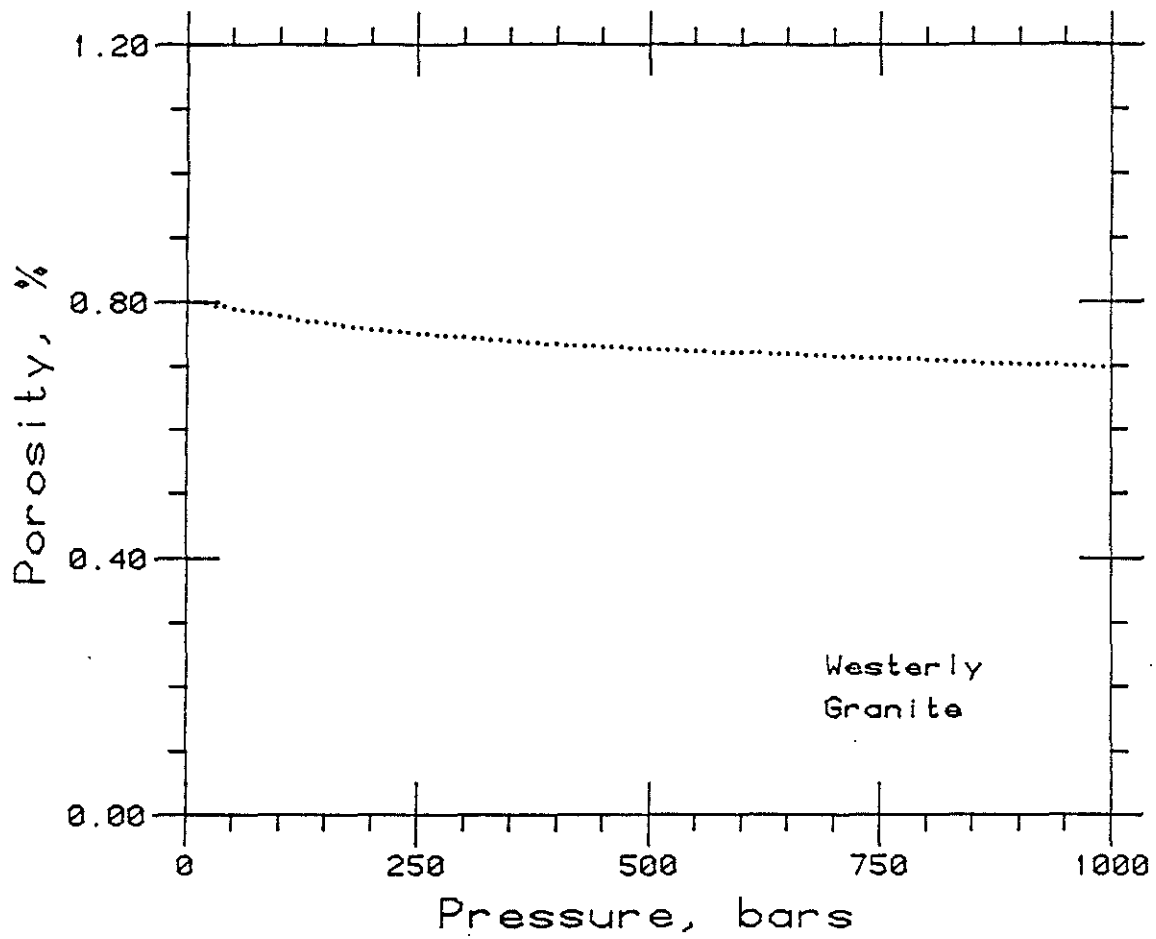


Figure 30. Porosity versus hydrostatic confining pressure for Westerly granite calculated fromunjacketed and jacketed stress-strain measurements.

