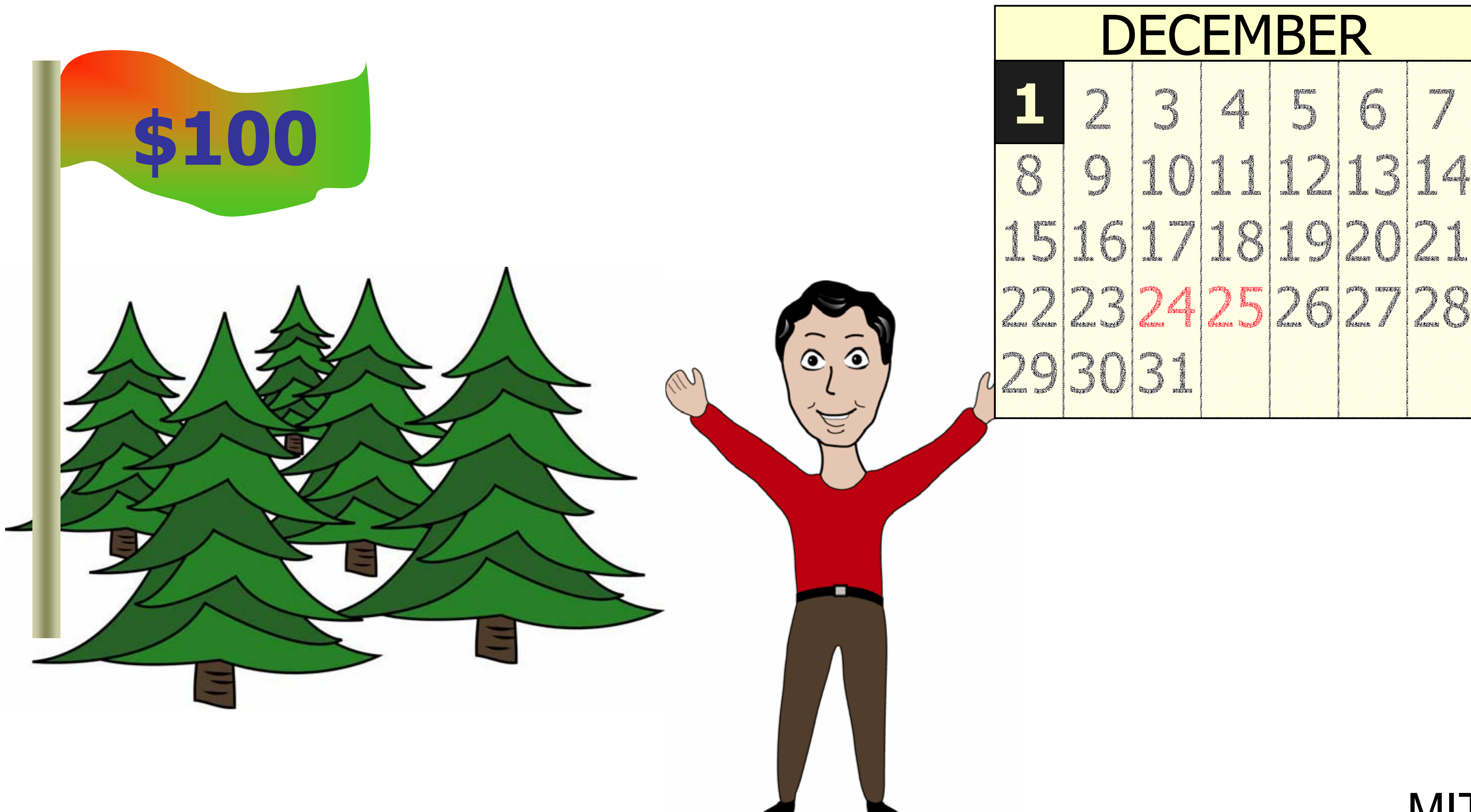
A Classical Trade Off

The Newsvendor Model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15 16 17 18 19 20 21
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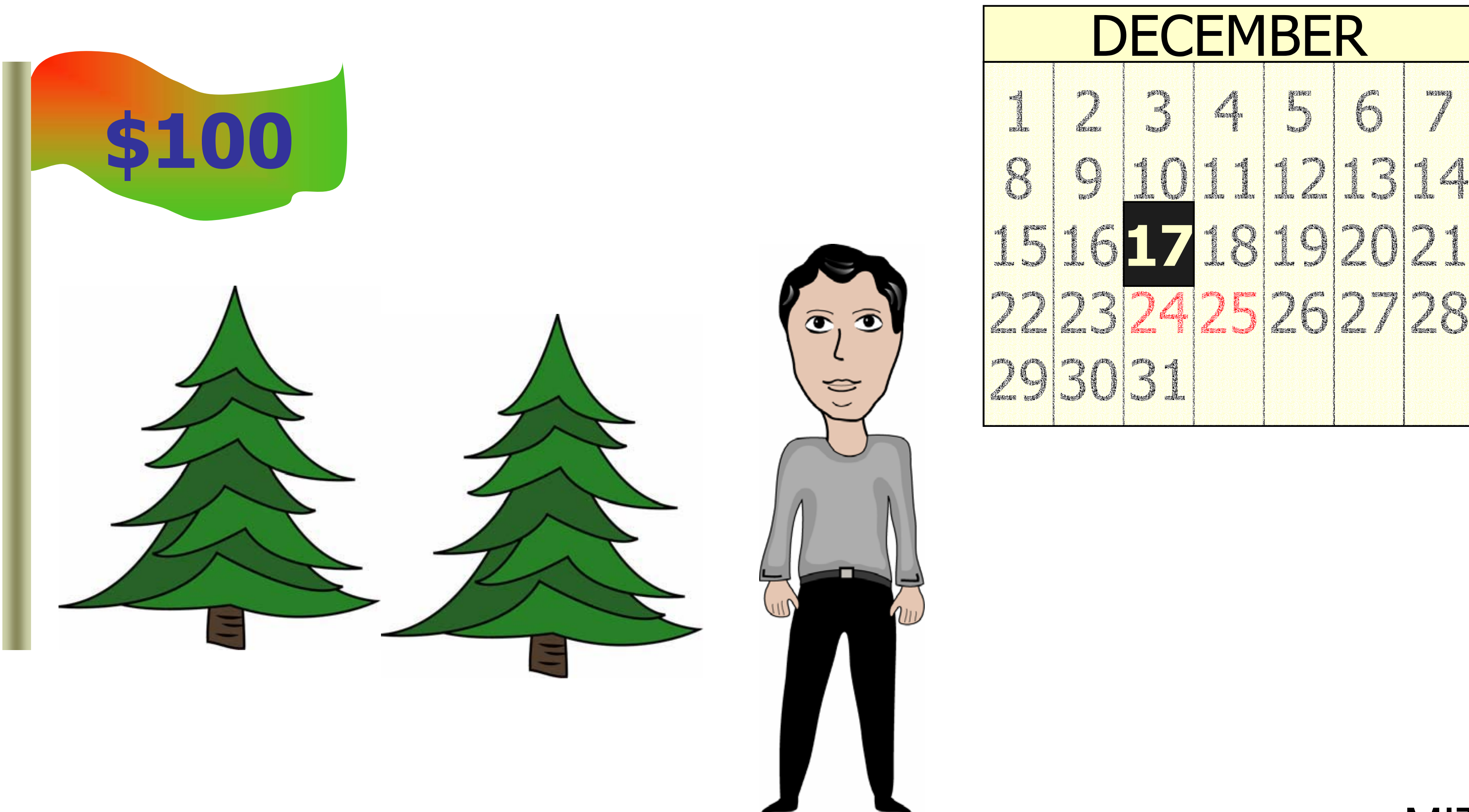
29 30 31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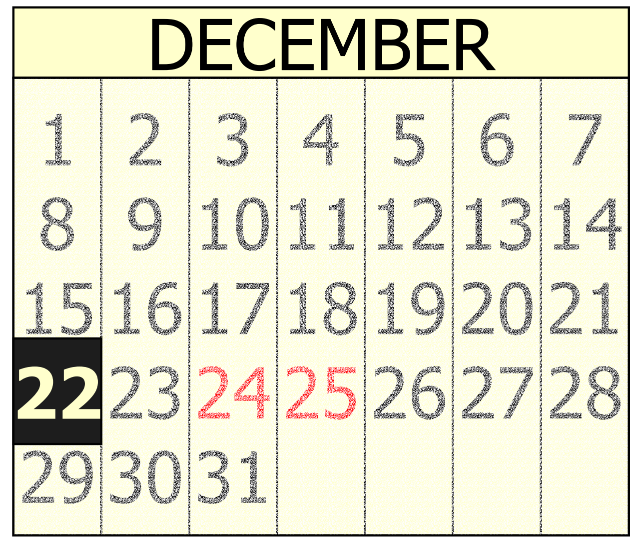
