### Dynamic Incentive Scheme for Rental Vehicle Fleet Management

by

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Submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degree of

Master of Engineering in Electrical Engineering and Computer Science **ARCHIVES** 

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#### Abstract

Mobility on Demand is a new transportation paradigm aimed to provide sustainable transportation in urban settings with a fleet of electric vehicles. Usage scenarios proposed by Mobility on Demand systems must allow one-way rentals. However, one-way rentals bring significant challenges to fleet management because areas of high demand will tend to lose their inventory, whereas areas of low demand will tend to accumulate inventory. Dynamic incentives can be provided to encourage different usage patterns and alleviate the problem of demand asymmetry. This thesis proposes a dynamic incentive scheme for rental vehicle fleet management in the context of Mobility on Demand. Simulation using Vienna taxi data shows the scheme to be effective at maintaining the equalibrium state of the fleet. It holds great promise to be incorporated in a real-world deployment of Mobility on Demand system.

Thesis Supervisor: Kent Larson Title: Principal Research Scientist, Media Laboratory

#### Acknowledgments

Many people have helped along my way as a researcher. First, I would like to thank my thesis advisor Kent Larson for guiding me along the topic and providing me with valuable feedback and creative ideas. He has always painted the bigger picture for me beyond the mundane transportation frameworks we have today.

I would also like to thank Ryan Chin in our research group for providing me with contacts of industry experts and other students working on the similar topics. In addition, Jakob Puchinger, Martin Rinthaler, and Katja Schechtner from Austrian Institute of Technology have helped me tirelessly to obtain crucial data from Vienna for my analysis.

My girlfriend Yu Linlin Huang has provided me constant care and support throughout. Without her, this thesis would see much more typos and would not have an organized bibliography.

Most importantly, I want to thank my family. My dad, Runping Zhou, has sacrificed too much for me. I owe much of my academic career in the US to his sacrifices. I hope my education path has made him proud. My mom, Hongyan Zhang, has always been my pillar of support for all these years away from home. I could not have made this far without the weekly calls to her and my grandmas.

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## Chapter 1

## Introduction

#### 1.1 Research Motivation

Urban population has steadfastly increased over the past decades, and much of the development has been fueled by the increasing ownership of automobiles [16]. While providing additional convenience and mobility, automobiles are also major sources of urban problems, such as greenhouse gas emissions, sound pollution, and traffic congestions [16]. Many of these problems can be alleviated by public transportation systems. However, they also impose significant inconveniences: the unpredictable wait time at the station, the long distance from the origin to the nearest public transportation station (the "First Mile Problem"), and the distance from the arrival station to final destination (the "Last Mile Problem") [21].

Changing Places group at MIT Media Laboratory has developed innovative vehicles, such as CityCar and RoboScooters, for a new approach to the urban mobility problem, broadly known as "Mobility on Demand" (MOD) systems. In combination with traditional vehicles and public transportation systems, the new compact and versatile vehicles provide a range of possibilities that offer urban dwellers new levels of mobility and convenience.

In order to be successful, CityCar must allow users to pick up and drop off the vehicles according to their needs. In other words, one-way trips must be allowed so that a user may pick up a CityCar right outside of his home, use it to get to the train station, and leave the vehicle at the train station to be utilized by the next user. This, however, creates the problem that some stations tend to "attract" cars and some stations tend to "lose" cars, depending on the time of the day and the location of the station. The stations will eventually reach a degenerative state and will require operators to haul or drive the cars to be re-balanced. To mitigate this problem, a dynamic pricing scheme can be implemented to give users appropriate incentives to pick up from low demand stations and drop off at high demand stations.

This thesis will explore the dynamics of this new pricing scheme using theoretical frameworks and software simulations. A specific algorithm, dynamic incentive scheme for Mobility On Demand (DISMOD), is proposed. Preliminary software has been developed to simulate the effects of DISMOD. Historical taxi data from the city of Vienna is used to evaluate the feasibility of such a scheme. Finally, the results of the simulation are compared against the historical data and other methods of transportation.

#### 1.1.1 Urban Mobility, Trends and Challenges

Urbanization is an integral part of human history, and in 2008 for the first time ever, more people–an estimated 3.3 billion people–live in urban areas than elsewhere. By 2030, the size of urban population is expected to reach 5 billion [27, 28]. Transportation needs in urban areas, or urban mobility, still remains one of the toughest problems to tackle.

At least in the United States, urban mobility options have diversified to include much more than personal automobiles in recent years [6]. Cycling is on the rise in many major cities, thanks to improved infrastructure dedicated to biking and increased awareness of the benefits of biking [6]. Car sharing services in various forms also adds to the diversification. For example, Zipcar allows its members to rent cars from its fleet on hourly or daily rates. Similarly, RelayRides allows members to pledge their own cars as part of the fleet, which can be then rented by other members. Figure 1-1 shows a Boston Hubway station, which is a successful bike-sharing provider.