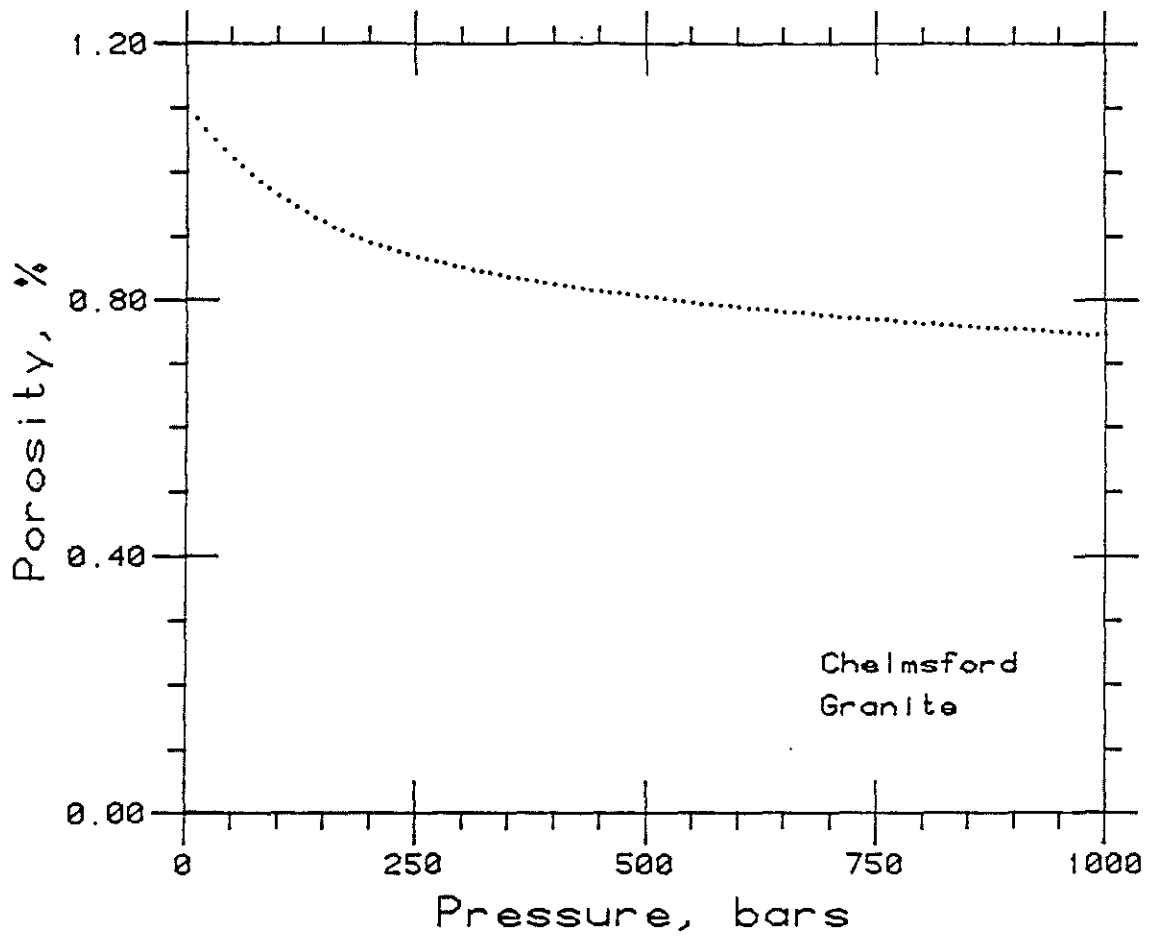


Figure 31. Porosity versus hydrostatic confining pressure for Chelmsford granite calculated fromunjacketed and jacketed stress-strain measurements.

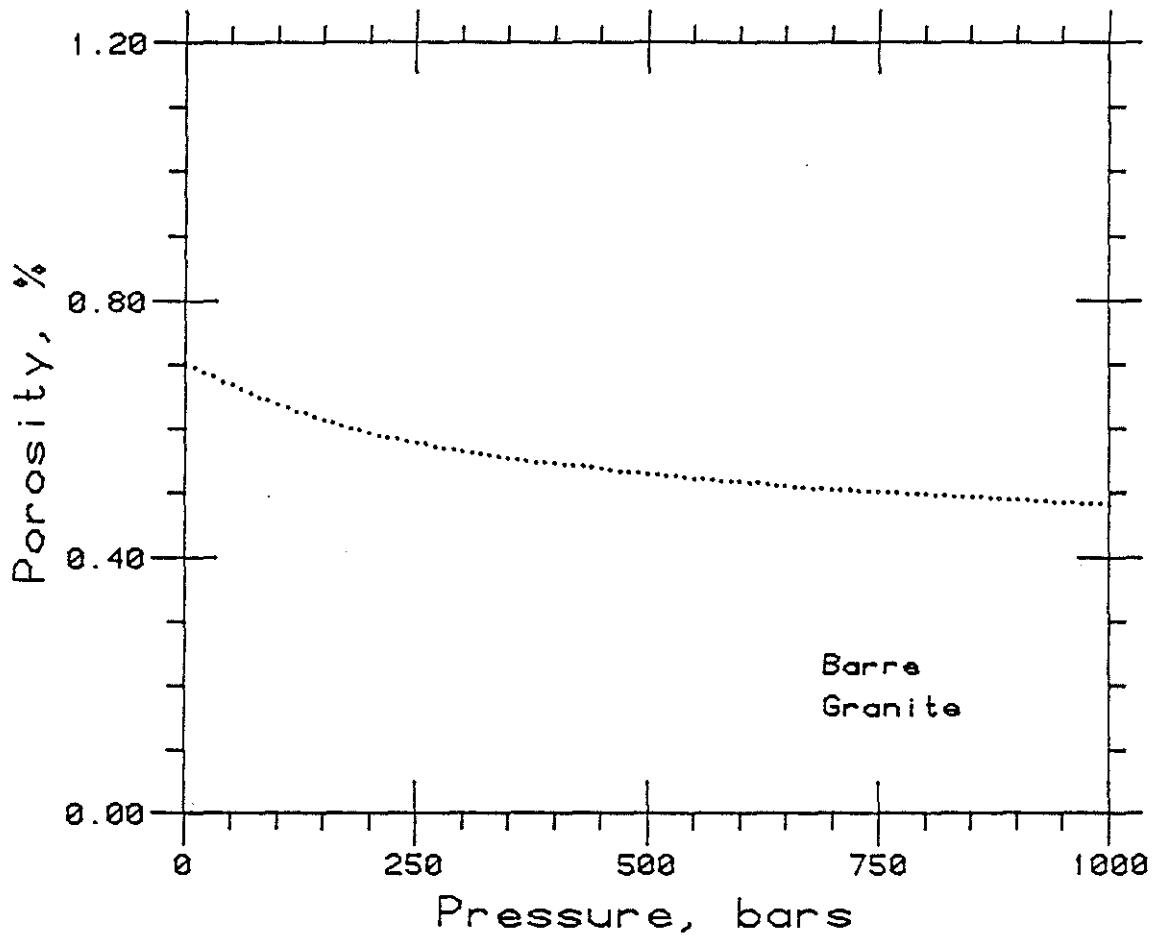


Figure 32. Porosity versus hydrostatic confining pressure for Barre granite calculated fromunjacketed and jacketed stress-strain measurements.