Preliminary Design and Viability Consideration of
External, Shroud-Based Stators in Wind Turbine Generators

by

Nathaniel Shoemaker-Trejo

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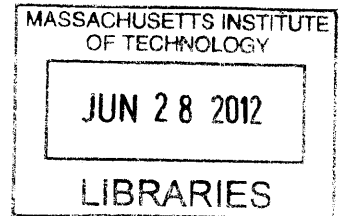Fulfillment of the Requirements for the Degree of

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ABSTRACT

Horizontal-axis wind turbine designs often included gearboxes or large direct-drive generators to compensate for the low peripheral speeds of the turbine hub. To take advantage of high blade tip speeds, an alternative generator configuration could employ a stator positioned in a shroud on the outside diameter of the turbine.[17] I performed a preliminary investigation into some of the main design concerns and costs of a shroud-stator turbine. To answer the basic questions of functionality and cost, I considered turbine-blade deflection, material costs, and stator configuration. Although I did not complete the entire investigation, I determined the approximate deflection of the loaded wind turbine blades and developed several relationships between the force and the current output of the turbine for use in designing the stator.

Thesis Supervisor: David Gordon Wilson
Title: Professor of Mechanical Engineering, Emeritus

INTRODUCTION: SHROUD STATOR CONFIGURATION

Background: Turbine generator technology

Wind turbines of the early 2000's required gearboxes to generate electricity efficiently. The compact electric generators suitable for operation in wind turbine nacelles typically required peripheral speeds on or near the order of 100m/s.[20] Wind turbine peripheral speeds at the hub could not meet this need. To provide an interface between the rotor and the generator, turbine designs often incorporated gearboxes which stepped up the rotational velocity of the rotor.

Most turbine gearboxes were large, heavy, expensive, and difficult to maintain. A 1MW horizontal axis turbine could require a gearbox weighing 7 tons or more. Installation of the gearbox in an elevated nacelle – more than 230ft. in the air – increases the difficulty and the cost of initial construction and follow-up maintenance.[25]

Around 2010, in an effort to move away from gearboxes, wind energy companies began to install direct-drive generators in their newer, larger turbine models. Direct drive generators eliminated the need for a gearbox by operating at higher torques and employing large-diameter rotors. To remain relatively lightweight, they also made use of lighter materials, permanent magnets, and a higher number of poles.[10] Combined with current controllers, they could adjust to varying wind speeds for maximum power production.[25]

Background: Turbine blade construction

As wind turbines grew bigger to take advantage of the greater efficiencies offered by larger rotor discs, the difficulties inherent in constructing long, thin blades increased. Most blades for megawatt-output turbines were constructed by forming a lightweight shell of a material such as balsa wood around a structural spar made of fiberglass.[22] Fiberglass became the spar material of choice because of its low density, low cost, and high tensile strength. Even larger turbines, such as those capable of generating tens of megawatts, required turbine blades of over 100m in length. Fiberglass did not possess the strength necessary to construct 100m spars without excessive cost and added weight.

To solve the spar strength problem, turbine designers began to consider other fiber reinforced materials, especially composites of carbon fiber. Carbon fiber was stronger than and as light as fiberglass, but prohibitively expensive. Consequently made of a combination of carbon fiber and fiberglass began to appear.[22]

Previous work: Shroud-Stator Concept

Although direct drive turbines were a significant improvement over geared turbines, the problem of weight, size, and cost reduction never disappeared. Direct-drive turbines were still relatively large, and could weigh nearly as much as a traditional generator and gearbox combined.[10] Other generator configurations might merit consideration.

As an alternative to direct- and geared-drive turbines, Professor David Gordon Wilson developed the concept of a generator with a stator mounted at the outer diameter of the turbine blades, expanding on the idea that a larger diameter is better. Rather than high torque and a moderately increased diameter, his design took advantage of the high tip speeds of the turbine

blades to achieve an appropriate generator output.[20] Magnets could be attached to the ends of the turbine blades, turning them into a rotor for the generator. The stator could be mounted on a shroud surrounding the turbine. The blade tips would sweep past the stator, inducing an electric current in the windings.[18]

Problem statement: Competitive Turbine Design

Can a turbine with a shroud-mounted generator produce the same power output as its hub-based counterparts while still remaining financially and logistically effective? To compete with current designs, the shroud-generator turbine must provide a significant financial advantage through higher efficiency or lower construction and maintenance costs. At the same time, it must remain structurally sound and capable of meeting or exceeding the anticipated lifetime of direct-drive and geared turbines.