Sold Out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DECEMBER						
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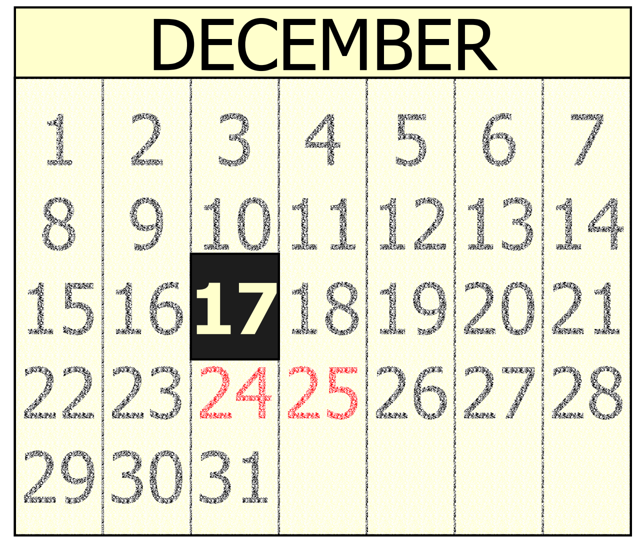
$100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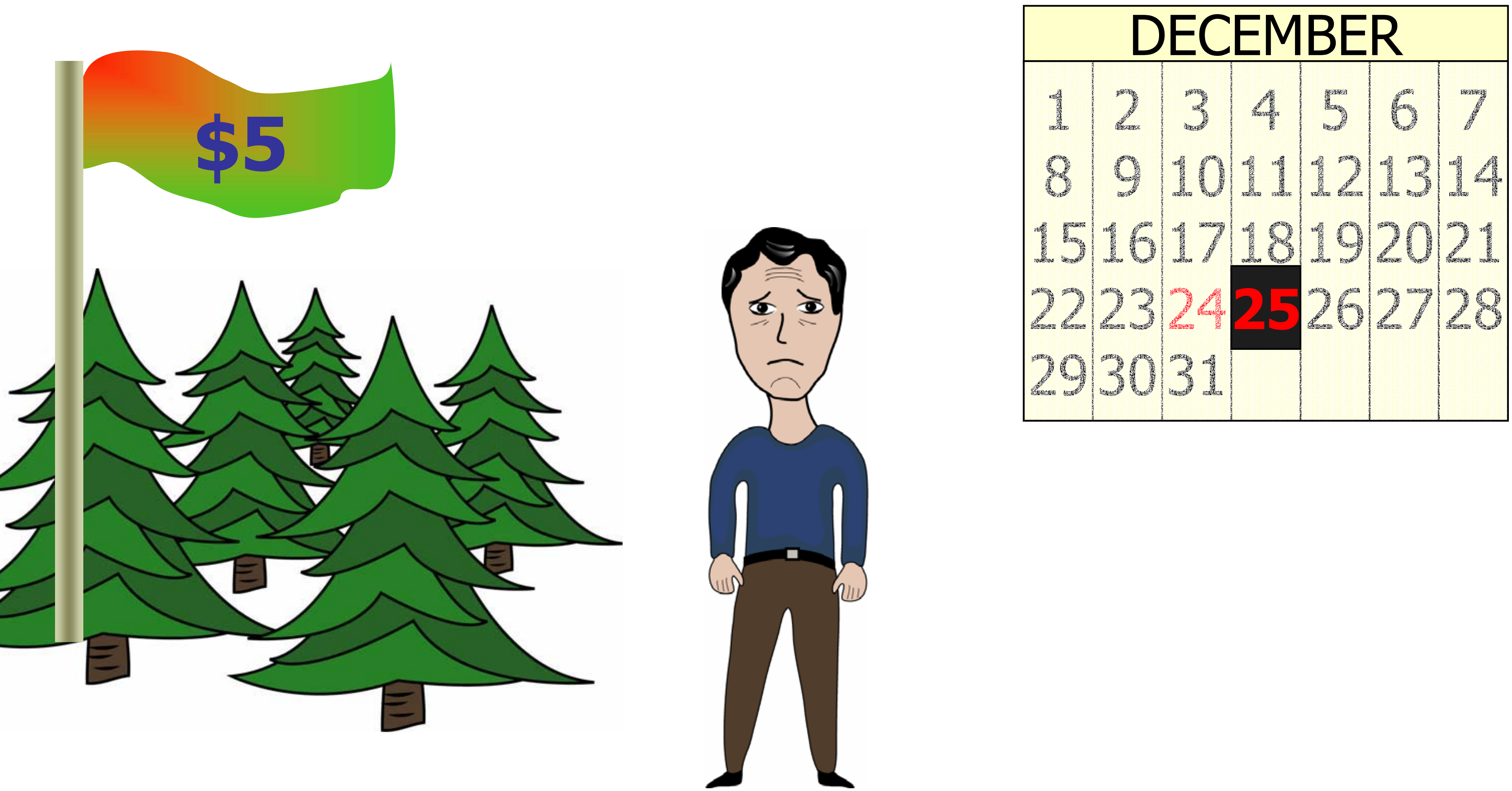
$5
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The Principal Trade-Off

