14.461 Problem Set 1

Fall 2009

Problem 1

In this problem you will replicate the simulations in the lecture notes for the standard stochastic growth model. Consider a stochastic growth model with preferences and technology given by

$$\begin{aligned} U\left(C_t, N_t\right) &= \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\eta} N_t^{1+\eta}, \\ A_t F\left(K_{t-1}, N_t\right) &= A_t K_{t-1}^{\alpha} N_t^{1-\alpha}. \end{aligned}$$

The process for A_t is as follows

$$A_t = e^{a_t},$$

$$a_t = \rho a_{t-1} + \epsilon_t.$$

Use parameters

$$\begin{aligned} \beta &= 0.99, \quad \delta = 0.025, \\ \eta &= 1, \quad \sigma = 1, \\ \alpha &= 0.36, \quad \rho = 0.95. \end{aligned}$$

(i) Setup the planner problem and derive the first order conditions.

(ii) Log-linearize the conditions characterizing the equilibrium and solve for the recursive equilibrium law of motion with the method of undetermined coefficients.

(iii) Plot impulse response functions for a_t, i_t, c_t, y_t, n_t (with small capitals denoting logs).

(iii) Replace the technology process with

$$a_t = \rho a_{t-1} + \epsilon_{t-3}.$$

Derive impulse response functions to ϵ_t for a_t, i_t, c_t, y_t, n_t in the new model.

(iv) Try to change the elasticity of intertemporal substitution σ and see how it affects equilibrium dynamics.

Problem 2

In this problem you analyze the role of wealth effects in the model with habit formation of Christiano, Ilut, Motto and Rostagno.

Consider a stochastic growth model as in Problem 1, except that preferences are given by:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \left(\log \left(C_t - bC_{t-1} \right) - \frac{1}{1+\eta} N_t^{1+\eta} \right)$$

and the law of motion of the capital stock is

$$K_t = (1-\delta) K_{t-1} + \left(1 - \frac{a}{2} \left(\frac{I_t}{I_{t-1}} \right)^2 \right) I_t.$$

(i) Setup the planner problem and derive first order conditions.

(ii) Log-linearize the conditions characterizing the equilibrium and solve for the recursive equilibrium law of motion with the method of undetermined coefficients.

(iii) Simulate the model with the anticipated technology process

$$a_t = \rho a_{t-1} + \epsilon_{t-3}$$

for different values of the parameters a and b. Try to replicate the impulse responses to ϵ_t obtained by Christiano et al.

Now consider the following partial equilibrium exercise. Consider the problem of a consumer who maximizes

$$\sum_{t=0}^{\infty} \beta^t \left(\log \left(C_t - bC_{t-1} \right) - \frac{1}{1+\eta} N_t^{1+\eta} \right)$$

subject to the budget constraint:

$$\sum R^{-t} \left(C_t - W N_t \right) = B_0.$$

Suppose $R\beta = 1$.

(iv) Setup the consumer problem and derive optimality conditions.

(v) Show that for the right initial condition $C_{-1} = \bar{C}$ the problem is in steady state with

$$\bar{C} = \frac{1}{1-\beta} W \bar{N} + B_0$$

(vi) Suppose there is an unexpected shock $\Delta > 0$ to non-labor income at date 0, so the consumer begins life with $B_0 + \Delta$. Compute the optimal path of consumption and labor supply for different values of b.

(vii) What is the impact response of labor supply? What is the long run response? Comment.