Present approach: Blade deflection and generator configuration

I performed a preliminary investigation into the power generation capabilities, structural design difficulties, and construction costs of a hypothetical 10MW turbine equipped with a generator with a shroud-mounted stator. To answer the basic question of financial viability, I focused on the quantity and type of material used in the generator, the structure of the blades, and the power output of the turbine.[20] Detailing the generator and turbine blades in turn required an examination of the deflection of the tips of the turbine blades and geometric configuration of the stator.[13]

To acquire an accurate picture of the turbine, I made approximate calculations of the forces applied to the blades of a shroud-generator turbine configured to 10MW. I used two different shroud stator configurations, one with a slot for the blade tip and one flat, and estimated the stresses that arose in the blades to check if additional reinforcement by tensioning wires was necessary. I checked for compatibility of the blade deflection with the stator designs and planned to determine the quantity of material required for each of the main components of the generator, which allowed me to estimate the cost of constructing the turbine.[13]

The results of the approximations showed that wind forces on the airfoil blades caused the turbine blade to deflect towards the rear of the turbine by 0.5m, rendering one of the stator configurations useless without the added support of tension wires. The turbine, assuming the blades did not break, could supply a force on the order of 0.4MN to the rotor magnets. I was unable to determine a size and configuration of the stator that would provide 10MW of power, and was consequently unable to determine construction costs of the turbine.

METHOD: APPROXIMATING A 10MW SHROUD-STATOR TURBINE

Theoretical background: Wind turbine airfoils and magnetic circuit forces

Beam stresses and strains: Forces and moments applied to a long, thin member, such as an I-beam or a wind turbine blade, caused it to bend according to the magnitude and location of the forces as well as the material properties and geometry of the member. Internal shear forces and moments of the beam could be found from the forces and used to calculate the deflection of the beam as a function of a point along its length. Deflection of a beam's centroidal axis, denoted by v, was given by the integral

$$v(x) = \frac{1}{EI} \iint M \, dx \qquad (1)$$

or by

$$v(x) = \frac{1}{EI} \iiint V \, dx \qquad (2)$$

where E, I, M, V and x were Young's modulus, the second moment of area about the axis perpendicular to the plane of symmetry of the bent beam, the internal moment of the beam, the internal shear force of the beam, and the coordinate along the length of the beam, respectively. The constants of integration associated with equations 1 and 2 could be found using fixed points of the beam where the position is known to be zero.

Finding the internal moments and shear forces also permitted the calculation of stresses present in the beam and a way to establish a maximum load for the beam. Like the deflection, the stress in a beam also depended on the geometry and material properties of the beam. The normal stress σ, shear stress τ, and von Mises stress σ' were

$$\sigma(x,y) = \frac{-My}{I} \qquad (3)$$

$$\tau(x,y) = \frac{VQ}{Ib} \qquad (4)$$

$$\sigma'(x,y) = \sqrt{\sigma^2 + 3\tau^2} \qquad (5)$$

where y, Q, and b were the axis perpendicular to the length and in the plane of symmetry of the bent beam, the first moment of the area between y and the maximum value of y, and the thickness of the beam at y, respectively. The remaining variables were defined as above for equations 1 and 2. By finding the location of maximum von Mises stress and comparing it to the tensile strength of the beam material, an upper bound on the beam loading could be found.

Lift and drag forces on turbine blades: Wind turbine blades operated much like airplane wings. They used lift forces generated by their movement through the air to drive their rotation. The asymmetrical airflow above and below the blade caused the pressure below and behind the blade to exceed that of the pressure above and in front, pushing it forward. The lift generated by the pressure difference was proportional to the airflow velocity, air density, and the circulation of air around the blade.[25]

The same airflow that produced lift on a blade was also responsible for drag forces. Drag was a product of both skin friction and a change in pressure. Skin friction operated the same as with any two surfaces sliding against each other. The rougher the surface of the blade, the more drag developed. Pressure drag came from the separation of the boundary layer of fluid from the surface of the blade before it reached the trailing edge.[25]

