14.461 Part II
Problem Set 1

Fall 2009

1 Wage Dispersion

This problem extends the search model with random matching and Nash bargaining seen in class to allow for match-specific productivity. This simple extension will be able to generate wage dispersion.

Time is discrete and horizon infinite. There is a continuum of risk-neutral ex-ante homogeneous workers of measure 1 and a continuum of larger measure of risk-neutral ex-ante homogeneous firms. They have common discount factor β. Workers can search freely, while firms have to pay a cost k to open a vacancy. At the beginning of the period firms post vacancies and workers search for a job. Then matching takes place according to a standard constant returns to scale matching function. Let $\mu(\theta_t)$ denote the probability a worker meets a firm and $\mu(\theta_t)/\theta_t$ the probability a firm meets a worker, where θ_t is the market tightness. Assume $\mu(\theta)$ is continuous and twice differentiable with $\mu'(\theta) > 0$ and $\mu''(\theta) < 0$ for all $\theta \in [0, \infty)$. Also, assume $\mu(\theta) \leq \min\{\theta, 1\}$. When a firm and a worker meet, they draw a match-specific productivity y from a distribution $F(\cdot)$ with full support on $Y \equiv [\underline{y}, \overline{y}]$, where y is observed by both and constant until separation, that happens with probability s. Assume that $F(.)$ is differentiable, with $f(.)$ denoting the associated density function. After observing y, the worker and the firm bargain the wage w_t according to generalized Nash bargaining. If a worker is unemployed (either because is not matched or because walk away from the match) he gets b (home production) and can search next period.

1. Write down the value functions for an unemployed worker (U_t), an employed worker in a match producing y ($V_t(y)$), a firm with an unfilled vacancy (W_t), and a firm with a filled vacancy and productivity y ($J_t(y)$). All the value functions are evaluated at the beginning of the period before matching and separation take place. Use $e_t(y) \in \{0, 1\}$ to denote whether a job is created after observing y.

2. Focus on the steady state equilibrium. Write down the generalized Nash bargaining problem, assuming that the worker has bargaining power η, and show how the surplus is split among the worker and the firm. Do all matches lead to job creation?

3. Find the steady state equilibrium. In particular, characterize (as far as you can) the steady state job creation and wages.

4. Recover the steady state wage distribution. How does it vary with an increase in b?

5. Write down the Planner problem. Is the equilibrium constrained efficient?

2 Firms' Superior Information

This problem consider a competitive search model with asymmetric information and firms' limited liability similar to the one we have seen in class. The main difference is that now firms have private information.

Consider a competitive search model with the same preferences and technology described above, with the difference that match-specific productivity is now private information of the firm. Moreover, we assume that there is limited liability on the firm side, that is, a firm can always decide to go bankrupt, fire the worker and stop paying them, with no punishment. At the beginning of each period t, firms can open a vacancy at cost k which entitles them to post an employment contract C_t. Then workers observe all the posted contracts and decide where to apply. Let $\Theta_t(C_t)$ be the market tightness associated to contract C_t. Let $\mu(\Theta_t(C_t))$ denote the probability a worker applying for C_t meets a firm and $\mu(\Theta_t(C_t))/\Theta_t(C_t)$ the probability a firm posting C_t meets a worker. After a match is formed, y is drawn and observed by the firm. Assume also that $F(.)$ satisfies the monotone hazard rate condition $d(F(y)/f(y))/dy > 0$. Given firms' limited liability, firms will never make payments to workers they do not hire. If a worker is hired the match is productive until separation which happens with probability s. Finally, if a worker is not matched or not hired, he gets b and search next period.

1. Firms can post unrestricted contracts. Using the revelation principle, you can restrict attention to incentive-compatible and individually-rational direct mechanisms. How can you represent such a contract? Write down the form of the contract and the IC and IR constraints (for whom?) that such a contract has to satisfy.

2. Define a Competitive Search Equilibrium, restricting attention to the set of incentive-compatible and individually-rational contracts.

3. Show if it is possible to characterize any competitive search equilibrium with a constrained optimization problem.

4. Show that there exists a CSE. Is this equilibrium unique?

5. Consider a social planner who does not observe y and cannot force firms to make transfers to the workers if they do not hire them (that is, the planner faces also a limited liability constraint on the firms' side). Write down the sequence problem that defines the region of the Pareto frontier where the firms receive zero utility.

6. **OPTIONAL.** Write down a recursive version of the planner problem. Is the equilibrium constrained efficient?

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