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Sheared toroidal flows can cause bifurcations to zero-turbulent-transport states in tokamak plasmas. The maximum temperature gradients that can be reached are limited by subcritical turbulence driven by the parallel velocity gradient. Here it is shown that q/ϵ (magnetic field pitch/inverse aspect ratio) is a critical control parameter for sheared tokamak turbulence. By reducing q/ϵ, far higher temperature gradients can be achieved without triggering turbulence, in some instances comparable to those found experimentally in transport barriers. The zero-turbulence manifold is mapped out, in the zero-magnetic-shear limit, over the parameter space (γ_E, q/ϵ, R/L_T), where γ_E is the perpendicular flow shear and R/L_T is the normalized inverse temperature gradient scale. The extent to which it can be constructed from linear theory is discussed.

Introduction.—The heat loss that occurs as a result of turbulence driven by the ion temperature gradient (ITG) is one of the main obstacles to a successful fusion reactor. A large body of experimental work has demonstrated the effectiveness of strongly sheared equilibrium-scale flows in reducing this turbulence [1–3]. Numerical models [4–7] have demonstrated that by reducing the strength of the ITG instability, which drives the turbulence, and by shearing apart the turbulent structures, the radial gradient of the flow component perpendicular to the magnetic field can indeed lead to a great reduction in the heat loss that results from a given temperature gradient. However, Ref. [6] also demonstrated that the instability associated with the parallel velocity gradient (PVG) [8–10] could start to drive turbulence at higher flow gradients and prevent the complete suppression of the turbulent transport.

More recent work has demonstrated that, at even higher flow shears, it is possible, at moderate temperature gradients, for the perpendicular velocity shear to overcome both the ITG and the PVG instabilities and completely quench the turbulence [11]. This quenching is most effective at zero magnetic shear [12–14], a regime which has been associated in experiments with high confinement of energy in the presence of strongly sheared flows [3,15]. References [12–14] also demonstrated the existence, at zero magnetic shear, of a bifurcation to a high-temperature-gradient reduced-transport state, driven by a toroidal sheared flow. However, the maximum temperature gradient that could be reached via such a bifurcation was found to be limited by the fact that turbulence was rekindled at high toroidal shear, in the form of subcritical fluctuations driven by the PVG [9–13]. The question arises, which parameter regime is most favorable to the suppressing effect of the perpendicular flow shear and least favorable to the ITG and PVG drives? In other words, how can the temperature gradient which results from the transport bifurcation described in Refs. [12–14] be maximized?

At zero magnetic shear, the turbulence is subcritical for all nonzero values of the flow shear: there are no linearly unstable eigenmodes, and sustained turbulence is the result of nonlinear interaction between linear modes which grow only transiently before decaying. A recent paper [10], which studied this transient growth in slab geometry, demonstrated that at large velocity shears the maximal amplification exponent of a transiently growing perturbation before it decays is proportional to the ratio of the PVG to the perpendicular flow shear. In a torus, this quantity is equal to the ratio of the toroidal to poloidal magnetic field components, or q/ϵ, where q is the magnetic safety factor and ϵ is the inverse aspect ratio. Therefore, if we conjecture that a certain minimum amplification exponent is required for sustained turbulence, Ref. [10] predicts that there should be a value of q/ϵ below which the PVG drive is rendered harmless. Below that value of q/ϵ, it should be possible to maintain an arbitrarily high temperature gradient without triggering turbulent transport provided a high enough perpendicular flow shear can be achieved.

In this Letter, motivated by the possibility of reduced transport at low values of q/ϵ, we use nonlinear gyrokinetic simulations to map out *the zero-turbulence manifold*, the surface in the parameter space that divides the regions where turbulent transport can and cannot be sustained. The parameter space we consider is (γ_E, q/ϵ, R/L_T), where γ_E is the normalized perpendicular flow shear:

$\gamma_E = u'/(q/\epsilon)$, where $u' = dR\omega/dr/(v_{\text{thi}}/R)$ is the toroidal shear, ω the toroidal angular velocity, r the minor radius of the flux surface, v_{thi} the ion thermal speed and R the major radius, and where R/L_T is the inverse temperature gradient scale length normalized to R. For brevity, we will refer to R/L_T as "the temperature gradient." We set the magnetic shear to zero, the regime we expect to be most amenable to turbulence quenching by shear flow [3,11–16].

