Tropical Cyclone Motion
Tropical cyclones move approximately with a suitably defined vertical vector average of the flow in which they are embedded.
Lagrangian chaos:
Vortices in PV gradients:
Baroclinic vortices in shear: A simple model

- Two layers, with zero effective PV gradient, but upper layer moving with respect to lower layer
- Lower layer contains point potential vortex, whose circulation projects outward and upward
- Upper layer has point source of zero PV air co-located with lower point vortex; zero PV air separated from surroundings by a single, expanding contour
- Flow owing to upper level PV anomaly solved by contour dynamics

(From Wu and Emanuel, 1993)
Lower (left) and upper (right) flows for zero shear:
Evolution of upper layer vortex patch when weak shear is present
Evolution of upper layer vortex patch when moderate shear is present
Evolution of upper layer vortex patch when strong shear is present.
Fig. 11. Trajectories (units of 500 km) of the lower-layer vortex for $\xi = 0.25$, $\gamma = 0.79$, and $x = 0.25$ (shown as "+"); $x = 1.25$ (shown as "*"), and $x = 5$ (shown as "O").

Fig. 12. The relation between the maximum induced vortex speed and the magnitude of the vertical shears ($u$) for $\xi = 0.25$ and $\gamma = 0.79$. 

Operational prediction of tropical cyclone tracks:

Tropical Prediction Center Performance Measures
yearly-average official track forecast errors and trend lines, Atlantic basin

120-hour
96-hour
72-hour
48-hour
24-hour
Example: 20 random tracks passing within 100 km of Boston
20 “worst” tracks:
Interaction of Tropical Cyclones with the Upper Ocean

- Resonance with near-inertial oscillations
- Mixed layer cooling by entrainment
- Coupled models
Change on SST needed to cancel increase in enthalpy in core:

\[
L_v q^* (T_a) H + c_p T_a = L_v q^* (T_a - \Delta T) + c_p (T_a - \Delta T)
\]

\[
L_v q^* (T_a - \Delta T) \approx L_v q^* (T_a) - L_v \frac{\partial q^*}{\partial T} \Delta T
\]

\[
= L_v q^* (T_a) - L_v \frac{L_v q^*}{R_v T_a^2} \Delta T
\]

\[
\rightarrow \Delta T \approx \frac{L_v q^* (1 - H)}{c_p + \frac{L_v^2 q^*}{R_v T_a^2}} \approx 2.5^\circ C
\]
Physics of near-inertial oscillations:

PEs linearized about a rotating stratified fluid at rest:

\[ \frac{\partial u}{\partial t} = -\alpha_0 \frac{\partial p}{\partial x} + fv \]
\[ \frac{\partial v}{\partial t} = -\alpha_0 \frac{\partial p}{\partial y} - fu \]
\[ \frac{\partial w}{\partial t} = -\alpha_0 \frac{\partial p}{\partial z} + B \]
\[ \frac{\partial B}{\partial t} = -N^2 w \]
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]
\[
\rightarrow \frac{\partial^2}{\partial t^2} \nabla_3^2 w + N^2 \nabla_2^2 w + f^2 \frac{\partial^2 w}{\partial t^2} = 0
\]

\[
w = w_0 e^{i(kx+ly+rz-\omega t)}
\]

\[
\omega^2 = N^2 \frac{\lambda^2}{\lambda^2 + r^2} + f^2 \frac{r^2}{\lambda^2 + r^2},
\]

\[
\lambda^2 \equiv k^2 + l^2
\]

\[
\omega^2 \approx f^2 \quad \text{for} \quad r^2 \gg \frac{N^2}{f^2} \lambda^2
\]
Mixing and Entrainment:

(a) 

(b) 

(c)
Entrainment Formulation:

Criticality of a Bulk Richardson Number:

\[ Ri \equiv \frac{gh\Delta\rho}{\rho u^2} \]

Assume that density jump is what would result from eroding a constant background stratification down to depth \( h \):

\[ Ri \equiv \frac{1}{2} \left( \frac{h^2 N^2}{u^2} \right) \]

Equivalently, \( Nh = R'u \) \( \Box \) \( (1) \)

\[ R' \equiv \sqrt{2Ri} \]
Criticality assumption: $R' = constant$. 

Mixed layer momentum conservation (neglecting Coriolis turning):

$$
\frac{\partial (\rho_w uh)}{\partial t} = \rho_a u_*^2.
$$

(2) 

$$
u_*^2 \equiv C_D \left| \mathbf{V} \right|^2
$$

Combine (2) with (1):

$$
\frac{\partial h^2}{\partial t} = R' \frac{\rho_a}{\rho_w} \frac{u_*^2}{N}
$$

Note: units of diffusivity
Comparison with same atmospheric model coupled to 3-D ocean model; idealized runs:
Full model (black), string model (red)
Mixed layer depth and currents

Full physics coupled run ML depth (m) and currents at t=10 days

$V_{\text{max}} = 1.774 \text{ms}^{-1}$
Independent column coupled run ML depth (m) and currents at t=10 days

$V_{max} = 1.6455 \text{m/s}^{-1}$
SST Change

Full physics coupled run \( \Delta \text{SST} \left(^\circ\text{C}\right) \) at \( t=10 \) days

\( \Delta \text{SST}_{\text{max}} = -3.9819^\circ\text{C} \)
Independent columns coupled run $\Delta$ SST ($^\circ$C) at t=10 days

$\Delta$ SST$_{max}$ = -3.4715 $^\circ$C
Define feedback factor:

\[ F_{\text{SST}} = \frac{\Delta p}{\Delta p \mid_{\text{SST}}} - 1, \]

where \( \Delta p \mid_{\text{SST}} \) is the central pressure drop at fixed SST. Do many, many numerical experiments, varying SST, Coriolis parameter, traslation speed, etc. Curve fit dependence of \( F_{\text{SST}} \) on these parameters. Result:
\[ F_{\text{SST}} = -0.87 e^{-z} \]

\[ z \equiv 0.55 \left( \frac{h_0}{30 \text{ m}} \right)^{1.04} \left( \frac{u_T}{6 \text{ m s}^{-1}} \right)^{0.97} \left( \frac{\Delta p}{50 \text{ hPa}} \right)^{-0.78} \times \eta^{-0.85} \left( \frac{f}{5 \times 10^{-5} \text{ s}^{-1}} \right)^{0.59} \left( \frac{\Gamma}{8 \times 10^{-2} \text{ K m}^{-1}} \right)^{-0.40} \left( \frac{1 - \mathcal{H}}{0.2} \right)^{0.46} \]

\( \eta = \text{storm size scaling factor} \)
Effects of Environmental Wind Shear

- Dynamical effects
- Thermodynamic effects
- Net effect on intensity
Upper level
Southwest wind at 50 MPH

Lower level
West wind at 20 MPH

Shear: 34 MPH
PV dynamics
Streamlines (dashed) and $\theta$ surfaces (solid)