Diffusion Driven Object Propulsion in Density Stratified Fluids

by

Conor Lenahan

Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degree of

Bachelor of Science in Engineering as Recommended by the Department of Mechanical Engineering

at the

Massachusetts Institute of Technology

February 2009

Signature of Author...
Department of Mechanical Engineering
January 30, 2009

Certified by..
Thomas Peacock
Atlantic Richfield Career Development Professor of Mechanical. Engineering
Thesis Advisor

Accepted by..
John H. Lienhard
Professor of Mechanical Engineering
Executive Officer, Department of Mechanical Engineering

Diffusion Driven Object Propulsion in Density Stratified Fluids

by

Conor Lenahan

Submitted to the Department of Mechanical Engineering on January 30, 2009 in Partial Fulfillment of the Requirements for the Degree of Bachelor of Science in Engineering as Recommended by the Department of Mechanical Engineering

Abstract

An experimental study was conducted in order to verify the appropriateness of a two dimensional model of the flow creating diffusion driven object propulsion in density stratified fluids. Initial flow field experiments studying the phenomena that drives the so called diffusion fish indicated that there may be an unaccounted for reliance on the width of the diffusion fish in relation to the width of the tank it moved in. Further study examining a full array of wide fish revealed that there is a nonlinear dependence on the width of the fish, and that for a two-dimensional flow field model to be accurately verified experimentally further experiments must take place to quantify and account for this discrepancy.

Thesis Advisor: Thomas Peacock
Title: Associate Professor of Mechanical Engineering

Summary: A unique new propulsion mechanism has been discovered whereby, in a stratified fluid, the geometry of a specially shaped object induces a diffusion driven flow that propels the object without the use of moving parts. Existing dimensional analysis is based around the assumption that this is primarily a two dimensional phenomena, but initial flow field experiments indicated a strong three-dimensional effect. Further experimentation clearly shows this effect and suggests that the current model only applies to objects of a certain width. This data serves as a strong foundation for the development of a theory that can accurately describe this propulsion mechanism, which in turn will have wide ranging applications in geophysical and biological flows.

Introduction:

The geometry of the fluid motion caused by diffusion driven flow that develops around these objects is not understood. The specially shaped object, referred to as a "diffusion fish" and shown in Fig. 1, is a wedge shaped object which is designed to float stably in fluid at a density somewhere between that of fresh water and fully saturated salt water. The sloped surfaces on the top and bottom of the fish induce a diffusion driven flow of fluid moving from the tip to the base of the wedge. The diffusion fish was conceived, created and experimentally verified by Prof. Peacock.

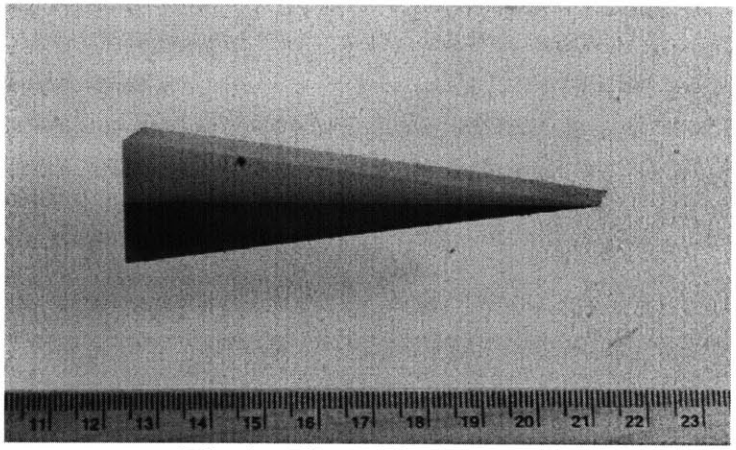

Fig. 1 – The Diffusion Fish

Initial experiments measured the diffusion fish velocity for varied lengths and angles. To be clear, the length of the fish is measured from the base of the triangle to its tip, the width is the distance the wedge is "projected", and the angle is defined as half the angle of the tip. The diffusion fish velocities obtained from these tests matched theory developed by Peacock et al (1) (2) in studying the diffusion driven flow over stationary surfaces in a salt water stratification. Following the collection of this data it was attempted to define the velocity profile of the flow field around the diffusion fish using particle-tracking methods. It was thought that, to minimize the appearance of stray three-dimensional effects, the widest possible diffusion fish that could easily move in the tank should be used. The initial data was found using 5cm wide fish, and the new fish were 15cm wide (in a tank that is 20cm wide).

