The Road to Multi-Dimensional Magnetic Levitation: Realizing Two-Dimensional Control in Classical Feedback

by

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Submitted to the Department of Electrical Engineering and Computer Science
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Abstract

A stable multi-dimensional magnetic levitator was characterized and implemented. This thesis contains a full analysis of the feedback specifications, a short summary of the circuits used in the design of the setup, and results from the physical device. Two and three dimensional magnetic levitation systems were analyzed and designed. A modular cell of the two-dimensional levitator was implemented.

Thesis Supervisor: James K. Roberge
Title: Professor of Electrical Engineering

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1 Introduction

Ernshaw's theorem, proved in 1842, states that a collection of point charges cannot be maintained in a stable configuration with static fields. In layman's terms, this theorem proves that the suspension of objects with static magnetic fields is impossible. The result does not show that magnetic levitation is inherently impossible, but rather it hands the problem to an area of engineering that can deal with changing fields, namely analog feedback. In this thesis, stable multi-dimensional magnetic levitation systems are theoretically designed for one, two, and three dimensions. A cell of the magnetic levitator for two dimensions is also practically implemented to show proof of concept. The combination of these two parts of this thesis show how to circumvent Ernshaw's theorem and realize stable magnetic levitators in all dimensions.

The goals of this thesis are to levitate an object purely with magnetic fields, to use the techniques of classical feedback to produce a stable output, and to allow movement of the levitated object between solenoids around an n-dimensional space, for $n \in \{1, 2, 3\}$. For the one dimensional levitator, the object will hover statically. The behavior of this system is analyzed in sections (2.1) and (3.1). The two-dimensional levitator produces a levitated object that maintains a stable, constant height while additionally able to move between solenoids arranged in a straight line. This system is analyzed in sections (2.2), (3.2) and (4). The three-dimensional levitator maintains the object's height while moving it in two dimensions. While the author was not able to design a true three-dimensional levitator for reasons that are explored in section (3.3), the system's dynamics are explored in section (2.3). A design of a three-dimensional levitator where the object can only move between lattice points, a "2.5-Dimensional Levitator", is designed in section (3.3).

This thesis is broken into several different themes. Section 2, "Theory", deals with modeling the ideal dynamics of the theoretical n-dimensional levitation system. The state equations are developed for $n \in \{1, 2, 3\}$. The stability and implied compensation are also analyzed, resulting in full feedback diagrams for each system. Section 3, "Design", analyzes the necessary sensing mechanisms needed to produce a practical design. Circuits that implement the feedback diagrams designed in Section 2 are produced. Section 4, "Implementation" deals with a modular cell of the two-dimensional levitator that was built to show proof of concept. The "Results" subsection contains data from the physical system.

2 Theory

In this section, we develop the theory behind one, two, and three dimensional levitation. We will derive the state equations, find ways to compensate the systems, and develop feedback diagrams to represent the closed loop systems. We begin with the theory of levitation in one dimension.

2.1 One Dimensional Levitation

The basic setup of a one-dimensional magnetic levitator is detailed in Figure 1. It consists of an electromagnet used to oppose the force of gravity, a position sensor to detect the height of the levitated object, the object to be levitated, a stand for mounting the circuit and electromagnet, and a circuit to provide the necessary compensation to produce a stable system. The direction of the current through the inductor i_M and the direction of the position of the object x_B are marked in the figure.

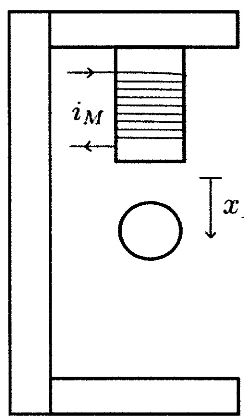


Figure 1: One-Dimensional Levitation Basic Setup

As we will see shortly, the open loop transfer function of this setup contains a right half plane pole. This results in an unstable system. However, we can design a compensator such that the system becomes stable, by linearizing the state equations and applying classical feedback techniques. The position sensor's output is subtracted from a DC voltage input, which determines the height at which the object levitates. This feedback scheme is shown abstractly in Figure 2.

2.1.1 Linearized State Equations

In order to understand how to stabilize the one-dimensional levitator, we must develop the state equations that govern the system. The full derivation of the linearized state equations involves complicated mechanical engineering techniques outside the scope of this thesis. Here we present a concise derivation, which borrows heavily from the original work of Woodson and Melcher and the summary in Roberge.

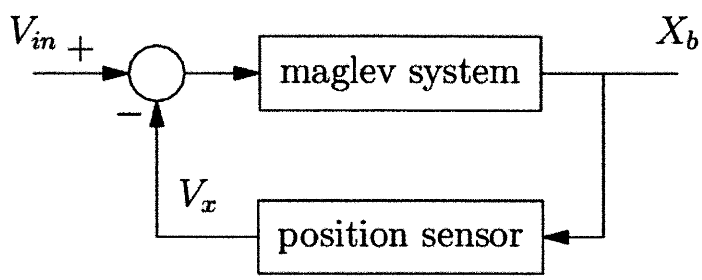


Figure 2: The Abstract Feedback System

To derive the transfer function, we make a number of assumptions about our system. We assume that the inductance of the electromagnet can be roughly approximated by a first order Taylor approximation. Given x_B as defined in Figure 1, the inductance is approximated as

$$L(x_B) \approx L_o + \frac{L_1}{1 + a x_B} \tag{2.1}$$

where $L(x_B)$ is defined as the function that gives the impedance of the inductor and $1/a$ is the characteristic length of the system. The coenergy of the system is given in Roberge as

$$W'_M(i_M, x_B) = \frac{1}{2} L(x_B) i_m^2 \tag{2.2}$$

which implies the force on the ball is the first derivative with respect to position, or

$$\begin{aligned} f_M(i_M, x_B) &= \frac{\partial W'_M}{\partial x_B} \\ &= \frac{-a L_1 i_M^2}{2(1 + a x_B)^2} \end{aligned} \tag{2.3}$$

Given the orientation of x_B, the force from the electromagnet pulls the ball upwards, which corresponds with our intuition. This state equation is non-linear and cannot be used with classical feedback techniques, which rely on linear time invariant (LTI) systems. In order to use this equation with classical feedback techniques, we choose an operating point (X_B, I_M, F_M) and linearize.

$$f_M = F_M + f_m \approx f_M(I_M, X_B) + \left(\left. \frac{\partial f_M}{\partial x_B} \right|_{I_M, X_B} \right) x_b + \left(\left. \frac{\partial f_M}{\partial i_M} \right|_{I_M, X_B} \right) i_m \tag{2.4}$$

Substituting equation (2.3) into (2.4) results in

$$f_M = \frac{-aL_1 I_M^2}{2(1+aX_B)^2} + \frac{a^2 L_1 I_M^2 x_b}{(1+aX_B)^3} - \frac{aL_1 I_M i_m}{(1+aX_B)^2} \tag{2.5}$$

Figure 3 shows the force relationships on the ball.

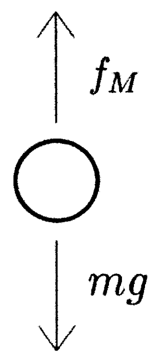


Figure 3: Forces on the Object in the One-Dimensional Case

Equating the forces,

$$m\frac{d^2 x_B}{dt^2} = mg + f_M \tag{2.6}$$

where m is the mass of the object and g is the force of gravitation, approximately $9.8m/s^2$. Combining equations (2.5) and (2.6), we have

$$m\frac{d^2 x_B}{dt^2} = mg - \frac{aL_1 I_M^2}{2(1+aX_B)^2} + \frac{a^2 L_1 I_M^2 x_b}{(1+aX_B)^3} - \frac{aL_1 I_M i_m}{(1+aX_B)^2} \tag{2.7}$$

We can simplify this equation by introducing another equation that must be satisfied: the operating point equation. The DC upward force is given by (2.3) and the downward force is given by mg, so in steady state, when the object is balanced, these two forces add to zero. This implies the equation

$$mg - \frac{aL_1 I_M^2}{2(1+aX_B)^2} = 0 \tag{2.8}$$

Substituting this result into (2.7) implies

$$\begin{aligned} m\frac{d^2 x_B}{dt^2} &= mg - \frac{aL_1 I_M^2}{2(1+aX_B)^2} + \frac{a^2 L_1 I_M^2 x_b}{(1+aX_B)^3} - \frac{aL_1 I_M i_m}{(1+aX_B)^2} \\ &= \frac{a^2 L_1 I_M^2 x_b}{(1+aX_B)^3} - \frac{aL_1 I_M i_m}{(1+aX_B)^2} \\ &= \frac{aL_1 I_M^2}{(1+aX_B)^2}\left(\frac{a x_b}{1+aX_B} - \frac{i_m}{I_M}\right) \end{aligned} \tag{2.9}$$

From (2.8), we have

$$2mg = \frac{aL_1 I_M^2}{(1+aX_B)^2} \tag{2.10}$$

Substituting back into (2.9), this gives

$$m\frac{d^2 x_B}{dt^2} = 2mg\left(\frac{ax_b}{1+aX_B} - \frac{i_m}{I_M}\right) \tag{2.11}$$

This equation gives us a good way for finding the transfer function. Taking the Laplace transform,

$$ms^2 X_b(s) = 2mg\left(\frac{aX_b(s)}{1+aX_B} - \frac{I_m(s)}{I_M}\right) \tag{2.12}$$

Solving for $\frac{X_b(s)}{I_m(s)}$, we have

$$\frac{X_b(s)}{I_m(s)} = \frac{-k_I}{ms^2 - k_X} \tag{2.13}$$

where $k_I = \frac{2mg}{I_M}$ and $k_X = \frac{2mg}{1+aX_B}$, $k_I, k_X > 0$. This system has two poles; $p_1, p_2 = \pm\sqrt{k_X/m}$. Unfortunately, one of these poles is in the right half plane. This results in an inherently unstable system.

As a consequence of this inherent instability, we need to design a compensator to improve the phase margin.

2.1.2 One Dimensional Compensation and Stability

In classical feedback, there are many different compensation strategies to choose from. For the purposes of this project, we want compensation that will prefer fast changes to slow ones, in order to quickly make small changes to the position of the object, rather than slow changes that will have large effects. This corresponds to lead compensation. Figure 4 shows the feedback diagram corresponding to this system.

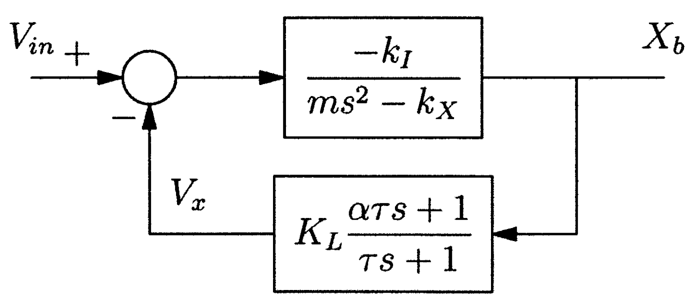


Figure 4: One Dimensional Compensated Feedback System

With lead compensation, our compensator is of the form $G_c(s) = K_L\frac{\alpha\tau s+1}{\tau s+1}$. If we consider this to be the feedback in a system with the plant given by (2.13), then we get

$$\begin{aligned}\frac{X_b(s)}{I_m(s)} &= \frac{\frac{-k_I}{ms^2-k_X}}{\frac{-k_I}{ms^2-k_X}K_L\frac{\alpha\tau s+1}{\tau s+1}+1} \\ &= \frac{-k_I}{-k_IK_L\frac{\alpha\tau s+1}{\tau s+1}+ms^2-k_X} \\ &= \frac{-k_I\left(\tau s+1\right)}{-k_IK_L\left(\alpha\tau s+1\right)+\left(ms^2-k_X\right)\left(\tau s+1\right)} \end{aligned} \tag{2.14}$$

Unfortunately, it is difficult to evaluate the stability from this equation. Instead, we use root locus techniques. The loop gain is given by

$$\frac{-k_I}{ms^2-k_X}K_L\frac{\alpha\tau s+1}{\tau s+1} \tag{2.15}$$

Which has a zero at $\frac{-1}{\alpha\tau}$ and poles at $\pm\sqrt{\frac{k_x}{m}}$ and $\frac{-1}{\tau}$. This system's root locus is given in Figure 5. Notice that if the gain is large, all the poles of the system are in the left half plane and the system is stable. This shows that lead compensation will suffice to stabilize this system, as long as the gain is large enough.

