INVERSION FOR PERMEABILITY FROM STONELEY WAVE VELOCITY AND ATTENUATION

by

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ABSTRACT

The in situ permeability of a formation is obtained by the inversion of Stoneley wave phase velocity and attenuation, which are evaluated by applying the Extended Prony's method to the array sonic logging data. The Maximum Likelihood inversion is used together with logarithmic parameterization of the permeabilities. Formation shear wave velocity is also inverted for. This process is tested on both synthetic and field data. Logarithmic parameterization contributes to rapid convergence of the algorithm. Permeabilities estimated from field data are in good agreement with core measurments.

INTRODUCTION

Ever since Williams et al. (1984) showed the strong correlation of in situ permeability with Stoneley wave velocity and attenuation, attempts have been made to obtain in situ permeability directly from full waveform acoustic logging data. Burns et al. (1988) applied the damped least square inversion to borehole Stoneley wave attenuation data to estimate in situ permeability. The forward model was based on the Biot-Rosenbaum model of wave propagation in a borehole in a porous formation (Biot, 1956a,b; Rosenbaum, 1974). The results are in reasonable agreement with the core measurement. The ultrasonic model laboratory experiments performed by Winkler et al. (1989) filled the gap between Biot theory and field application. The laboratory measured Stoneley wave velocity and attenuation in a permeable borehole are in excellent agreement with the predictions of the Biot-Rosenbaum model.

In this paper, we try to improve on the results of Burns et al. (1988) by using both Stoneley wave velocity and attenuation. In addition, instead of a straightforward spectral ratio estimate of attenuation from two receiver data, we use data from a multi-receiver array tool. In this particular case, the tool has two sources and twelve receivers, leading to an effective array length of 24 receivers. We process the array data using the Extended Prony's method (Lang et al., 1987; Ellefsen et al., 1989) to estimate borehole Stoneley wave velocity and attenuation as a function of frequency. The inversion is then formulated using the Maximum Likelihood Inversion algorithm together with logarithmic parameterization of the model. The velocity and attenuation data are inverted simultaneously to determine in situ permeability as well as formation shear wave velocity. The method is first tested with synthetic data and then applied to actual field data and the results compared with core measured permeability.

METHOD

Data Analysis

We process full waveform acoustic logging data by the Extended Prony's method to estimate borehole Stoneley wave phase velocity and attenuation (Ellefsen et al., 1989). This method transforms the data from time domain into frequency domain, and then at each frequency, the spectral data at each receiver is fitted to a propagating wave mode (pseudo-Rayleigh or Stoneley) of the following form:

$$A(\omega)e^{-\alpha(\omega)z}e^{i(\phi(\omega)+k(\omega)z)} \tag{1}$$

where ω = angular frequency
A(ω) = incident amplitude at the first receiver
$\phi(\omega)$ = incident phase at the first receiver
k = wavenumber of the propagating mode
alpha = attenuation coefficient of the propagating mode
z = distance between source and receiver.

In this way, we can find A(ω), k(ω) and $\alpha(\omega)$ which best fitted the data in the least square error sense. The phase velocity is given by

$$c(\omega) = \frac{\omega}{k(\omega)}. \tag{2}$$

The attenuation coefficient, α, is sometimes alternately expressed as the imaginary part of the wavenumber k. Using the Extended Prony's method, the velocity dispersion and attenuation of the Stoneley wave as a function of frequency can be easily determined.

Logarithmic Parameterization

The parameter of primary interest is permeability. The range of its magnitude is about a factor of 10^4 from core measurements. A standard linear parameterization scheme will lead to very uneven weighing of the different parameters. Here we adopt a logarithmic parameterization scheme in both the data and the model space to set up our inverse problem. Our original inversion problem consists of a set of linear equations:

$$\sum_{j=1}^{M} \frac{\partial D_i}{\partial P_j} \Delta P_j = D_i^o - D_i \qquad (i = 1, 2, \ldots.., N) \tag{3}$$

where D_i, D_i^o are calculated and observed data respectively. P_j are parameter to be estimated with initial value P_{jo}. The new value of P_j is

$$P_j = P_{jo} + \Delta P_j. \tag{4}$$

With logarithmic parameterization we have

$$\sum_{j=1}^{M} \frac{P_j \partial D_i}{D_i \partial P_j} \Delta \ln P_j = \ln \frac{D_i^o}{D_i}. \tag{5}$$

The new value of P_j is

$$P_j = P_{jo} \exp(\Delta \ln P_j). \tag{6}$$

One advantage of the logarithmic parameterization scheme is its ability to deal with large changes in the paramters in one iteration. In this case, changes in the permeability of one order of magnitude will only result in a change of unity in the actual parameter vector. This parameterization really helps in stabilizing the inversion.