In these tests it was found that this wider diffusion fish moved much slower than its skinnier counterpart. The geometry of fluid motion around the fish of different widths was also found to vary. It appeared that the proximity of the tank wall to the fish affected the geometry of the flow. It became clear that experiments varying the width of the diffusion fish are needed, with special emphasis on the size of the fish as compared to the size of the tank they are tested in. This showed a clear need for analysis of the movement of a full set of wide fish. These fish were constructed and tested to compare speeds to skinny fish of similar length and angle.

The ultimate goal of this work is to develop a model that describes the motion of both the diffusion fish and the flow around it. Isolating a case that is perturbed by experimental conditions as little as possible it will make development of this model much simpler. The data found in the course of these experiments lays the ground work for understanding the importance of the width of the fish relative to the width of the tank it is moving in, and will allow for isolation of an appropriate case for flow field analysis that can test the two dimensional predictions for its formation.

Background:

Diffusion driven flow is an important mechanism for fluid motion in density-stratified fluids. This mechanism was first experimentally observed by O. M. Phillips at John Hopkins (3). He discovered that a tilted surface in a stratified fluid will generate a flow up the surface. In a stable density stratified fluid, fluid particles of a given density are kept on a horizontal isopycnal (line of constant density) by a balance between gravity forces pulling downward and diffusive forces pointing upward. These isopycnals must meet physical barriers in the fluid at a ninety degree angle as a result of there being a no flux condition across a physical barrier. Therefore, if the barrier is sloped, the density gradient must bend down to be parallel to the barrier on the surface of the barrier. Gravity forces and diffusive forces no longer balance each other, and salt begins to diffuse up the barrier, consequently creating a fluid flow as well.

Later work by Professors Peacock and Stocker (1)(2) expands on the theories developed by Phillips. The first paper examines the dependence of the fluid velocity to the slope of the barrier (1). This work showed that for very small angles, approximately 2.8 degrees, the velocity of the flow is maximized. For angles less than this the velocity drops of sharply until it reaches no velocity at zero degrees, and it drops off much less quickly as the angle increases, eventually reaching no velocity at ninety degrees. The second paper examines diffusion driven flow in a fissure (2). It found that the optimal fissure angle varied a great deal with the fissure size and showed just how dependant on geometry diffusion driven flows can be.

Professors Stocker and Roman also carried out initial analysis of predicted velocities of a free floating object using diffusion driven flow as propulsion. These initial calculations, based heavily on the work done previously by Hopkins, found the following relation between velocity and wedge angle

$$U = \frac{\cot^2 \alpha D^2 \gamma \cos\alpha}{4Q}$$

where

$$\gamma^4 = \frac{N^2 \sin^2 \alpha}{2Q}$$

and

$$Q = D \cot \alpha$$

for diffusivity D, buoyancy frequency N and diffusion fish angle alpha. Initial experimental verification (unpublished) was carried out by Professor Peacock and Michael Allshouse, and showed promising results that matched the predicted speed very closely for diffusion fish angles above 5 degrees. These results are purely two-dimensional, for a given density stratification, the speed is determined only by the diffusion fish angle alpha. The two dimensional argument was made expecting that boundary conditions should be sufficiently small as to be able to discount their effect.

Methods:

All experiments were run in a 20cmx40cmx40cm tank made of acrylic. This tank has a port at the bottom corner of one of its sidewalls that allows water to be pumped into it through a sponge "diffuser". In this tank the motions of the fish were clearly visible from all angles. Both types of test required the creation of a specific density stratification and the stable introduction of the diffusion fish, after which the procedure deviated.

The diffusion fish itself is machined from two pieces of polyurethane of dissimilar density. By making the lower half of the wedge a denser material the upper, the fish becomes dynamically stable. This is due to the fact that the buoyant force on a submerged object acts at that objects center of volume, while the gravitational force acts on its center of mass. The higher density lower portion of the diffusion fish simply lowers the center of mass a small amount, placing it directly below the center of volume. As a result, and disturbance in the level of the fish creates a torque that rotates the diffusion fish back to a level position. This ensures that, so long as no bubbles form on its surface, the diffusion fish will always float levelly and not affect the angle of its surfaces relative to the direction of the stratification.

The Density Stratification is created by using a double bucket system to fill a tank from the bottom with water that increases in density linearly with time as the tank is filled. Before the tanks are filled, a MATlab script was used to determine the parameters needed to set up the double bucked system. Using this script it is possible to set up stratification of not only the correct strength, but of the correct range. Each of the diffusion fish are at slightly different densities, and it is important that the stratification created allows the fish to float at an appropriate level in the tank (not too close to the surface or the bottom). Using the calculated parameters, the two buckets are each filled with a specific volume of a specific density salt water. It was found that the water should be left to sit in the buckets for a period of at least one day to allow the water to de-aerate. If not allowed to sit for a proper amount of time bubbles will form during the test and cling to the fish, causing the fish to change its orientation in the water and invalidating the run.

