18.100C. Problem Set 2

Due date: February 23 (Thursday) in lecture or in my office before noon on due date (except for the writing assignment). Late homeworks will be accepted only with a medical note or for some other MIT approved reason. You may work with others, but the final write-up should be entirely your own and based on your own understanding.

Each problem is worth 10 points.

Problem 1: Rudin: ex. 2 and 3 page 43 (extra credit: Can you construct a real number which is not algebraic?)

Problem 2: Recall the notation from lecture: if A is a set, $P(A)$ denotes the set of all subsets of A (the *power set*).

a) Show that the set $S = \{E \subset \mathbb{N} : E \text{ is finite}\}$ is countable. (Hint: write $S = \cup_{n=1}^{\infty} P(\{1, 2, \ldots, n\})$. (Here $\mathbb{N}$ is the set of positive integers.)

b) Show that the set $P(\mathbb{N})$ is equivalent to $\mathbb{R}$. (From lecture, we know that $P(\mathbb{N})$ is uncountable, the question is how to put this set in 1-1 correspondence with $\mathbb{R}$.)

Problem 3: Rudin: ex. 11, page 44.

Problem 4: Rudin: ex. 10, page 44 (except the question about compact sets).

Problem 5: Rudin: ex. 9 (a),(b),(d),(f), page 43.

Writing assignment: Due Wednesday, February 22. Hand in a second draft of the previous assignment. Make corrections, if necessary, to the proofs (especially to part (d)). Include a short introduction. The second draft should look more as a short paper than as homework.

The following problems are recommended for additional practice. They should *not* be turned in with the homework and they will not count towards the homework score. Chapter 2: 5, 7, 8, rest of 9.