ACOUSTIC LOGGING GUIDED WAVES IN TRANSVERSELY ISOTROPIC FORMATIONS

by

K. J. Ellefsen, C. H. Cheng, and D.P. Schmitt[1]

Earth Resources Laboratory
Department of Earth, Atmospheric, and Planetary Sciences
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

We have studied the velocity dispersion of guided waves in transversely isotropic formations. Theoretical velocity dispersion curves were calculated with elastic constants based on laboratory and field measurements and compared to dispersion curves for isotropic formations having the same vertical P- and S-wave velocities. The symmetry axis for the transverse isotropy was parallel to the borehole. The differences between the phase velocities for the transversely isotropic and isotropic formations depend on the type of wave, its frequency, and the amount of anisotropy, and can be as high as 7 to 10 percent.

The changes in the phase velocity due to changes in the elastic constants of the formation (c_{11}, c_{13}, c_{33}, c_{44}, and c_{66}) and the bulk modulus of the borehole fluid (λ) vary with frequency. In a hard formation, the tube wave's velocity is sensitive to c_{66} at low frequencies, to c_{44} at high frequencies, and to λ at all frequencies. The pseudo-Rayleigh wave is affected by c_{44} near its cutoff frequency and by λ at high frequencies. The flexural wave, which is generated by a shear wave logging tool, is similarly affected by c_{44} at low frequencies and by λ at high frequencies. As the formation becomes soft, the effect of the elastic constants upon the phase velocity gradually changes. Like a hard formation, the tube wave's velocities in a moderately soft formation are primarily affected by c_{66} and λ at low frequencies, but the influence of c_{44} is much greater at high frequencies.

Since array processing methods can accurately estimate the velocity dispersion of the guided waves over a wide range of frequencies, some elastic constants can be estimated. In a hard formation, the refracted P- and S-wave velocities uniquely determine c_{33} and c_{44}, and an inversion can be used to estimate c_{66} and λ. In a moderately soft formation, the refracted P-wave velocity determines c_{33}, the flexural wave from the

[1] Now at Etudes et Productions Schlumberger, 26 Rue de la Cavée, 92142 Clamart, France

shear wave logging tool determines c_{44}, and the tube wave's velocity dispersion can be used to estimate c_{66} and λ.

INTRODUCTION

Knowledge of the mechanical properties of a formation is useful in several areas of hydrocarbon exploration and production. In the context of elastic wave propagation, the mechanical properties are described by elastic constants. These constants are currently used to estimate a formation's strength, which is important information in fracturing and preventing collapse during production, and to tie acoustic logging and seismic data, which is necessary for reservoir characterization and stratigraphic analysis away from the borehole.

The refracted P- and S-waves from acoustic logging data are used frequently to determine the bulk and shear moduli, which adequately describe the dynamic behavior of isotropic rocks. However, when a rock is transversely isotropic, which is quite common for many sedimentary rocks (Thomsen, 1986), the information from these two waves just is not enough to completely characterize the mechanical properties. To overcome this problem, the guided waves might be used. By applying array processing methods to data collected from multi-receiver tools, accurate velocity dispersion curves for the guided waves can be estimated. The important question is whether the dispersion curves for the guided waves can be used to estimate the elastic constants of a transversely isotropic formation.

Previous authors had conducted numerical studies to determine the effects of a transversely isotropic formation upon the acoustic logging waveforms. White and Tongtaow (1981) studied the waves from monopole and dipole sources and determined how the formation affects the velocities and amplitudes of the refracted waves. They also determined what elastic constants affect the tube wave at zero frequency and the pseudo-Rayleigh wave at its cutoff frequency and suggested that this information could be used to help determine the elastic constants. Chan and Tsang (1983) investigated acoustic logging in radially layered, transversely isotropic formations, and their work focused on the refracted waves as White and Tongtaow's (1981) did.

The purpose of this paper is to show how the phase velocities of the guided waves are affected by the mechanical properties of transversely isotropic formations. Their effects upon the dispersion curves are emphasized by comparing the curves to those for isotropic formations having the same vertical P- and S-wave velocities. Normalized partial derivatives of the phase velocity with respect to each elastic constant are calculated over a broad range of frequencies. These derivatives show the relationship between the dispersion curves and the elastic constants and suggest which constants can be estimated. The analysis was conducted for a hard formation and a soft formation, which have typical velocity dispersion curves.