Maximum Likelihood Inversion

We can rewrite inversion problem in a more compact form

$$\mathbf{Gx} = \mathbf{b} \tag{7}$$

where

$$b_i = \ln \frac{D_i^o}{D_i},$$

$$x_j = \Delta \ln P_j,$$

$$g_{ij} = \frac{P_j \partial D_i^o}{D_i^o \partial P_j}.$$

G is also called the sensitivity matrix. It consists of the sensitivities of the phase velocity and attenuation of the Stoneley wave with respect to the inversion parameters. In an elastic or anisotropic formation, these sensitivities can be obtained analytically (Cheng et al., 1982; Ellefsen and Cheng, 1989). Here we calculate the sensitivities by solving the period equation for the Stoneley wave for small changes in the parameters and then use finite differences in the resulting wavenumber and attenuation.

The observed data **b** consist of Stoneley wave velocity and attenuation estimated from the Extend Prony's method analysis of the array waveform data. In this paper, we considered only two parameters in the inversion: the shear wave velocity of the dry rock and the permeability. In Biot's equations, the shear wave modulus is essentially independent of the fluid saturation. Thus given the porosity and density of the pore fluid, we can calculate the saturated shear wave velocity and the inverted dry shear wave velocity.

The other consideration is data quality. In general, attenuation is much more difficult to estimate than velocity. In other words, we have data with different quality. We have sought to keep the uncertainty in the estimate of attenuation to a minimum by using the estimated attenuation coefficient, which is just the imaginary part of the estimated wavenumber, as our data. If we had used other measures of attenuation such as $1/Q$ instead, we would have introduced the error associated with the velocity estimation into these estimates also.

Under the usual assumption of Gaussian statistics, we seek a solution which minimize

$$\mathbf{G}^{\mathbf{T}}\mathbf{R}_{\mathbf{dd}}^{-1}\mathbf{G} + \mathbf{x}^{\mathbf{T}}\mathbf{R}_{\mathbf{mm}}^{-1}\mathbf{x} \tag{8}$$

where $\mathbf{R}_{\mathbf{dd}}$ is the covariance matrix for data, $\mathbf{R}_{\mathbf{mm}}$ is the covariance matrix for parameter, and the superscript T stands for transposition. The maximum likelihood least squares solution is then given by:

$$\mathbf{x} = (\mathbf{G}^{\mathbf{T}}\mathbf{R}_{\mathbf{dd}}^{-1}\mathbf{G} + \mathbf{R}_{\mathbf{mm}}^{-1})^{-1}\mathbf{G}^{\mathbf{T}}\mathbf{R}_{\mathbf{dd}}^{-1}\mathbf{b}. \tag{9}$$

With this formulation, it is easy to deal with different errors associated with different quality of data and put some knowledge of the parameters into inversion.

RESULTS

Synthetic Data

The method mentioned above is first tested on synthetic data. The test model is the borehole surrounded by porous formation with permeable wall. We assume saturated fluid is the same as borehole fluid. The radius of the borehole is 0.1 m. The other parameters are listed in Table 1. Twelve traces of the full waveform sonic logging data are generated by the discrete wavenumber method (Figure 1). Formation permeability and dry shear wave velocity are chosen as inversion parameters. Their sensitivities are shown in Figure 2, generated with the same parameters used in calculating the synthetic microseismograms. The sensitivity of attenuation with respect to permeability and dry V_s is high, and the former increases as frequency decreases. Sensitivity of phase velocity with respect to permeability is low, especially at high frequencies. We have thus limited our inversion to frequencies below 6 kHz.

It is necessary at this point to remark about the relatively high sensitivity of Stoneley wave attenuation to formation shear velocity. At first glance this is not intuitive. What is actually happening is that $1/Q$ is very insensitive to changes in the formation shear wave velocity, as it should be. However, since $1/Q = 2Im(k)/Re(k)$, changes in $Im(k)$ are about twice that of $Re(k)$. Hence the sensitivity of the Stoneley wave attenuation coefficient to formation shear wave velocity is about twice that of the velocity.

Stoneley wave phase velocity and attenuation are estimated by Prony's method from the synthetic microseismograms. Data below 6 kHz are used. Because this is a nonlinear inverse problem, we linearize it with an initial model and then solve it by iteration. Permeability, with initial value 500 md, converges to 190.8 md. Dry V_s, with initial value 1600 m/s, converges to 2126.0 m/s. The error in the solution amounts to 5% for the permeability estimate and 2% for the shear wave velocity. This can be attributed to the error in the estimated velocity and attenuation from the Extended Prony's method, as well as inaccuracies inherent in calculating the sensitivity matrix by finite differencing the dispersion curves.

Figure 3 shows how the predictions from the initial model and the final model fit the velocity and the attenuation data. Figure 4 plots the values of permeability and dry V_s at each iteration step. Convergence is very rapid. After 3 iterations permeability and dry V_s are already very close to the final values. This is an advantage of logarithmic parameterization. In this synthetic case the parameters of interest are well resolved. We tested another initial model, with permeability 50 md and dry V_s 1600 m/s, and these two parameters converge to the same values as the first initial model. The inversion results are stable with the different initial model.