In the coming years, urban mobility will continue to see innovative improvements. Vehicles will be increasingly aware of their own states via on-board computers and GPS locations [10]; they will be increasingly aware of their surroundings via Internet connections and near range communications [18]; and they will be increasingly aware of the drivers and passengers within them via behavior monitoring software [13]. All of these innovations aim to solve the problem: how to transport people in a sustainable, convenient, and reliable way in the context of increasing urban densities.



Figure 1-1: A Hubway Bike Station in Boston

#### 1.1.2 Mobility on Demand Systems

Researchers in the Changing Places group at MIT's Media Laboratory have focused on developing a sustainable model for urban mobility called Mobility on Demand. MOD systems consist of a coordinated fleet of Lightweight Electric Vehicles (LEVs) including CityCar, RoboScooter, GreenWheel electric bicycle, and Persuasive Electric Vehicles (PEVs). Each fulfills a subcategory of urban mobility needs: CityCar offers medium-range travel and ample cargo space; RoboScooter and GreenWheel are both more versatile and fulfills short-range travel; and PEV encompasses more aspects of traveling compared to the simple, traditional concerns of time and cost. For example, the ubiquity of smart phones can be leveraged to persuade users toward more healthy and sustainable transportation practices. Figure 1-2 shows some of the rendering of the vehicles developed.

New electric charging infrastructures and technologies also accommodate the success of MOD systems. Area of explorations include rapid charging, inductive charging, static and mobile energy storage, and Vehicle-to-Grid strategies. In addition, a comprehensive travel planning system is being developed to include multiple transportation modes for a given

![](_page_15_Picture_0.jpeg)

Figure 1-2: New Vehicles to be Used in MOD Fleet

trip, thus leading to a more fluid and personalized travel experience.

Similar to the hardware innovations, the accompanying software must also adapt to the particular needs and challenges of MOD systems. It must meet all the demands created by the multi-modal usage scenario of a modern mobility system. Central to the software is the management algorithm that provides the most availability and convenience for all the components in the system.

#### **1.2 Demand Asymmetry Problem**

The MOD system, at its foundation, promises availability, convenience, and reliability to the consumers. Therefore, the envisioned usage scenarios all involve the possibility of oneway trips. However, transportation needs follow fixed patterns given the location and time. For example, stations in suburban residential areas will likely have a high level of demand in the morning from people commuting to work, who will use the cars near their homes to go to the public transportation hub spot. Similarly, the stations near commercial districts will also have a high demand in the morning from these same commuters once they get off from the public transportation. If unmanaged, these demand patterns will result in empty stations where people need to pick up cars and full stations where people need to drop off, negatively affecting MOD system's availability, convenience, and reliability.

#### **1.2.1** Current Limitations

In contrast to MOD systems, existing rental services traditionally emphasize on picking up and dropping off from the same location. Although some already allow one-way trips, these services have many unattractive drawbacks that impose significant inconvenience and financial strain on the consumer. It is worthwhile to compare the limitations of location options due to the demand asymmetry problem.

The limitations can be summarized into three broad categories: different location drop off never allowed, different location drop off always penalized with premium, and different location drop off charged with unpredictable amount.

- Zipcar and RelayRides are two prominent examples in the first category. All cars are picked up and dropped off to the same location; no exceptions allowed.<sup>1</sup>
- Most enterprise car rental companies fall into the second category. A high premium is imposed on any car returned to a different location. As an illustration, Table 1.1 shows the reservation prices of a one day rental from and to Boston and New York locations of Budget Rent a Car System, Inc.<sup>2</sup>

		Origin		
		Boston	New York	
Destination	Boston	\$87.99	\$150.99	
Destination	New York	\$119.99	\$101.99	

Table 1.1: Example Prices of Renting Cars from Different Locations

Going from Boston and New York to a different city cost \$32.00 and \$49.00 more, respectively. Furthermore, New York seems to be a place where demand is higher based on its higher quoted rate, so it should benefit from additional cars driven from other places to New York. However, this intuitive strategy is not reflected from this simple example, as prices Boston to New York price is still higher.

<sup>&</sup>lt;sup>1</sup>It is potentially possible to return to a different location since RelayRides users arrange pick up and drop off locations with the car owner. However, the car owner must have other means to get to that different location, so such arrangements are uncommon.

<sup>&</sup>lt;sup>2</sup>Prices are retrieved from budget.com on May 15, 2012 for reservations of a Standard class vehicle from Aug 15, 2012 to Aug 16, 2012.

• U-Haul is one of the largest cross-country moving truck providers, and their quoted prices are in fact based on origin and destination locations. However, they do not provide details beyond that, and the price fluctuation is unpredictable.

#### **1.2.2** Possible Mitigations and Solutions

The over majority of fleet operators still rely on manual labor to re-balance their fleets, in order to satisfy demand. For enterprise car rental companies, this process involves operators driving passenger vans to different locations carrying multiple drivers, who will then relocate the cars. For bike sharing providers such as Bixi and Hubway, who allow customers to pick up and drop off at any docking station, the re-balancing process is similarly labor-intensive; employees have to drive trucks around to pick up excess bikes from some station and drop them off in empty stations. For example, for every one thousand bikes, Bixi requires one truck with two employees spending sixteen hours per day to re-balance the fleet in Montreal, Canada [20].