METHOD

The borehole model used to study the problem of acoustic logging in a transversely isotopic medium consists of a fluid-filled, infinitely long, cylindrical hole in an infinite formation (Figure 1). The fluid is described by its bulk modulus (λ) and density (ρ_f), an isotropic formation by two elastic constants (c_{33} and c_{44}) and density (ρ), and a transversely isotropic formation by five elastic constants (c_{11}, c_{13}, c_{33}, c_{44}, and c_{66}) and density (ρ). The axis of symmetry for the transverse isotropy is parallel to the borehole. The fluid and formation are assumed to be homogeneous and linearly elastic. The effect of the logging tool will be ignored to simplify the analysis.

Relating the elastic constants to various velocities will help us study the velocity dispersions of the guided waves. In an isotropic formation, the P-wave velocity and consequently the refracted P-wave velocity are given by $\sqrt{c_{33}/\rho}$. Similarly the S-wave velocity and the refracted S-wave velocity are $\sqrt{c_{44}/\rho}$. In a transversely isotropic formation, the relationship between the velocities and the elastic constants becomes quite complicated (see e.g., White, 1983), but in some special directions the mathematical relations simplify. For example, the velocities for vertically propagating (parallel to the axis of symmetry) P- and S-waves are $\sqrt{c_{33}/\rho}$ and $\sqrt{c_{44}/\rho}$, respectively. These velocities also apply to the refracted P- and S-waves in the acoustic logging situation (White and Tongtaow, 1981). In the horizontal direction, the P-wave velocity is $\sqrt{c_{11}/\rho}$, and the S-wave velocity is either $\sqrt{c_{44}/\rho}$ or $\sqrt{c_{66}/\rho}$ depending upon its polarization.

An important issue is the extent to which the velocity dispersion curves for the guided waves would change if a transversely isotropic formation is incorrectly assumed to be isotropic. The elastic moduli in isotropic rocks are determined by the refracted P- and S-waves in hard formations and by the refracted P-wave and flexural wave in soft formations. In cases where the shear wave logging tool is not available and thus there is no flexural wave measurement, formation shear wave velocities are sometime obtained indirectly from the tube wave velocities (Chen and Willen, 1984; Stevens and Day, 1986; Cheng and Toksöz 1983). With only this information, distinguishing between the isotropic and transversely isotropic formations is not possible. Therefore, an equivalent isotropic formation was defined as having the same vertical P- and S-wave velocities as the transversely isotropic formation. The phase velocity for guided waves in this equivalent formation was calculated using the period equation, and the group velocity using Rayleigh's principle.

The phase and group velocities were calculated over a broad frequency range for tube, pseudo-Rayleigh, flexural, and screw (quadrupole) waves in hard and soft, transversely isotropic formations and are compared to the velocities for the equivalent isotropic formations. The hard and soft formations are the Green River shale and shale (5000), respectively, for which the properties are tabulated by Thomsen (1986). These two examples were chosen because their velocities exhibit the general characteristics of many hard and soft transversely isotropic formations.

The effects which the elastic constants have upon guided wave phase velocity are shown by the partial derivatives of velocity with respect to the elastic constants. The derivatives are computed at constant frequency with Rayleigh's principle to avoid numerical differentiation. After normalization, the derivatives, which will be called sensitivities, have the form $\frac{m}{c}\frac{\partial c}{\partial m}$ in which c is the phase velocity and m is any elastic constant. The sensitivities can be interpreted as the percent change in phase velocity due to a one percent change in the elastic constants. The sensitivities are computed for the hard and soft, transversely isotropic formations.

RESULTS AND DISCUSSIONS

Hard Formation

The phase and group velocities for the guided waves in a transversely isotropic, hard formation and an equivalent isotropic formation are shown in Figure 2. In general, the shapes of the two sets of curves are very similar. The phase and group velocities of the guided waves in the transversely isotropic formation are generally higher than those for the equivalent isotropic formation, the only exception being the group velocity of the high frequency pseudo-Rayleigh wave. The consistently higher velocities are caused by the higher rigidity of the transversely isotropic formation in the horizontal direction.

