1 Balance sheets and asset prices

- Kiyotaki and Moore (1997)

- Mechanism: balance sheet effects + forward looking prices $\Longrightarrow$ amplification

- Risk neutral consumers and entrepreneurs with preferences

$$\sum \beta^t c_t$$

- Two goods: consumption good, capital in fixed supply $\bar{k}$, never depreciates

- Relative price of the capital good q_t

- Entrepreneurs ("farmers") flow of funds

$$c_t^E + q_t k_{t+1} \leq n_t + \beta b_{t+1}$$

- Net-worth dynamics

$$n_t = (a + q_t)\, k_t - b_t$$

- Collateral constraint

$$b_{t+1} \leq q_{t+1} k_{t+1}$$

- Inalienable human capital of entrepreneurs necessary to produce a (a form of limited enforcement)

- Alternative use for capital: concave production function controlled by the consumers ("gatherers")

$$\tilde{y}_t = G\left(\tilde{k}_t\right)$$

- Market clearing

$$k_t + \tilde{k}_t = \bar{k}$$

- Optimality condition for the use of capital in the G sector (unconstrained)

$$q_t = \beta\left[q_{t+1} + G'\left(\tilde{k}_{t+1}\right)\right]$$

- Initial conditions: k_0 and b_0

- Suppose initial conditions such that entrepreneurs repay, i.e. $\exists$ equilibrium with

$$q_0 k_0 \geq b_0$$

Some results:

- the entrepreneurs are constrained and consume $c_t^E = 0$ for the first T periods (T could be zero)

- After T they are unconstrained and the price is equal to

$$q_t = q^* = \frac{\beta}{1-\beta} a$$

and capital stock invested in entrepreneurial firms is $k_{t+1} = k^*$, such that

$$a = G'\left(\bar{k} - k^*\right)$$

- In all previous periods $k_{t+1} < k^*$ and $q_t < q^*$

- Find sequence that satisfies

$$q_t = \beta \left[q_{t+1} + G' \left(\bar{k} - k_{t+1} \right) \right]$$

  and

$$q_t k_{t+1} = (a + q_t) k_t - b_t + \beta q_{t+1} k_{t+1}$$

  up to period $T - 1$, and the second as $\geq$ from T onwards

Check optimality

$$V_t(n_t) = \max_{c_t^E, k_{t+1}, b_{t+1}} c_t^E + \beta V_{t+1}\left(\left(a + q_{t+1}\right) k_{t+1} - b_{t+1}\right)$$

$$\begin{aligned} c_t^E + q_t k_{t+1} &\leq n_t + \beta b_{t+1} \\ b_{t+1} &\leq q_{t+1} k_{t+1} \end{aligned}$$

- FOC

$$\begin{aligned} 1 &\leq \lambda_t \\ \lambda_t q_t &= \beta\left(a + q_{t+1}\right) V'_{t+1} + \mu_t q_{t+1} \\ \lambda_t \beta &= \beta V'_{t+1} + \mu_t \end{aligned}$$

- Envelope

$$V'_{t+1} = \lambda_t$$

- Decreasing sequence of λ_t that converges to $\lambda_t = 1$ in finite time

$$
\begin{aligned}
q_t \lambda_t &= \beta \left(a + q_{t+1}\right) \lambda_{t+1} + \mu_t q_{t+1} \\
\mu_t &= \beta \lambda_t - \beta \lambda_{t+1}
\end{aligned}
$$

$$
q_t = \beta \left(a \frac{\lambda_{t+1}}{\lambda_t} + q_{t+1} \right) < \beta \left(a + q_{t+1}\right)
$$

fine as long as

$$
\beta \frac{a + q_{t+1}}{q_t} > 1
$$

and delivers

$$
\lambda_t = \frac{\beta a}{q_t - \beta q_{t+1}} \lambda_{t+1} = \frac{\beta a}{q_t - \beta q_{t+1}} \frac{\beta a}{q_{t+1} - \beta q_{t+2}} \dots \frac{\beta a}{q^* - \beta q_{T-1}}
$$

Finding an equilibrium

- Balance sheet relation:

$$k_1 = \frac{(a + q_0)\, k_0 - b_0}{q_0 - \beta q_1} = \frac{(a + q_0)\, k_0 - b_0}{\beta G'\left(\bar{k} - k_1\right)}$$

increasing relation between asset price q_0 and investment in entrepreneurial sector