- Ordering Too Much
  - Inventory left over at Period End
  - Inventory sold at Loss
- Ordering Too Little
  - Not all Demand is served
  - Loosing out on Revenue

Newsvendor Problem

- How much to Produce or Buy
- One Time Shot
- “Perishable Good”
- Excess Demand is Lost

Model Parameters

- c = Unit Cost
  - to produce or buy
- r = Unit Revenue
- s = Unit Salvage Value
  - Unit Revenue at end of Period
- d = Demand (unknown)
- q = Order Quantity

Model Derivation…

	IF d > q	IF d ≤ q
Profit:	$q \cdot (r - c)$	$d \cdot (r - c) + (q - d) \cdot (s - c)$
Incremental Analysis:	$q \to q + 1$:	
Δ Profit:	*r - c*	*s - c*
EAP:	$P(d > q) \cdot (r\text{-}c) + P(d \le q) \cdot (s\text{-}c)$	

As long as the *Expected Additional Profit* [EAP] is positive, it's lucrative to increase *q* to *q + 1* !!!

…Model Derivation

Determine *q*, such that the Expected Additional Profit is just 0.

$$P(d > q)\cdot(r\text{-}c) + P(d \le q)\cdot(s\text{-}c) = 0 \quad !!!$$

$$P(d \le q)\cdot(s\text{-}c) = -P(d > q)\cdot(r\text{-}c)$$

$$P(d \le q)\cdot(s\text{-}c) = -[1 - P(d \le q)]\cdot(r\text{-}c)$$

$$P(d \le q)\cdot[(s\text{-}c) - (r\text{-}c)] = -(r\text{-}c)$$

The Newsvendor Formula

$$P\left(d \le q\right) = \frac{r - c}{r - s}$$

An Interpretation

$$P(d \le q) = \frac{r-c}{r-s} = \frac{r-c}{r-c+c-s}$$

$r-c$: Cost of Having too little, *Underage*. k_U

$c-s$: Cost of Having too much, *Overage*. k_O

$$P(d \le q) = \frac{P(d \le q)}{P(d \le q) + P(d > q)} = \frac{k_u}{k_u + k_o}$$

Fast-Clockspeed Components

Digital Devices Incorporated (DDI) is in an intensely competitive market for high-speed signal processing chips. They have perennially been an industry leader, but now find that within ten weeks of the release of a new chip, a swarm of copycat competitors rush in and kill any further profitability for their product. Thus DDI basically launches a product that has an expected economic life of ten weeks. DDI is about to launch a new version of its *Fruitfly* TM signal-processing chip. They estimate that the demand until the swarm arrives has a normal distribution with a mean of 150,000 and a standard deviation of 45,000. The margin on the chip (price minus variable cost) is expected to be about $100 for the first ten weeks; after which the market completely dries up for DDI's product. (The price charged is $150 and the cost is $50.) They must commit now with their foundry supplier as to how many *Fruitfly* TM chips to produce. What number should they choose?

