LN–9 IDLE MIND SOLUTIONS

1. Given: E_a = 315 kJ/mole
D_o = 1.70 cm^2/s

At what T will D be 5 x 10^{-11} cm^2/s?

$$D = D_o \exp(-E_a/RT)$$

$$5 \text{ x } 10^{-11} = 1.70 \text{ exp} \frac{-315,000}{8.3 \text{ T}}$$

$$\ln \frac{5 \text{ x } 10^{-11}}{1.7} = -\frac{E_a}{RT}$$

$$T = \frac{E_a}{24.2 \text{ x } R}$$

$$\boxed{T = 1563\text{K} = 1290^{\circ}\text{C}}$$

2. D for Li into Si is given as:
10^{-5} cm^2/s at 1100°C (1373K)
10^{-6} cm^2/s at 692°C (965K)

Thus:

$$10^{-5} = D_o \, e^{-E_a/RT_1}$$

$$10^{-6} = D_o \, e^{-E_a/RT_2}$$

$$10 = e^{-\frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} \qquad 2.3 = -\frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$E_a = -\frac{2.3 \text{ x } R}{\frac{1}{T_1} - \frac{1}{T_2}} = \boxed{62 \text{ kJ/mole}}$$

$$10^{-5} = D_o \, e^{-E_a/RT}$$

$$\boxed{D_o = 10^{-5} \text{ x } e^{E_a/RT} = 2.3 \text{ x } 10^{-3}}$$

3. Since electrons do not electrostatically interact with the electron shells of the atoms, they encounter “virtually empty space” and accordingly the diffusion constant for neutrons must be expected to be **significantly larger than even the diffusion constants of liquids**.

4. $$\frac{c}{c_o} = 0.5 = \mathrm{erfc}\frac{x}{2\sqrt{Dt}} = \mathrm{erf}\frac{x}{2\sqrt{Dt}}$$

From the tables we find $\frac{x}{2\sqrt{Dt}} = 0.48$

$$t = \frac{x^2}{(0.96)^2 \text{ x } D} = \frac{9 \text{ x } 10^{-8}}{0.92 \text{ x } 8 \text{ x } 10^{-12}}$$

$$\boxed{t = 12228 \text{ sec} = 3.4 \text{ hours}}$$

5. The diffusion profile in Fe is given by $\frac{c}{c_o} = \mathrm{erfc}\frac{x}{2\sqrt{Dt}}$ with c/c_o = const.

Correspondingly: $$\frac{x}{2\sqrt{Dt}} = \text{const}$$

$$\text{or} \quad \frac{x}{\sqrt{t}} = K \quad \text{and} \quad \frac{x^2}{t} = K'$$

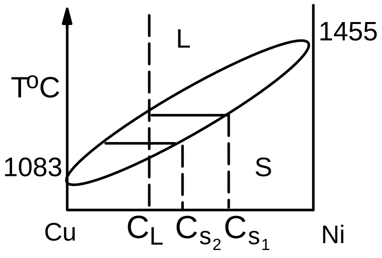
This constant can be determined with the information given:

$$K' = \frac{(0.04)^2}{10} = 1.6 \text{ x } 10^{-4} \text{ cm}^2/\text{h}$$

The time for diffusion to a depth y (0.08) is then given as:

$$\boxed{t = \frac{y^2}{K'} = \frac{(0.08)^2}{1.6 \text{ x } 10^{-4}} = 40 \text{ hours}}$$

6. First a clarification: What is the origin of the indicated composition gradient across grains of a Cu–Ni alloy? It is explained by a look at the phase diagram. Melt of composition C_L freezes into grains of steadily increasing radius; obviously the growth stops when all the melt around this grain has solidified (onto this and other grains).



With increasing radius the freezing alloy first has the composition C_{S_1}, which steadily increases to C_{S_2} and even higher (why?). This phenomenon is referred to as “coring”.