Another way to circumvent the problem of asymmetry in demand is to provide enough vehicles in the fleet such that they effectively saturate the market all the time. Car2Go in Austin, Texas and DriveNow in Germany are both examples that employ this strategy. Cars are ubiquitous throughout the city such that there will be at least one vehicle within walking distance.

In addition, researchers at Changing Places have become increasingly aware of the holistic nature of urban transportation, especially in the MOD context. Traveling should not be solely confined to focus on time, distance, or cost. In contrast, travel plans may take into account of one's fitness goals, community needs, and everyday errands. These personal aspects could be leveraged to coordinate with the system on the whole, such that both personal and system goals are reached. Namely, personalized recommendations that favor a balanced fleet distribution can be presented to users for mutual benefits. This can be achieved via a combination of using Persuasive Elective Vehicles, mobile interfaces, and integrated multi-mode trip planning.

Looking into the future, autonomous vehicles will most definitely solve the asymmetry problem efficiently and reliably. The MOD system will be able to coordinate the locations of the its fleet centrally, and each vehicle can be directed to its desired location at the desired time.

#### **1.3 Variants of Dynamic Pricing Schemes as Solutions**

Among different schemes to help solve the asymmetry problem in MOD fleet management, dynamic incentive remains one of the most promising solutions. It gives the most freedom to operators since the pricing signals can be adjusted constantly. This thesis explores the most basic version focusing on the end-consumer price, but more possibilities remain viable.

![](_page_18_Figure_2.jpeg)

Figure 1-3: Illustration of a Dynamic Pricing Scheme

Figure 1-3 shows a simple illustration of the idea of dynamic pricing. Faced with two viable options, the top path is to be encouraged with incentives because the trip would be taking a car from a low demand station to a high demand station. In comparison, the bottom path is to be discouraged with penalties because the trip would be taking a car from a high demand station to a low demand station.

#### 1.3.1 Dynamic Pricing

Price is a strong and effective signal to the consumers that influences their behavior. From the customers' perspective, they are more likely to opt for the cheaper priced stations. From the operators' perspective, the distribution of the fleet will approach a more favorable condition if appropriate prices are set. Specifically, price should be high if the trip is from a low-inventory station to a high-inventory station, and price should be low if the trip is from a high-inventory station to a low-inventory station. In the meantime, the desired level of inventory, historical demand pattern, and conditions of nearby stations should also be taken into account. This creates a dynamic station-to-station price matrix whenever a customer is considering to use MOD system, and in aggregate, the fleet distribution should maintain its equilibrium.

#### 1.3.2 Zone Based Pricing

One tweak can be made to the dynamic pricing model to simplify the price matrix by sectioning different stations into zones. Prices then can be calculated based on the origin and destination zones, similar to many public transportation systems and flat-fare taxis. These zones can be dynamic or static, based on analysis of demand patterns. This could potentially be a more simplified experience for the consumer. However, it aggregates detailed stationto-station pricing, so its granularity may be too high to manage the MOD fleet effectively.

#### **1.3.3** Tiered Customer Segmentation

Borrowing from the experiences of airlines, who have long tiered their customers to utilize unused capacity on the low end and maximize profits on the high end, MOD systems could also promise different levels of service to different tiers. In the context of fleet management, the upper tier could be promised with the full range of stations, whereas the lower tier would have limited choice in pick up and drop off locations. Specifically, pick up limits will be imposed on low-inventory stations and drop off limits will be imposed on high-inventory stations. With a full range of customers from different tiers, the management of the fleet can be achieved by dynamically adjusting the limits based on the current conditions of the fleet.

#### **1.3.4 Gamification of Fleet-balancing Process**

Other than monetary incentives, recent trends have shown that using game techniques in non-game context ("gamification") can be a power tool to influence behavior [5]. Thus, it is easily imaginable to make the re-balancing process a game, in which participants gain points by completing a suggested "re-balance" trip.<sup>3</sup> The whole city becomes a game board and many gamers participate to help MOD systems stay in equilibrium. However, the cost of employing or mobilizing these gamers remain difficult to estimate.

#### **1.4** Literature Review

The literature in the transportation area has long established the benefits of car-sharing. Report by Millard-Ball et al. looked at car-sharing across the United States and found that the impacts of car-sharing are overwhelmingly positive; these impacts reduced travel, induced travel, lower emissions, increased transit ridership, cost savings, and greater mobility [17]. Shasheen and Meyn compiled a comprehensive analysis of twenty-eight car-sharing service providers in North America and recommends increased support from public and private sectors to use car-sharing for a more sustainable and efficient transportation network [23]. Similarly, Cervero draws from the experience of car-sharing in San Francisco and found users benefit from substantial time and cost savings [4]. Moreover, the users are willing to pay market rate for such services [4]. In comparison to new road scheme, Fellows and Pitfield found that car-sharing provides comparable benefits while incurring a fraction of the cost [7]. However, with its overwhelming benefits, car-sharing also introduces new challenges in operation, including vehicle security, user convenience, trip recording accuracy, vehicle management, accounting methods, and system efficiency [2].