Field Data

Field data are recorded by twelve receivers with two sources, resulting in a total of twenty-four traces. The receivers are 6 inches apart. We select the data from a sandstone section, with porosity around 30%. Tool effect is considered by multiplying wavenumber with factor 0.94 (Cheng and Toksöz, 1981). This number is obtained by best fitting the pseudo-Rayleigh dispersion from a section of low porosity, low permeability rock (Ellefsen, 1990). We assume the borehole wall is permeable, the saturating fluid is the same as the borehole fluid and viscosity is 0.1 centipoise. The main contribution to Stoneley wave attenuation comes from borehole fluid attenuation Q_f and the fluid flow between borehole and formation (permeability effect). Q_f is determined at a depth with known permeability, and then is fixed througout the inversion process (Q_f=40.0). The other parameters are well constrained, and are obtained directly from available logs. The compressional wave velocity of the borehole fluid, which has a significant effect on the Stoneley wave velocity, is measured to be 1.6 km/s (Tubman, personal communications). Only data less than 4 kHz are used in order to emphasize the dependence on permeability.

Figure 5 shows twelve traces of the field data used in the inversion. Figure 6 shows the attenuation and phase velocity of the Stoneley wave obtained from the data and predictions from the initial and final models from the inversion. The measured attenuation of the field data has a lot of variations even with 24 traces. Velocity estimation is more stable than attenuation, but there are still variations. This points out the difficulty in estimating formation permeability using only two traces, as done by Burns et al. (1988). In this case inversion parameters are well resolved. Convergence is reached after 4 to 5 iterations. The inversion results of permeability are plotted against the core measurements (Figure 7). Values of dry and saturated V_s are plotted in Figure 8. In general, the inversion values of permeability are in good agreement with the core measurements. The scatter in the inversion results is within about half an order of magnitude, which can be considered good. More importantly, the trend in the permeability variations agrees well with the core data. The inverted saturated shear wave velocity is around 1.5 km/s, less than the compressional wave velocity of the borehole fluid, consistent with the lack of shear/pseudo-Rayleigh wave arrival in the data.

CONCLUSIONS

We have presented a method of estimating in situ permeability from full waveform acoustic logs. This method consists of the estimation of Stoneley wave phase velocity and attenuation by the Extended Prony's Method and then simultaneous inversion using the Maximum Likelihood least squares algorithm. This method provides a stable estimation of in situ permeability which compares well with core permeability. With logarithmic parameterization, convergence is reached after only a few iterations. The inversion parameters, in situ permeability and dry V_s, are well resolved in both the synthetic and field cases.

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ρ_s (kg/m^3)	α_m (m/s)	β_m (m/s)	K_s (pa)	ϕ %	κ (darcy)	ρ_f (kg/m^3)	α_f (m/s)	η (cp)	Q_f	Q_p	Q_s
2650	3670	2170	3.79×10^{10}	0.19	0.2	1000	1500	1	30	60	60

Table 1. Synthetic model parameters.