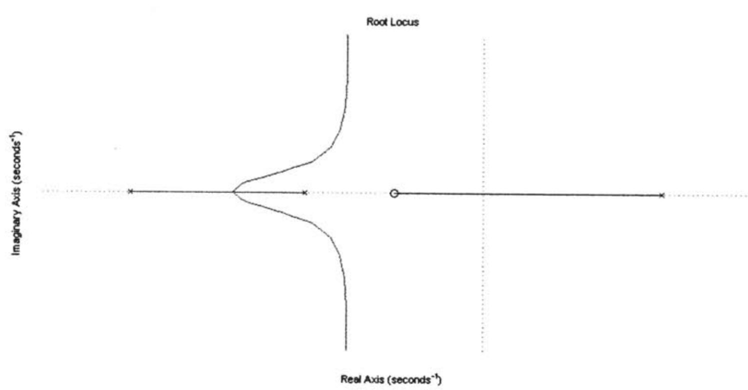


Figure 5: The Root Locus Plot of the Compensated System

We have developed the state equations and compensation strategy for the one dimensional system. This resulted in a design for a compensator to stabilize the system. These results are enough to engineer a stable one-dimensional system, so we turn our attention to the two-dimensional model.

2.2 Two-Dimensional Levitation

2.2.1 Two Dimensional State Equations

Figure 6 shows the basic setup for the two dimensional levitation system. There are two solenoids, each of which exerts a force on our object.

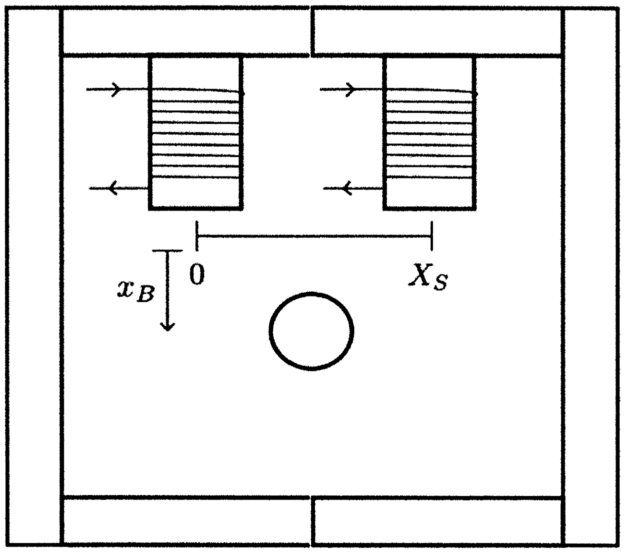


Figure 6: Basic Setup for Two Dimensional Levitation

Figure 7 shows the forces on the levitated object. Each of the solenoids is run at the same current, although they may have different inductances because of imperfect matching. Our algorithm for moving the object from $x_H = 0$ to $x_H = X_S$ has two parts:

1. Stabilize the sum of the vertical components of f_{M_1} and f_{M_2} against the force of gravity mg
2. Stabilize the horizontal components of f_{M_1} and f_{M_2} against each other

The two dimensional levitator could be built with any number of solenoids within reason. To design an arbitrarily sized system, it suffices to show how to create stable levitation and movement between two solenoids. This is because we can use a circuit to switch between pairs of solenoids: Using one pair to transfer the object from point A to B and another to transfer it from point B to C. The circuits for switching are provided in section (3.2).

Unfortunately, the equations from the one-dimensional levitator do not readily apply to the horizontal balancing that needs to happen in the two dimensional levitator. In the following section, we derive the transfer function for the horizontal components and show how to stabilize both the vertical and horizontal components.

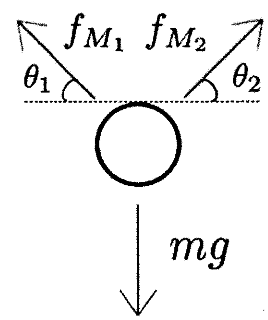


Figure 7: Forces on the Object During Two Dimensional Levitation

We can use many ideas from the last section. The model of an individual solenoid's force is the same as in section (2.1), which means only slight changes need to be made to our previous expression for f_M to make the previous state equations model this system accurately.

To start, the total force in the vertical direction is the sum of the vertical components of f_{M_1} and f_{M_2}, or $f_{M_{net,v}} = f_{M_1} sin(\theta_1) + f_{M_2} sin(\theta_2)$. If $f_{M_{net,v}} + mg = 0$, then our levitated object is stable. In other words, the same equations model the vertical components of the two dimensional system and the one-dimensional levitator, with $f_{M_{net,v}}$ substituting for f_M in the state equations from the previous section. We can control $f_{M_{net,v}}$ by sensing the vertical position of the object. The fact that the previous equations apply to this system means that we can stabilize the vertical component of the force by utilizing lead compensation, as analyzed in (2.1).

Having found a way to stabilize the vertical component, we turn our attention to the horizontal component. Our state equation for the horizontal position x_H, is

$$m\frac{d^2 x_H}{dt^2} = f_{M_1} cos(\Theta_1 + \theta_1) + f_{M_2} cos(\Theta_2 + \theta_2) \tag{2.16}$$

If the scale of x_H is defined as in Figure 6, then $cos(\Theta_1) = \dfrac{X_H}{\sqrt{X_H^2 + X_B^2}}$ and $cos(\Theta_2) = \dfrac{X_S - X_H}{\sqrt{X_H^2 + X_B^2}}$. In order to simplify our equations, we will assume that θ_1 and θ_2 do not change appreciably during the course of the horizontal transients. We justify this approximation by the fact that the solenoids will be as close as we can make them, the object will be as far away as we can allow, and the change in the angles will be very small compared to the change in the current and voltage passing through the electromagnet.

Earlier, we derived $f_M = \dfrac{-aL_1 I_M^2}{2(1 + aX_B)^2} + \dfrac{a^2 L_1 I_M^2 x_b}{(1 + aX_B)^3} - \dfrac{aL_1 I_M i_m}{(1 + aX_B)^2}$. The same expression holds for the horizontal force, with a few changes. The inductances of the solenoids can be different, so we will have a L_1 and a L_2. Also, we are no longer dealing with the variable X_B, so we must directly substitute X_H. This implies

$$
\begin{aligned}
m\frac{d^2x_H}{dt^2} = & \left[\frac{-aL_1I_M^2}{2(1+aX_H)^2} + \frac{a^2L_1I_M^2x_h}{(1+aX_H)^3} - \frac{aL_1I_Mi_m}{(1+aX_H)^2}\right]\left(\frac{x_h - X_H}{\sqrt{X_H^2+X_B^2}}\right) \\
- & \left[\frac{-aL_2I_M^2}{2(1+aX_H)^2} + \frac{a^2L_2I_M^2x_h}{(1+aX_H)^3} - \frac{aL_2I_Mi_m}{(1+aX_H)^2}\right]\left(\frac{X_S - X_H - x_h}{\sqrt{X_H^2+X_B^2}}\right) \qquad (2.17)
\end{aligned}
$$

The operating point equation is

$$
0 = \frac{-aL_1I_{M_1}^2}{2(1+aX_H)^2}\left(\frac{X_H}{\sqrt{X_H^2+X_B^2}}\right) - \frac{-aL_2I_{M_2}^2}{2(1+aX_H)^2}\left(\frac{X_S - X_H}{\sqrt{X_H^2+X_B^2}}\right) \qquad (2.18)
$$

so we may simplify (2.17) to

$$
\begin{aligned}
m\frac{d^2x_H}{dt^2} = & \left[\frac{a^2L_1I_M^2x_h}{(1+aX_H)^3} - \frac{aL_1I_Mi_m}{(1+aX_H)^2}\right]\left(\frac{X_H}{\sqrt{X_H^2+X_B^2}}\right) \\
- & \left[\frac{a^2L_2I_M^2x_h}{(1+aX_H)^3} - \frac{aL_2I_Mi_m}{(1+aX_H)^2}\right]\left(\frac{X_S - X_H}{\sqrt{X_H^2+X_B^2}}\right) \qquad (2.19)
\end{aligned}
$$

Factoring out common terms and simplifying gives

$$
\begin{aligned}
& \left[\frac{a^2L_1I_M^2x_h}{(1+aX_H)^3} - \frac{aL_1I_Mi_m}{(1+aX_H)^2}\right]\left(\frac{X_H}{\sqrt{X_H^2+X_B^2}}\right) - \left[\frac{a^2L_2I_M^2x_h}{(1+aX_H)^3} - \frac{aL_2I_Mi_m}{(1+aX_H)^2}\right]\left(\frac{X_S - X_H}{\sqrt{X_H^2+X_B^2}}\right) \\
= & \frac{aI_M}{(1+aX_H)^2\sqrt{X_H^2+X_B^2}}\left[\left(\frac{aL_1I_Mx_h}{1+aX_H} - L_1i_m\right)\Big(X_H\Big) + \left(\frac{aL_2I_Mx_h}{(1+aX_H)} - L_2i_m\right)\Big(X_S - X_H\Big)\right] \\
= & k_c\left(x_h\left[\frac{aL_1I_MX_H}{1+aX_H} - \frac{aL_2I_M(X_S-X_H)}{1+aX_H}\right] - i_m\Big[L_1X_H - L_2(X_S-X_H)\Big]\right) \\
= & k_c(x_hk_h + i_mk_m) \qquad (2.20)
\end{aligned}
$$

where

1. $k_c = \dfrac{aI_M}{(1+aX_H)^2\sqrt{X_H^2+X_B^2}}$
2. $k_h = \dfrac{aL_1I_MX_H}{1+aX_H} - \dfrac{aL_2I_M(X_S-X_H)}{1+aX_H}$
3. $k_m = L_1X_H - L_2(X_S - X_H)$

From these last few equations, we can derive the transfer function.

$$
\begin{aligned}
& m\frac{d^2x_H}{dt^2} = k_c(x_hk_h + i_mk_m) \\
\Rightarrow\ & ms^2X_h(s) = k_cX_h(s) + k_mI_m(s) \\
\Rightarrow\ & \frac{X_h(s)}{I_m(s)} = \frac{k_m}{ms^2 - k_c} \qquad (2.21)
\end{aligned}
$$

This equation is of the same form as the transfer function that modeled the one-dimensional dynamics. It has two poles, at $\pm\sqrt{\frac{k_c}{m}}$. One is a right half plane pole, so the system is inherently unstable. The only differences between the one-dimensional equations and these are the constants. However, due to the fact that it has the same form as the equation from (2.1), the root locus analysis from before applies to this system. Therefore, we may use lead compensation to stabilize this component of our forces as well.

We now have ways to stabilize the two components that make up the forces on the object. However, the solenoids can only take a single input. Therefore, we must find a way to sense, compensate, and combine the two components that make up our forces.

2.2.2 Two Dimensional Compensation and Stability

One way to make the system stable is to control the most important component the fastest, namely the vertical component. Then, making sure the object is stable in the vertical dimension, slowly change the object so that the horizontal position changes. This corresponds to minor loop compensation, with the vertical component feedback being the minor loop and the horizontal section serving as the major loop. Each of these individual loops also has lead compensation, as discussed earlier, to produce a stable system. Figure 8 shows this compensated feedback system.