With respect to the asymmetry problem inherent in one-way rental services, many methods have been proposed to model, simulate, or alleviate the problem. The Vehicle Routing Problem with Backhauls is an established problem faced by truck operators, where a set of goods need to be delivered but the vehicles must return to the central depot; Hassan and Osman proposes self-organizing features maps algorithm that solves for a set of minimum cost routes [8]. In a one-way rental scheme, Nair and Miller-Hooks developed a stochastic, mixed-integer program involving join-chance constraints, which will solve for a set of minimum cost redistribution trips while satisfying near-term demand; the benefits of using the algorithm has been demonstrated in a real world deployment in Singapore [19]. Tra proposes another method to circumvent the asymmetry problem [26]. Users are encouraged to either join trips or split trips such that the combined vehicle movement will result in

<sup>&</sup>lt;sup>3</sup>Presumably the gamer would have to make the return trip via other transportation methods.

redistribution of the fleet [26]. Lastly, Barth and Todd designed a simulation model that explored the vehicle availability, vehicle distribution, and energy management aspects of one-way rental fleet [1].

In other aspects of rental fleet management, several articles have outlined methods for locating stations or transit terminals. Taniguchi et al. have introduced a mathematical model that determines the size and locations of public transit terminals, taking into account traffic conditions for the system; the model has been successfully applied for a public transportation system in the Kyoto-Osaka region in Japan [25]. In another aspect, Kaltenbrunner et al. have analyzed biking data obtained in Barcelona areas to establish demand and traffic patterns in the rental vehicle context; the data have shown clear temporal and spacial patterns and the results were applied to predict vehicle availability and demand better [9].

Using economic levers to attain equilibrium is a well researched area in economics. Smith and van Ackere incorporated system dynamics models within economic equilibrium analysis [24]. May et al. designed a "transport marketplace" and studied the user motivations, constraints, and requirements in a car-share transportation framework; they have found the potential motivators to include perceived benefits of reduced cost, environmental benefit, social benefit, and reception of location based information [12]. Meijkamp looked further into the environmental aspects of car-sharing usage and found that the potential for environmental improvement through car-sharing services is very large [14].

#### **1.5** Research Contribution

This thesis proposes a generalized dynamic incentive scheme for rental vehicle management under the context of MOD. It designs a deployable user interface to be used in trip planning employing such incentive scheme. It proposes software architecture to accommodate the deployment of the scheme in MOD systems. It simulates the effects of using the scheme with historical data. It explores the feasibility of such scheme based on the results. Lastly, it provides a benchmark for future real-world deployment.

#### **1.6** Thesis Organization

This thesis is organized into five chapters. Chapter 2 will explain the proposed algorithm. Chapter 3 will detail the methodology used to test the algorithm via simulation. Chapter 4 explains the results of the simulation. Lastly, Chapter 5 offers discussions of the result and suggestions for future works.

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### Chapter 2

# Dynamic Incentive Scheme for Mobility on Demand (DISMOD) Algorithm

#### 2.1 Feasibility

Among the possibilities for solving demand asymmetry problem, dynamic pricing holds the most promise. Similar ideas have already been deployed, such as in traffic congestion control and parking management. Specifically in the MOD context, dynamic incentives are advantageous in many ways: favorable perception by consumers, flexible terms available to operators, and responsive feedback loop on the system level.

#### 2.1.1 Congestion Control via Dynamic Pricing

Dynamic pricing has been deployed in a variety of situations to help reduce traffic congestion. Wie and Tobin developed two dynamic congestion pricing models, and following the models, they have found optimal solutions to congestion control via dynamic pricing [29]. Experiences employing dynamic congestion pricing in San Diego and Bangalore have shown that users are willing to pay to save time; the overall effect is largely positive with reduction in commuter travel time [3, 15].

#### 2.1.2 Parking Management via Dynamic Pricing

Almost all parking places adjust their pricing to some extent according to demand. The "Event Parking" sign is all too common near a baseball or basketball stadium. Whereas the traditional strategies are fairly static and unresponsive, new developments bring the parking prices to a more dynamic, and hopefully more efficient, experience. Texas A&M University has proposed using RFID to collect real-time data on parking spots and auction spare units [30]. The city of San Francisco has piloted a program call SF*park*. The system uses a variety of sensors to analyze demand of both on- and off-street parking units and incorporates the results to set parking prices in segments of time periods [22].