The sensitivities for the guided waves are shown in Figure 3. For the tube wave, the effects of λ and c_{66} on the phase velocity are important at low frequencies but diminish as the frequency increases. Although the effect of c_{44} is negligible at low frequencies, it increases at higher frequencies. By comparison, in an isotropic formation the effect of c_{44} is important at low frequencies and decreases with increasing frequency (Cheng et al., 1982). Elastic constants c_{11}, c_{13}, and c_{33} have little influence upon the tube wave's phase velocity. This behavior is similar to the isotropic case in which the formation P-wave velocity has little effect on the guided waves. The phase velocities of the pseudo-Rayleigh, flexural, and screw waves are entirely controlled by c_{44} near their cutoff frequencies. This relationship is expected because the velocity of the refracted S wave, associated with these guided waves at their cutoff frequencies, is determined by c_{44}. As the frequency increases, the influence of c_{44} diminishes and that of λ increases, the latter becoming dominant at high frequencies. The phase velocities are virtually unaffected by the other elastic constants.

Soft Formation

The phase and group velocites for the guided waves in the transversely isotropic, soft formation and the equivalent isotropic formation are shown in Figure 4. As in the case of the hard formation the shapes of the two sets of curves are similar, and the velocities in the transversely isotropic formation are higher than those for the equivalent isotropic formation. The differences in the two sets of velocities are larger than those in the hard formation because the guided waves in a soft formation are more sensitive to the formation shear modulus (rigidity).

The sensitivities are shown in Figure 5. For the tube wave, changes in λ and c_{66} greatly affect the phase velocity at low frequencies, but their influence decreases as frequency increases. The effects of c_{44} are exactly the opposite, eventually dominating at high frequencies. The influence of c_{11}, c_{13}, and c_{33} on the phase velocity are again quite small. Near the cutoff frequencies for the flexural and screw waves, the phase velocities are controlled by c_{44}. As the frequency increases, the effect of c_{44} diminishes and that of λ increases. The other elastic constants have little effect on the phase velocities of these guided waves

Similarities exist in the behavior of the guided waves in the hard and soft formations. In both formations, the phase velocities are mostly affected by the bulk modulus of the borehole fluid, λ, and the two formation shear moduli, c_{44} and c_{66}. The other elastic constants, those mainly associated with P-wave propagation, have little influence over the guided waves. At high frequencies the tube, flexural and screw waves have similar phase and group velocities and sensitivities because they behave like a Stoneley wave along a planar, fluid-solid interface.

The differences in phase velocities between the transversely isotropic and equivalent isotropic formations in our examples are sometimes as large as 10 percent. Differences of this magnitude generally occur when the velocity changes between vertically and horizontally propagating P- and S-waves are about fifteen percent. Thomsen's (1986) tabulation of the elastic constants for sedimentary rocks shows that a large minority have velocity variations of this size.

Because the phase velocity differences are large enough to be detected with array processing of multi-receiver data, some of the elastic constant might be estimated. In a hard formation, the refracted P- and S-waves uniquely determine c_{33} and c_{44}. The remaining elastic constants could be estimated by inverting the phase velocities at many frequencies. Since the phase velocities of the guided waves are primarily sensitive to λ and c_{66}, the inversion would only resolve these two constants well. The sensitivities for the flexural and screw waves appear similar to that for the pseudo-Rayleigh wave tentatively suggesting that the additional data provided by these two waves would not be very helpful in the inversion. In a soft formation, the refracted P wave determines

c_{33} and a shear logging tool with its flexural wave would establish c_{44}. The dispersion curves for the tube wave could be inverted for λ and c_{66}.

CONCLUSIONS

We have studied the propagation of tube, pseudo-Rayleigh, flexural and screw waves in a borehole in a transversely isotropic formation. We have also studied the sensitivities of these guided waves to the different elastic constants and compared them with those to an isotropic formation. It was found that the tube wave is more sensitive to the horizontal shear modulus (c_{66}) at low frequencies whereas the psuedo-Rayleigh, flexural and screw waves are sensitive to the vertical shear modulus (c_{44}), at least near their cutoff frequencies. A combined inversion of the phase velocity dispersions of two or more of these guided waves will allow us to determine both c_{44} and c_{66}, and thus the degree of shear wave anisotropy. By the same token, formation shear wave velocities obtained indirectly from tube wave velocities assuming isotropy could be significantly higher than those measured by a shear wave tool in a transversely isotropic formation. The degree of P-wave anisotropy cannot be determined from the phase velocities of these guided waves.