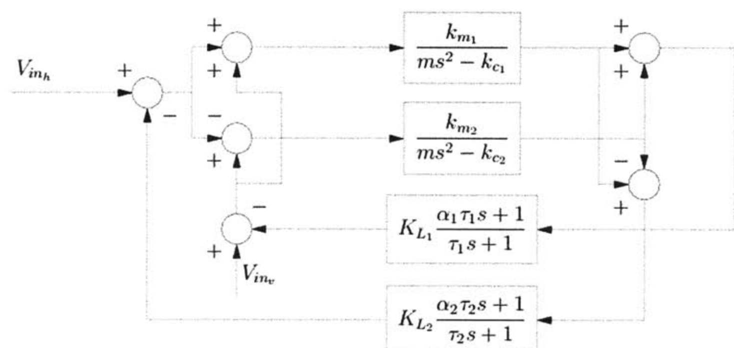


Figure 8: Two Dimensional Compensated Feedback System

It is easiest to understand this system in terms of superposition. If we only consider the vertical voltage input, the feedback diagram becomes

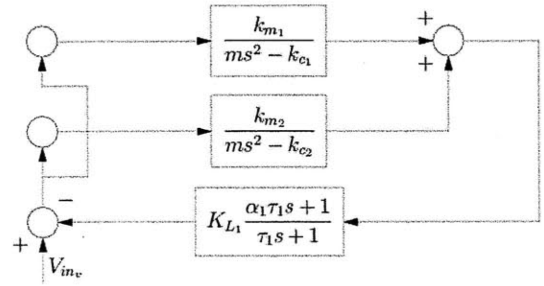


Figure 9: Two Dimensional Compensated Feedback System - The Vertical Component

In this feedback diagram, the author has neglected to add the sensors. This is to reduce the complexity of the already complex diagram. However, in Figure 9, the height of the object is sensed from the output. It is then fed through a compensator, compared to the DC voltage input, and used to drive both solenoids equally. From a feedback perspective, this system is the same as the system that governs the one-dimensional levitator, with the slight exception that the loop has a larger gain. However, the system has the same poles and zeros, so by the analysis in (2.1), we can safely say that we can stabilize the vertical component of the force on the object using the scheme outlined in Figure 8.

Figure 10 shows the other substate of the system.

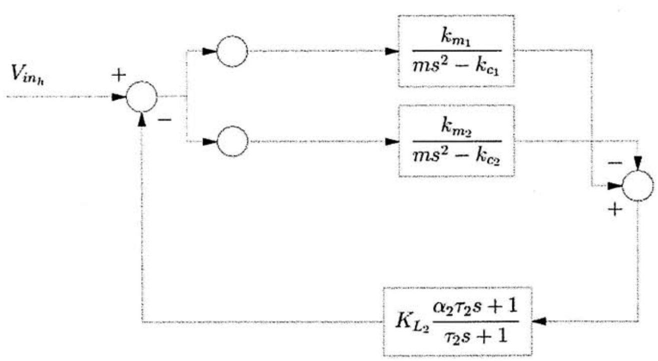


Figure 10: Two Dimensional Compensated Feedback System - The Horizontal Component

In this feedback diagram, the horizontal distance of the object from each solenoid is sensed from the object's position. The difference is taken, and then fed through a compensator. This is the exact same model we used earlier in this section. As a result, it has the same transfer function of the equations analyzed earlier in this section and may be stabilized using lead compensation.

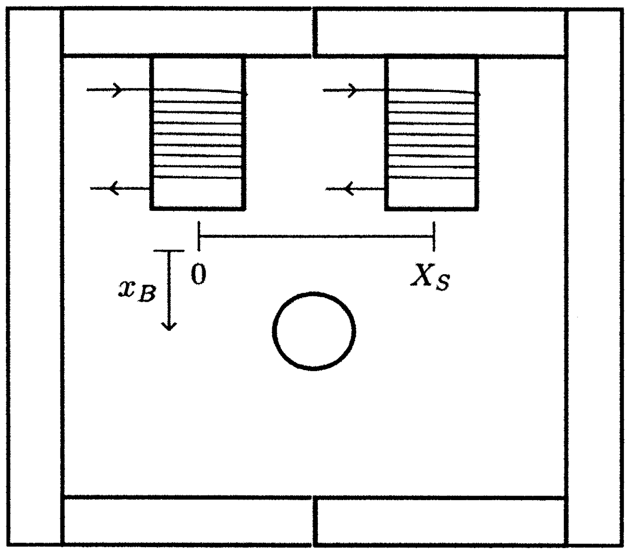
Because the two-dimensional levitator model is linear, the solution is the sum of the superimposed substates. Given that each substate produces a stable output, the overall output must also be stable. This is true for a finite number of substates. Therefore, the scheme outlined in Figure 8 is able to stabilize the position of an object between two solenoids.

We now have effective feedback diagrams for one and two dimensional systems. In order to complete our modeling, we turn our attention to the three-dimensional system.

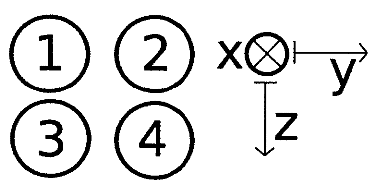
2.3 Three-Dimensional Levitation

2.3.1 Three Dimensional State Equations

Figure 11 shows the basic setup for the three dimensional case. It is a logical extension of the two dimensional case, with solenoids laid out in a grid to allow two directions of movement, plus the dimension of levitation. Subdiagram (a) of Figure 11 shows the side view of the solenoids, while subdiagram (b) shows a top view. There are four solenoids and they are laid out in a square pattern.



(a) Side View



(b) Top View

Figure 11: Basic Three Dimensional Levitation System

Figure 12 shows the forces on the object. There are now four directions in which the object is tugged, one for each of the solenoids. To stabilize the vertical components, we can use the same strategy as the two dimensional system: Control $f_{net,v}$, the force that represents the sum of the normal components. The analysis that this method is able to stabilize the vertical component is the same as in (2.2).

Let us call the horizontal directions y and z as labeled in Figure 11b. The force in the direction of y does not affect the force in the direction of z. This is because they are perpendicular, so the

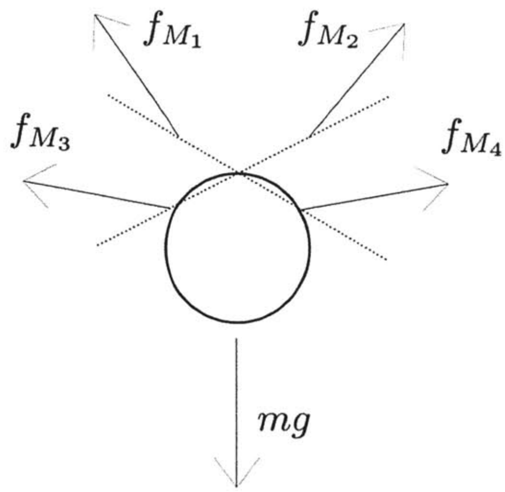


Figure 12: Forces on the Object in Three Dimensions

projection of the vector corresponding to the force in the z direction onto the y force vector is 0. Therefore, we can stabilize these directions separately. In addition, the state equations for z are the same as the state equations for y, aside from directional changes, because they have the same setup and the same components. Their transfer functions are the same. As a result, we can stabilize each direction separately using lead compensation. The feedback diagram is a logical extension of the feedback diagrams of (2.1) and (2.2), but it becomes very complicated, because the solenoids still have only one input, which now must contain the information for all three directions.

2.3.2 Three Dimensional Compensation and Stability

In order to stabilize all three components of the force vector on the object, we take the logical extension of the minor loop compensation, which the author calls micro loop compensation. The fastest direction we need to stabilize is the vertical loop, so that component forms the micro loop. The second direction to stabilize is chosen arbitrarily, but we will choose the y direction. This component forms the minor loop. Finally, the z direction component forms the major loop. We can again analyze this feedback by utilizing superposition. Figure 13 shows the full feedback diagram. Much like section (2.2), it is easier to understand the substates, shown in Figures 14, 15, and 16. Each one of the substate feedback diagram's plants can be simplified to a multiple of the plant in section (2.2), so each substate feedback diagram's plant can be stabilized with a lead compensator. We must take care to make sure that the forces we add to get a net force are correct, but besides that minor issue, the feedback diagrams are provably stable by the analysis from section (2.2). Because each substate produces a stable solution, the overall solution must be stable as well. This completes the analysis of the dynamics of the one, two, and three dimensional levitators.

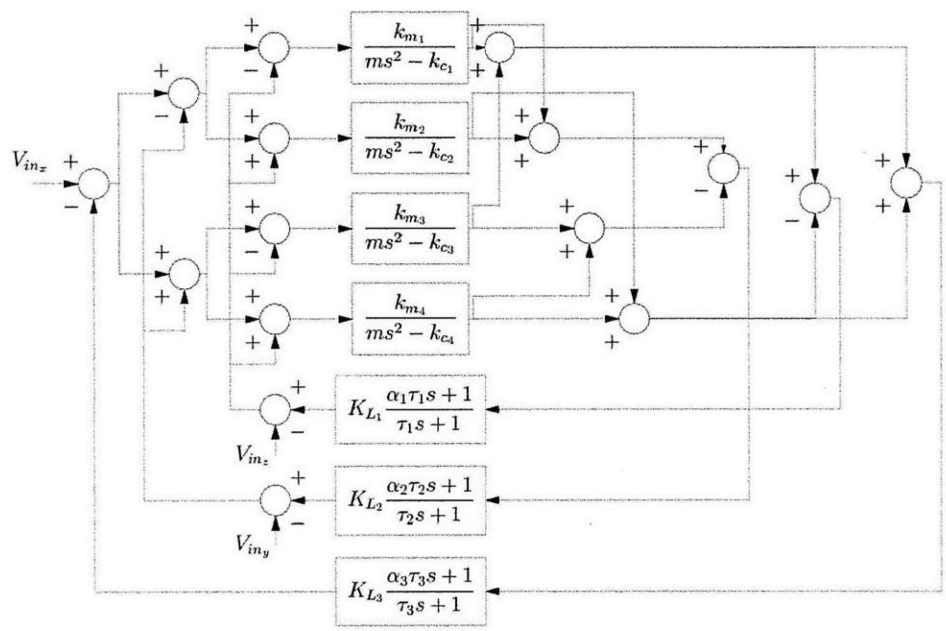


Figure 13: Three Dimensional Compensated Feedback System

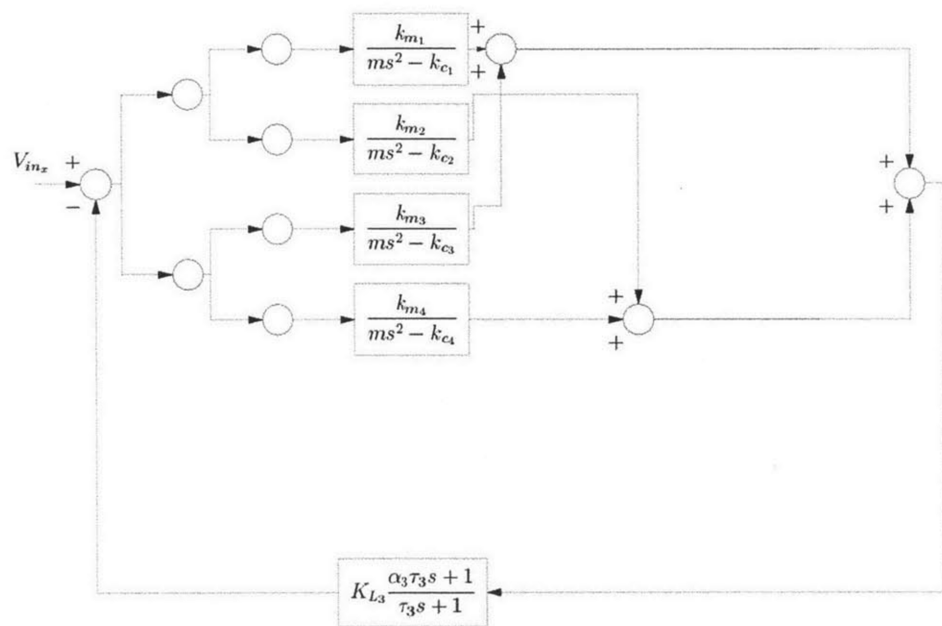


Figure 14: Three Dimensional Compensated Feedback System - X Component

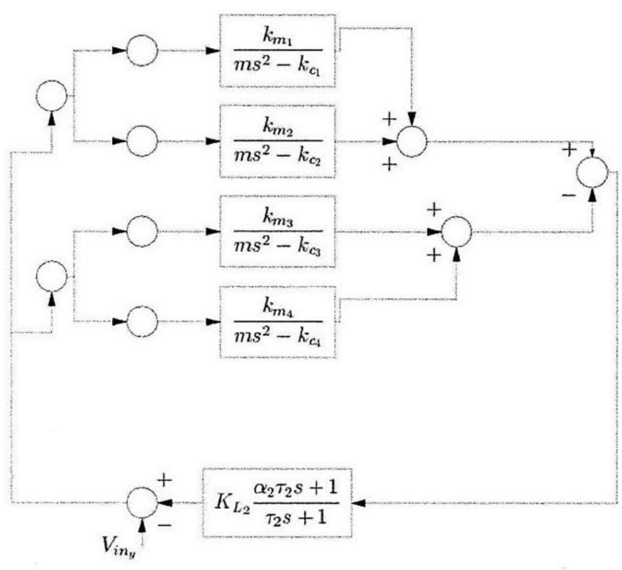


Figure 15: Three Dimensional Compensated Feedback System - Y Component

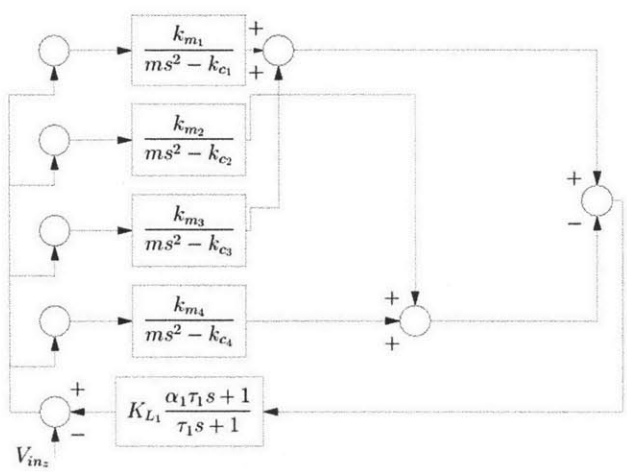


Figure 16: Three Dimensional Compensated Feedback System - Z Component

3 Design

3.1 One Dimensional Levitator Design

In this section, we describe a design for a one dimensional levitation system. This design is based on the 6.302 final project.