We discover that reducing q/ϵ is indeed uniformly beneficial to maintaining high temperature gradients in a turbulence-free regime, and that values of R/L_T can be achieved that are comparable to those experimentally observed for internal transport barriers [3,17].

In the next sections, having presented our numerical model and methodology, we will describe these results and discuss their physical underpinnings, as well as their implications for confinement in a toroidal plasma. We will show that linear theory of subcritical fluctuations [10] can, with certain additional assumptions, provide good predictions of the nonlinear results.

Numerical model.—To model the turbulence, we use the gyrokinetic equation [18] in the high-flow, low-Mach limit [19] (i.e., the toroidal rotation velocity is ordered to be smaller than the sound speed but much larger than the diamagnetic velocity; Coriolis and centrifugal effects are neglected [20], but velocity gradients are retained). We take the electrostatic limit and assume a modified Boltzmann electron response. The model used is identical to that in Ref. [13]. The gyrokinetic system of equations is solved using the local nonlinear simulation code GS2 [23–25]. As in Ref. [13], we take the Cyclone Base Case parameter regime [26], i.e., concentric circular flux surfaces with $\epsilon = 0.18$, inverse ion density scale length $R/L_n = 2.2$ and ion to electron temperature ratio $T_i/T_e = 1$ [27]. The magnetic shear is $\hat{s} = 0$. The ratio q/ϵ is varied by varying q alone. Collisions are included by means of a model collision operator, which includes scattering in both pitch angle and energy and which locally conserves energy, momentum and particles [30,31]. The resolution of all simulations was $128 \times 128 \times 40 \times 28 \times 8$ (poloidal, radial, parallel, pitch angle, energy). Note that relatively high parallel resolution was needed to resolve the PVG modes [13].

Method.—We wish to determine, in a three-dimensional parameter space $(\gamma_E, q/\epsilon, R/L_T)$, the boundary between the regions where turbulence can and cannot be sustained nonlinearly. We cover this space using four scans with constant q/ϵ [Fig. 1(b)], three scans with constant γ_E [Fig. 1(c)] and one scan with constant R/L_T [Fig. 1(d)]. For each of these cases, we consider multiple values of a second parameter and find the value of the third parameter corresponding to the zero-turbulence boundary. The boundary is defined as the point where both the turbulent heat flux and the turbulent momentum flux vanish. Thus,
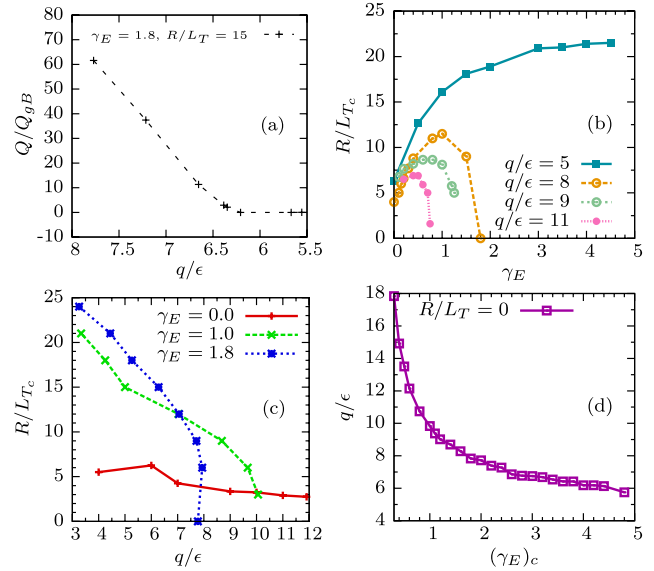