ACKNOWLEDGEMENTS

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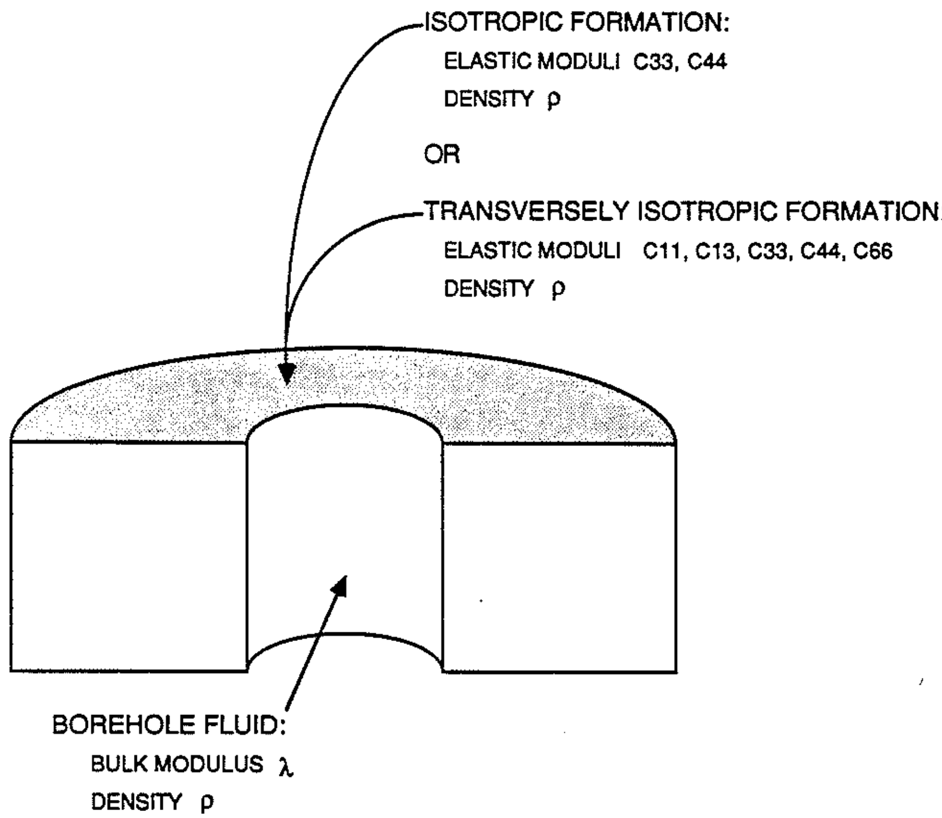

Figure 1: Borehole model used to study the guided waves.

