18.100C. Problem Set 5

Due date: April 5 (Wednesday) in recitation or in my office before 3 on due date. Late homeworks will be accepted only with a medical note or for some other MIT approved reason. You may work with others, but the final write-up should be entirely your own and based on your own understanding.

Each problem is worth 10 points.

Problem 1: Rudin: Chapter 4, ex. 11 (without the "alternative proof of theorem in exercise 13").

Problem 2: Rudin: Chapter 4, ex. 13 (in addition, answer only the question: "Can $\mathbb{R}$ be replaces by any complete space?").

Problem 3: Rudin: Chapter 4, ex. 14.

Problem 4: Rudin: Chapter 4, ex. 15.

Problem 5: Rudin: Chapter 4, ex. 23. (Prove only that every convex function is continuous).

Problem 6: Rudin: Chapter 4, ex. 24.

Problem 7: A function $f : X \to Y$ between two metric (more generally, topological) spaces is called a *homeomorphism* if f is continuous, bijective, and the inverse f^{-1} is also continuous. Theorem 4.17 in the textbook says that if f is continuous, bijective, and X is compact, then f is necessarily a homeomorphism (see also example 4.21).

(a) Prove (by explicitly constructing homeomorphisms) that any interval in $\mathbb{R}$ is homeomorphic to one of $(0, 1)$, $[0, 1]$, $[0, 1)$.

(b) Prove that there exist surjective continuous maps $(0, 1) \to [0, 1]$, but NOT *bijective* continuous. (The same result, and proof, applies to maps $(0, 1) \to [0, 1)$, $[0, 1) \to [0, 1]$. Note that this is a stronger result than saying that $(0, 1)$ is not homeomorphic to $[0, 1]$.)

The following problems are recommended for additional practice. They should *not* be turned in with the homework and they will not count towards the homework score. Chapter 4: 6, 16, 17, 19.