#### 2.2 Design Goals

In designing DISMOD, several goals were considered important:

- Generalized: Although devised and focused in the context of MOD, the algorithm should be generalized to other situations of rental fleet management. With minimal adaptations, experiences and results gained from DISMOD could be applied elsewhere straightforwardly.
- Accommodating to Different Traffic Patterns: Different cities have vastly different traffic patterns. Those who have a heavy concentration of residential districts and business districts experience the asymmetry problems acutely; whereas those who have a very even distribution of demand experience very little of the asymmetry. DIS-MOD should not make assumptions about the particular traffic pattern.
- **Consistent Price Attribution**: The heart of DISMOD is to assign a tangible price, whether incentive or penalty, based on the current conditions of the MOD system. DISMOD should do so in a uniform way that assesses the same amount of monetary value given two identical system states.
- Adjusting for Consumer Expectation: DISMOD aims to be a commercially viable scheme, therefore it has to deal with real consumer expectations, habits, and demands. It should have adjustable parameters such that the algorithm can be adjusted based on varying consumer expectations, from city to city or from time to time.

• Adhering to Physical Constraints: Again, in order for DISMOD to be a commercially viable scheme, it has to account for physical constraints such as how fast people can walk and how many passengers can fit in one car.

#### 2.3 Overview of the Algorithm

DISMOD tries to address the design goals by encompassing these key parameters in a MOD system:

- **Incentive level** that allows the adjustment of overall incentive level, which reflects seasonal and temporary fleet demands
- Base operating cost that allows the adjustment of a base price, which reflects a "safe" operating cost and minimizes the risks associated with dynamic pricing
- **Desired inventory level** of each station that can be adjusted based on historical analysis of demand pattern
- Maximum and minimum inventory level of each station which are the physical limitations of each station
- Effective Distance within which different stations will correlate with others since stations should not be isolated and nearby stations should be accounted when considering overall demand of a region

Various strategies were considered to incorporate these parameters in a meaningful way which are discussed in Section 2.3.2. The one that was considered the most promising models the fleet after Coulomb's Law for electric charges, because many similarities are shared between the dynamics of electric charges and MOD fleet system.

#### 2.3.1 Model with Coulomb's Law

The high level principle of DISMOD follows a simple heuristic: cars should be evenly dispersed among all the stations. A high concentration or lack of cars in any station is undesirable. In other words, it would be a desirable property that cars *repel* each other, which will make them disperse. This is very similar to electric particles of the same charge

in the physical world, so it is a natural start to model the cars in the MOD after electric particles.

The next step is to incorporate the notion that stations have a optimal level of inventory, determined by historical analysis of demand patterns. The level is a highly dynamic property of the station based on its location, the time of the day, the day of the week, etc. Having modeled the cars as electric charges is obviously not adequate, since particles would eventually disperse into an evenly spaced configuration with no notion of stations.

Therefore, DISMOD proposes the following model:

**Proposal.** The notion of electric field in the DISMOD context composes of stations which are the actual particles. They carry negative charges that represent their optimal level of inventory; each desired car is one negative charge. Each car in the fleet is one positive charge.

When a station has exactly the same number of cars as its optimal level, the station particle has *neutral* charge; it neither attracts nor repels other positive particles. When a station has less than the optimal number of cars, it is *negative*, it attracts other cars, which are positive.

In addition to the influence of a station's own charge, the attraction force is a result of its surrounding stations as well. The force of attraction is directly proportional to the *net force* experienced by the station, which is the sum of the individual forces between this station and its surrounding stations.

The incentive scheme then follows naturally from this model: when a car moves from an origin to a destination, the cost of the trip should be directly proportional to the difference between the net forces experienced by the origin and the destination. In other words, the cost of a trip is directly proportional to the work needs to be done for this positive particle to be moved in the electric field.

Formally, the cost C is defined as:

$$C = I\Delta F \tag{2.1}$$

where I is the incentive level constant and,

$$\Delta F = F_{destination} - F_{origin} \tag{2.2}$$

$$F_{station} = \sum F_{electric} \tag{2.3}$$

$$F_{electric} = \frac{q_1 q_2}{r^2} \tag{2.4}$$

 $q_1$  and  $q_2$  are electric charges as defined before. r is the radius or distance between two stations.

#### 2.3.2 Alternative Algorithm Designs Considered

Several other natural phenomena were also considered as the basis of DISMOD algorithm, but each has its own shortcomings.

- **Gravity**: The notion of attraction and repulsion of cars lead to the idea of gravity. A simple way to model is to have each station act as an gravitational body and each car is a unit of mass if it's in the station. However, this is inconsistent with the desire that more cars at a station should cause strong repulsion, rather than attraction as the gravity model would dictate. In addition, it is hard to account for the optimal level of inventory at each station.
- Air molecules: Dispersion naturally lead to the thought of air molecule behaviors because they follow a very well defined entropy pattern that results in uniform distribution. This model would be consistent to the desire that cars should repulse each other and result in even distribution. However, the notion of stations is difficult to incorporate, as MOD does not want cars to be everywhere, but just in specific stations.
- Valleys and hills: The cost of a trip can also be thought to correlate with some form of potential energy change. In other words, stations with a lot of cars are on top of a conceptual hill, and stations with very few cars are at the bottom of the hill. Therefore, it's easier (encouraged) to take cars from the top to the bottom and more difficult (discouraged) the other way around. This model has some intuitive merits, but again, it is hard to incorporate the notion of optimal inventory at a station.

