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Suggested topics for 18.100C. Spring 2006

The topics below are compiled mainly from the following sources:

W. Rudin *Principles of mathematical analysis*, 3rd edition, McGraw-Hill
R. Boas *A primer of real functions*, 4th edition, Carus Mathematical Monographs, MAA, 1996
R. M. Young *Excursions in Calculus*, Dolciani Mathematical Expositions, MAA, 1992
E. Artin *The Gamma Function*, Athena Series, 1964
The American Mathematical Monthly, MAA, www.maa.org/pubs/monthly.html (in Hayden Library: QA.A5125)

1. **Riemann's zeta function**:
- ex 16/p.141, ex. 13,14/p. 198-199 in Rudin.
- the values of $\zeta(2n)$ using Bernoulli numbers: ex. 18 (and references) p. 352-353 in Young.
- T. Osler: *Finding $\zeta(2n)$ from a product of sines*, AMM. vol 111, no 1, 2004, p. 52-54
- H. Tsumoru: *An elementary proof of Euler's formula for $\zeta(2m)$*, AMM vol 111, no 5, 2004, p. 430-431
- G. Rzadkowki: *A short proof of the explicit formula for Bernoulli numbers*, AMM vol 111, no 5, 2004, p. 432-434.
- N. Elkies: *On the sums* $\sum_{k=-\infty}^{\infty} \frac{1}{(4k+1)^n}$, AMM vol 110, no 7, 2003, p. 561-573.

2. **Baire's Theorem**:
- ex. 30/p. 46, ex. 21,22/p.82 in Rudin.
- sec 9 (Nested sets and Baire's theorem) and sec. 10 (some applications of Baire's theorem) in chapter 1 of Boas (see also the references)

3. **Cauchy sequences, completions, p-adic numbers**:
- ex. 23,24,25/p.82 in Rudin.
- sec. 3.1, 3.2 in F. Gouvea's *p-adic numbers: an introduction*

4. **The Cantor set**:
- examples in chapter 2 of Rudin, ex.19/p.81, ex.6/p.138, ex.14/p.168.
- sec 6 (Dense and nowhere dense sets) in chapter 1 of Boas (also references)

5. **The Gamma function**:
- chapter 8 in Rudin
- approximations, Stirling's formula: ex.20/p.200-201 in Rudin, sec. 3 p.20-24 and sec.5 p.29-31 (the error in Stirling's formula) in Artin

6. **Fourier series, Fejér's theorem, Gibbs phenomenon**
- ex. 15,16/p.199 (maybe also 17) in Rudin.
- J. Walker: *Fourier analysis* 1988

7. **Convex functions**
- ex. 23,24/p.101 in Rudin.
- sec. 23 in Boas (and references)
- (maybe) log-convexity of the Gamma function, in Artin.
- P. Roselli, M. Willem: *A convexity inequality*, AMM vol 109, no 1, p. 64-70

8. **Newton's method**
- ex. 25/p.118 in Rudin.
- Suli-Meyers or Hildebrandt: *An introduction to numerical analysis*
- Burden-Faires *Numerical analysis*
- Tjalling J. Ypma: *Historical development of the Newton-Raphson method*, SIAM Review 37, p.531-551, 1995.

9. **Applications of the intermediate value property and fixed points**
- fixed points: ex. 22,23,24/p.117-118 in Rudin.
- end of sec 14 in Boas (and references)
- bisecting curves, areas, volumes (see the references in Boas sec 14)
- R. Brown, J. Girolo: *Isolating fixed points* (and references), AMM vol 109, no 7, 2002, p. 595-611

10. **Derivatives**: Boas p. 153-158 and references therein
- when is the product of two derivatives a derivative?
- on the differentiability of the ruler function (and other exotic functions).
- L. Wen : *A nowhere differentiable continuous function constructed by infinite products*, AMM vol 109, no 4, 2002, p. 378-380.

11. **Lebesgue's differentiation theorem**
- R. Boas
- M. Botsko: *An elementary proof of Lebesgue's differentiation theorem*, AMM vol 110, no 9, p. 834-838

12. **Rearrangement of a conditionally convergent series**
- U. Elias: *Rearrangement of a conditionally convergent series*, AMM vol 110, no 1, 2003, p. 57
- J. Stefansson: *Forward shifts and backward shifts*, AMM, vol 111, no 10, 2004, p. 913-914
- S. Krantz, J. McNeal: *Creating more convergent series*, AMM vol 111, no 1, 2004, p. 32-38.