3.1.1 Hall Sensor Feedback

In order to stabilize an object in one dimension, we require a way to sense its position. If we were to put a magnet, or make the levitated object of a conducting material, then the magnetic flux through the solenoid would be a function of the position of the object. When the object was close, there would be a lot of magnetic flux. When the object was far away, there would be little. This is a convenient way to measure position, because a Hall sensor is capable of measuring the amount of magnetic flux that passes through it. Figure 17 shows a circuit configuration that uses a Hall sensor to measure the position of the levitated object.

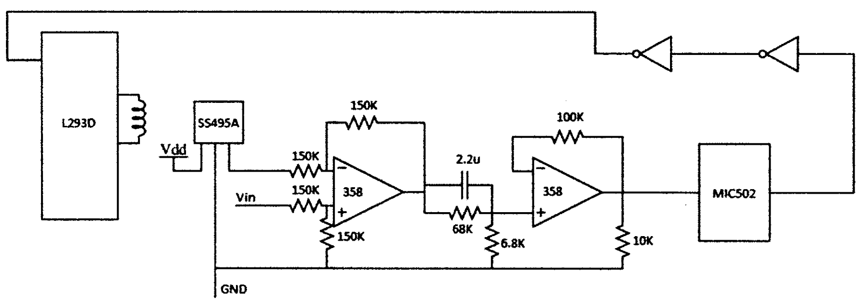


Figure 17: One-Dimensional Levitator Design

The SS495A is the Hall sensor in the circuit of Figure 17. It is placed suggestively close to the inductor, to indicate that it reads the magnetic flux passing through the solenoid. The Hall sensor's output is a voltage. That voltage is subtracted from a DC voltage input, and then passed through a lead compensator, which is drawn in the diagram as a resistor and capacitor in parallel and another resistor in series. The voltage signal is then fed through a gain stage, to add more loop gain to the circuit.

We need a way to drive the solenoid based on this voltage. The L293D is a bridge driver, a chip that is perfect for this job. The chip takes a variable duty cycle square wave on its input, and drives its output in proportion to the duty cycle of the wave. In Figure 17, the L293D drives the inductor.

Unfortunately, we still need a way to convert from a DC voltage to a duty cycle square wave. The MIC502 has exactly that function, and is used to bridge the gap between the compensated sensed voltage and the L293D. The inverters are there to make sure that the square wave is clean and to buffer the signal.

This circuit has a unfortunate problem. Because the inductor is being controlled with voltage, instead of with current as our analysis went in the "Theory" section, there is an additional pole from the inductor at its time constant. This makes the circuit harder to control. However, it is still fine for the purposes of one dimensional control. The additional pole is unacceptable in the two dimensional case, however, because it adds extra jitter to the levitated object, which is much worse when two directions of force are inherently unstable. We will show how to eliminate this problem in the next section.

3.2 Two Dimensional Levitator Design

In this section we describe a design for a two dimensional levitation system. The system overcomes the challenge of sensing vertical position over a range of horizontal positions by using infrared LEDs in the feedback path. In order to fully realize a two dimensional levitator, we need ways of measuring the vertical component, the horizontal component, and switching between pairs of solenoids.

3.2.1 Sensing the Vertical Component

In order to create the feedback system outlined earlier, we need a way of sensing the vertical component over a range of different horizontal positions. One way to do this is to use an infrared LED and two detectors. When the object blocks the light from the LED, the detectors are able to sense the change in light. The difference in the light levels will be approximately linear with position, making an appropriate position sensor. Given that the distance between two separate solenoids is small, the difference in the light sensed by the detectors over different horizontal positions should be small, making this system a useful vertical position sensor over a range of horizontal positions.

Figure 18 shows a possible implementation. It uses two detectors, one to sense when the object is far away from the solenoid, and the other to detect when it's near. The circuit then uses the difference between the two signals to drive a MOSFET. Driving a MOSFET allow us to control the current in the inductor, which gets rid of the inductor pole. The system still has a right half plane pole, however, so compensation is necessary.

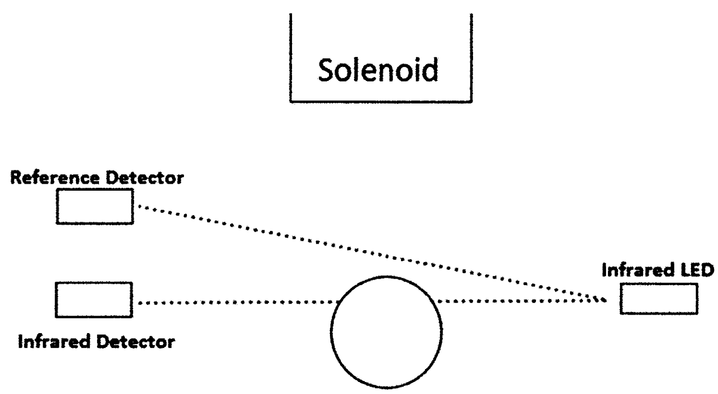


Figure 18: Vertical Component Sensing - Method

Figure 19 shows a possible circuit implementation of the system. The first LM358 takes the difference between the two signals. The result is fed through a compensation and a gain stage, and then a second LM358 drives the MOSFET to deliver current to the inductor. If the difference is large, the inductor will have more current and pull more strongly. If the difference is small, the inductor will have less current and pull more weakly. This gives us a way to deliver the vertical component to the solenoid.

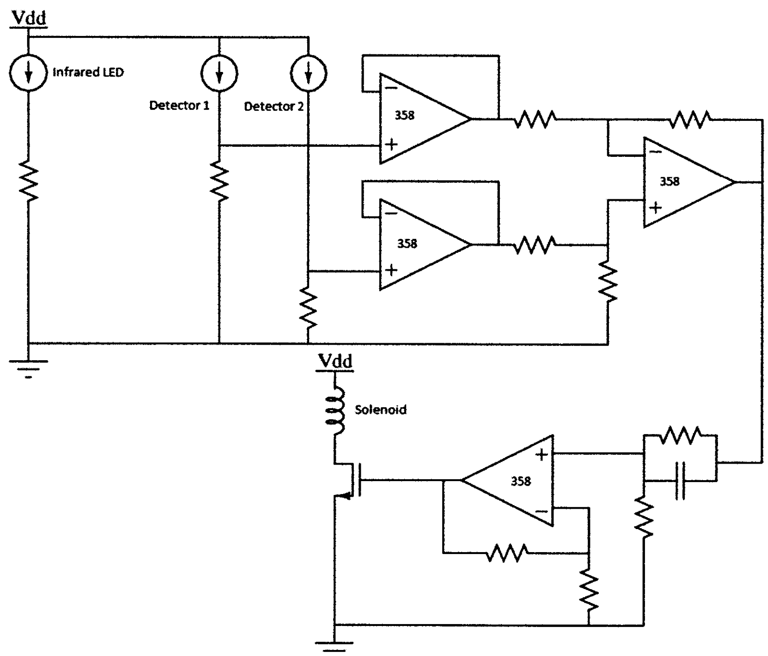


Figure 19: Vertical Component Sensing - Circuit

At this point we can stabilize the vertical component. We need to find a way to sense and stabilize the horizontal component.

3.2.2 Sensing the Horizontal Component

To sense the horizontal component, we use two Hall sensors, each attached to the bottom of a solenoid as in section (3.1).

A single Hall sensor's output depends roughly on horizontal position. If we consider the two solenoids that are switched on, the two Hall sensors that correspond to these solenoids are the ones we will use to measure position. Figure 20 shows a circuit that follows the plan of Figure 10. It measures the two horizontal positions (the Hall sensor voltages), takes their difference, feeds it through a compensator, and then produces an output voltage that can be used to drive the two solenoids differentially. We want to drive the solenoids differentially to produce movement of the object.

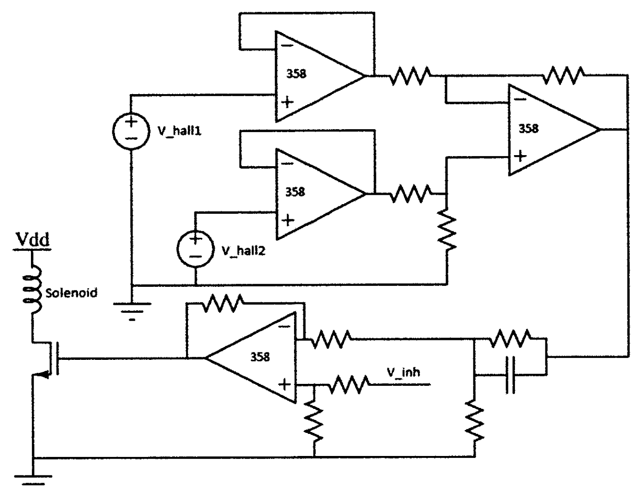


Figure 20: Horizontal Component Sensing - Circuit

We now have good methods for sensing both the horizontal and vertical positions. However, in order to follow the feedback scheme outlined in Figure 8, we need a way to combine the signals to drive the two solenoids. Figure 21 shows such a circuit.

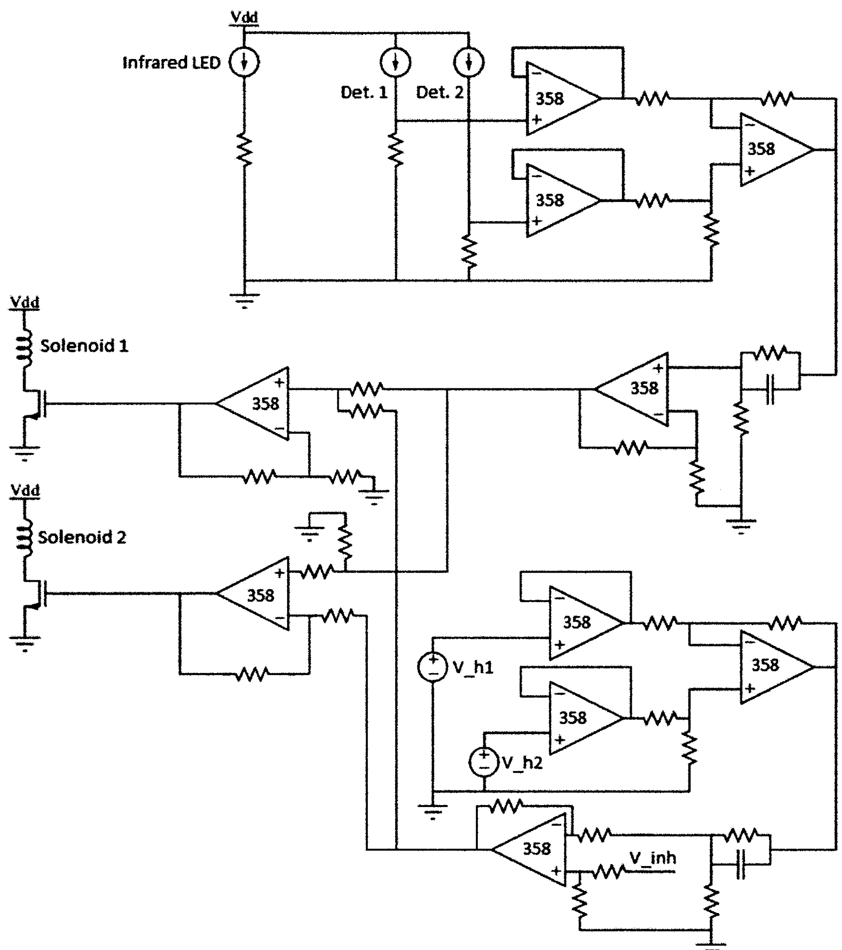


Figure 21: Two Dimensional Levitator Control Circuit

V_{in_h}	S_1	S_2	S_3	S_4
$0 - 2.5$	1	0	1	0
$2.5 - 5$	0	1	0	1

Table 1: Solenoid Switching Logic

This circuit combines the two circuit halves from figures 19 and 20, but uses their output differently. The vertical output is used to drive both equally, in order to oppose gravity, while the horizontal output drives the outputs differentially. As a result, two different op-amp circuits are needed to add together the signals. Additionally, an input signal for the horizontal component is added to the sensor output, so that the feedback can stabilize the object over a range of horizontal positions.