### Chapter 3

# Testing DISMOD with Vienna Taxi Data Simulation

#### 3.1 Testing Strategy

To validate DISMOD and explore its effects and implications, a suite of software was developed to simulate the dynamics and a user interface was proposed for visualization. The simulation utilizes data from a Vienna, Austria taxi operator as the basis of demand pattern, and it follows a Monte Carlo approach to explore the dynamics when used in real-world situations. The simulations are run multiple times with different number of stations, different number of cars in fleet, and different incentive levels. The results can be compared to see the effects of DISMOD.

#### **3.2 Data Source**

Researchers at the Mobility Department from Austrian Institute of Technology are also researching in the dynamic transportation space. They are in collaboration with a taxi company in Vienna who has been collecting trip information about its fleet of about 800 taxis. About a month of this data, ranging from September 30, 2011, to October 31, 2011, were obtained for analysis. The data was used as the basis of demand from imaginary users in Vienna if MOD were deployed. It assumes that taxis are ubiquitous and every trip made with a taxi represents one usage point that would result in similar demand in MOD.

The data covers information such as time, GPS location reading, and status of the taxi

(hired, waiting, etc.). For the duration, there were a total of 829, 161 trips recorded, of which 252, 930 trips were trips carrying paid passengers. The trips were further filtered to only cover trips within the city proper region<sup>1</sup> and with a traveled distance of longer than 1.5 kilometers. This reduced the total sample size of the data to be 184, 654 trips.

One important aspect of the simulation is to guess whether users will change their pick up and drop off decisions based on the incentives given. Thus the price elasticity must be known to make a prediction. A small scale survey was conducted among students at MIT. A total of 47 people gave answers to the following question:

Imagine you want to use an hourly rental car for your one-hour grocery shopping trip. You see that the station closest to you has a car available for \$10/hr; another station, an extra 10 min walk from the closest station, also has cars available but charges differently. Holding all else equal except the price, how much should the other station charge in order for you to consider it worthwhile to take the car by walking farther?

The re	esults	$\operatorname{are}$	summarized	in	Table	3.1:
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Responses
1
4
2
7
6
9
2
9
0
4
3

Table 3.1: Survey Responses to Price Elasticity

Price elasticity can be inferred from the survey data. A simple linear regression gives a price elasticity of \$0.21559/kilometer<sup>2</sup>. The simulation assumes this elasticity to make probabilistic decisions on how people react to different prices.

<sup>&</sup>lt;sup>1</sup>The region was limited to the bounding box of 48.1564°N, 16.2598°E and 48.2671°N, 16.5214°E for convenience.

<sup>&</sup>lt;sup>2</sup>The calculation makes the assumption that people walk 1 kilometer in 10 minutes.

#### 3.3 Software Architecture

The testing software is developed in Python and is composed of three components: DISMOD algorithm component, demand generation component, and the metrics logging component. DISMOD algorithm takes in a list of stations and computes the costs from relevant origin stations and destination stations. Demand generation component generate user's trip demands. Metrics logging component logs the health condition of the system as well as aggregate statistics about users.

#### **3.3.1** Algorithm Component

The algorithm component implements the DISMOD algorithm. In addition to computing the cost matrix between relevant stations in the system, it also makes a probabilistic decision on which route a given user will take. The component takes all the stations and trip information, which includes its origin and destination coordinates, and it outputs a reservation with origin station, destination station, and reservation price. The reservation represents the decision by a customer to commit the pick up and drop off at the indicated times.

At each step, it will first find the stations within walking distance to the origin and destination. It will then compute a matrix of cost between each pair of these possible origin and destination stations. The cost calculation follows DISMOD algorithm as detailed in Section 2.3.

Based on the cost, a probabilistic decision will be made for which origin and destination stations will be used by the user. The likelihood of a pair being chosen is inversely related to the cost of the trip, and the inverse coefficient is the price elasticity found by the survey described in Section 3.2.

#### 3.3.2 Demand Generation Component

The demand generation component is responsible for generating user demands. In this simulation, demand information is read directly from the Vienna taxi data. In other words, each time a taxi is hailed, MOD also assumes a potential customer with the same request. Once the component reads the demand, it hands off the demand to DISMOD to determine what reservation will be made exactly.

#### 3.3.3 Metrics Logging Component

At the end of each round of simulation, the conditions of the stations as well as reservations are recorded by the metrics logging component. Demand not satisfied and stations out of balance<sup>3</sup> are two major pieces of statistics logged to determine the effects of DISMOD.

