14.461 Problem Set 2

Fall 2009

Problem 1: Imperfect information on technology

[Based on Angeletos and La'O, 2009]

Consider an economy with a continuum of sectors $[0,1]$. In each sector there is a continuum of producers. The representative producer in sector i has preferences given by

$$E\left[C_i - \frac{1}{2}N_i^2\right]$$

where the consumption $C_{i,t}$ has the usual CES form

$$C_i = \left(\int {C_{i,j}}^{1-\rho} dj\right)^{\frac{1}{1-\rho}}$$

with $\rho \geq 0$. The timing is as follows. First, in the *production stage*, each producer i produces good i according to the production function

$$Y_i = A_i N_i.$$

Second, in the *trading stage*, they sell Y_i on a competitive market and consume at the same time (no production can take place in this stage). Use P_i to denote the price of the good produced by sector i and P to denote the appropriate CPI.

The sectoral productivities take the form

$$A_i = \exp\{\varepsilon + \eta_i\}$$

where η_i are idiosyncratic shocks (i.e., they satisfy $\int_0^1 \eta_i di = 0$) and ε is an aggregate shock. The shocks ε and η_i are independent normal random variables with zero mean and variances σ_ε^2 and σ_η^2.

(i) Suppose first that producer i observes both the aggregate shock ε and the sectoral shock η_i when choosing N_i. Setup the individual optimization problem. Derive first order conditions for labor supply in the production stage and for the consumption of each good in the trading stage.

(ii) Define $C = \int C_j dj$ and show that the total output of sector j must be equal to $Y_i = (P_i/P)^{-1/\rho} C$. Using market clearing in each sector i and the agents' budget constraints show that aggregate consumption C in the trading stage satisfies

$$C = \left(\int_0^1 (A_i N_i)^{1-\rho} \right)^{\frac{1}{1-\rho}}.$$

(iii) Derive a relation between the relative price in sector i, P_i/P, and the cross sectional distribution of productivities and labor supplies $\{A_j, N_j\}_{j\in[0,1]}$.

(iv) Show that there is an equilibrium with labor supply in each sector given by

$$n_i = \bar{n} + \frac{1-\rho}{1+\rho} a_i + \frac{2\rho}{1+\rho} a,$$

or, equivalently, by

$$n = \bar{n} + \varepsilon + \frac{1-\rho}{1+\rho} \eta_i,$$

where lowercase denote logs and $\bar{n}$ is a constant (absolutely no need to derive the constant term $\bar{n}$ explicitly). How does aggregate labor supply responds to the shocks? What's the effect of idiosyncratic shocks? How different is this economy from an economy with only aggregate shocks?

(v) Now consider the case where agents do not observe ε and η_i separately, they only observe their sectoral productivity A_i and a public signal

$$s = \varepsilon + \nu,$$

where ν is an additional random shock, normal with mean zero and variance σ_ν^2. The signal s is the same for all agents. Find a log-linear equilibrium where aggregate labor supply takes the form

$$n = \bar{n} + \psi_a a + \psi_s s,$$

(Hint: use the fact that $E[a|a_i, s] = \beta_a a_i + \beta_s s$ for some coefficients β_a and β_s and use a method of undetermined coefficients to find ψ_a and ψ_s).

(vi) Using analytical tools or numerical examples see how aggregate employment and aggregate output responds to the aggregate shocks ε and ν for different values of ρ, σ_ε^2, σ_ν^2, σ_η^2. Can the response of employment to ε be negative? If yes, explain. Can the response of employment to ν be negative?

1 Problem 2

Change the matlab file provided online to deal with the general case treated in Woodford (2001), where the money supply follows the process

$$\Delta m_t = \rho \Delta m_t + u_t.$$

The Kalman filtering part needs minor changes, but the fixed point for ϕ needs to be expanded.

Replicate the right panels of Figure 5 in Woodford (2001), that is show that for ρ large enough the model with imperfect information can generate a peak in inflation that follows the peak in output.

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