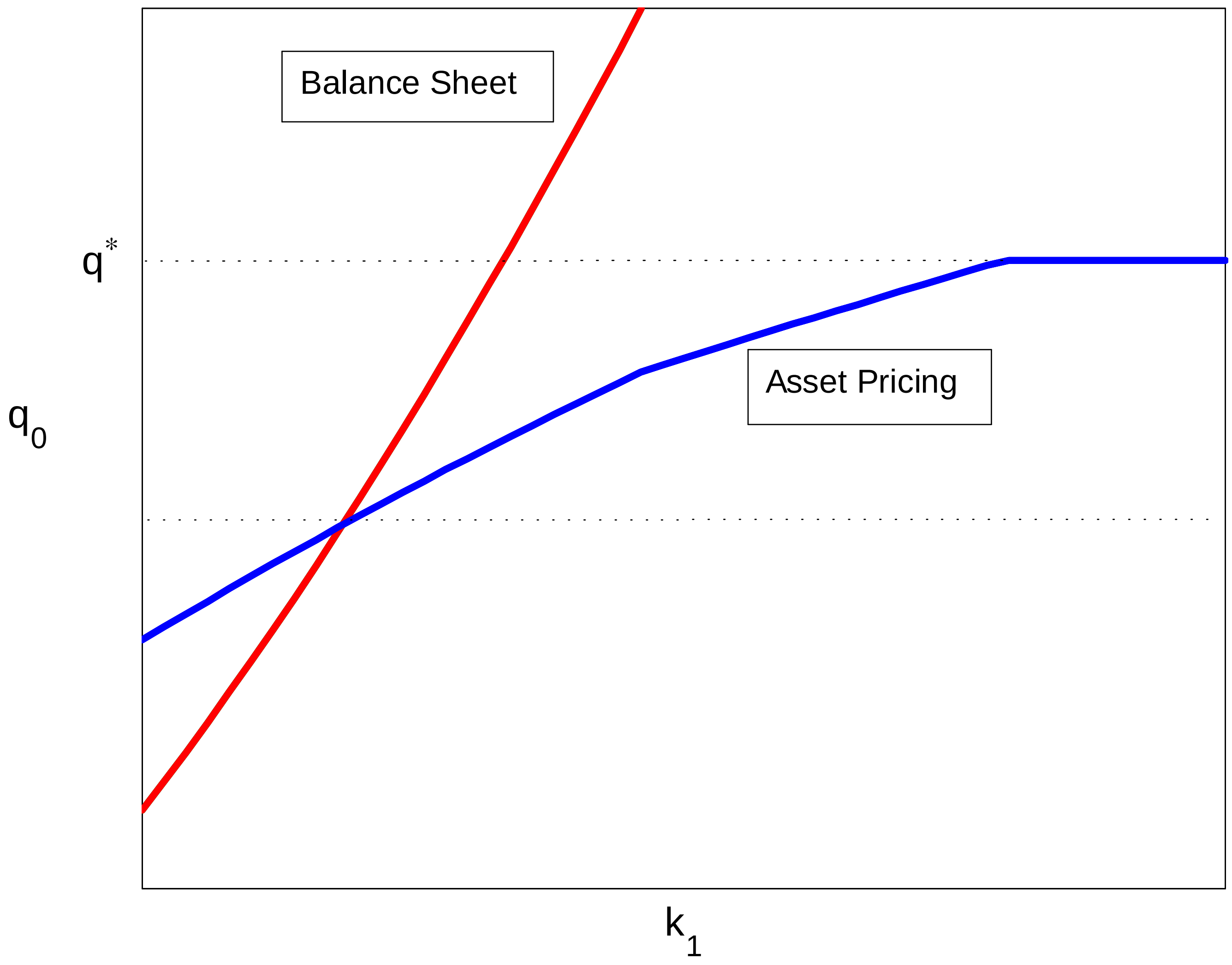
- Asset pricing relation: given k_1 find sequence $\{k_t\}_{t=2}^{\infty}$ that satisfies

$$k_{t+1} = \frac{(a + q_t)\, k_t - b_t}{q_t - \beta q_{t+1}} = \min\left\{\frac{a k_t}{\beta G'\left(\bar{k} - k_{t+1}\right)}, k^*\right\}$$

and find

$$q_0 = \sum_{t=0}^{\infty} \beta^{t+1} G' \left(\bar{k} - k_{t+1} \right)$$

increasing relation between investment in entrepreneurial sector and asset price q_0