Figure 21 corresponds directly to Figure 8. By analogy, as long as the compensators are designed correctly, the output will be fully stable.

There is still one problem to be solved for this circuit. We need a good way to switch between different pairs of solenoids.

3.2.3 Switching Between Solenoids

The vertical component input is the reference detector voltage, which acts as a non-adjustable input. The horizontal component input, however, is an adjustable voltage. We would like to engineer our system so that changing this voltage from 0 to 5V corresponds to the full range of horizontal positions. We have up to this point designed a system that works given two solenoids and an input range from 0 to +V, where V is an unspecified value that corresponds to the the object being at X_S. So as long as we can switch in the correct solenoids, switch in the correct sensors, and offset the voltage appropriately depending on the region the object resides in, our system should be fully stable. To accomplish that goal, we use a switching network and a microcontroller.

As an example, consider three solenoids. The switch arrangements for the solenoids and sensors are shown in figures 22 and 23. We want to use a microcontroller to turn the switches in these circuit on or off depending on which region the object is in. Assuming that a V_{in_h} of 0 produces a fully left justified object, and a V_{in_h} produces a fully right justified object, we want to add in an additional offset to the voltage signal, and switch which inputs are being used midway through the input voltage range. Therefore, The switching logic should be

This will switch the correct inputs to the feedback network, and the correct solenoids to the output MOSFETs.

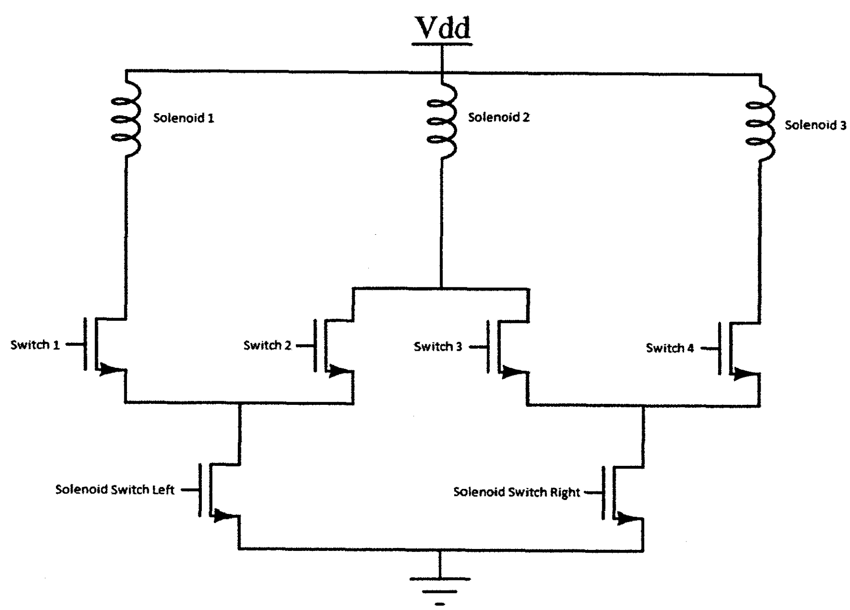


Figure 22: Solenoids Switching - Solenoids

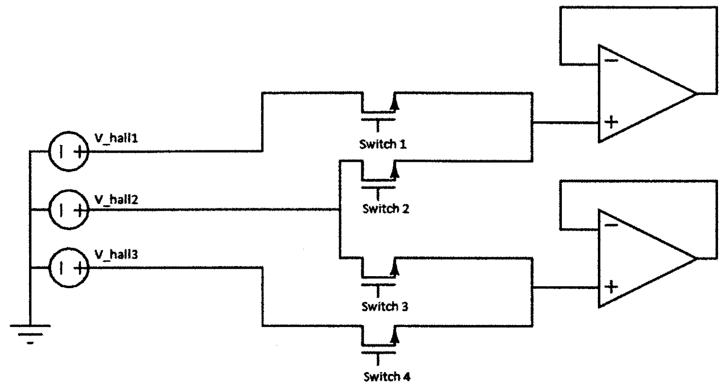


Figure 23: Solenoid Switching - Sensors

However, as mentioned before, we also need to make sure that V_{in_h} is offset by the correct value for each region. When V_{in_h} becomes 2.5V, we need to add in an offset of -2.5V to the input of the feedback system, to offset the input to the feedback system. Otherwise, the object would not be left justified. Instead, the feedback system, having switched from the solenoid 1/solenoid 2 pair to the solenoid 2/solenoid 3 pair, would assume that we wanted the object underneath solenoid 3, which would make the object unstable. Figure 24 shows how the offset voltage has to change. When V_{in_h} becomes large enough, the switch logic shown in Table 1 toggles the switches and different offset voltages are sent to the feedback system's input.

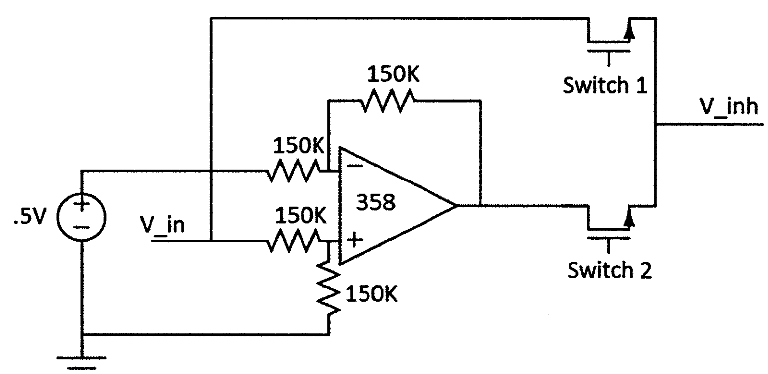


Figure 24: Solenoid Switching - V_{in_h}

The microcontroller is not shown, but produces the values in Table 1, taking V_{in} as input.

In order to show how all these pieces fit together, the next section includes a full circuit of a two dimensional levitator with three solenoids.

3.3 Three Solenoid Example

All the elements, excepting the microcontroller, are shown connected in Figure 25.

All of the subcircuits in this diagram are taken from the preceding sections, and therefore function correctly. If we use the switching logic shown in Table 1, we have produced a design for a stable, two dimensional levitator. In the next section, we will go over the design to implement the three dimensional levitator.

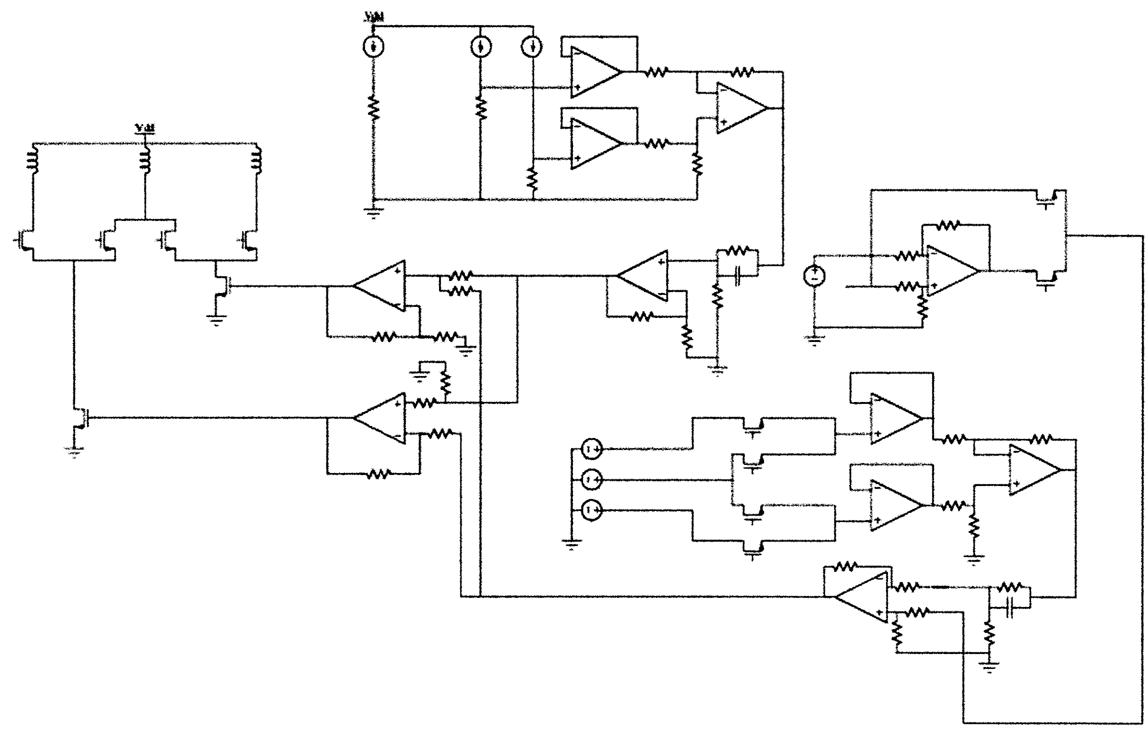


Figure 25: A Two-Dimensional Levitator

3.4 Three Dimensional Levitation Design

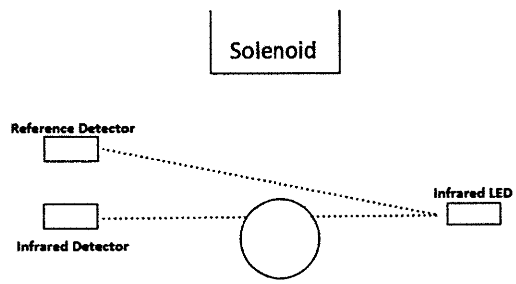
In order to build a three dimensional levitator, we need to develop a way to sense all three dimensions at once, combine the information, and use that to control the solenoids. Given the previous work, we need a separate way to sense the third dimension. The author tested a number of different chips, in order to find one sensitive enough to use on this project. Unfortunately, the author was not able to find one. Given that the author only wishes to present designs which are sure to work, this thesis instead presents the design of a "2.5-D" levitator: a levitator that moves the object along grid lines. This setup explores most of the difficulties inherent in the three dimensional design, while building on the work in the two dimensional design section. In order to build such a levitator, we need a way to sense the components, switch the correct sensors to the inputs, switch the correct solenoids to the output, and offset the input voltages by the appropriate amounts. In the three dimensional case, we have two voltage inputs to our system. However, we will start by sensing the X component.

3.4.1 Sensing the X Component

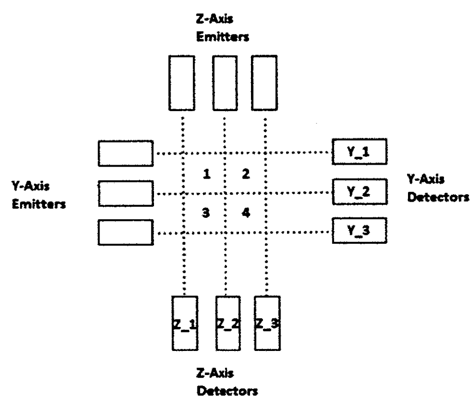
In order to sense the X component, we refer to the feedback diagram from figures 13 and 14. While Figure 13 shows the overall system, Figure 14 shows the parts that need to be implemented just for the X component sensor. As in the two dimensional case, we will use the output from two infrared emitter/detector pairs. Figure 26 shows the method.