FIG. 1 (color online).   (a) The simulations used to find the point on the manifold $\gamma_E = 1.8$, $R/L_T = 15$, $q/\epsilon = 6.3$, showing the heat flux vs q/ϵ at ($\gamma_E = 1.8$, $R/L_T = 15$). The point on the manifold is the point where the heat flux drops to zero. (b)–(d) Sections through the critical manifold with parameters as indicated. Turbulence cannot be sustained for $R/L_T < R/L_{Tc}$ in (b),(c), or for $\gamma_E < \gamma_{Ec}$ in (d). The data points were found as illustrated in (a), and used to generate the manifold shown in Fig. 2.

the location of each single point on the boundary is determined using on the order of ten nonlinear simulations. An example of this procedure is shown in Fig. 1(a). In total, we performed more than 1500 simulations to produce the results reported below.

Because the turbulence that we are considering is subcritical, there is always a danger that a simulation might fail to exhibit a turbulent stationary state because of an insufficient initial amplitude [32,33]. As we are not here concerned with the question of critical initial amplitudes we will consider a given set of parameters to correspond to a turbulent state if such a state can be sustained starting with a large enough perturbation. Therefore, all simulations are initialized with high-amplitude noise. They are then run to saturation; close to the boundary a simulation may need to run for up to $t \sim 1000R/v_{\text{thi}}$ to achieve this.

The critical curves obtained in this manner are plotted in Figs. 1(b)–1(d). These curves, which effectively give the critical temperature gradient R/L_{Tc} as a function of γ_E and q/ϵ, are then used to interpolate a surface, the zero-turbulence manifold, plotted in Fig. 2. The interpolation is carried out using radial basis functions with a linear kernel [34] (see also Ref. [13]).

Results.—The results of the scan described above are displayed in Figs. 1(b)–1(d). These three figures show, at fixed values of either γ_E, R/L_T or q/ϵ, the threshold in either R/L_T or q/ϵ below which turbulence cannot be

sustained; they are, in effect, sections through the zero-turbulence manifold.

Considering first Fig. 1(b), we see that, at fixed q/ϵ, the critical gradient R/L_{Tc} first rises with γ_E, as the perpendicular flow shear suppresses the ITG-driven turbulence, and then falls—in most cases to 0—as the PVG starts to drive turbulence instead. This phenomenon was discussed at length in Refs. [11–13] (indeed the curve for $q/\epsilon = 8$ is taken from Ref. [13]). Thus, for every q/ϵ, there is an optimum value of the perpendicular flow shear γ_E (and hence of the toroidal shear u') for which the critical temperature gradient R/L_{Tc} is maximized. We see that reducing q/ϵ increases the maximum R/L_{Tc} that can be achieved without igniting turbulence. Figure 1(c) shows that this rule applies for all considered values of flow shear [35]. This is to be expected, because lower q/ϵ means weaker PVG relative to the perpendicular shear, allowing higher values of the perpendicular flow shear to suppress the ITG before the PVG drive takes over.

Last, Fig. 1(d) shows the threshold in γ_E above which the PVG can drive turbulence alone, without the help of the ITG; in other words, even configurations with a flat temperature profile would be unstable. At very high q/ϵ, already a very small flow shear will drive turbulence; as q/ϵ decreases, higher and higher values of γ_E are required for the PVG turbulence to be sustained. It cannot be conclusively determined from this graph whether, as suggested by linear theory [10], there is a finite critical value of q/ϵ below which PVG turbulence cannot be sustained, i.e., a nonzero value of q/ϵ corresponding to $\gamma_{Ec} \to \infty$. However, for $q/\epsilon \lesssim 7$, the critical γ_E is far above what might be expected in an experiment [37], and so the $\gamma_E \to \infty$ limit is somewhat academic. A definite conclusion we may draw is that at experimentally relevant values of shear, pure PVG-driven turbulence cannot be sustained for $q/\epsilon \lesssim 7$.