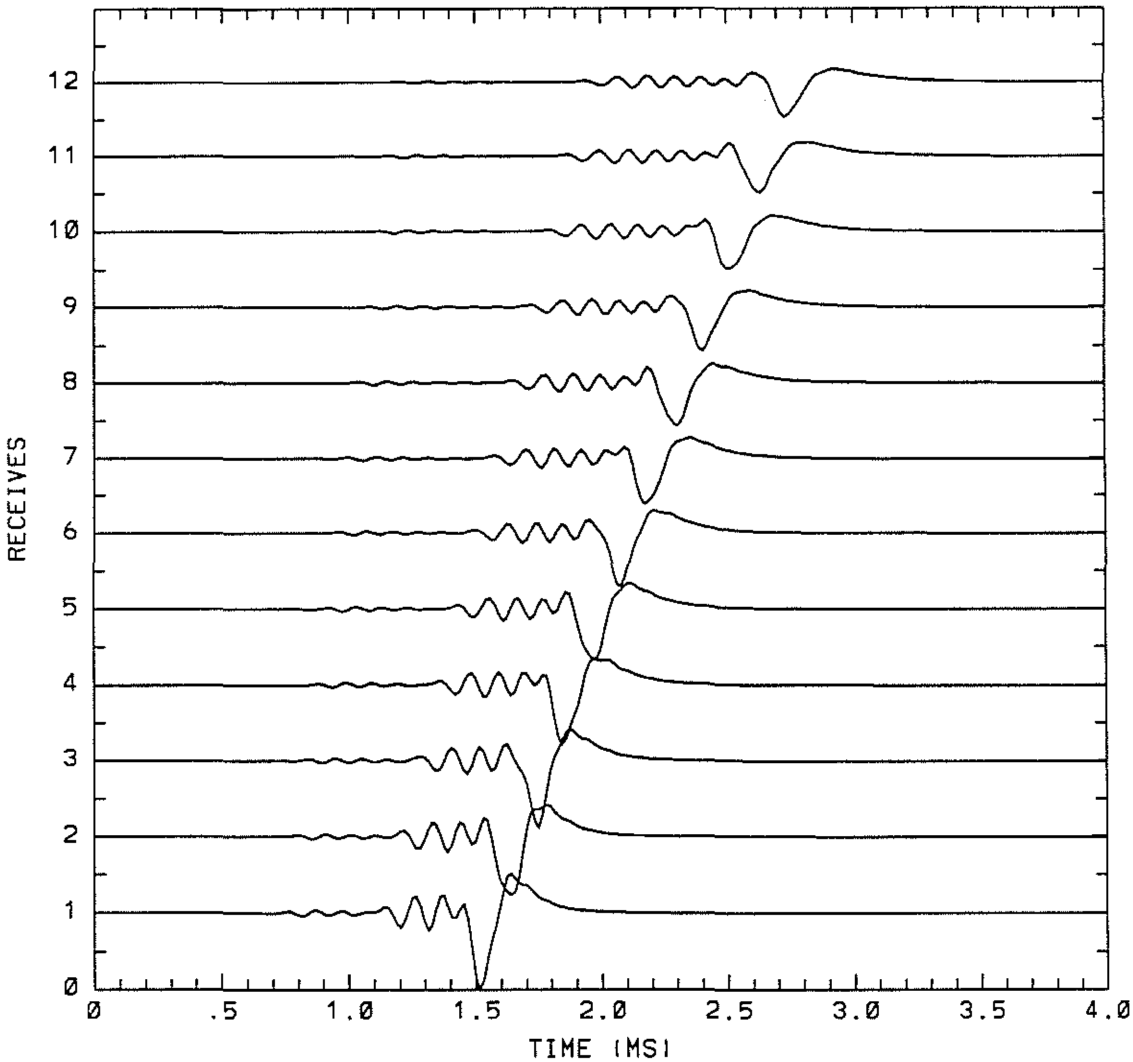

Figure 1: Twelve traces of synthetic full waveform with model parameters listed in Table 1.

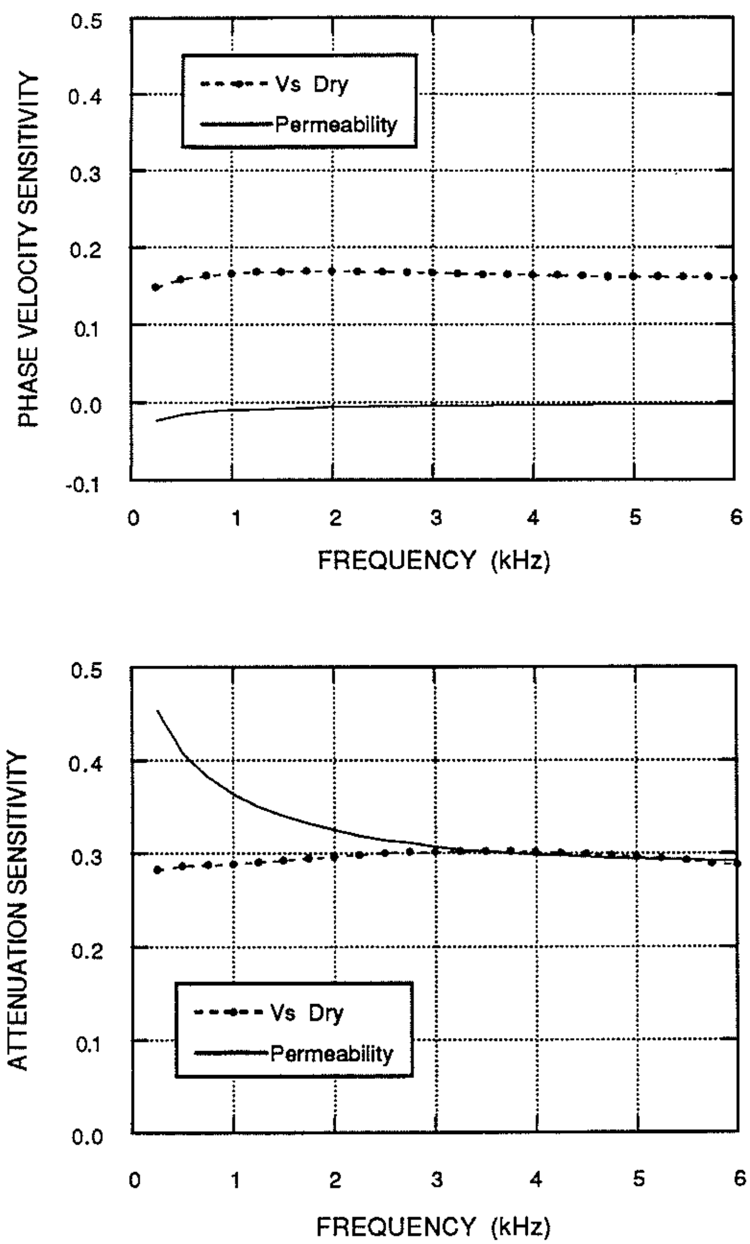

Figure 2: Sensitivity of Stoneley wave phase velocity and attenuation with respect to permeability and dry V_s. The model is the same as our synthetic model.