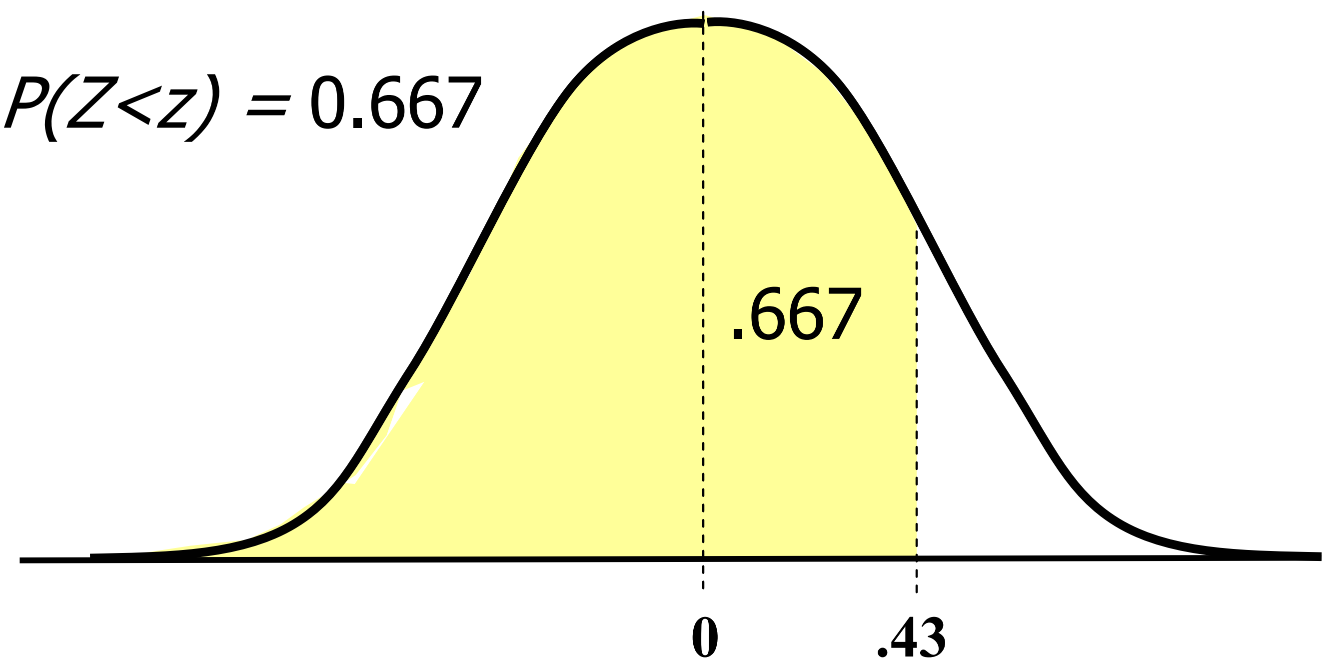
Problem 2

$r = 150, \quad c = 50, \quad s = 0$

$$P(d < q) = \frac{r - c}{r - s} = \frac{150 - 50}{150} = \frac{2}{3} = .667$$

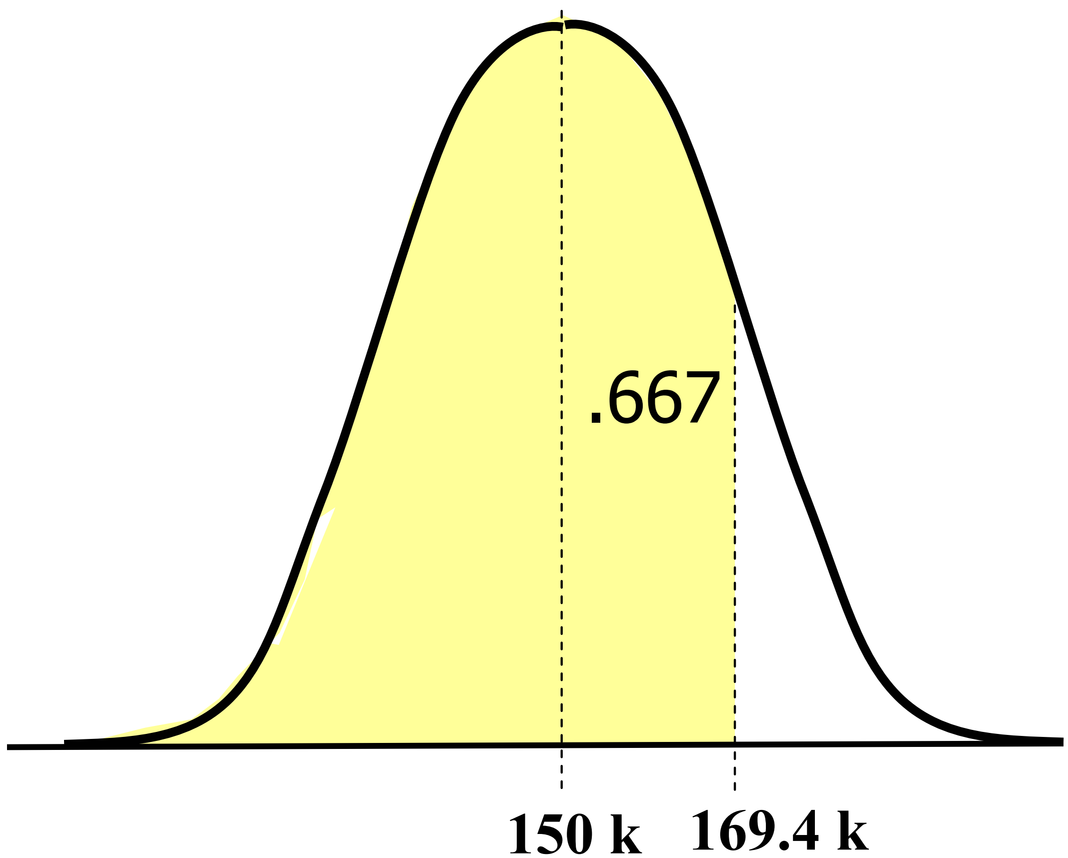