Water is pumped from the low-density bucket directly into the bottom of the experimental tank at a carefully controlled flow rate using a peristaltic pump. Using another peristaltic pump, water is pumped from the high-density bucket to the low-density bucket at half the flow rate of the first pump. Thus, the water being pumped into the experimental tank will increase in density linearly. Once full, the tank is left to sit for at least one day to allow any final aeration to take place

Once the experimental tank is full the stratification is checked. This is accomplished by removing a small amount of liquid (5-15ccs) from the tank at 6-10 depths using a long thin syringe. The density of these samples was found using a digital densitometer. Plotting the density vs depth, a calculation of the final stratification strength could be made by finding the slope of a linear best-fit line to the curve. Visually, this graph would also show if the final stratification was nonlinear due to some error in the filling process.

Once the stratification is created, the diffusion fish is introduced. The diffusion fish is prepared ahead of time for entry by applying a thin layer of polyethylene to it using a spray can. This helps to combat the formation of bubbles on the surface of the diffusion fish, which as discussed above can invalidate a test. A "cage" made of steel rod is lowered into the tank through holes in the lid using a linear motor driver. The lid is carefully lifted, and the diffusion fish is slid into the cage and allowed to fall into the density stratification. The cage orients the fish so that it moves straight across the tank and holds it in place while any currents generated by its insertion die out. After at least 1 hour of waiting for the tank to settle, the linear motor driver lifts the cage out of the tank.

The procedure splits here depending on the test being preformed. A camera above the tank, looking down on the diffusion fish is used to measure the velocity. A Cannon EOS digital SLR was used. Using Cannon's camera software, images were recorded every 5-10 minutes for a duration of 5-48 hours (depending on the expected speed of the diffusion fish). After the completion of the test, the pixel co-ordinate of a corner of the fish was manually found and recorded using Microsoft Paint. The pixel "Length" of the fish was also found manually by the same method. Using the pixel length and the known length of the diffusion fish, a conversion scale was made for each individual run. The data was processed assuming that the diffusion fish traveled in a straight line, and the velocity found by finding the slope of a linear best-fit line of the distance traveled by the fish over time.

Data was discarded if the diffusion fish did not travel in a straight line, ran into a wall or had visible bubble formation on its surface. Generally, bubble formation caused the diffusion fish to not travel in a straight path, and not traveling in a straight path would cause the diffusion fish to run into a wall. In some cases the diffusion fish ran into a wall due to an initial misalignment of the fish as it was released from the cage, though these cases were rare as the test was usually reset if it was noticed that the diffusion fish was not correctly aligned at the outset of the test.

To examine the motion of the fluid around the diffusion fish, a laser particle tracking system is used. A density stratification is created as described previously, with the exception of the addition of a special set of particles. These particles are very small glass spheres of densities ranging from lighter than fresh water to denser that fully saturated salt water, and are specially made to reflect laser light of the wavelength used in this experiment. The particles are added to the low-density bucket once it is initially filled, and settle during the aeration periods both in the bucket and in the experimental tank. A special digital camera made for the laser tracking is used in conjunction with a powerful laser illuminate and track the motion of these particles. The laser shines a sheet of light across the tank, giving a planar slice of the flow. A special computer synchronizes laser pulses and camera recordings to store pictures of this flow, and to then track the motion of the particles in the flow over time.

In order to "sweep" the position of the laser sheet from test to test an 80/20 structure was made that allowed the laser sheet to be moved horizontally and vertically without disrupting the alignment of the mirrors that directed the laser. In order to get the most accurate possible data, the lens on the camera observing the diffusion fish was zoomed in as far as possible, although only approximately 1/5$^{\text{th}}$ the total length of the diffusion fish would be visible at one time. In order to accurately align the multiple tests that had to be taken to observe the entirety of the diffusion fish, the camera that observed the was mounted on a linear motor driver that allowed the camera to be moved precise distances along the length of the fish.