The zero-turbulence manifold interpolated from the numerical data points is displayed in Fig. 2. The manifold comprises three main features: a "wall" where the critical temperature gradient increases dramatically at low q/ϵ; a "spur" at low γ_E, jutting out to high q/ϵ (where, as γ_E increases, the ITG-driven turbulence is suppressed somewhat before the PVG drive becomes dominant), and finally the curve where the manifold intercepts the plane $R/L_T = 0$, whose shape is described above.

Practical implications.—In order to illustrate better the implications of our findings for confinement, we plot, in Fig. 3, contours of R/L_{Tc} vs q/ϵ and the toroidal flow shear $u' = dR\omega/dr/(v_{\mathrm{thi}}/R)$. The basic message is clear: the lower the value of q/ϵ, the higher the temperature gradient that can be achieved without igniting turbulence. Once we have obtained the lowest possible value of q/ϵ, there is an optimum value of u' which will lead to that maximum R/L_{Tc}. We note that the dependence of this optimum value of u' on q/ϵ is not as strong as the dependence of the optimum value of γ_E on q/ϵ (clearly this must be so because $u' = (q/\epsilon)\gamma_E$). In a device with an optimized value of q/ϵ, a near maximum critical temperature gradient would be achievable for $u' \gtrsim 5$, shears comparable to those observed in experiment [3,16,17].

While simulation results obtained for Cyclone Base Case parameters are not suitable for detailed quantitative comparison with real tokamaks, it is appropriate to ask whether our results are at all compatible with experimental evidence. A recent study [38] suggests that certainly the qualitative shape of the dependence, and possibly also the quantitative values, obtained here for R/L_{Tc} vs u' and q/ϵ are in agreement with the temperature gradients measured in MAST [39].

Relation to linear theory.—Since the mapping of the zero-turbulence manifold using nonlinear simulations is computationally expensive, we may ask whether linear theory can predict marginal stability. The question is also
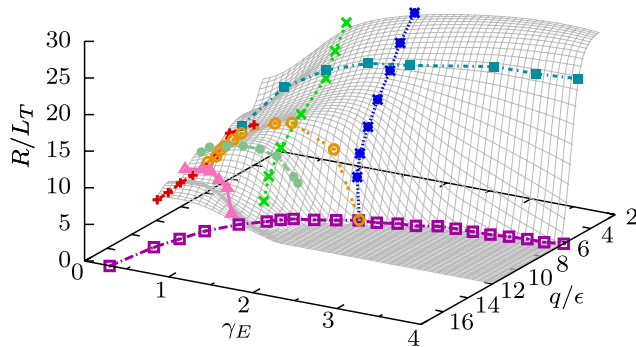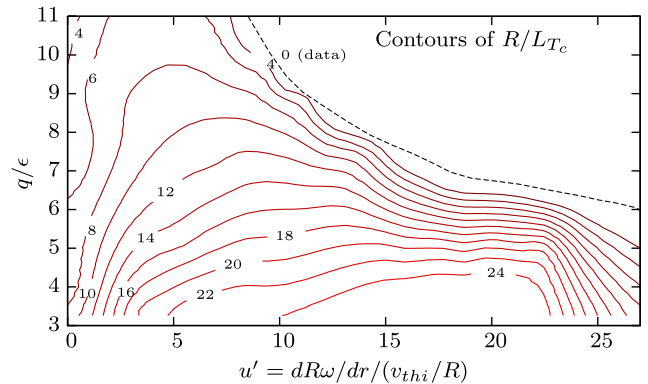

FIG. 2 (color online). The zero-turbulence manifold. Turbulence can be sustained at all points outside the manifold (that is, at all points with a higher temperature gradient and/or higher value of q/ϵ than the nearest point on the manifold). This plot is made up from the sections shown in Figs. 1(b)–1(d) (heavy lines) and the manifold interpolated from them (thin grey mesh).