Look up in Table $P(Z<z) = 0.667$

$\Rightarrow z = 0.43$



Problem 2

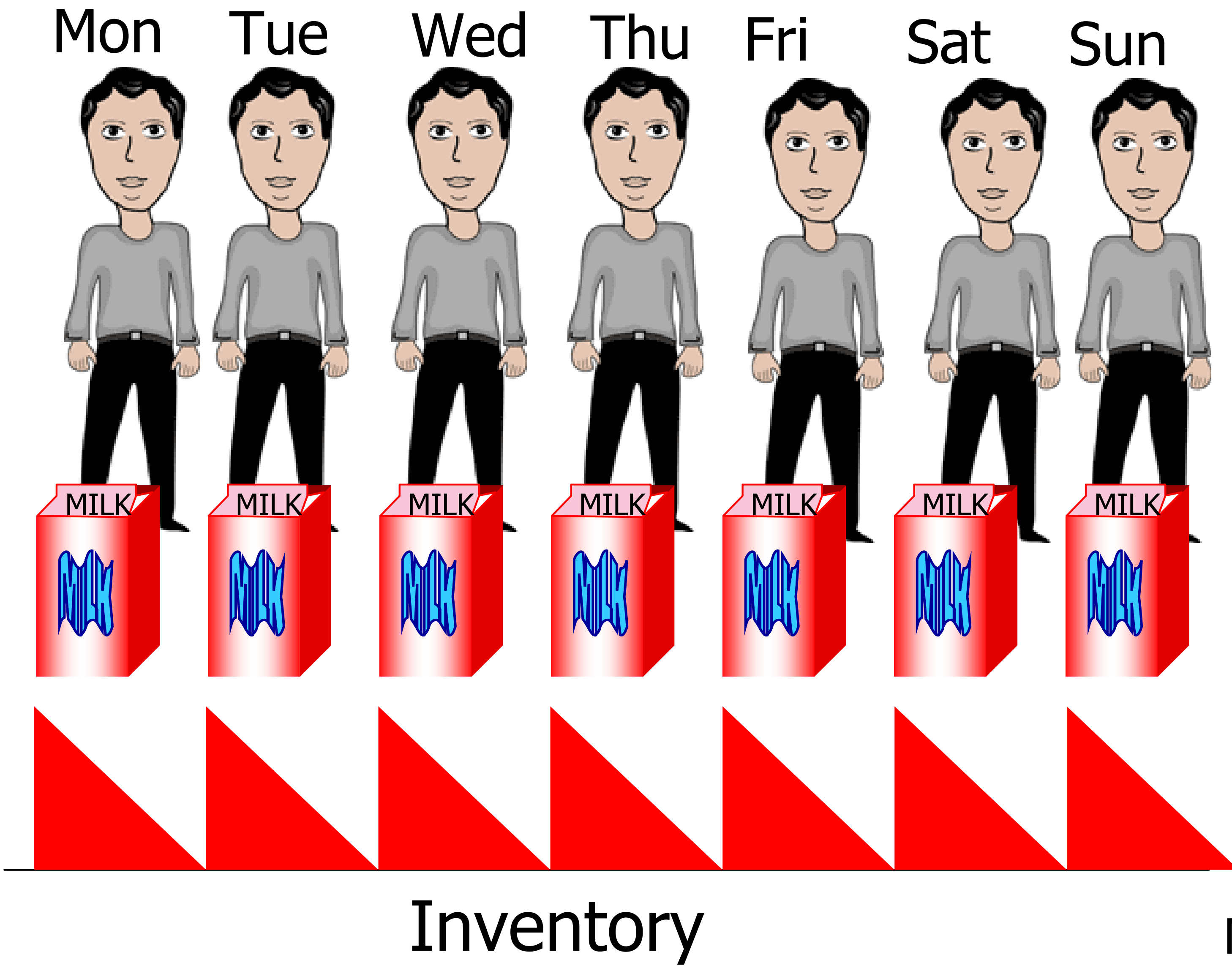
$$Q = \mu + z * \sigma = 150{,}000 + .43 * 45{,}000 = 169{,}350$$