In order to sense the height, we use multiple pairs of infrared emitter/detectors, as shown in Figure 26(b) for nine solenoids. The object will always be blocking the light from one emitter to detector, so we will be able to detect the height of the object. The object may be in one or two sensors pairs at a time. As a result, we will use one pair at a time. In order to determine which pair to use, we construct a separate circuit to detect which pair sees the largest difference between its reference detector and infrared detector and use that pair. We do this because the pair with the largest difference has the object closest to it, so we can be guaranteed that the object is in the path between the detector and emitter of the pair. This setup allows us to use the same voltage reference for height as before, namely the built in reference detector voltages. It also means that our circuit implementation differs slightly from Figure 14. However, while Figure 14 would work, it is more practical to implement this system in the manner described.

In any case, given the two sensors to detect the object, Figure 27 shows how to construct a circuit to give the correct output voltage. It is essentially the same as in the two dimensional case, because we are only using two detectors at a time.



(a) Individual Pair



(b) Overall Array

Figure 26: X Component Sensing - Method

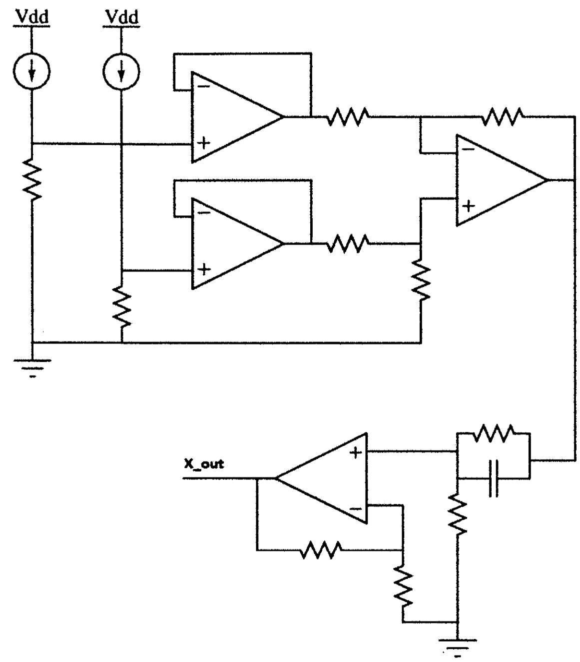


Figure 27: X Component Sensing - Circuit

In order to ensure that we only are using one emitter/detector pair at once, we will design a circuit to switch in the correct sensor pair. This will be detailed in a later section.

3.4.2 Sensing the Y Component

We also need to sense the Y component. Sensing the Y component is analogous to sensing the horizontal position in the two dimensional case. Figure 15 shows the feedback system for the three dimensional case. To sense our position, we use the hall sensors placed underneath each solenoid, in groups of four. In the nine solenoid case, we would have four area designations. Figure 26(b) shows these four areas. While on the lines near each of these grids, we will use the difference between the sums of pairs of solenoids in these groups to determine position. This maintains a constant sensed Y-component while the Z component changes. As in the two dimensional case, if we assume that Y_{ref} is fully left justified when it is 0V and fully right justified when it is 5V, then we need to add in an offset voltage depending on how large Y_{ref} is. However, the basic principle remains.

Referring to Figure 15, and assuming Y_{ref} is set up correctly, we obtain Figure 28.

This circuit implements the diagram from Figure 15. It adds together the signals in an appropriate way as to generate a Y-component (see Figure 11(b) for the directions and corresponding solenoid numbering), feeds the signal through a compensator, and generates an output by subtracting the result from the input voltage.

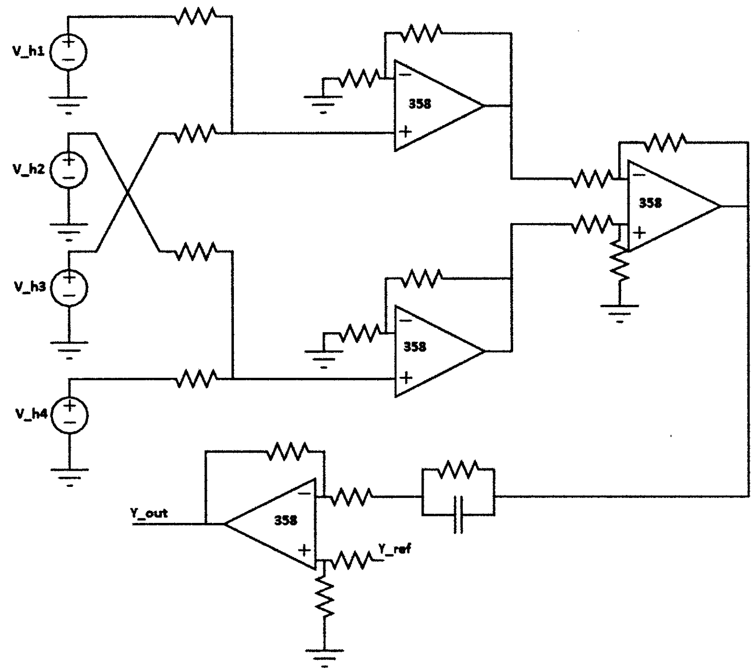


Figure 28: Y Component Sensing - Circuit

V_{hi} corresponds to the Hall sensor on solenoid i, as numbered in Figure 11(b), $i \in \{1, 2, 3, 4\}$.

3.4.3 Sensing the Z Component

To sense the Z component, we follow the same plan as we did sensing the Y component. However, we need to add together different signals (see Figure 11(b)). This creates the circuit of Figure 29.

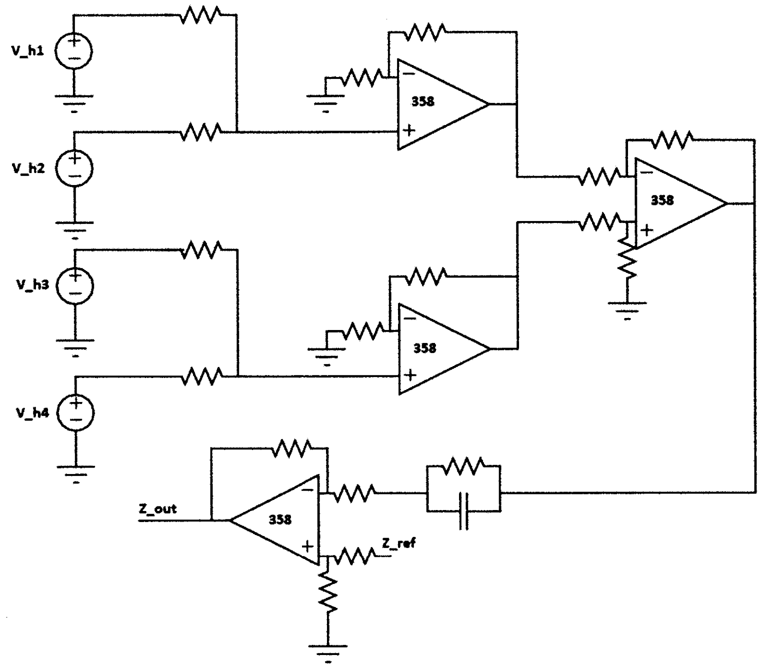


Figure 29: Z Component Sensing - Circuit

At this point, if we are given the correct input signals and are able to switch the correct components into place, the object should be stable in three dimensions and be able to move along the gridlines. In order to complete the project, we need to realize circuits that switch in the correct sensors, solenoids, and offset voltages.

3.4.4 X Component Sensor Switch

When sensing the X component, we need to make sure that we're reading the correct sensors. In order to ensure this, we will design a circuit to switch in the correct pair, based on which one has the largest signal. In the nine solenoids case, we take the six infrared detectors, as defined in Figure 26(b), and use them as inputs to a microcontroller. The microcontroller runs code to decide which one is the largest, and then selects that pair as the one that is used to balance the vertical height of the object. This is to pick the correct reference detector, (term defined in Figure 26(a)), to use to balance the object. Figure 30 shows this circuit for a nine inductor setup.dif

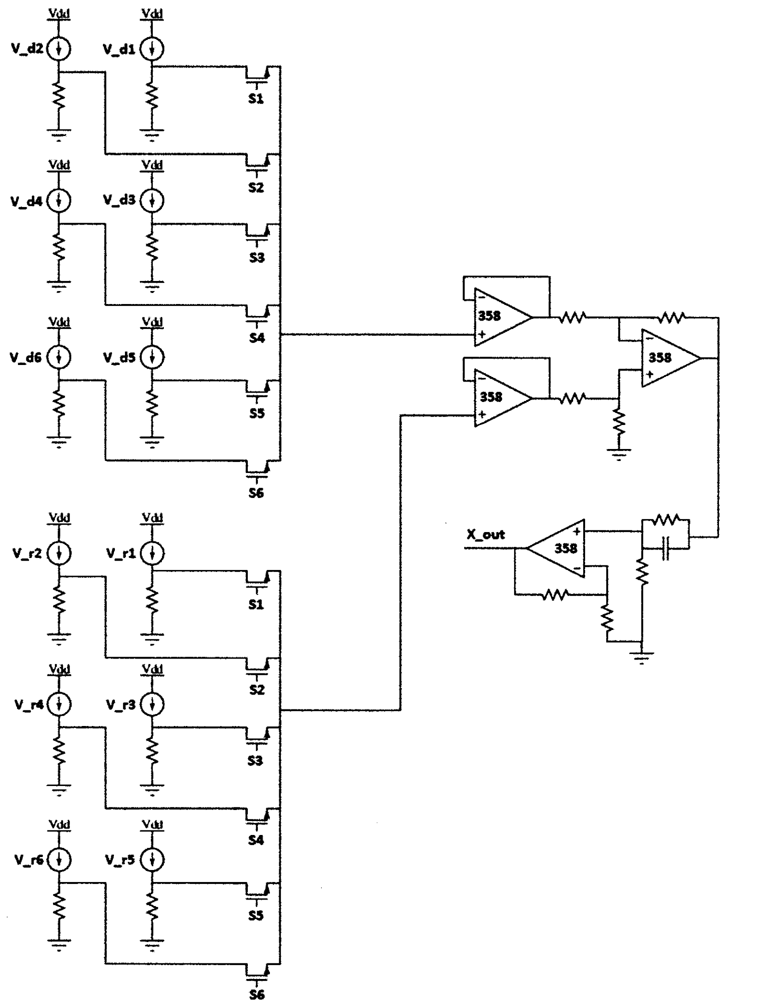


Figure 30: X Component Switching Circuit

3.4.5 Y and Z Component Sensor Switch

The Y component sensor also needs to make sure it's using the correct sensors for input. We use a similar approach as to the two dimensional case to switch the correct sensors to the input. For the nine inductor case, there will be two regions for each component, and the logic we follow is the logic table shown in Table 2.

Y_{ref}	S_1	S_2
$0 - 2.5$	1	0
$2.5 - 5$	0	1

Table 2: Y Component Switching Logic

A similar table exists for the Z component.

Z_{ref}	S_3	S_4
$0 - 2.5$	1	0
$2.5 - 5$	0	1

Table 3: Z Component Switching Logic

Neither of these tables are especially complex. The two regions of Y and Z variation mean that there exist four regions that the object can levitate within, as shown in Figure 26(b). These tables simply assign switches to the different regions. With these tables, we can combine the two circuits to produce Figure 31, which shows how to arrange the switch so that the logic is satisfied.

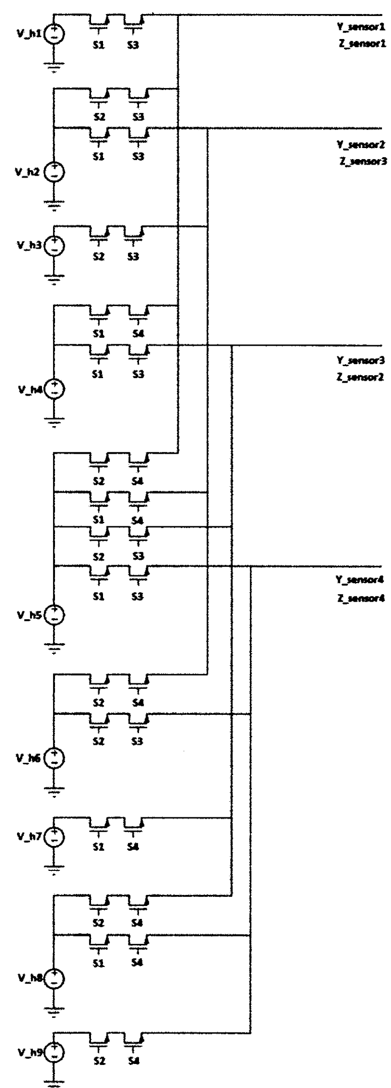


Figure 31: Y and Z Component Switching Circuit

This circuits switch the correct sensor the output based on where the object is located. And the Y and Z components use the same sensors, but in a different order, so we can use the same four outputs for both sets of inputs.