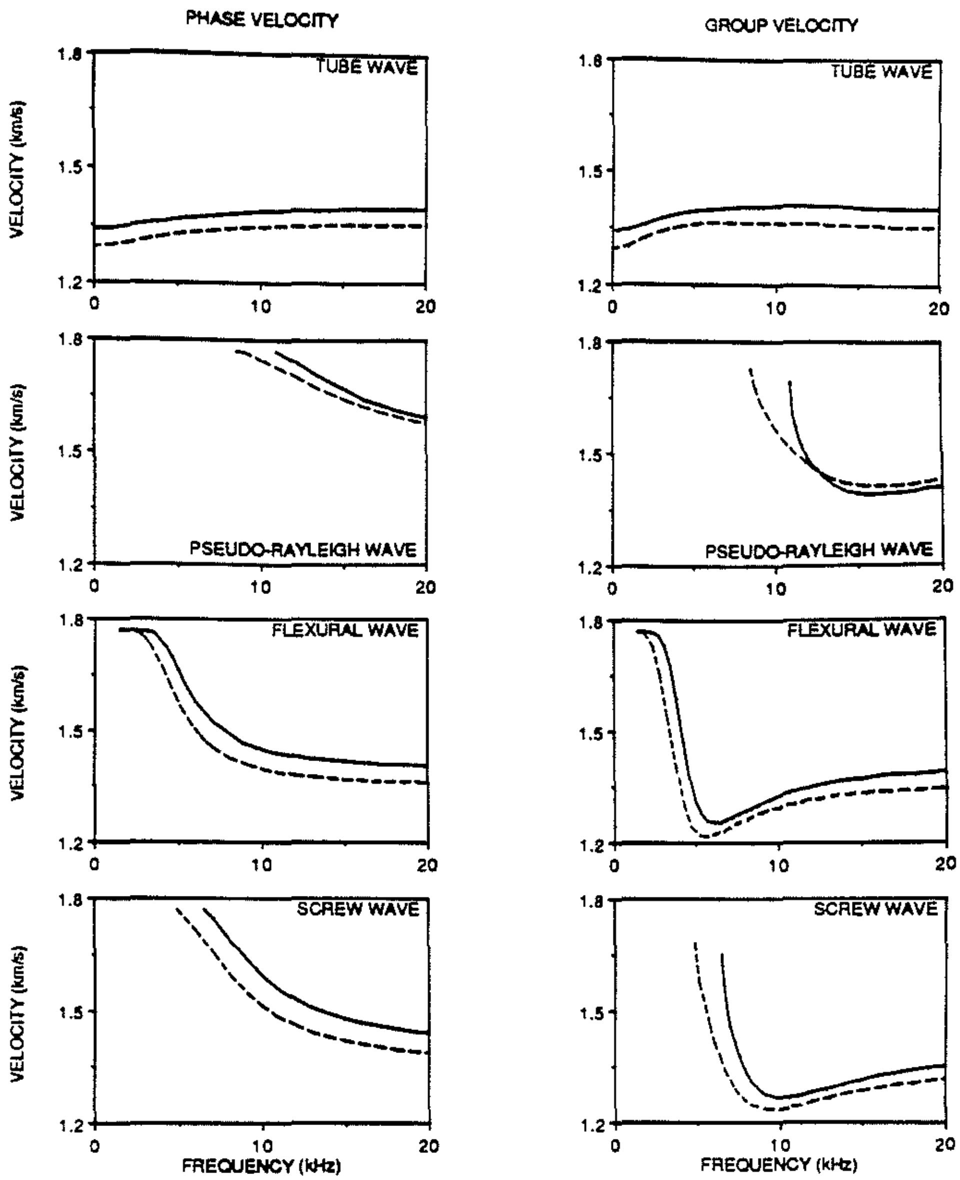

Figure 2: Phase and group velocities for the tube, pseudo-Rayleigh, flexural, and screw waves in the transversely isotropic, hard formation (solid lines) and the equivalent isotropic formation (dotted lines).