For engineering purposes, drag and lift forces were calculated from correlations derived from experimental data. The data were incorporated into equations in the form of lift and drag coefficients, usually denoted C_L and C_D, associated with particular shapes, airfoil surfaces, and wind speeds. The lift forces, L, and drag forces, D, were then given by the equations

$$L = \frac{1}{2}\rho_{air}A_{blade}v_{wind}^2 C_L \tag{6}$$

$$D = \frac{1}{2}\rho_{air}A_{blade}v_{wind}^2 C_D \tag{7}$$

where ρ_{air}, A_{blade}, and v_{wind} stood for the density of air, the area – area parallel to the chord and to the span – of the blade, and the velocity of the wind, respectively.[25]

Substitution of a simplified trapezoidal blade geometry and appropriate coefficients resulted in lift and drag forces of

$$L(x) = \rho_{air}\pi \sin\alpha \left(\frac{x}{l}(d-c)+c\right)(v_{wind}^2 + (\omega x)^2) \tag{8}$$

$$D(x) = \frac{1}{2}\rho_{air}C_D\left(\frac{x}{l}(d-c)+c\right)(v_{wind}^2 + (\omega x)^2) \tag{9}$$

where α is the angle of attack of the blade, l is the length of the blade, d is the diameter of the support spar, and ω is the rotational velocity of the blade, respectively. The remaining variables are defined as in previous equations. I used the lift and frag forces to calculate internal stress and deflection of a turbine blade by decomposing them into their radial and axial components relative to the axis of the turbine.

Magnetic circuits: Electric generators used induction to convert from mechanical to electrical power. Direct drive generators consisted of permanent magnets or electromagnets mounted on a rotor. The changing magnetic field generated by the rotation of the rotor induced a voltage and current in the low-resistance windings of the stator.

To amplify the magnetic flux passing through the stator windings, materials with high magnetic permeability were used as cores inside the stator windings. The field of the rotor magnets magnetized the cores much more strongly than empty space. Because the cores passed through the windings, they increased the flux by increasing the field intensity through the loops.

Viewing the iron cores as wires, flux as current, and magnetomotive force as potential, generators could be analogized to electrical circuits. Matter such as air, which had a low magnetic permeability, was assigned a reluctance and acted like a resistor. Inductors acted like voltage sources.[11]

The generator configurations under consideration for the turbine, one slotted to allow the blade to pass through a larger number of coils and one flat to allow the blade to flex back and forth, have two air gaps, one magnet, and one inductor coil arranged in series. Based on common generator construction materials, I assumed that the magnets would be neodymium-iron-boron, the coils would be copper, the core would be iron, and that air would fill the gaps between rotor and stator. [18]

Balancing the magnetic flux and magnetomotive force and using the material constants of air and neodymium resulted in an equation for the flux linkage λ in the stator coil as a function of the stator current i_c and the air gap width w_g (where gap width is a dimension parallel to the separated faces, not the intervening distance):

$$\lambda(i_c, w_g) = \frac{N_c^2\, i_c}{R_{total}} - \frac{l_m H_c N_c}{R_{total}} \tag{12}$$

$$R_{total}(w_g) = \frac{l_m}{\mu_m w_m d_m} - \frac{2l_g}{\mu_0 d_g w_g} \tag{13}$$

where N_c, R_{total}, l_m, l_g, w_m, w_g, d_m, d_g, μ_m, μ_o, and H_c are the number of coil windings through which the magnetic field passes, the reluctance of the circuit, the length of the magnet and the air gap, the width of the magnet and the air gap, the depth of the magnet and the air gap (such that the width times the depth equal the cross-sectional area), the permeability of the magnet and free space, and the coercivity of the magnet, respectively.[11]

Assuming that the magnetic circuit of the generator worked as a black box converter of mechanical and electrical energy, I decided to use coenergy of the system to calculate the forces on the magnet. The differential of coenergy W' was defined as

$$dW'(x, i) = \lambda\, di + f\, dx \tag{13}$$

where x, i, and f are the position of the magnet with respect to the coil, the current in the coil, and the force on the magnet respectively. The derivative of the coenergy with respect to magnet position determined the force on the magnet, so I integrated the coenergy along a zero flux path that allowed me to set forces in the circuit to zero, then took the derivative of the resulting function. The resulting equation provides the force f on the magnet as a function of magnet position and coil current:

$$F_c(i_c, x) = \frac{-\mu_0 d_g (N_c i_c - l_m H_c)^2}{4 l_g \left[1 + \frac{\mu_0 d_g l_m}{\mu_m d_m l_g}\left(1 - \frac{x}{w_m}\right)\right]^2} \tag{15}$$

where the gap width w_g has been substituted for a function of the magnet position coordinate, x.[11]

Procedure: Plugging the Numbers into the Equations

To get real numbers out of the equations above, I coded the formulas I found into the symbolic math toolbox of Matlab. I obtained material constants for all of the turbine components, obtained approximate geometry of a 10MW turbine by scaling a 3MW turbine to size, plugged the numbers in, and let the code run.