3.4.6 Switching between Solenoids

In addition to switching the correct sensors to the circuit, we need to switch the correct solenoids to the output. The circuit used is a direct extension of the one used for the two dimensional case. If the solenoids are numbered according to Figure 11(b), then the circuit is shown in Figure 32.

With the correct solenoids switched to the output and the correct sensors switched to the input, the only remaining circuit to produce is the circuit that generates the correct offset for each horizontal direction.

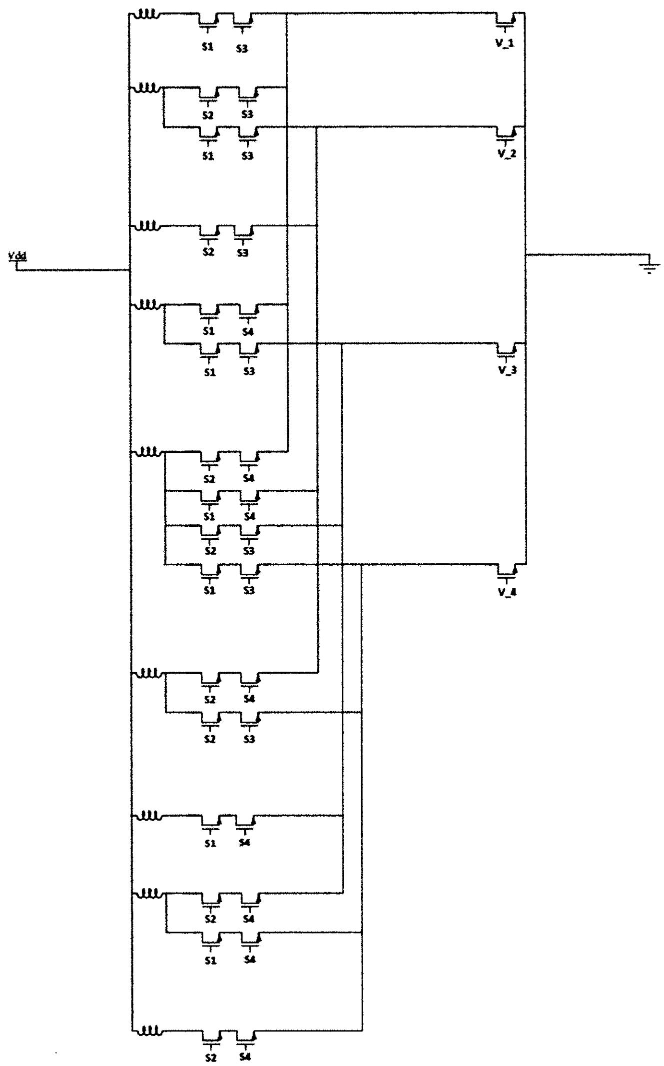


Figure 32: Solenoid Switching Circuit

3.4.7 Adding in the Offset

Much like the two dimensional case, we need to switch in the correct offset to the input voltage to make sure that the input voltage is zero when it switches to a new region. Like most of the circuits in this section, the circuit is a generalization of the two dimensional circuit, and is shown below.

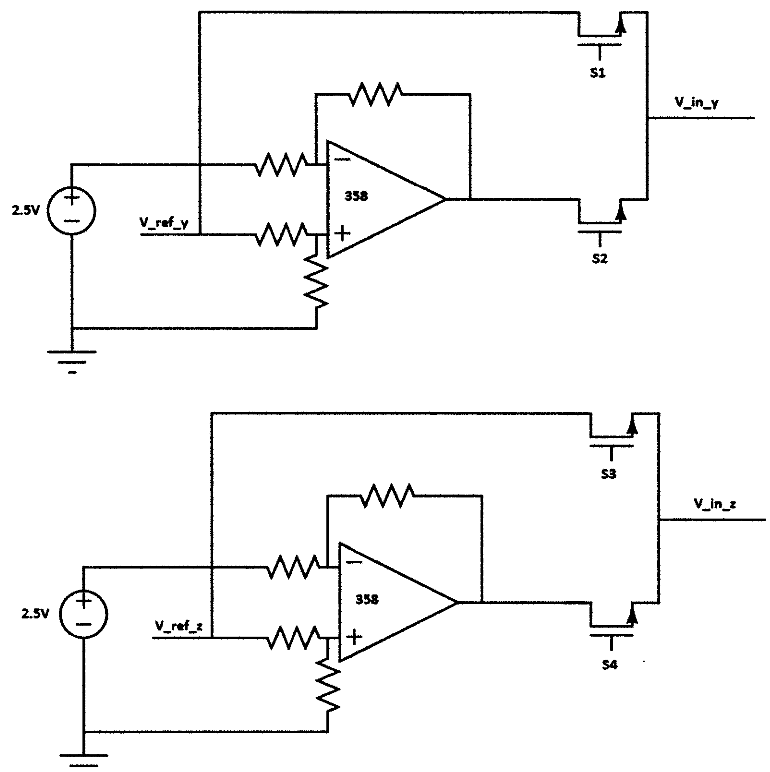


Figure 33: Solenoid Switching Circuit

3.4.8 Nine Inductor Example

At this point, we have created all of the subcircuits of a "2.5-D" levitator, with the exception of one. There is a need to add together the signals, as in the upper left of Figure 13. However, this is not a difficult circuit, and the function is well understood, so it is represented in the overall pictures with a block labeled "Adder". The rest of Figure 34 combines the subcircuits from the previous sections to create a full design of the "2.5-D" levitator, and completes the design portion of this thesis.

In the next section, we implement a modular cell of the two dimensional levitator.

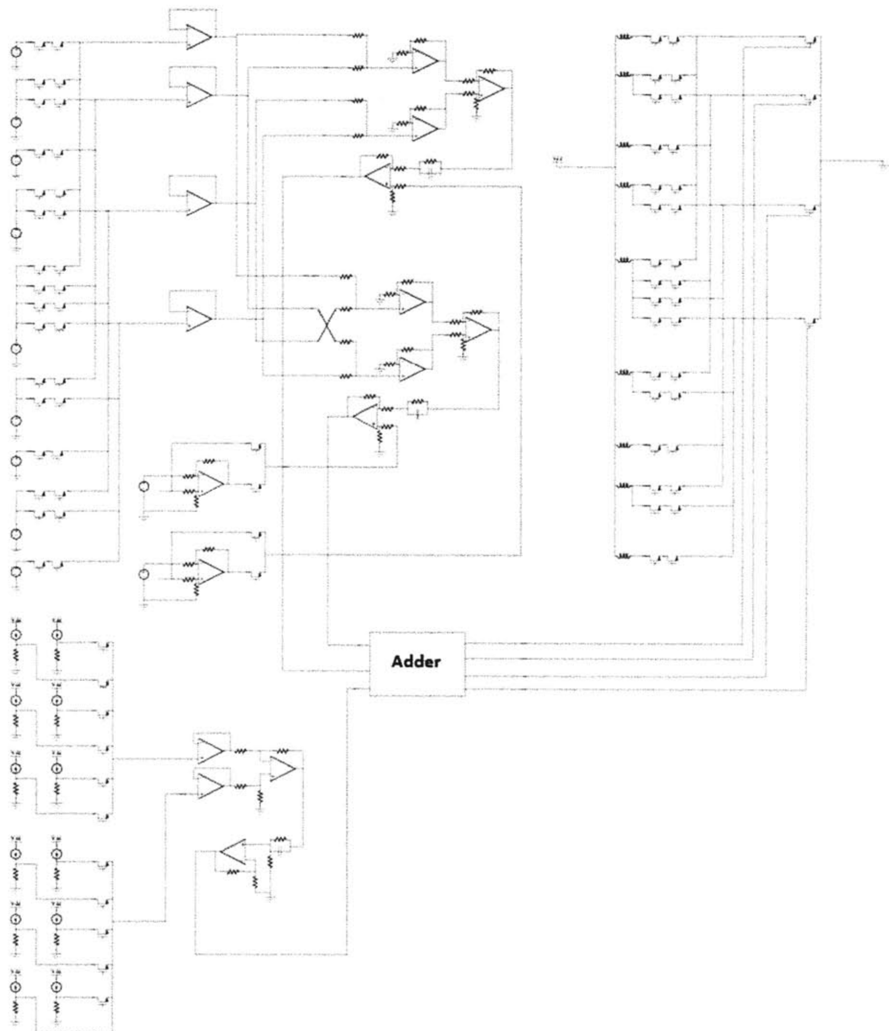


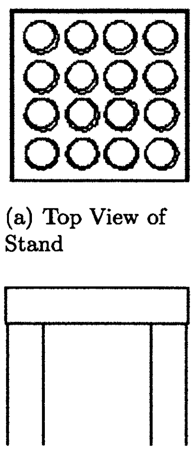
Figure 34: Solenoid Switching Circuit

4 Implementation

A modular cell of the two dimensional levitator was implemented. This section deals with the details of the implementation and the results from the physical device.

4.1 Building the Stand

The stand was built out of wood, using a standard and simple design. Figure 35 shows the side and top views for the stand.

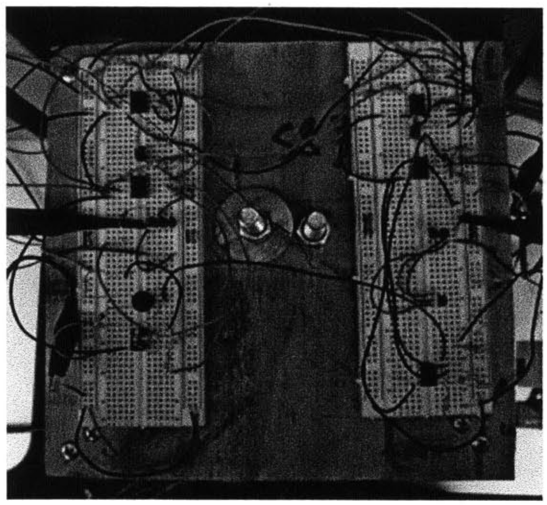


(a) Top View of Stand

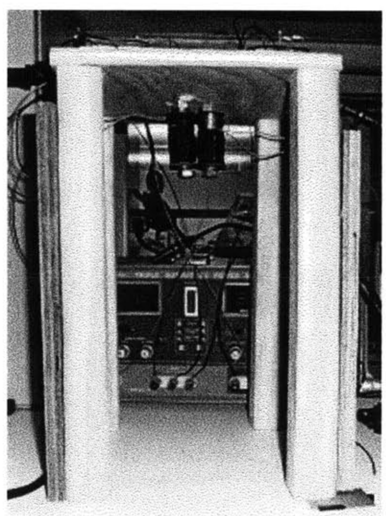
(b) Side View of Stand

Figure 35: Stand Design

Holes are drilled into the top of a piece of plywood to provide appropriate mounting holes for the solenoids. The stand is on legs to allow plenty of space below the solenoids for the object to levitate. And there are pieces of plywood on the sides to provide appropriate mounting for the infrared emitters and detectors. An actual picture of the setup is shown in Figure 36.



(a) Top View of Stand



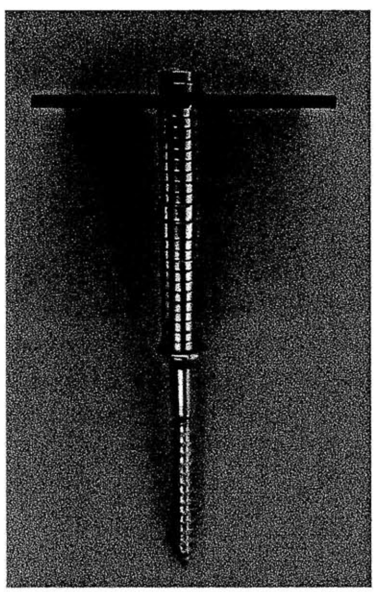
(b) Side View of Stand

Figure 36: Stand Implementation

4.2 Building the Levitated Object

The levitated object was chosen to be a metal screw. Once the circuit was fully constructed, a flat, wide, and weak magnet was added to the top to provide integration of the magnetic fields across the object, which helps to increase stability. Finally, magnets were added until the object levitated at a height appropriate to passing the object from one solenoid to the other. A picture of the object is shown below.