FIG. 3 (color online). Contours of the zero-turbulence manifold plotted against the toroidal flow shear $u' = dR\omega/dr/(v_{\mathrm{thi}}/R) = \gamma_E/(q/\epsilon)$. The contours indicate the value $R/L_T = R/L_{Tc}$ below which turbulence is quenched.

interesting in terms of our theoretical understanding of subcritical plasma turbulence. It is clear that in a situation where perturbations grow only transiently, existing methods based on looking for marginal stability of the fastest growing eigenmode will not be applicable. In Ref. [10], we considered these transiently growing modes in a sheared slab and posited a new measure of the vigor of the transient growth: N_{max}, the maximal amplification exponent, defined as the number of e foldings of amplification a perturbation experiences before starting to decay, maximized over all wave numbers. It appears intuitively clear that in order for turbulence to be sustained, transient perturbations must interact nonlinearly before they start to decay. We may then assume that a saturated turbulent state will exist if $N_{max}(\gamma_E, q/\epsilon, R/L_T)$ is greater than some threshold value of order unity.

We now test this idea by calculating N_{max} for linear ITG/PVG-driven transient perturbations, using the GS2 code to solve the linearized gyrokinetic equation. Figure 4(a) shows N_{max} vs γ_E for a range of values of R/L_T at constant $q/\epsilon = 6.0$. In line with the prediction of Ref. [10], N_{max} tends to infinity as the flow shear tends to zero, and tends to a constant as the flow shear tends to infinity. In the intermediate region, N_{max} decreases as the perpendicular shear suppresses the ITG drive; for lower values of the ITG, it may even decrease to zero before the PVG drive makes it increase it once again.

We find that, in fact, there is no constant critical value of N_{max} that may be used to reproduce the zero-turbulence manifold: the critical amplification exponent $N_c(\gamma_E, q/\epsilon) \equiv N_{max}(\gamma_E, q/\epsilon, R/L_{Tc}(\gamma_E, q/\epsilon))$ varies in general with both flow shear [see Fig. 4(a)] and q/ϵ. However, we find that at high flow shear, N_c is virtually independent of q/ϵ, and at low flow shear it is only weakly dependent on q/ϵ. In addition, at low flow shear, R/L_T is only weakly dependent on N_{max} [i.e., the N_{max} vs γ_E curves

corresponding to each given R/L_T are nearly vertical; see Fig. 4(a)]. A consequence of this is that we can find a fitted function $N_c^f(\gamma_E)$, independent of q/ϵ [and also shown in Fig. 4(a)], such that the equation $N_{max}(\gamma_E, q/\epsilon, R/L_T) = N_c^f(\gamma_E)$ gives a good reconstruction of the zero-turbulence manifold.

The difference between $N_c(\gamma_E, q/\epsilon)$ and $N_c^f(\gamma_E)$ for any given q/ϵ does not appear to be very large. Thus, the practical conclusion of this exercise is that it may be possible to obtain a rough approximation to the zero-turbulence manifold via a nonlinear scan at a single value of q/ϵ.

Discussion.—We have presented two key results. First, and principally, we have calculated the shape of the zero-turbulence manifold, the surface that divides the regions in the parameter space (γ_E, q/ϵ, R/L_T) where subcritical turbulence can and cannot be nonlinearly sustained. We have described the shape of this manifold and its physical origins, and presented its two implications for confinement in toroidal plasmas: that reducing the ratio q/ϵ, i.e., increasing the ratio of the poloidal to the toroidal magnetic field, improves confinement at every nonzero value of γ_E, and that at fixed q/ϵ, there is an optimum value of γ_E [that is, an optimum value of the toroidal flow shear $u' = dR\omega/dr/(v_{thi}/R)$] at which the critical temperature gradient is maximized, in some instances to values comparable to those observed in internal transport barriers [3,17]. How to calculate the heat and momentum fluxes that would need to be injected in order for such optimal temperature gradients to be achieved was discussed in Ref. [14].