CONCLUSIONS: BLADE DEFLECTION

The calculations carried out by Matlab's symbolic toolbox indicated that the blades of the turbine should deflect by 0.59m under pressure from oncoming wind. Such a large amount of deflection prohibited the use of a grooved stator due to the fact that the blade would not maintain proper clearance as it passed through. As a result, the shroud stator turbine would have to make use of a flat stator or use tensioning wires to stabilize the tips of the blades.

With a chosen wind velocity of 8m/s, the turbine blades generated approximately 0.43MN of force. Given a turbine tip speed in the hundreds of meters per second, this is more than enough force to generate 10MW of power. I was unable to determine the conditions under which the generator could collect this power. The stator dimensions and coil windings proved difficult to approximate or determine through research.

Because I was unable to determine a size and configuration of the stator that would provide 10MW of power, the question of turbine cost remained unresolved. The stator

configuration also played a role in determining the operating forces on the turbine blades, leaving the question of turbine durability open as well.

In future, important subjects to consider include a final stator configuration, fatigue wear of the blade tips, aerodynamic considerations of tension wires, total power output, establishing a stator configuration for most efficient power production, consideration of alternative designs such as full-shroud rotors, and comparison with competing designs such as direct drive generators.[20]

APPENDICES

List of variables
F_c = generator induction forces
ρ_{air} = density of air
l_b = length of blade
d_b = diameter of blade structural rod
c_b = chord of blade at root
v_{air} = ground velocity of air
ω_b = rotational velocity of blade
x = linear distance from root of blade
α = angle of attack of blade
C_D = approximate coefficient of drag of blade
V = shear forces in blade
M = internal moment in blade
σ = normal stress in blade
y = distance from neutral axis of blade
τ = shear stress in blade
σ' = von Mises stress in blade
L_b = lift force on blade
D_b = drag force on blade
μ_{air} = permeability of free space
d_m = depth of magnet
N_c = number of turns of coil
i_c = coil current
l_m = length of magnet perpendicular to magnetic circuit
H_c = coercivity of magnet
l_g = distance between magnet and windings mum=permeability of magnet,
x = position of magnetic field relative to winding
w_m = width of magnet coil windings
E = tensile modulus of turbine blade core
I = second moment of area about axis parallel to bending moment axis of beam, typically z-
 axis