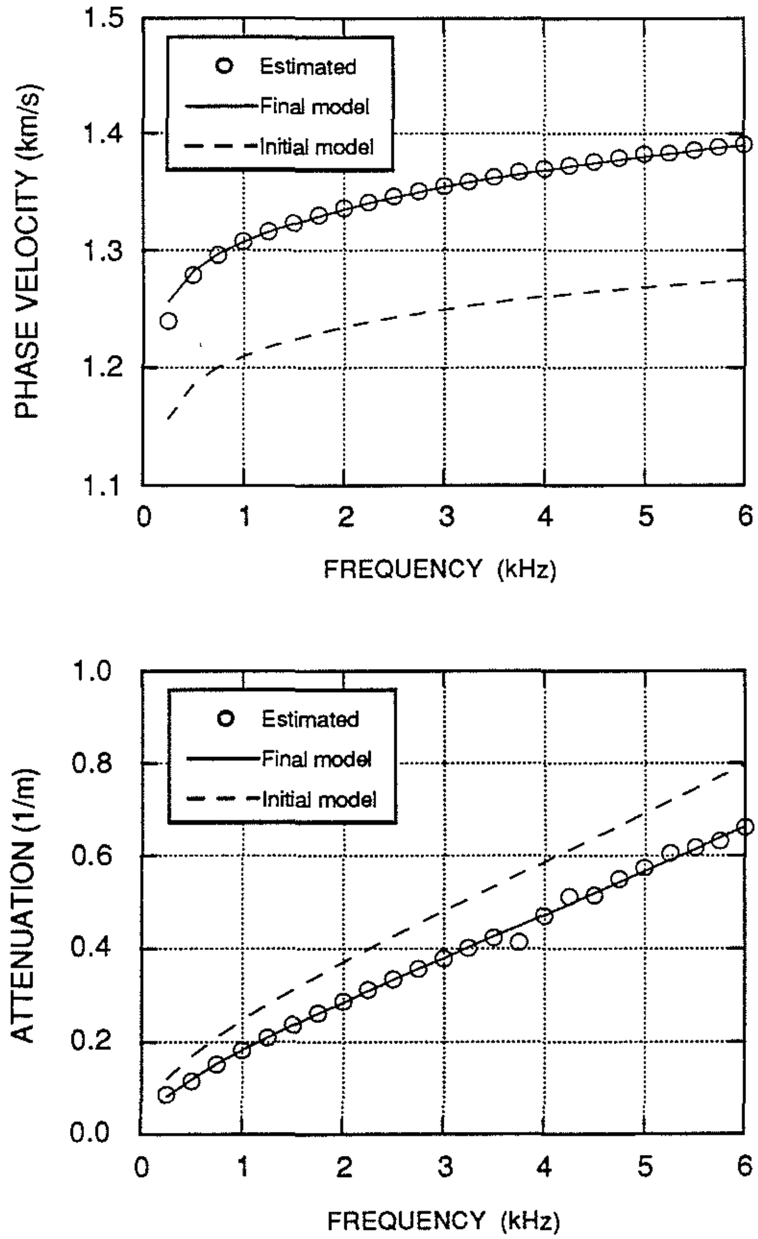

Figure 3: Stoneley wave phase velocity and attenuation from synthetic data, initial model prediction and final model prediction. Poiints are for synthetic data, dashed line for initial model and solid line for final model.