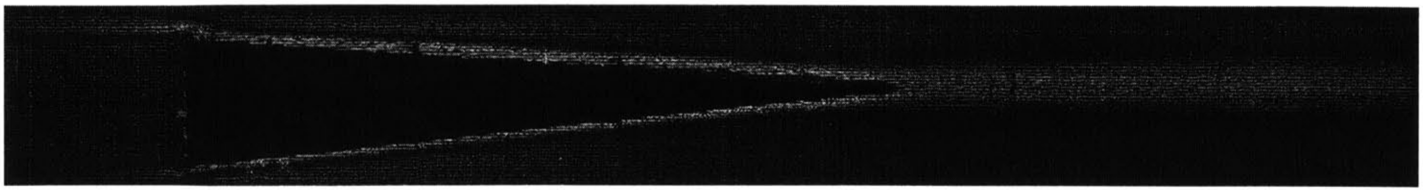

Fig 2 – Velocity Profile from laser tracking system.

Results and Discussion:

For the scope of this discussion, the important result of studying the flow field around the diffusion fish was to notice that it moved an order of magnitude slower than its skinnier counterpart. Given the understanding of the author it would only be speculation to point to any features of the flow field as indicative of fluid flow perpendicular to the plane of interest.

As initially indicated by the particle tracking experiments, the wider diffusion fish ran at a speed significantly slower than their thinner counterparts, as can be seen in Fig. 3. The predicted speed very closely tracks the results found by Peacock and Allshouse for angles above 5 degrees. However, for the exact same tank conditions, the wider fish ran at much slower speeds at small angles. Clearly, there is an unaccounted for discrepancy between the wide and skinny diffusion fish. Not only do the results show a clear reduction in speed, but they also indicate that no curve of the same form as the previously found result will describe the speed vs. angle relation of the wider diffusion fish, as seen in Fig. 4. Another polynomial factor is required to account for the drop in velocity. Looking at that simple case, it is clear that, for the wider diffusion fish, the velocity drops of exponentially faster than predicted from the model based on the data from the less wide diffusion fish.

This data alone, unfortunately, does not answer the question as to which case is a more appropriate example of the two dimensional flow case, or whether any two dimensional experimental analysis is appropriate. However, it is also clear that the data in no way invalidates the results found with less wide fish as an appropriate two-dimensional solution. As was initially thought in the motivation behind creating a wider diffusion fish for flow field study, it could be the case that the proximity to the side walls of the experimental tank prevents flows that move perpendicular to the sides of the fish, or it could be the case that the perpendicular flow is a result of a close interaction between the diffusion fish and the side wall. Given the success of prior work done with less wide diffusion fish it is probable that three dimensional effects become more prevalent for wider fish, but to fully answer this question, further research must be completed that either examines the flow field created in a horizontal plane by both of these cases. This would illuminate which diffusion fish creates the greatest amount of "three-dimensional" flow, and provide a base to choose a case to study in more detail in the vertical plane of interest.