When we consider diffusive effects, we like to talk about the “effective diffusion distance”: the distance over which the original surface concentration has decreased to 50% of its value ($c = c_o/2$). Thus:

$$\frac{c}{c_o} = 0.5 = \operatorname{erfc}\frac{x}{2\sqrt{Dt}}$$

The answer to the question:

$$D = D_o\, e^{-E_a/RT} = 2.7\, e^{-\frac{(235 \times 10^3)}{(8.314 \times 1373)}} = 3.1 \times 10^{-9}\ \text{cm}^2/\text{s}$$

$$0.5 = \operatorname{erfc}\frac{x}{2\sqrt{Dt}} \cong \frac{x}{2\sqrt{Dt}}$$

$$x = \sqrt{Dt}$$

$$t = \frac{x^2}{D} \qquad (x = \text{radius of the grain} - 0.005\ \text{cm})$$

$$\boxed{t = 8065\ \text{sec} = 2.2\ \text{hours}}$$

7. T = 1600K
D = 8 x 10^{-12} cm^2/s

$$\frac{c_2 - c(x,t)}{c_2} = \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

want c(x,t) = 1/2(c_2)

$$\frac{c_2 - \frac{c_2}{2}}{c_2} = \frac{1}{2} = \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Interpolating from Table II:

we find for erf(Z) = 0.5000, Z = 0.4772

$$\frac{x}{2\sqrt{Dt}} = 0.4772$$

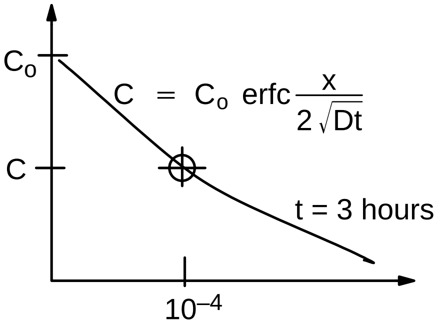
$$x^2 = 0.4772^2\ 2^2\ Dt = 0.911 \text{ x } Dt$$

$$t = \frac{(3 \text{ x } 10^{-4})^2}{0.911\ (8 \text{ x } 10^{-12})}$$

$$t = 1.23 \text{ x } 10^4 \text{ sec} = \boxed{3.43 \text{ hrs}}$$

8. To solve this type of problem, it is best to sketch the conditions:

$$\frac{C}{C_o} = 1 - \text{erf}\frac{x}{2\sqrt{Dt}}$$



(a) $$C = 10^{18}\left(1 - \text{erf}\frac{10^{-4}}{2\sqrt{7 \times 10^{-13} \times 3 \times 3600}}\right)$$

$C = 10^{18}$ (1 – erf 0.575)

$C = 10^{18}$ (1 – 0.58)

$$\boxed{C = 4.2 \times 10^{17} \text{ /cm}^3}$$

(b) $C = 10^{18}$ [1 – erf (0.575 x 3)]

$C = 10^{18}$ (1 – 0.98)

$$\boxed{C = 2 \times 10^{16} \text{ /cm}^3}$$

9. $$\frac{C}{C_o} = \left(1 - \text{erf}\frac{x}{2\sqrt{Dt}}\right)$$

$$0.3 = 1 - \text{erf}\frac{x}{2\sqrt{Dt}}$$

$$0.7 = \text{erf}\frac{x}{2\sqrt{Dt}}$$

From the tables we have: $$\frac{x}{2\sqrt{Dt}} = 0.74$$

$$\sqrt{Dt} = \frac{x}{1.48}$$

$$t = \frac{(8 \text{ x } 10^{-4})^2}{1.48^2 \text{ x } 8 \text{ x } 10^{-12}}$$

$$\boxed{t = 36529 \text{ s} = 10.1 \text{ hours}}$$

10. $$0.5 = \text{erf}\frac{x}{2\sqrt{Dt}}$$

$$\frac{x}{2\sqrt{Dt}} = 0.48$$

$$\sqrt{Dt} = \frac{x}{0.96}$$

$$t = \frac{x^2}{0.96^2 \text{ x } 8 \text{ x } 10^{-12}}$$

$$\boxed{t = 37.7 \text{ hours}}$$

11.
$$C = C_o \operatorname{erfc} \frac{x}{2\sqrt{Dt}}$$

$$0.35\ C_o = C_o \operatorname{erfc} \frac{x}{2\sqrt{Dt}}$$

$$0.35 = \operatorname{erfc} \frac{x}{2\sqrt{Dt}}$$

$$0.35 = 1 - \operatorname{erf} \frac{x}{2\sqrt{Dt}}$$

$$0.65 = \operatorname{erf} \frac{x}{2\sqrt{Dt}}$$

From the table: $\frac{x}{2\sqrt{Dt}} = 0.65$

$$\sqrt{Dt} = \frac{x}{1.3}$$

$$t = \frac{(0.01)^2}{1.3^2 \times 10^{-10}}$$

$$\boxed{t = 164 \text{ hours}}$$

12. Carburization is a diffusion controlled process; therefore:

$$X = k\sqrt{t}$$

$$X^2 = kt \qquad k = \frac{X^2}{t} = \frac{(0.04)^2}{10} = 1.6 \times 10^{-4}\ \text{cm}^2/\text{h}$$