Balance Sheet
Asset Pricing
q*
q0
k1

- Introduce a temporary shock to productivity

- Productivity is

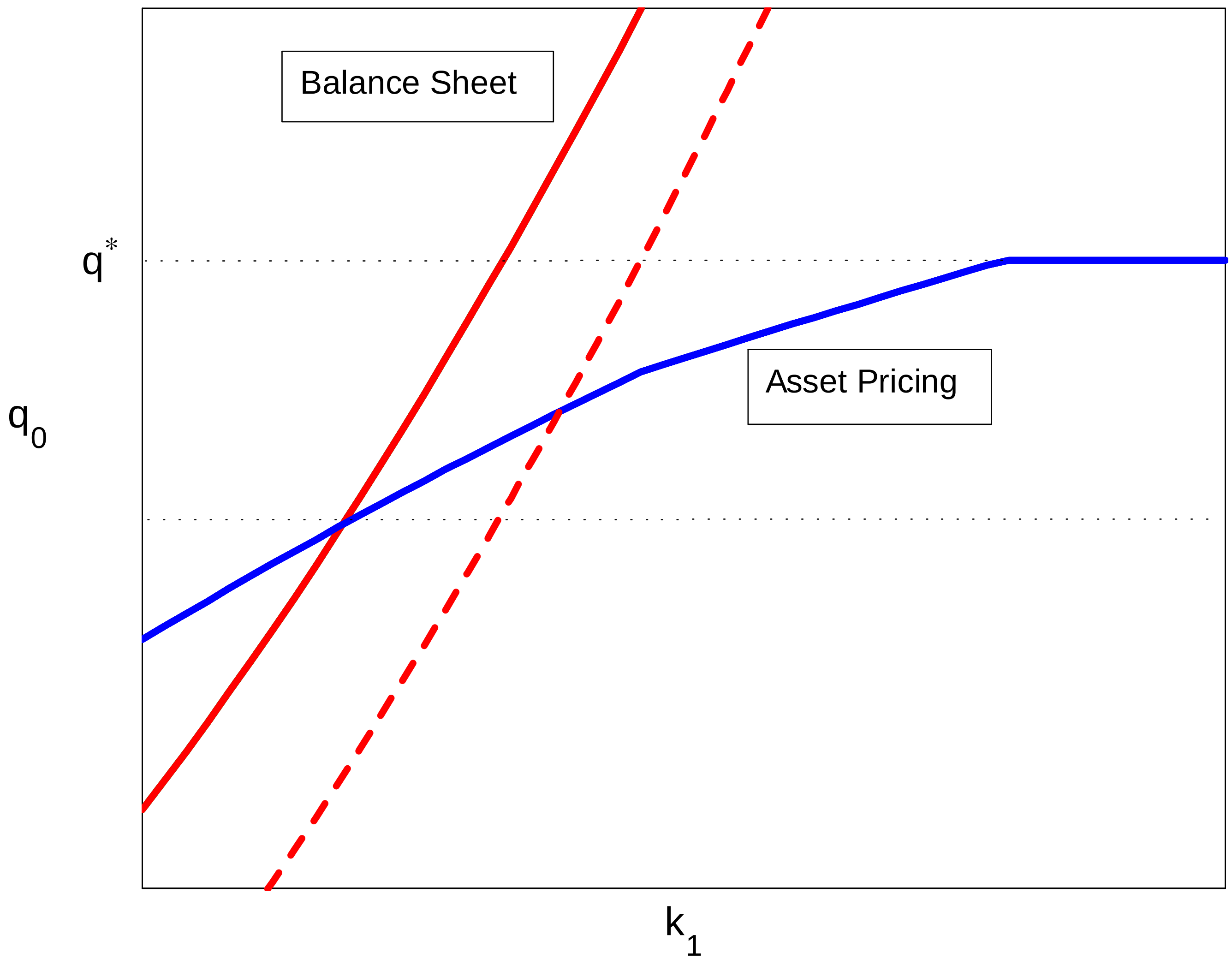
$$a + \Delta a$$

  for first period only

- This would have no effect in frictionless benchmark (purely forward looking)

- Here it shifts the BS relation to the right

$$k_1 = \frac{(a + \Delta a + q_0)\, k_0 - b_0}{q_0 - \beta q_1}$$


Balance Sheet
Asset Pricing
q*
q0
k1

- Backward looking effect of net worth on investment

- ...+amplification due to forward looking element

- Questions:
  - here shock is completely unexpected
  - what happens if state contingency allowed?
  - do entrepreneurs want to insure (hedge)?
  - if yes, why they do not do it?