#### 3.3.4 Graphical User Interface

In addition to the back end code, a graphical user interface (GUI) has also being developed for visualizing the health status of the MOD system. The goal of the GUI is to show the key properties of all stations in an efficient and straightforward manner. For each station, two concentric circles are drawn over its mapped location. The inner circle denotes the optimal level of inventory, and the outer circle denotes the maximum level of inventory. Each circle is also shaded. The size of the shading shows the current level of inventory, whereas the color of the shading indicates the health state of this station. Red means out of balance, and green means very close to the optimal level of inventory.

![](_page_33_Figure_4.jpeg)

Figure 3-1: Operator GUI Screen, Good Health

<sup>&</sup>lt;sup>3</sup>defined as any station with more than 2 cars away from its optimal level.

![](_page_34_Figure_0.jpeg)

Figure 3-2: Operator GUI Screen, Bad Health

Figure 3-1 shows the initial stage of the stations, which are all at the optimal level of inventory. In comparison, Figure 3-2 shows the system at a degenerative state where almost all stations are either full or empty.

Another feature for the operator GUI is to inspect the price easily. When hovering over a station, the cost from this station to all other stations are indicated by lines, where thicker lines represent cheaper cost and thinner lines represent more expensive cost, shown in Figure 3-3.

#### **3.4** Station Allocation

With the Vienna taxi data, the demand pattern can be deduced easily. However, the simulation still requires stations to be located throughout the city. Station allocation is not a trivial matter in real deployment. Aside from the obvious factor of location, numerous other factors are also extremely important to the allocation. For example, space has to allow the stations to be built, zoning permits have to be obtained, preferences of local residents have to be respected, and more. To simplify the allocation process for the sake of analysis,

![](_page_35_Figure_0.jpeg)

Figure 3-3: Operator GUI Screen, Showing the Cost to Other Stations

the physical properties of the city are ignored. Stations are simply located as if they are on a featureless flat land.

The k-means clustering algorithm is used to determine the exact locations of the stations. K-means algorithm is a well established algorithm in statistics for clustering observations [11]. It finds the centroid point among observation vectors that minimizes the sum of square distances from the centroid to each observation point. In the context of MOD, each pick up and drop off location in the taxi data is like an "observation" point of one demand of a station. Therefore, using k-means algorithm is equivalent to trying to satisfy demand with the least distance separating customers in need of cars and stations.

Once stations are allocated onto specific latitude and longitude points, each is assigned an optimal inventory level proportional to the number of demand points in its cluster. In other words the centroid points found by k-means are the stations, and the corresponding cluster indicate the popularity of that stations. Therefore, the optimal level of inventory should be directly related to the number of points in the cluster.

## Chapter 4

## Simulation Results

The simulations were run with different values for the various parameters in DISMOD. Furthermore, the simulations were run with different hypothetical setups. Since there is no concrete plan or data on the deployment of a MOD system in Vienna, the amount of stations as well as amount of cars in the fleet can only be speculative. Lastly, one crucial factor in DISMOD is the mapping from calculated "field forces" to dollar amounts under specific settings, and the mapping cannot be reliably predicted in a hypothetical setup, absent of operational, personnel, and material costs analysis.

The reference parameters were set at 100 stations and 800 cars in the fleet, which is similar to the size of the taxi fleet at about 800 taxis. 100 stations were chosen because on average, eight cars per station should provide a reasonable buffer against peaks and troughs in the demand pattern

#### 4.1 Increasing Cars in Fleet

Holding the number of stations and incentive level constant, a series of simulations were run with increasing number of cars in the fleet. The increasing number of cars in fleet shows an increased efficiency overall. Figure 4-1 shows the number of unsatisfied trips on the left axis and the average percentage of unbalanced stations on the right axis.

The trend indicates that increasing number of cars in the fleet lead to a more efficient fleet, since more demands are being satisfied while less stations are becoming unbalanced. This is an intuitive result in a normal fleet, where more cars obviously mean better service. DISMOD, in this case, did not hinder the performance and was able to scale along with the

![](_page_37_Figure_0.jpeg)

Figure 4-1: Metrics of the Fleet with Increasing Number of Cars

increasing number of cars.

#### 4.2 Increasing Incentives

Holding the number of stations and cars constant, a series of simulations were run with increasing level of incentive. The increasing level of incentive does not have a linear relationship with the efficiency level of the fleet. Figure 4-2 shows the number of unsatisfied trips on the left axis and the average percentage of unbalanced stations on the right axis.

While showing a correlation between the number of missed trips and average percentage of station out of balance, DISMOD seems to introduce additional complexities into the fleet management in our simulation methodology. In a real world scenario, consumers who have changed their minds because of DISMOD's pricing signal will create a slightly offset demand pattern, which, intuitively, should favor equilibrium of the fleet. However, in the simulation scenario, the demand pattern is static and has been pre-populated. It entirely overlooks the possibility of "helpers" that may arise in a dynamic pricing scheme, where new demand appears based on the pricing signal. Therefore, simulating DISMOD with the simple assumptions appear to have oversimplified the dynamics.