$$0.08^2 = 1.6 \times 10^{-4}\ (t_x)$$

$$\boxed{t_x = 40 \text{ hours}}$$

13. $$\frac{C}{C_o} = 0.2 = \operatorname{erfc}\frac{x}{2\sqrt{Dt}}$$

$$0.8 = \operatorname{erf}\frac{x}{2\sqrt{Dt}}$$

From the table: $$\frac{x}{2\sqrt{Dt}} = 0.91$$

$$t = \frac{0.02^2}{1.82^2 \text{ x } D} \qquad D = 0.47\ e^{-\frac{332 \text{ x } 10^3}{8.314 \text{ x } 1273}}$$

$t = 1.1 \text{ x } 10^{10}$ s $\qquad D = 1.1 \text{ x } 10^{-14}$ cm^2/s

$$\boxed{t = 348 \text{ years}}$$

14. $$\frac{C_x}{C_2} = \operatorname{erfc}\frac{x}{2\sqrt{Dt_x}} = 1 - \operatorname{erf}\frac{x}{2\sqrt{Dt_x}}$$

To solve this problem you need to know D_{1375K} which can be read from the accompanying graph as $D_{1375K} = 8 \text{ x } 10^{-13}$ cm^2/s^{-1}.

(a) $$c_x = c_o\left(1 - \operatorname{erf}\frac{1 \text{ x } 10^{-4}}{2\sqrt{3 \text{ x } 3600 \text{ x } 8 \text{ x } 10^{-13}}}\right) = c_o(1 - \operatorname{erf} 0.538)$$

From LN9–12 we have = $c_o(1 – 0.553)$

$c_x = 10^{18}(1 – 0.553)$

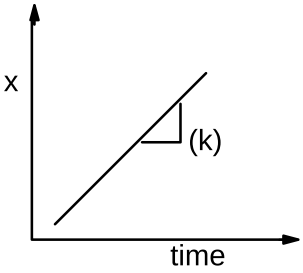
$$\boxed{c_x = 4.5 \text{ x } 10^{17} \text{ at/cm}^3}$$

(b) $$c_x = c_o\left(1 - \operatorname{erf}\frac{3 \text{ x } 10^{-4}}{2\sqrt{3 \text{ x } 3600 \text{ x } 8 \text{ x } 10^{-13}}}\right)$$

$= c_o(1 – \operatorname{erf} 1.62)$

$= c_o(1 – 0.978) = \boxed{2.2 \text{ x } 10^{16} \text{ at/cm}^3}$

15. If a metal oxidizes according to a linear rate law, it means the thickness (x) of the oxidation product (rust) is proportional to time (x = kt).



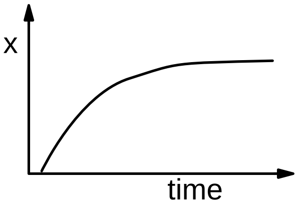
For such a process, (dx)/(dt) = k = const. This is typically the case if the oxidation product is “non–coherent” and oxygen has direct access to the metal surface.

If aluminum is added, the aluminum will form a coherent film of Al_2O_3 through which oxygen cannot readily diffuse.

Further oxidation is therefore very slow. This behavior can be expressed as

x = k log (at + b)

Hence the oxidation follows a logarithmic rate law.



16. A solution to Fick’s 1st law for the given boundary conditions was presented in class:

$$\frac{c}{c_o} = 1 - \operatorname{erf}\frac{x}{2\sqrt{Dt}} \qquad \operatorname{erf}\frac{x}{2\sqrt{Dt}} = 1 - 0.018 = 0.982$$

From the error function tables we find that the argument which yields an error function value of 0.982 is given by 1.67. This means:

$$\frac{0.002}{2\sqrt{Dt}} = \frac{0.001}{\sqrt{Dt}} = 1.67 \quad ; \quad D = D_o \cdot e^{\frac{-286 \cdot 10^5}{8.314 \cdot 1253}} = 6.45 \cdot 10^{-13}\,\frac{cm^2}{sec}$$

$$\boxed{t = \frac{0.001^2}{1.67^2 \cdot 6.45 \cdot 10^{-13}} = 5.56 \cdot 10^5 \ \text{sec} = 6.4 \ \text{days}}$$