A Different Trade-Off

The EOQ Model

Running to the Store a Lot…



...Running to the Store a Little

The Principal Trade-Off

- Ordering Too Much
  - High average Inventory
- Ordering Too Little
  - High Ordering Costs

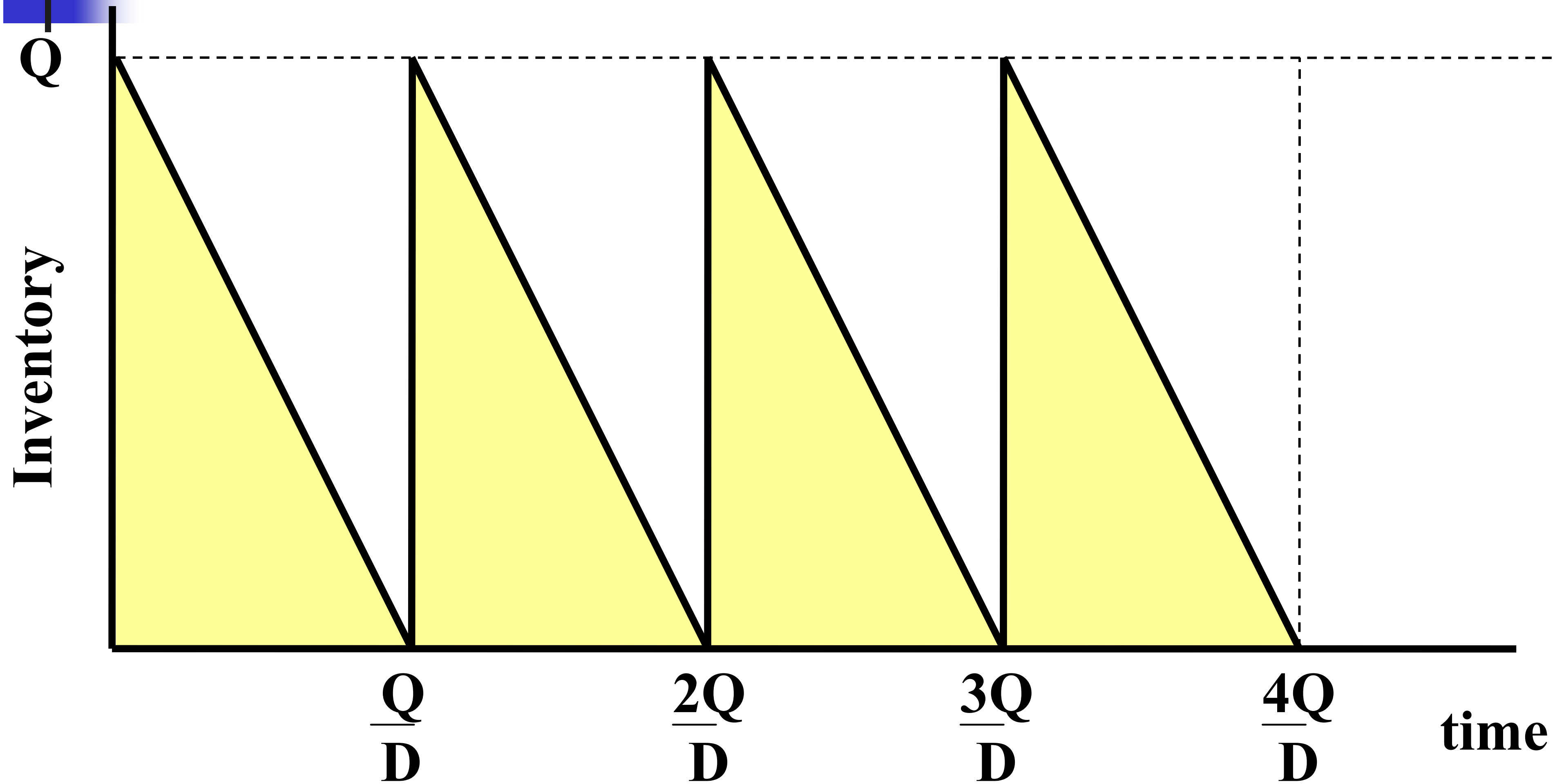
Economic Order Size (EOQ)

- How much to Produce or Order
- Repeated Ordering
- Known and Constant Demand
- Fixed Cost per Order
- Inventory Holding Costs
- Instantaneous Replenishment

Model Parameters

- D = Demand Rate [units/time]
- C = Cost per Unit [$]
- K = Inventory Carrying Costs [% p.a.]
- S = Cost per Order or Setup
- Q = Order Quantity (Batch Size) [units]

Inventory Build-Up Diagram



Inventory Costs

Inventory Costs $K \cdot \frac{C \cdot Q}{2}$ Average Inventory

Order Costs: $S \cdot \frac{D}{Q}$ Number Of Orders

Total Costs: $TVC = S \cdot \frac{D}{Q} + C \cdot K \cdot \frac{Q}{2}$

EOQ Formula

Set First Derivative to 0:

$$\frac{\partial TVC}{\partial Q} = -\frac{DS}{Q^2} + \frac{CK}{2} = 0$$

Yields:

$$Q^* = \sqrt{\frac{2 \cdot DS}{CK}}$$

Dellpaq predicts a steady demand of 300,000 of its AMX-based systems each year for the next three years Dellpaq purchases the AMX processor and motherboards from its leading Taiwanese supplier at the cost of $200 per set, and assembles them into complete systems in a Dellpaq-Taiwan factory. The average total cost of an assembled system is approximately $3000. Dellpaq ships assembled systems by boat to its central warehouse in Portland. The loading plus shipping costs are $100,000 per trip plus $25 per system. The ship capacity is 10,000 sets. Placing a shipment order takes about 5 hours of direct labor, and shipping takes a week to arrive at Dellpaq's central warehouse facility in Portland.