Suppose state contingent contracts allowed at date -1

State s realized at date 0:

- s_g : productivity$= a$

- s_b : productivity$= a + \Delta a$ $(\Delta a < 0)$

State contingent enforcement constraint

$$b_0(s) \leq q_0(s)\, k_0$$

- Question 1: what happens to effect of shocks if firms decide to choose max borrowing in all s?

- Qustion 2: will firms even choose max borrowing?

Question 1: Now no feedback effect (vertical BS curve)

$$\beta G'\left(\bar{k}-k_1\right)k_1$$

so total effect is

$$\frac{\Delta k_1}{k_0}=\frac{1}{\beta G'-G''k_1}\Delta a$$

instead of

$$\frac{\Delta k_1}{k_0}=\frac{1}{\beta G'-G''k_1}\left(\Delta a+\Delta q_0\right)$$

Question 2:

Check optimality

Entrepreneurs problem at $t = -1$

$$\max_{k_0, b_0(s)} \sum_s \pi(s) \beta V_0 ((a(s) + q_0(s)) k_0 - b_0(s))$$

$$\begin{aligned} q_{-1} k_0 &\leq n_{-1} + \beta \sum_s \pi(s) b_0(s) \\ b_0(s) &\leq q_0(s) k_0 \end{aligned}$$

- FOC

$$\begin{aligned} \lambda_{-1} q_{-1} &= \beta \sum \pi(s) (a(s) + q_0(s)) V_0'(s) + \sum \pi(s) \mu(s) q_0(s) \\ \lambda_{-1} \pi(s) \beta &= \beta \pi(s) V_0'(s) + \pi(s) \mu(s) \end{aligned}$$

- Can we have $\mu(s) = 0$?

- Answer: yes if

$$V_0'(s) = \lambda_{-1}$$

- Substituting

$$\lambda_{-1} q_{-1} = \beta \sum \pi(s)(a(s) + q_0(s))\lambda_0(s) + \beta \sum \pi(s)(\lambda_{-1} - \lambda_0(s)) q_0(s)$$

yields

$$\lambda_{-1} = \frac{\beta \sum \pi(s) a(s) \lambda_0(s)}{q_{-1} - \beta \sum \pi(s) q_0(s)}$$

- Recall that

$$V_0'(s) = \lambda_0(s) = \frac{\beta a}{q_0(s) - \beta q_1(s)} \frac{\beta a}{q_1(s) - \beta q_2(s)} \cdots \frac{\beta a}{q^* - \beta q_{T(s)-1}(s)}$$

- Equilibrium construction: try

$$b_0(s) = q_0(s) k_0 \text{ for all } s$$

- If shocks $\Delta a < 0$ realized then $q_0(s)$ lower

- Check if

$$\lambda_0(s_1) < \frac{\beta \sum \pi(s) a(s) \lambda_0(s)}{q_{-1} - \beta \sum \pi(s) q_0(s)}$$

- If not then look for equilibrium where

$$\begin{aligned} b_0(s_g) &= q_0(s_g)\, k_0 \\ b_0(s_b) &\leq q_0(s_b)\, k_0 \end{aligned}$$

  Equilibrium with spare debt capacity

- Crucial point: Entrepreneurs risk neutral $\neq$ no hedging demand

- Value function $(V_0'(s))$ depends on asset prices!

- Even if $\pi(s_b)$ is very small we have a lower bound on how big can be the capital destruction in a crisis

- Broader question

- Why "entrepreneurs" (i.e. potentially financially constrained agents) do not insure?

- Classic example of lack of state contingency (actually of "wrong" state contingency): dollarization of liabilities in emerging economies

- In model leverage tends to be countercyclical

$$\downarrow \frac{q_0 k_1}{n_0} = \frac{q_0}{q_0 - \beta q_1} = \frac{\sum_{t=0}^{\infty} \beta^{t+1} G'\left(\bar{k} - k_{t+1}\right) \uparrow}{\beta G'\left(\bar{k} - k_1\right) \uparrow\uparrow}$$

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