Second, we have shown that the zero-turbulence manifold can be parameterized as $N_{max}(\gamma_E, q/\epsilon, R/L_T) = N_c^f(\gamma_E)$, where N_{max} is the maximal amplification exponent of linear transient perturbations (calculated from linear theory) and N_c^f must be fit to the data. Obviously, the need to fit $N_c^f(\gamma_E)$ indicates a limitation of our current theoretical understanding of the criterion for sustaining subcritical turbulence in a sheared toroidal plasma. The results reported here provide an empirical constraint on future theoretical investigations.

Another avenue for future investigations is determining the dependence of the zero turbulence boundary on some of the parameters that were held fixed in this work: T_i/T_e, magnetic shear, and, more generally, the shape of the flux surfaces, density gradient, inverse aspect ratio ϵ (separately from q), etc. Mapping out the dependence just on γ_E, q/ϵ and R/L_T took approximately 1500 nonlinear simulations at a total cost of around 4.5 million core hours. Adding even two or three more parameters to the search would take computing requirements beyond the limit of resources today, but not of the near future.
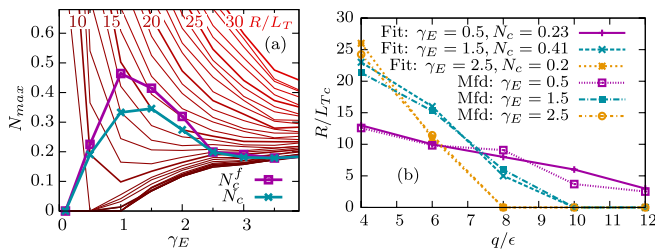
FIG. 4 (color online). (a) N_{max}, the transient amplification exponent maximized over all wave numbers, vs the flow shear γ_E, at different values of the ion temperature gradient R/L_T and at constant $q/\epsilon = 6.0$. Also shown is the critical amplification exponent $N_c(\gamma_E, q/\epsilon)$ for the same value of q/ϵ, and the fitted critical amplification exponent $N_c^f(\gamma_E)$. (b) The critical temperature gradient R/L_{Tc} vs q/ϵ for different values of γ_E, showing R/L_{Tc} obtained both from the interpolated manifold (Fig. 2), and from the equation $N_{max}(\gamma_E, q/\epsilon, R/L_T) = N_c^f(\gamma_E)$, with $N_c^f(\gamma_E)$ [shown in (a)] chosen to produce the best fit.

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[1] K. H. Burrell, Phys. Plasmas **4**, 1499 (1997).

[2] R. C. Wolf, Plasma Phys. Controlled Fusion **45**, R1 (2003).

[3] P. C. Vries et al., Nucl. Fusion **49**, 075007 (2009).

[4] R. E. Waltz, G. D. Kerbel, and J. Milovich, Phys. Plasmas **1**, 2229 (1994).

[5] A. M. Dimits, B. I. Cohen, W. M. Nevins, and D. E. Shumaker, Nucl. Fusion **41**, 1725 (2001).

[6] J. E. Kinsey, R. E. Waltz, and J. Candy, Phys. Plasmas **12**, 062302 (2005).

[7] C. M. Roach et al., Plasma Phys. Controlled Fusion **51**, 124020 (2009).

[8] P. Catto, M. Rosenbluth, and C. Liu, Phys. Fluids **16**, 1719 (1973).

[9] S. L. Newton, S. C. Cowley, and N. F. Loureiro, Plasma Phys. Controlled Fusion **52**, 125001 (2010).

[10] A. Schekochihin, E. Highcock, and S. Cowley, Plasma Phys. Controlled Fusion **54**, 055011 (2012).

[11] M. Barnes, F. I. Parra, E. G. Highcock, A. A. Schekochihin, S. C. Cowley, and C. M. Roach, Phys. Rev. Lett. **106**, 175004 (2011).

[12] E. G. Highcock, M. Barnes, A. A. Schekochihin, F. I. Parra, C. M. Roach, and S. C. Cowley, Phys. Rev. Lett. **105**, 215003 (2010).