Matlab code: Symbolic functions for solution of blade and generator forces.

```
% ASSUMPTIONS---------------------------------------------------------------

% rotation of blade is slow enough to disregard effects of
% acceleration-->can approximate blade as beam

% load of beam carried by central core member made of carbon fiber-->
% approximate blade as rod (better would be hollow rectangular tube, will
% change if get time)

% VARIABLES-----------------------------------------------------------------

% mg=mass of generator, g=gravity, theta=angle of turbine blade from
```

```
% vertical, Fc=generator induction forces, fb1, 2, or 3=distributed forces
% along lengths of blades, rhoair=density of air, lb=length of blade,
% db=diameter of blade structural rod, cb=chord of blade at root,
% vair=ground velocity of air, omegab=rotational velocity of blade,
% xb=distance from root of blade, alphab=angle of attack of blade,
% CD=approximate coefficient of drag of blade, mub=linear density of blade,
% Vb=shear forces in blade, c1=constant of integration, Mb=internal moment
% in blade, sigmab=normal stress in blade, yb=distance from neutral axis of
% blade, taub=shear stress in blade, vms=von Mises stress in blade,
% Lb=lift force on blade, Db=drag force on blade, vmsmaxxpts and
% vmsmaxypts=zeros of derivatives of and edges of von Mises stress,
% maxes=vector of all von Mises max values, Fg=total forces from generator,
% muair=permeability of free space (well, air really) dm=depth of magnet,
% Nc=number of turns of coil ic=coil current lm=length of magnet
% perpendicular to magnetic circuit, Hcm=coercivity of magnet, ls=length of
% space between magnet and windings, mum=permeability of magnet,
% xs=position of magnetic field relative to winding, wm=width of magnet and
% coil windings, rhobcore and rhobshell=density of blade core and shell
% materials, Fb1Eff, 2, or 3=effective force on turbine blade,
% Tb1Eff=effective torque on turbine blade, hb=height of blade,
% hshell=thickness of blade shell, LbTheta and DbTheta=lift and drag forces
% in the direction of theta around axis of turbine, E=tensile modulus of
% turbine blade

syms mg fb1 xb mub Vb c1 c2 Mb sigmab yb taub vms Lb Db vmsmaxxpts ...
    vmsmaxypts maxes dm Nc ic lm ls xs wm fb2 fb3 c1r c2r c3r c4r
% vars to solve for dm Nc ic lm ls wm


% KNOWN VALUES--------------------------------------------------------------


% note: units are in SI
muair=4*pi()*10^7; % H/m
mum=1.38*10^-6; % H/m
Hcm=8.36*10^5; %A/m
g=9.81; % m/s^2
theta=pi()/3; % radians; chose blade position just before one blade
    % emerges from stator because that is where the highest stress in the
    % blade will happen
rhoair=1.1885; % kg/m^3 at 10^5Pa and 20C
lb=130; % m
db=5; % m; approximate, based off of linear scaling a 3MW turbine
cb=9; % m; approximate, based off of linear scaling a 3MW turbine
vair=8; % m/s
omegab=pi()/3; % radians per second; working on getting better number
alphab=pi()/12; % radians; typical angle of attack is inside +-15 degrees
CD=0.04; % arbitrary value from Engineer's Toolbox which matches wind
    % energy textbook value; alternative estimated value was way too high
rhobshell=0.9; % kg/m^3 (balsa wood)
rhobcore=18.1; % kg/m^3 (carbon fiber)
hb=5; % m; approximate, based off of linear scaling a 3MW turbine
hshell=0.5; % m; approximate, based off of linear scaling a 3MW turbine
E=242*10^9; % Pa


% TURBINE BLADE FORCES------------------------------------------------------


% force on tip of blade
```

```
Fg=mg*g*sin(theta)+Fc;

% drag and lift forces
Db(xb)=1/2*rhoair*CD*(xb/lb*(db-cb)+cb)*(vair^2+(omegab*xb)^2);
Lb(xb)=rhoair*pi()*sin(alphab)*(xb/lb*(db-cb)+cb)*(vair^2+(omegab*xb)^2);