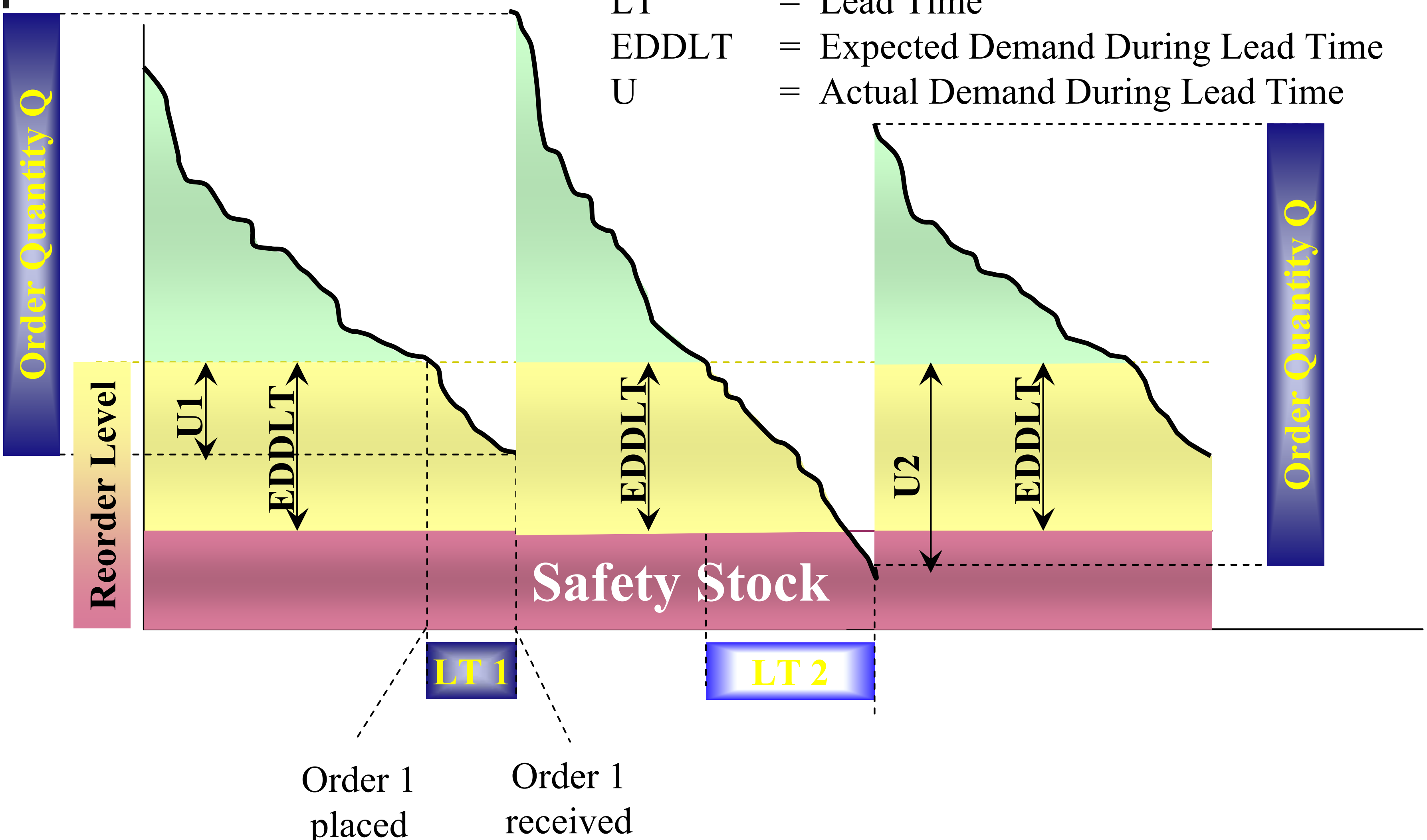
Unloading at the warehouse occurs at the rate of about 10 systems per employee hour, and requires rental equipment that costs $50/hour. The warehouse storage capacity is 15,000 systems, and every system must be routed via the warehouse before being routed by air freight to its final customer. At the warehouse, the employees have several tasks:

- arranging the systems in storage according to the options packages at the rate of 20/employee/hour;
- removing the sets for shipment upon receipt of an order requires 1/8th of an hour per set;
- surveillance/security equipment at the facility costs about 10,000 hours per year.

All the workers in the chain cost the same: $10/hour of labor. The cost of shipping a system from the warehouse to a customer is $20/system for packaging materials and $45 per system for delivery. Dellpaq computer expects a pre-tax rate of return of 20% on its investment. What ordering policies would you recommend to Dellpaq?