![](_page_38_Figure_0.jpeg)

Figure 4-2: Metrics of the Fleet with Increasing Level of Incentive

## 4.3 Presence of Incentive for Various Fleet-Station Configurations

Another set of simulations were run with different configurations of stations and fleet, while setting the incentives to be either on or off. Figure 4-3 shows the two measured metrics with both incentives on and off, number of unsatisfied trips on the left axis and the average percentage of unbalanced stations on the right axis.

![](_page_38_Figure_4.jpeg)

Figure 4-3: Metrics of the Fleet with Incentives On and Off

Having incentives turned on shows a small but consistent improvement over having incentives turned off. Again, this is related to the fact that the mapping from DISMOD values to real dollar values are only hypothetical and that the possibility of induced travel is entirely overlooked.

## Chapter 5

## Conclusion

#### 5.1 Discussion

While showing promise to alleviate the demand asymmetry problem, DISMOD showed only moderate improvements in simulation results with the Vienna taxi data. The trend is clearly indicating that DISMOD works as intended, helping stations to stay more balanced while not incurring additional operating cost. In addition, DISMOD is demonstrated not to interfere with the traditional model of redistribution; it only brings net benefit, albeit only marginally.

The simulation can be further improved to make additional assumptions on user behavior. Most important, making the assumption of additional travel usage induced by decreased price. Taxi data provides a basis of the demand pattern but proves to be inadequate to encompass all aspects of travel.

Lastly, the mapping from DISMOD values to real dollar values proves to be a crucial process to make the algorithm successful. This process, however, requires ground data on operational, personnel, and material cost for each specific deployment scenario.

#### 5.2 Future Works

The DISMOD algorithm holds promise to be a commercially viable scheme in the context of MOD fleet management. Future works should be done to understand further the dynamics and effects of DISMOD. It is especially important to conduct real user tests to validate assumptions about user behavior, because consumer behavior is very hard to predict from

a purely theoretical standpoint.

Vienna taxi data provides a solid start for the validation of DISMOD. However, data in different cities and mobility providers will help further the understanding of DISMOD. For example, Bixi and Hubway have been operating with one-way rental schemes for a number of years. Their data on user demand patterns, user behaviors, pricing schemes, operational challenges and costs will provide much value for a more comprehensive study of DISMOD.

Aside from the data aspect of the simulation, additional parametrization should also be simulated. Incentive level, number of stations, and number of cars in the fleet have been parametrized to validate the hypothesis. However, more parameters are available to be managed in DISMOD and their effects can be studied. Further, given a detailed deployment scenario, DISMOD can be simulated to find the optimum values for all the parameters.

Going beyond the narrow scope of fleet management using price signals, further studies regarding other aspects of travel should be incorporated into the fleet management strategies. As mentioned earlier, other forms of monetary incentives such as coupons and points can be used. In addition, non-monetary incentives such as environmental aspirations can also be used.

Further, DISMOD is developed in the context of MOD systems, which incorporates much more than just personal on-demand vehicles. Public transportation, bikes, and walking are also integral components that make MOD a sustainable system, so DISMOD should be incorporated into the larger ecosystem and provide maximum benefits with minimum impact and hassle.

Lastly, DISMOD must be deployed and tested in a real world environment to be fully utilized. Assumptions about users, operation procedures, and vehicle dynamics will be tested. The data and feedback from real deployment will lead to the final success of DISMOD and provide a sustainable framework for MOD fleet management.

# Appendix A

# Figures

![](_page_42_Picture_2.jpeg)

Figure A-1: Example Illustration of Station Allocation Using K-Means, 5 Stations

![](_page_43_Picture_0.jpeg)

Figure A-2: Example Illustration of Station Allocation Using K-Means, 10 Stations

Appendix B

Tables

Number	Number	Incentive Level	Average Percentage	Total Missed
of Stations	of Cars		Out of Balance	Trips
50	800	0	94.52%	48528
50	800	1000	87.76%	37465
60	800	0	92.87%	51852
60	800	1000	90.21%	46678
70	800	0	93.03%	51062
70	800	1000	91.70%	51962
80	800	0	92.00%	58740
80	800	1000	89.11%	55398
90	800	0	92.98%	54961
90	800	1000	91.39%	55222
100	800	0	90.14%	58910
100	800	100	89.04%	53863
100	800	200	88.20%	55491
100	800	300	91.02%	61162
100	800	400	89.17%	52390
100	800	500	89.66%	53962
100	800	600	86.98%	55919
100	800	700	90.85%	60460
100	800	800	88.53%	58017
100	800	900	88.81%	59533
100	500	1000	84.93%	67817
100	600	1000	87.91%	61868
100	700	1000	87.38%	59898
100	800	1000	90.55%	59911
100	900	1000	89.97%	50203
100	1000	1000	90.59%	46375
100	1100	1000	91.13%	45425
100	1200	1000	88.93%	42597
100	1300	1000	89.95%	38656

Table B.1: Simulation Results

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