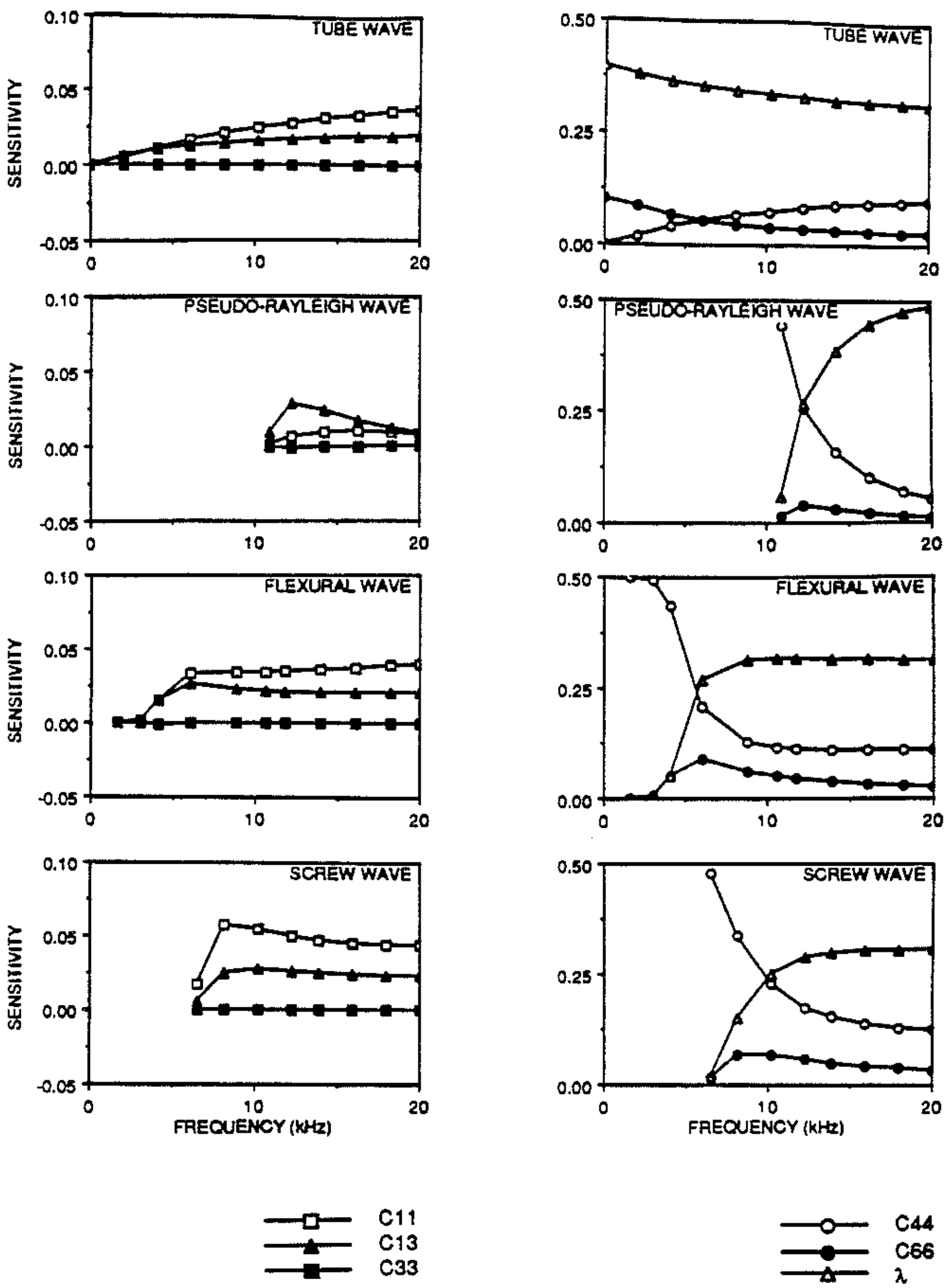

Figure 3: Sensitivities for the tube, pseudo-Rayleigh, flexural, and screw waves in the transversely isotropic, hard formation. Because the sensitivities due to c_{44}, c_{66}, and λ are generally much larger than the other sensitivities, they are grouped together and are plotted at a different scale.