[13] E. G. Highcock, M. Barnes, F. I. Parra, A. A. Schekochihin, C. M. Roach, and S. C. Cowley, Phys. Plasmas **18**, 102304 (2011).

[14] F. I. Parra, M. Barnes, E. G. Highcock, A. A. Schekochihin, and S. C. Cowley, Phys. Rev. Lett. **106**, 115004 (2011).

[15] A. C. C. Sips et al., Plasma Phys. Controlled Fusion **40**, 1171 (1998).

[16] P. Mantica et al., Plasma Phys. Controlled Fusion **53**, 124033 (2011).

[17] A. R. Field, C. Michael, R. J. Akers, J. Candy, G. Colyer, W. Guttenfelder, Y. Ghim, C. M. Roach, and S. Saarelma, Nucl. Fusion **51**, 063006 (2011).

[18] E. A. Frieman and L. Chen, Phys. Fluids **25**, 502 (1982).

[19] I. G. Abel, G. G. Plunk, E. Wang, M. A. Barnes, S. C. Cowley, W. Dorland, and A. A. Schekochihin, arXiv:1209.4782 [Rep. Prog. Phys. (to be published)].

[20] The impact of these effects on turbulence is studied in Refs. [21,22].

[21] A. G. Peeters, C. Angioni, and D. Strintzi, Phys. Rev. Lett. **98**, 265003 (2007).

[22] F. Casson, A. G. Peeters, C. Angioni, Y. Camenen, W. A. Hornsby, A. P. Snodin, and G. Szepesi, Phys. Plasmas **17**, 102305 (2010).

[23] M. Kotschenreuther, G. W. Rewoldt, and W. M. Tang, Comput. Phys. Commun. **88**, 128 (1995).

[24] F. Jenko, W. Dorland, M. Kotschenreuther, and B. N. Rogers, Phys. Plasmas **7**, 1904 (2000).

[25] W. Dorland et al., http://gyrokinetics.sourceforge.net/ (2009), URL Gyrokinetic Simulations Project.

[26] A. M. Dimits et al., Phys. Plasmas **7**, 969 (2000).

[27] A temperature ratio of 1 is appropriate for both lower power and future reactor-like conditions, but not for high-performance shots in current devices [16,28,29].

[28] The JET Team, Nucl. Fusion **39**, 1619 (1999).

[29] C. C. Petty, M. R. Wade, J. E. Kinsey, R. J. Groebner, T. C. Luce, and G. M. Staebler, Phys. Rev. Lett. **83**, 3661 (1999).

[30] I. G. Abel, M. Barnes, S. C. Cowley, W. Dorland, and A. A. Schekochihin, Phys. Plasmas **15**, 122509 (2008).

[31] M. Barnes, I. G. Abel, T. Tatsuno, A. A. Schekochihin, S. C. Cowley, and W. Dorland, Phys. Plasmas **16**, 072107 (2008).

[32] J. Baggett, T. Driscoll, and L. Trefethen, Phys. Fluids **7**, 833 (1995).

[33] R. R. Kerswell, Nonlinearity **18**, R17 (2005).

[34] M. Buhmann, Acta Numer. **9**, 1 (2001).

[35] The increase at $\gamma_E = 0$ cannot, of course, be due to reduction of the PVG; we assume that this occurs because of the simultaneous reduction of the maximum parallel length scale qR in the system, leading to weaker ITG turbulence; see Ref. [36].

[36] M. Barnes, F. I. Parra, and A. A. Schekochihin, Phys. Rev. Lett. **107**, 115003 (2011).

[37] By order of magnitude, $\gamma_E \sim M/q$, where M is the Mach number of the toroidal flow. Thus, values of γ_E much above unity are unlikely to be possible.

[38] Y. Ghim et al., arXiv:1211.2883 [Phys. Rev. Lett. (to be published)].

[39] Making such a comparison relies on the expectation that R/L_T measured in a real plasma will always be close to the zero-turbulence manifold ("stiff transport"). Some evidence that this is the case is also presented in Ref. [38].