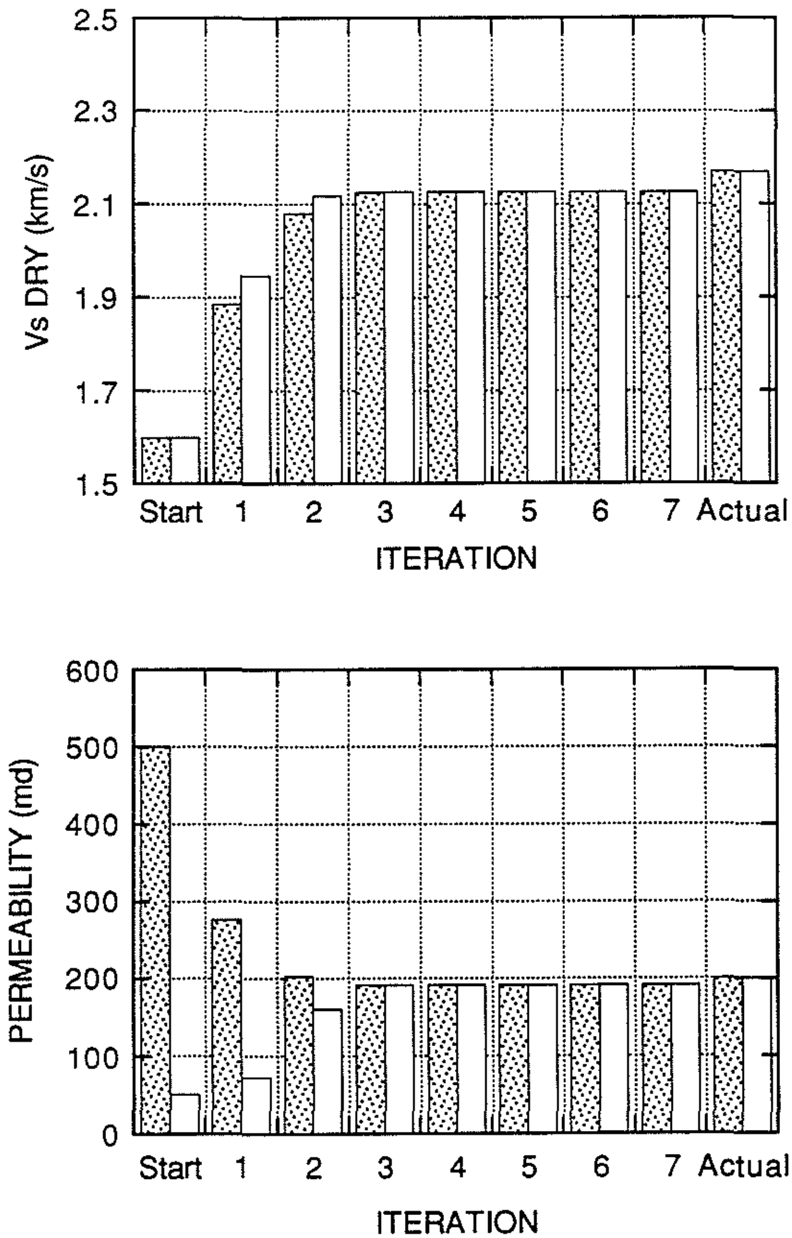

Figure 4: Values of permeability and dry V_s at each iteration step.

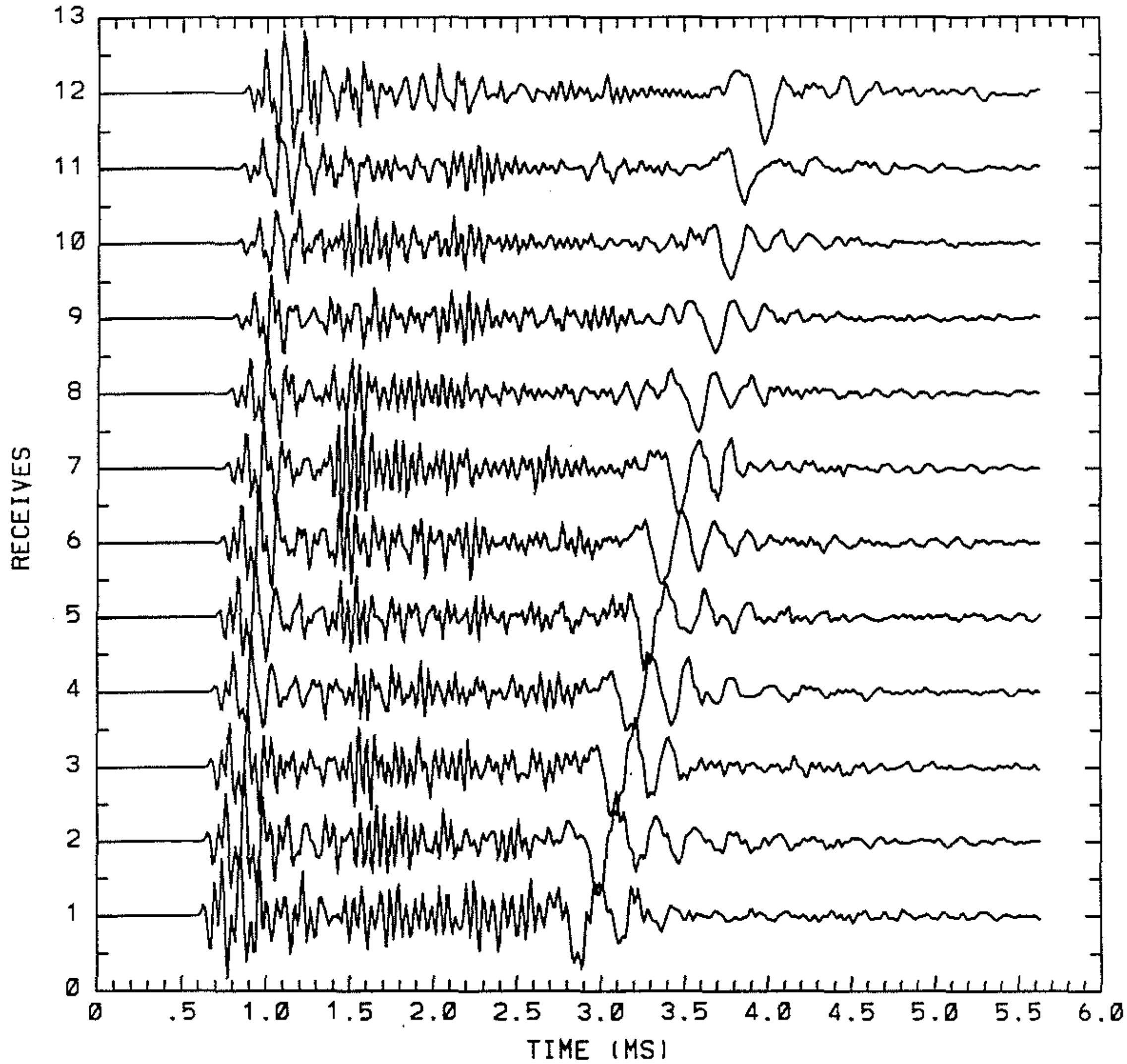

Figure 5: Typical field data set used for inversion. Only 12 of 24 traces are shown.