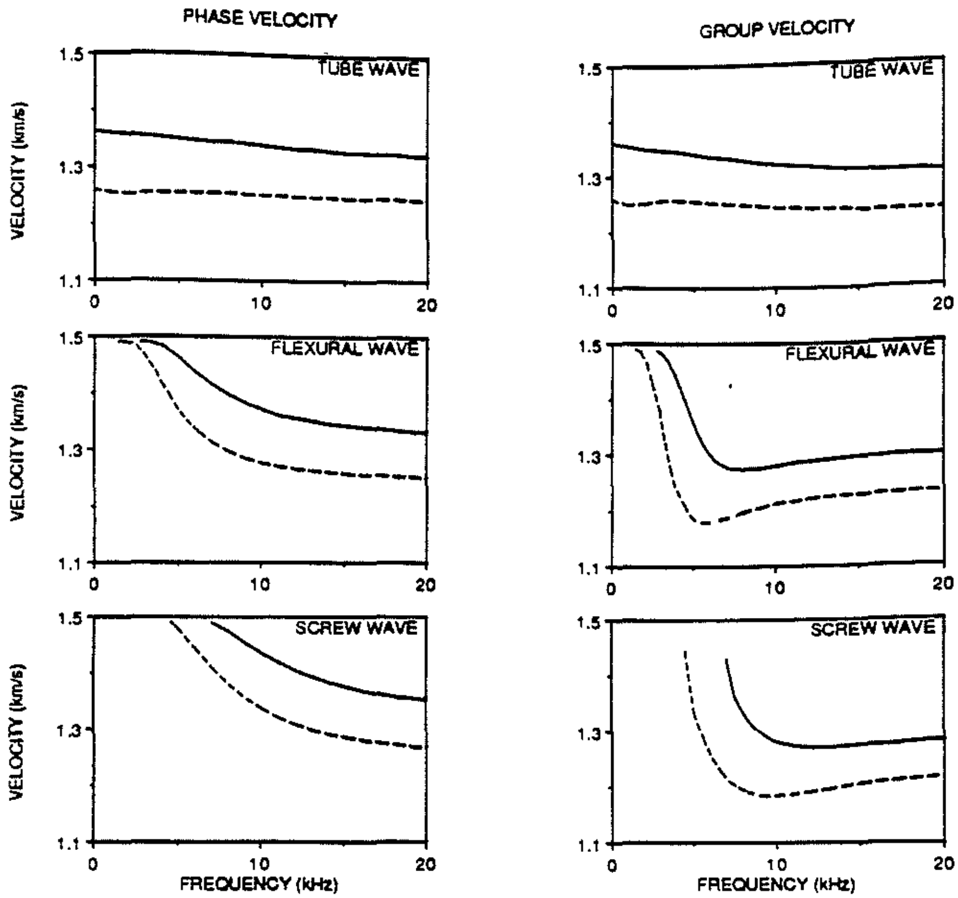

Figure 4: Phase and group velocities for the tube, flexural, and screw waves in the transversely isotropic, soft formation (solid lines) and the equivalent isotropic formation (dotted lines).

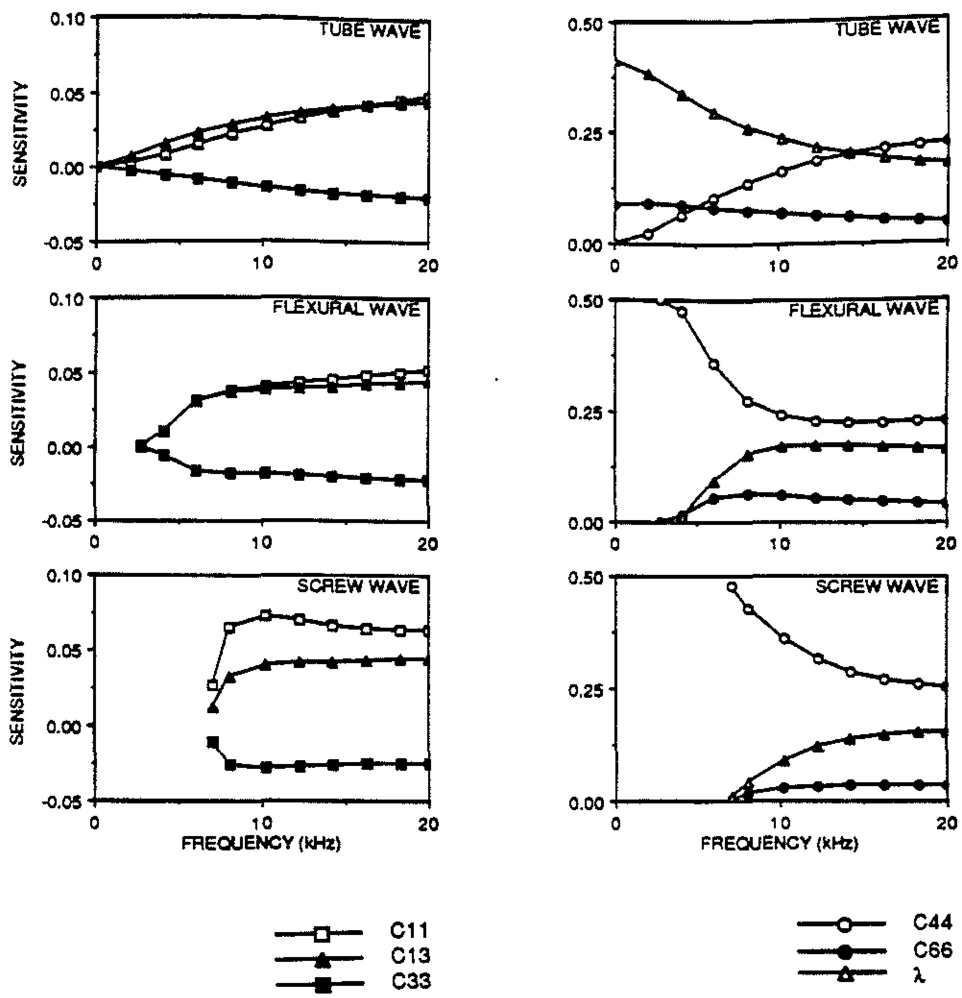

Figure 5: Sensitivites for the tube, flexural, and screw waves in the transversely isotropic, soft formation.