(a) Top View of Levitated Object



(b) Side View of Levitated Object

Figure 37: Object Implementation

4.3 Building the Circuit

The circuit, not being particularly sensitive to variations of the signal, was built on a grounded breadboard. The circuit topology is a slightly modified version of the circuit presented in Section 3.2. The values used are listed with the topology in Figure 38. Due to the low range of the Hall sensors, the Hall sensors were not included in this version of the circuit. Future versions of the circuit will need a larger range Hall sensor in order to provide full feedback.

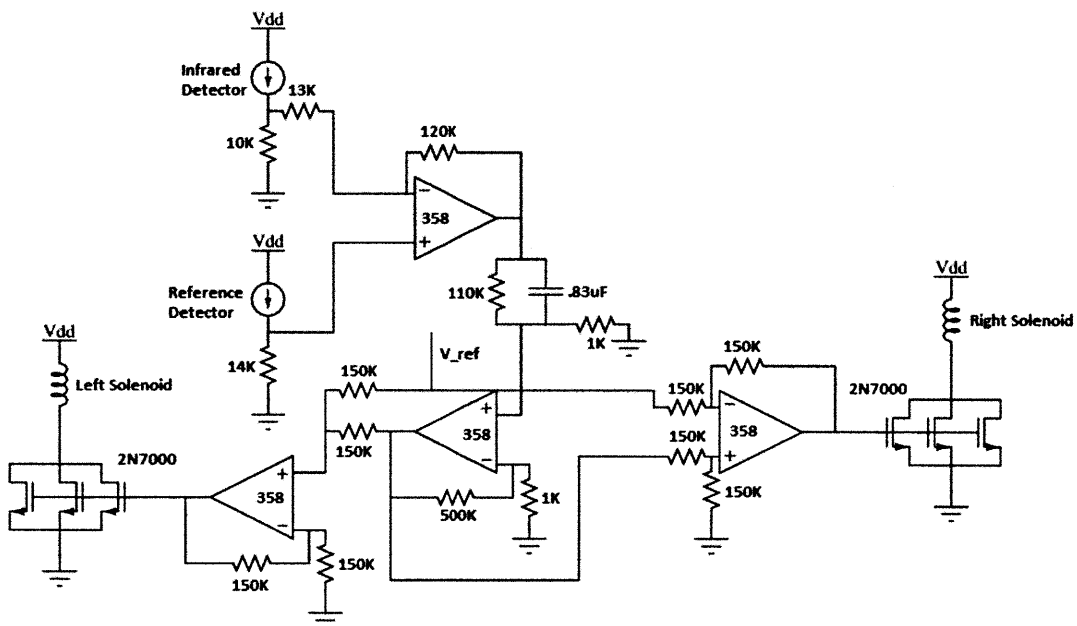


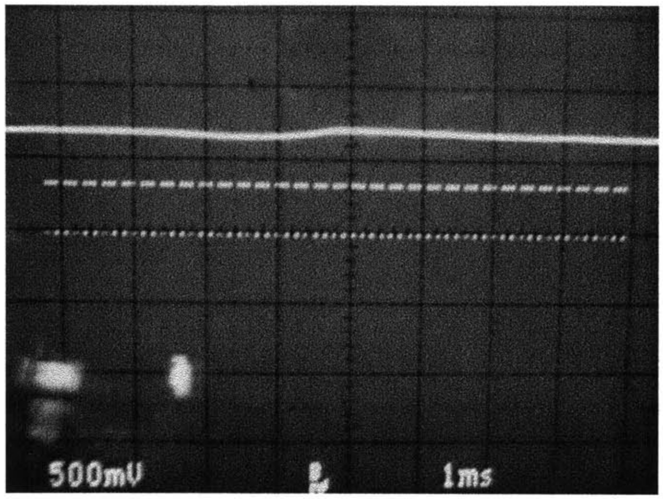
Figure 38: The Built Circuit

This circuit also does not provide the switching circuitry designed earlier, but does provide values for a two dimensional levitator. Its purpose it to prove that passing from one solenoid to another by changing the magnetic fields is not impossible.

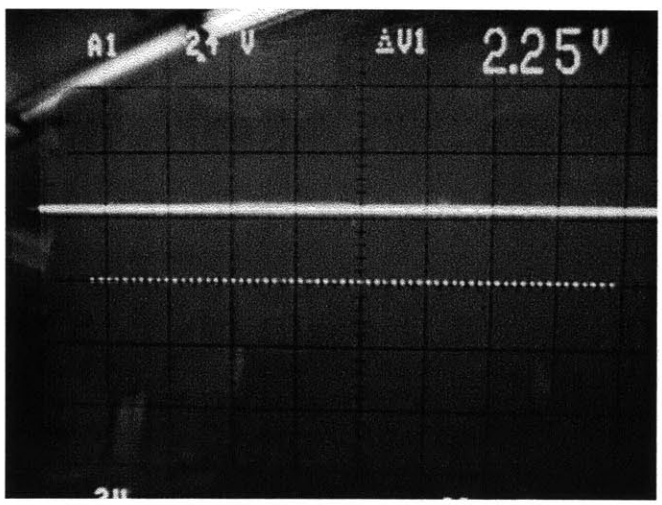
4.4 Results

The circuit outlined in the previous section was build and was able to levitate the previously mentioned object and move it between the two solenoids. Of interest to the designed is the amount of ripple and stability. The author measured the ripple on both voltage inputs to the 2N7000s at three different points: When the object was left justified, when it was centered, and when it was right justified. Figure 39 shows the ripple measurements for the left justified case.

The graphs show the measurements taken in a solid line. These measurements are the voltage on the gate of the MOSFET driving the left solenoid and the voltage on the gate of hte MOSFET driving the right solenoid. The dashed lines are measurement lines taken from the oscilloscope. The dashed lines are not part of the signal and should be ignored.



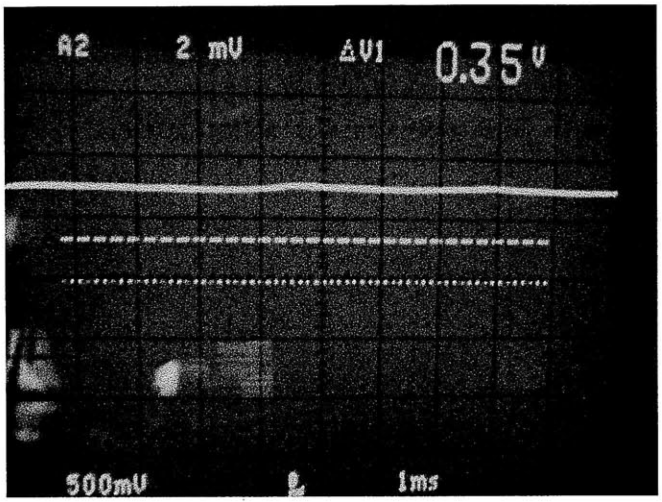
(a) Left Solenoid



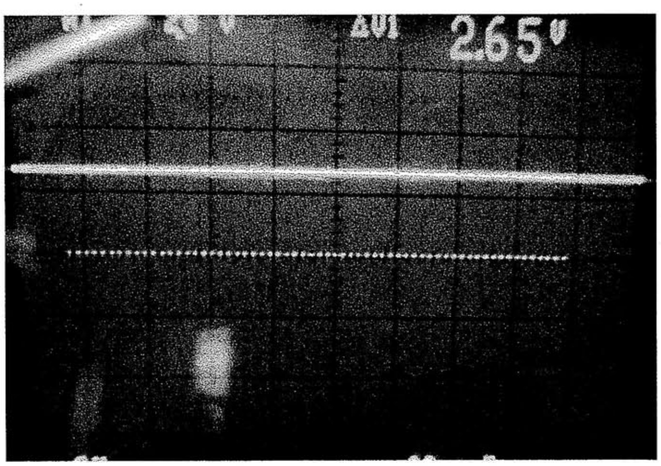
(b) Right Solenoid

Figure 39: Left Justification

These results shows that the ripple is very small in both cases, but it is larger on the left solenoid. Figure 40 shows the measurements from the center.



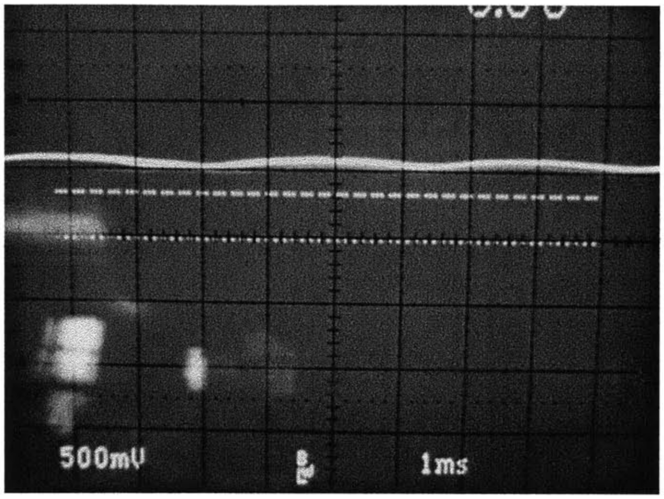
(a) Left Solenoid



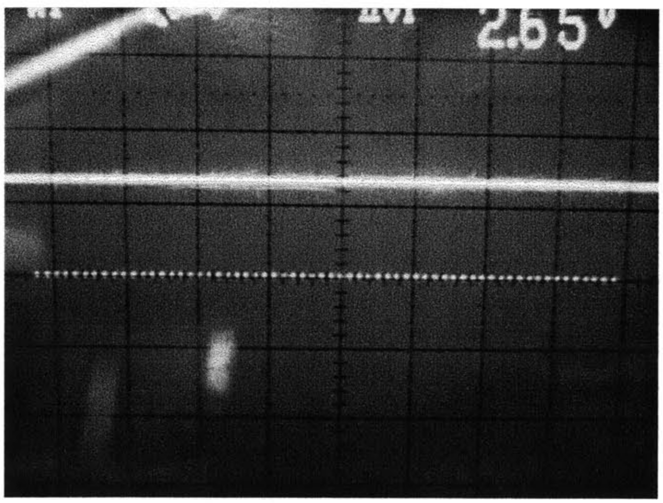
(b) Right Solenoid

Figure 40: Center Justification

The ripple here is small for both solenoids, but larger for the left solenoid. Figure 41 shows the ripple for the right justified case.



(a) Left Solenoid



(b) Right Solenoid

Figure 41: Right Justification

This ripple is still larger on the left solenoid. For some reason, the left solenoid is less stable than the right one. This is due to the differences in the plants of the two systems, which use the same compensation as their feedback function. However, this is not a large concern, because a stable output is maintained in all three cases. The object is levitated and can be moved between the two solenoids. This means that the modular cell of the two dimensional levitator works as intended.

5 Conclusion and Further Work

In this thesis, we have designed systems for one, two, and three dimensional levitating systems. Those systems had both feedback systems created to assess their stability and circuit implementations to show their feasibility. After all of this, we constructed a cell of the two dimensional system, to show that our scheme was possible.

Further work would include building the full two dimensional system for a small number of solenoids, as in the example of a three solenoid, two dimensional levitator explained earlier. In addition, building the nine solenoid, three dimensional levitator (actually a "2.5-D" levitator) is very possible given the work done here. Other work would include finding a way to accurately sense the third dimension, designing the same systems with a planar object as opposed to a point mass, and building a system that moves the solenoid as opposed to changing the fields.

This thesis attempts to unravel the mysteries of magnetic levitation. This thesis provides analysis of magnetic levitation systems in multiple dimensions, and the state equations developed at the beginning of this section have applications to many problems besides the one implemented within.

In the end, we have shown that three dimensional magnetic levitation is not only theoretically possible, but a practical possibility as well. Ernshaw's law may prevent us from achieving levitation with static magnetic fields, but using a little ingenuity, there is much we can accomplish with classical feedback and appropriate circuit topologies.

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