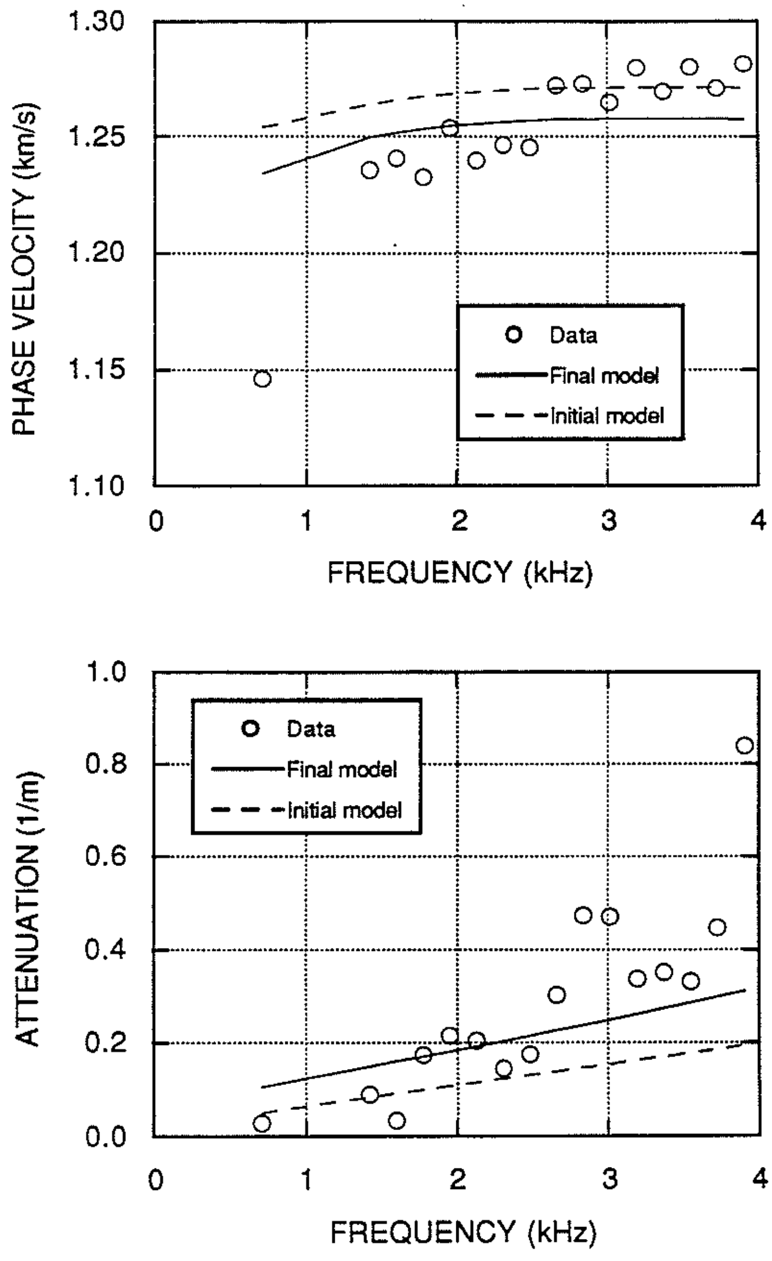

Figure 6: Stoneley wave phase velocity and attenuation from field data, initial model prediction and final model prediction. Points are for field data, dashed line for initial model and solid line for final model.