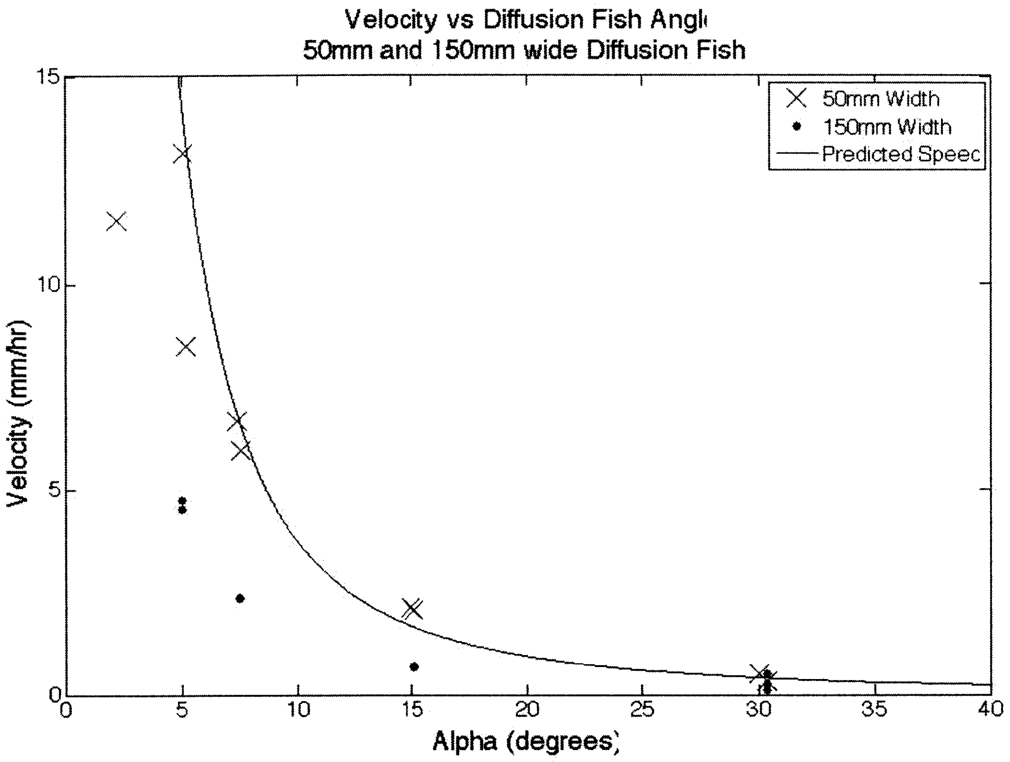

Fig 3 – Speed vs Diffusion Fish Angle , 50mm and 150mm Diffusion fish. Unpublished 50mm Diffusion fish data from Peacock and Allshouse

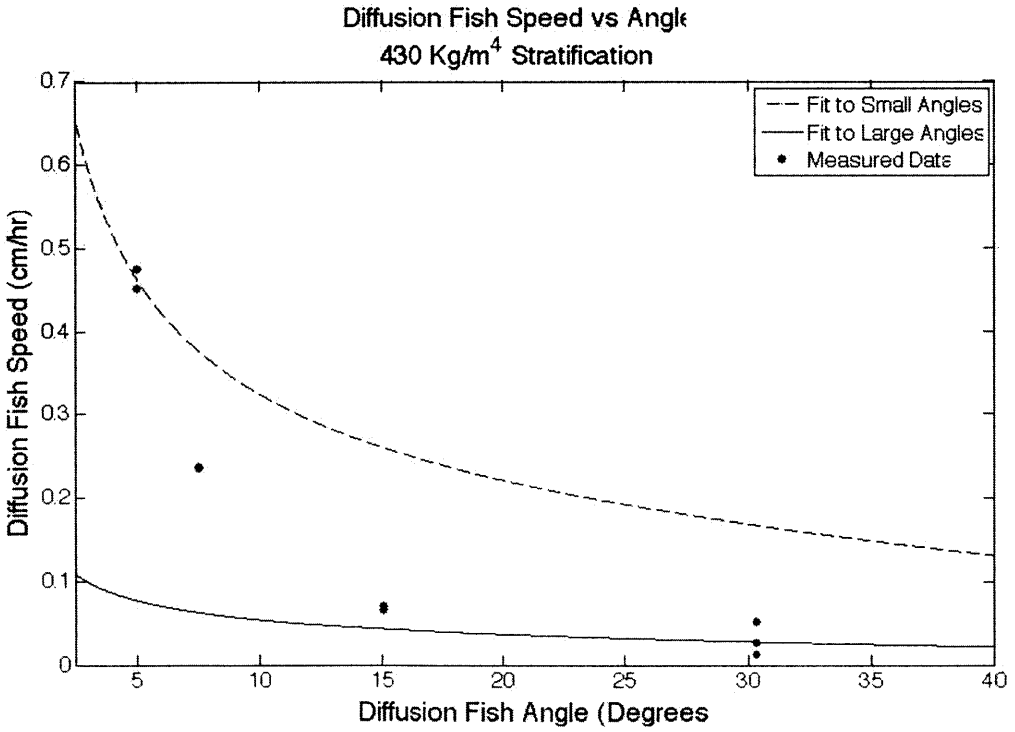

Fig 4 – Showing misfit curves

Conclusions:

This Experiment shows clearly that the width of the diffusion fish in relation to the width of the experimental tank plays a large role in the speed at which the diffusion fish moves. These preliminary experiments indicate that three dimensional effects in the flow become more prevalent as the width of the diffusion fish approaches the width of the tank, but do not show that the two dimensional approach is invalid in cases where the tank width is appropriately larger. In order to definitively prove that three-dimensional effects become prevalent only in cases where the fish is too wide, further investigation should be made using diffusion fish of widths both in-between and smaller than the currently studied cases.

References:

1 – T. Peacock, R. Stocker, and J. Aristoff, "An experimental investigation of the angular dependence of diffusion-driven flow," Phys. Fluids **16** 3503 (2004).

2 – T. Heitz, T. Peacock, and R. Stocker, "Optimizing diffusion-driven flow in a fissure," Phys. Fluids **17** 128104 (2005).

3 - O. M. Phillips, "On flows induced by diffusion in a stably stratified fluid," Deep-Sea Res. 17, 435 (1970).

4- T. Peacock and R. Stocker, "Diffusion Driven flow," 18th Congres Francais de Mecanique, Grenoble, 27-31 (2007).