% drag and lift forces in terms of radial and axial coordinates
LbTheta(xb)=-Lb(xb)*vair/((vair^2+(omegab*xb)^2)^(1/2));
DbTheta(xb)=Db(xb)*omegab*xb/((vair^2+(omegab*xb)^2)^(1/2));

Lbr(xb)=-Lb(xb)*omegab*xb/((vair^2+(omegab*xb)^2)^(1/2));
Dbr(xb)=Db(xb)*vair/((vair^2+(omegab*xb)^2)^(1/2));

% linear approximations of Lb's and Db's
LbTheta(xb)=xb*(LbTheta(lb)-LbTheta(0))/(lb-0)+LbTheta(0);
DbTheta(xb)=xb*(DbTheta(lb)-DbTheta(0))/(lb-0)+DbTheta(0);

Lbr(xb)=xb*(Lbr(lb)-Lbr(0))/(lb-0)+Lbr(0);
Dbr(xb)=xb*(Dbr(lb)-Dbr(0))/(lb-0)+Dbr(0);

% linear density of blade
mub(xb)=rhobshell*(2*hshell*(cb+hb-2*hshell)-(2*hshell*xb*(cb-db))/lb)+ ...
    rhobcore*pi()*db^2/4;

% convert drag, lift, and density of blades into force distributions over
% each blade, then calculate effective torque on each blade

% blade 1
fb1(xb)=LbTheta(xb)+DbTheta(xb)+mub(xb)*g*sin(theta);
fb1(xb)=simplify(fb1(xb));

% Fb1Eff=int(fb1(xb), xb, 0, lb)+Fg;
% Fb1Eff=simplify(Fb1Eff);

Tb1Eff=int(xb*fb1(xb), xb, 0, lb)+Fg*lb;
Tb1Eff=simplify(Tb1Eff);

% blade 2
fb2(xb)=LbTheta(xb)+DbTheta(xb);
fb2(xb)=simplify(fb2(xb));

% Fb2Eff=int(fb2(xb), xb, 0, lb);
% Fb2Eff=simplify(Fb2Eff);

Tb2Eff=int(xb*fb2(xb), xb, 0, lb);
Tb2Eff=simplify(Tb2Eff);

% blade 3
fb3(xb)=LbTheta(xb)+DbTheta(xb)-mub(xb)*g*sin(theta);
fb3(xb)=simplify(fb3(xb));

% Fb3Eff=int(fb3(xb), xb, 0, lb)+mg*g*sin(theta);
% Fb3Eff=simplify(Fb3Eff);
```

```
Tb3Eff=int(xb*fb3(xb), xb, 0, lb)-lb*mg*g*sin(theta);
Tb3Eff=simplify(Tb3Eff);


% note: many forces should cancel, but it would take more work for me to go
% go back and change than to just let my computer figure it out. Also, I
% need them for the stress analysis section
Fc=solve(0==Tb1Eff+Tb2Eff+Tb3Eff,Fc)


% GENERATOR FORCES-------------------------------------------------------

% force from generator current by induction
Fc(xs)=muair*dm*(Nc*ic-lm*Hcm)^2/(4*ls*(1+muair*lm/(2*mum*ls)*(1-xs/wm)))^2;


% constraint: omegab*lb=dxs/dt


% BLADE TIP DEFLECTION---------------------------------------------------

% solve for shear forces
Vbr(xb)=int(Lbr+Dbr, xb)+c1r;
c1r=solve(Vbr(lb)==0,c1r);
Vbr(xb)=int(fb1(xb))+c1r;
Vbr(xb)=simplify(Vbr(xb));


% solve for moments
Mbr(xb)=int(Vbr(xb),xb)+c2r;
c2r=solve(Mbr(lb)==0,c2r);
Mbr(xb)=int(Vbr(xb),xb)+c2r;
Mbr(xb)=simplify(Mbr(xb));


% solve for beam slope
sloper(xb)=int(Mbr(xb),xb)/(E*pi*db^4/64)+c3r;
c3r=solve(0==sloper(0),c3r);
sloper(xb)=int(Mbr(xb),xb)/(E*pi*db^4/64)+c3r;
sloper(xb)=simplify(sloper(xb));


% solve for beam deflection
deflectr(xb)=int(sloper(xb),xb)+c4r;
c4r=solve(deflectr(0)==0,c4r);
deflectr(xb)=int(sloper(xb),xb)+c4r;
deflectr(xb)=simplify(deflectr(xb));


TipDeflection=deflectr(lb)


% BLADE STRESS ANALYSIS--------------------------------------------------

% solve for shear forces
Vb(xb)=int(fb1(xb))+c1;
c1=solve(Vb(lb)==Fg,c1);
Vb(xb)=int(fb1(xb))+c1;
Vb(xb)=simplify(Vb(xb));


% solve for moments
Mb(xb)=int(Vb(xb),xb)+c2;
c2=solve(Mb(lb)==0,c2);
```

```
Mb(xb)=int(Vb(xb),xb)+c2;
Mb(xb)=simplify(Mb(xb));

% find normal stress
sigmab(xb,yb)=-Mb(xb)*yb/(pi()*db^4/64);
sigmab(xb,yb)=simplify(sigmab(xb,yb));

% find shear stress
taub(xb,yb)=Vb(xb)*(1/12*(db^2-4*yb^2))/(pi()*db^4/64);
taub(xb,yb)=simplify(taub(xb,yb));

% calculate von Mises stress
vms(xb,yb)=(sigmab(xb,yb)^2+3*taub(xb,yb)^2)^(1/2);
vms(xb,yb)=simplify(vms(xb,yb));

% find max von Mises stress
vmsmaxxpts=[solve(diff(vms(xb,yb),xb)==0,xb);0;lb];
vmsmaxypts=[solve(diff(vms(xb,yb),yb)==0,yb);0;db/2];

maxes=[];
for xzero=1:length(vmsmaxxpts)
    for yzero=1:length(vmsmaxypts)
        maxes=[maxes; vms(vmsmaxxpts(xzero),vmsmaxypts(yzero))];
    end
end

% constraint: do not exceed yield/break/snap strength of blade material
```

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