Perpetual Review System

LT = Lead Time
EDDLT = Expected Demand During Lead Time
U = Actual Demand During Lead Time



MIT Sloan

Perpetual Review System: Parameter Choice

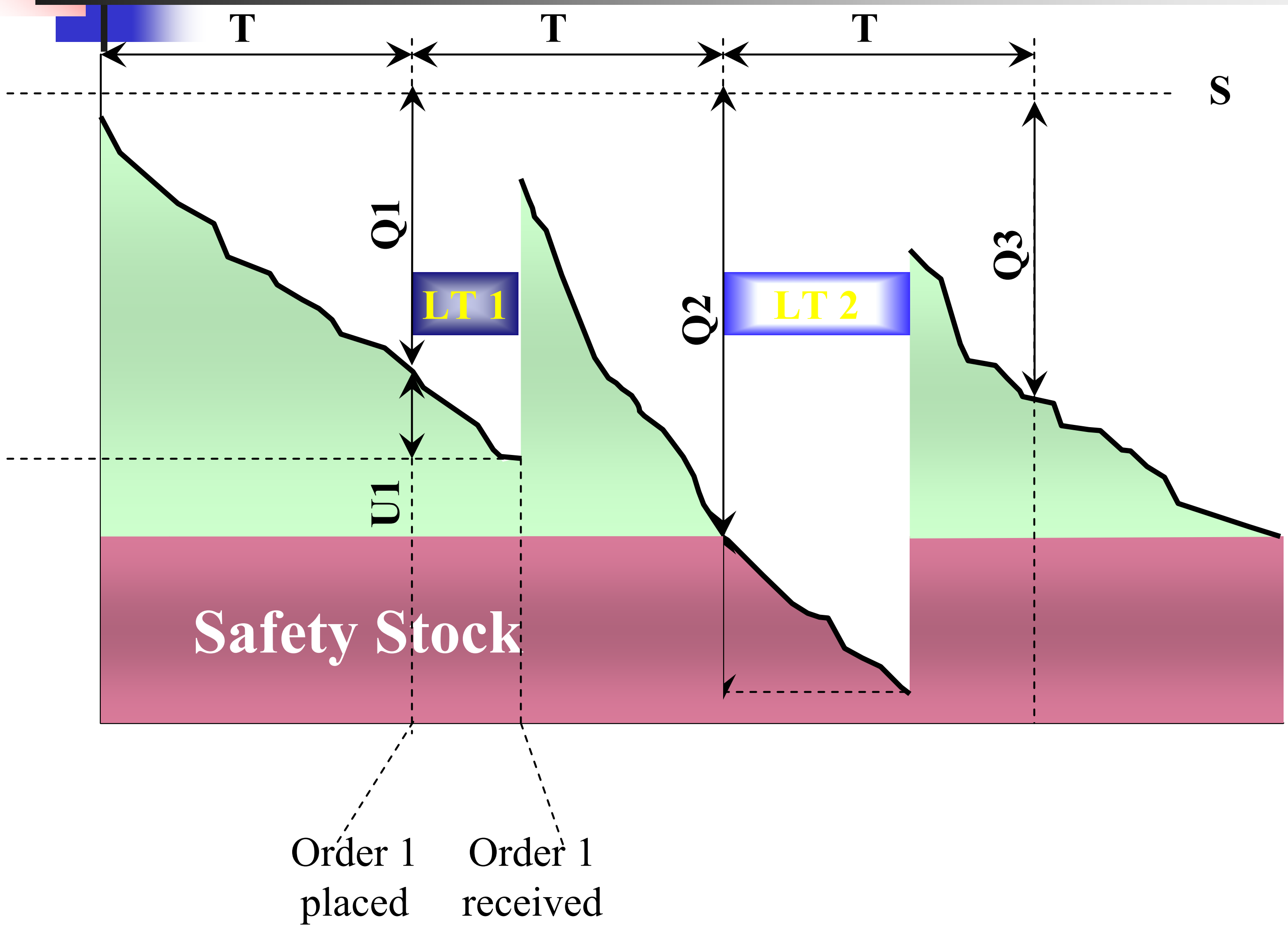
1. Order Size *Q:* Use *EOQ*

- Choose Reorder Point *R,* such that $P(DDLT<R) = f$
- Fractile *f*:
  - A given Service Level or
  - $f = k_U/(k_U+k_O)$ (from Newsboy)
- $DDLT \cong$ Demand During Lead Time
- Safety Stock: $SS = R - E[DDLT]$
- Average Inventory Position*: $AIP = R+Q/2$
- Average Inventory (on Hand)**: $AI \approx SS + Q/2$

* Inventory Position = Inventory on Hand + Outstanding Orders – Backorders

** The formula is an approximation since it incorporates backorders when stockouts occur. If service levels are high, (and stockouts rare) then the approximation is quite good.

Periodic Review System



LT = Lead Time
T = Cycle Time or Review Period
U = Actual Demand During Lead Time
Q = Order Size
S = Order Up To Level

Periodic Review System: Parameter Choice

1. Average Order Size: $Q = EOQ$
2. Time Between reviews: $T = Q/D$
3. Choose Order Up To Level S such that $P(DD(T+LT)<S) = f$
4. Fractile f as defined before
5. $DD(LT+T) \cong$ Demand dur. Lead Time + Rev. Period T
6. Safety Stock: $SS = S - E[DD(T+LT)]$
7. Average Inventory Position*: $AIP = S\text{-}Q/2$
8. Average Inventory (on Hand)**: $AI \approx SS + Q/2$

* Inventory Position = Inventory on Hand + Outstanding Orders – Backorders

** The formula is an approximation since it incorporates backorders when stockouts occur. If service levels are high, (and stockouts rare) then the approximation is quite good.

Wrap Up

- "The Entire Life is a Newsboy Problem"
  - One Shot Deals
  - Too much vs. too little
- EOQ
  - Repeatability
  - Ordering (Setup) vs. Inventory Costs
- Together, meaningful Ordering Policies