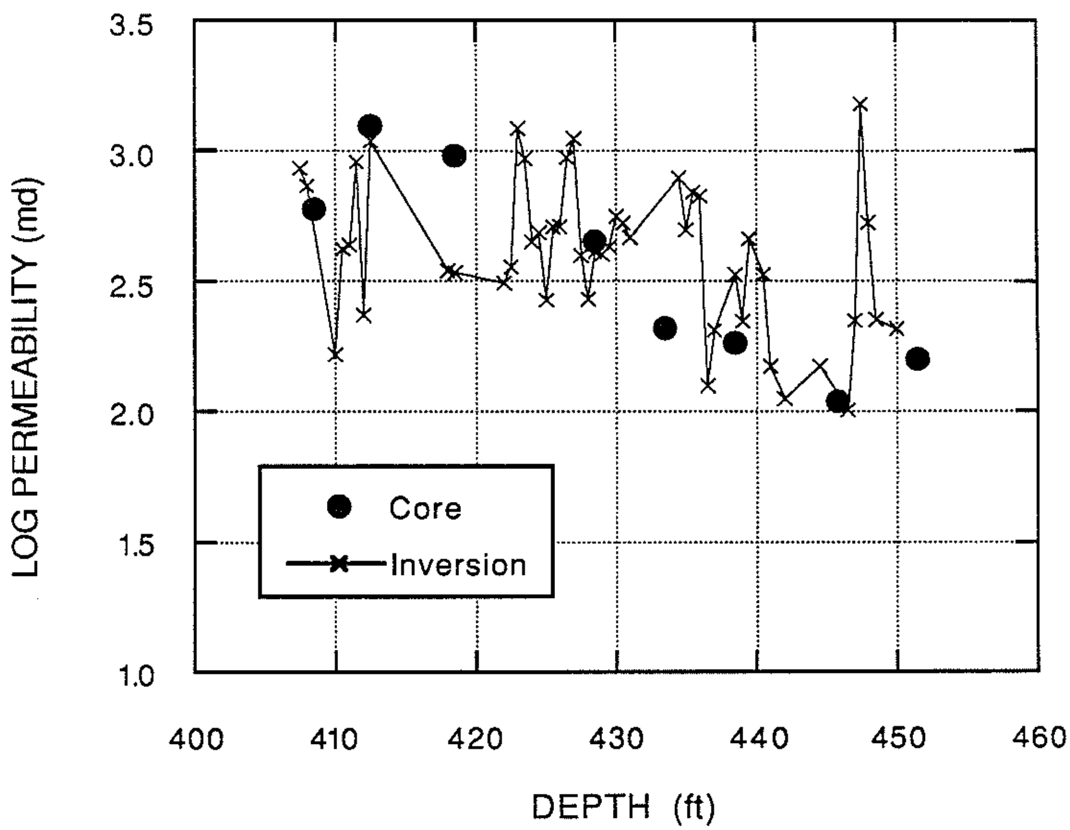

Figure 7: Inversion results of in situ permeability (crosses) versus core measurements (solid circles).

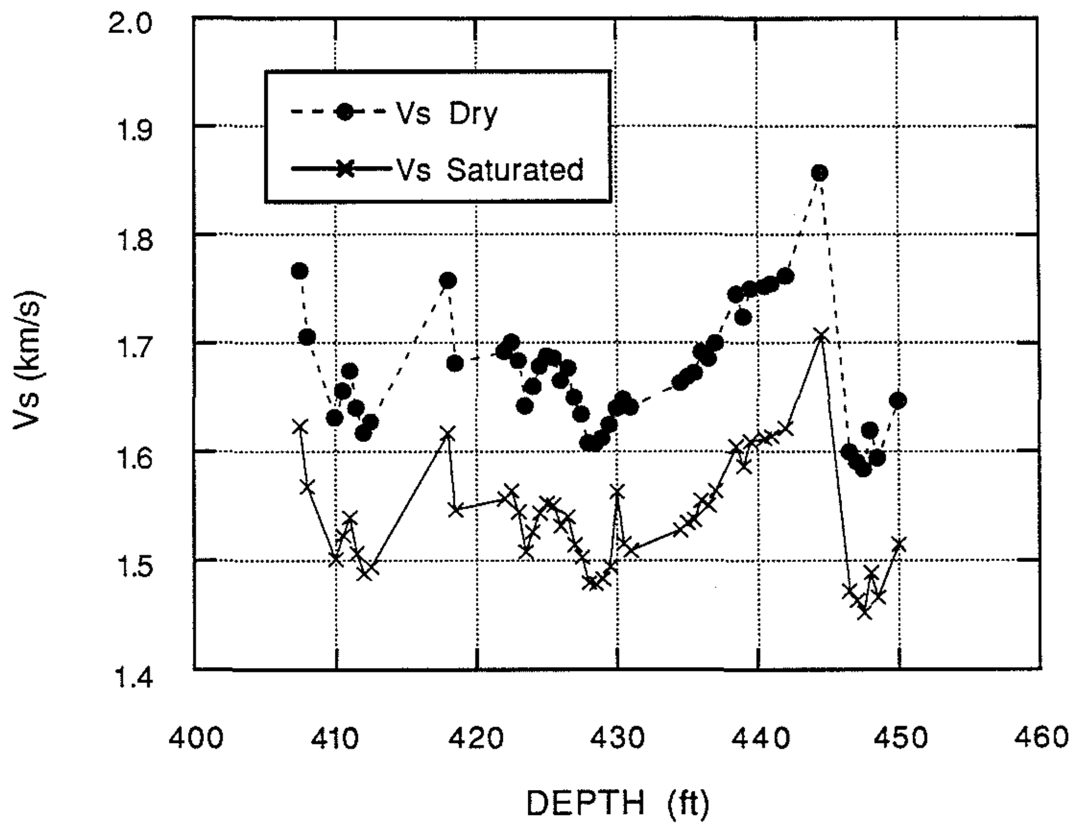

Figure 8: Inversion results of dry and saturated formation shear wave velocity.