

# 14.461 Lecture 6

## Wage Dispersion

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### 1 Wage Dispersion

See Mortensen (2003) for a review of the empirical findings on wage dispersion.

Mainly, two facts are uncontroversial:

1. workers' earnings are associated with observable workers' characteristics (education, experience, job tenure, marital status, race, location). However, the standard human capital wage equation does not explain much of the wage variation
2. workers' earnings are associated with employers' characteristics, particularly size and industry. Some attribute that to unobservable workers' characteristics. Other to different pay policy for identical workers. Recent effort to measure directly firm effects in wages by looking at matched employer-employee new data sets. Results: at least half of the industry differential and 70% of the size differential can be attributed to the fact that firms pay different wages to identical workers.

Different models can rationalize this behavior. Here, I present a classic model that delivers wage-dispersion without any heterogeneity in firms or workers. In Pset 1, we have seen that the standard Mortensen-Pissarides model can explain wage differentials among ex-ante identical workers because of heterogeneity in firms' productivity. One may think at other stories, such as efficiency wage (firms are heterogeneous in the monitoring costs), or sorting.

## 1.1 Burdett and Judd

Assume there is a large countable number of firms  $N$  and a large countable numbers of workers  $N\mu$  so that  $\mu$  denotes the worker/firm ratio. All agents are risk-neutral and the model is static. Firms can produce  $y$  per worker. If a worker is unemployed he gets  $b$ .

Sketch of the game:

- all firms simultaneously post a wage  $w$ , generating a distribution  $F(w)$ ;
- each worker ( $j$ ) decides how many wages to sample ( $n_j$ );
- each worker applies to the highest wage sampled, conditional on that being not smaller than his outside option  $\tilde{w}$ ;
- all firms hire all applicants and employ them at the promised wage  $w$ .

## 1.2 Firms

Assume the workers follow the following strategy: sample  $n$  wages and apply for the highest one iff it is no lower than the reservation wage  $\tilde{w} = b$ . Hence the workers behavior can be summarized by  $(\langle q_n \rangle_{n=1}^{\infty}, \tilde{w})$ , where  $q_n$  denotes the probability a randomly selected worker samples  $n$  wages. The reservation wage is the same for all workers, but the sampling behavior is allowed to be different.

**Definition 1** *Given  $(\langle q_n \rangle_{n=1}^{\infty}, \tilde{w})$ , a firm equilibrium is a pair  $(F(\cdot), \Pi)$  where  $F(\cdot)$  is a distribution function and  $\Pi$  is a scalar, such that (i)  $\Pi = \Pi(w)$  for all  $w$  in the support of  $F(\cdot)$  and (ii)  $\Pi(w) \leq \Pi$  for all  $w$ .*

The first condition imposes profit maximization, that is, all firms in equilibrium make the same profits. The second condition requires that there is no incentive for any firm to deviate.

**Lemma 1** *If  $(\langle q_n \rangle_{n=1}^{\infty}, \tilde{w})$  is such that  $q_1 \neq 1$  and  $(F(\cdot), \Pi)$  is an associated firm equilibrium,  $F(\cdot)$  is either continuous with connected support, or concentrated at  $y$ .*

**Sketch of the proof.**

Step 1. Here we prove that  $F$  is continuous. Suppose  $F$  has a discontinuity at  $w' \in [\tilde{w}, y)$ , that is,  $F(w'+) > F(w'-)$ . If  $q_1 \neq 1$ , there is a positive probability that a worker sample  $w'$  twice. Hence, if the firm posting  $w'$  decreases the wage infinitesimally, her expected profits increase because they would get more workers, that is,  $\Pi(w' + \varepsilon) > \Pi(w') = \Pi$ , a contradiction. The only possibility is that  $F$  is concentrated at  $y$ , in which case a firm cannot increase the wage without making negative profits.

Step 2. Here we prove that  $F$  has connected support. Suppose not, that is,  $F$  is constant over an interval  $[w_1, w_2]$ . This implies that  $\Pi(w_2 - \varepsilon) > \Pi(w_2)$  as long as  $\varepsilon < w_2 - w_1$ , given that  $w_2 - \varepsilon$  is accepted by all workers accepting  $w_2$  but the firm has to pay lower wages!

Given the above Proposition, we can write the expected profits of a firm posting  $w$

$$\Pi(w) = \begin{cases} (y - w) \sum_{k=1}^{\infty} q_k k \mu F(w)^{k-1} & \text{if } w \geq \tilde{w} \\ 0 & \text{if } w < \tilde{w} \end{cases} .$$

What is the probability that a worker apply for  $w$ ? It is  $\sum_{k=1}^{\infty} q_k k \mu F(w)^{k-1}$ . With prob.  $q_k$  a worker sample  $k$  wages and hence a firm meets a worker with prob.  $k\mu$ . Moreover, a worker who samples  $k$  wages is going to choose  $w$  if it is the highest wage sampled, that is, with prob.  $F(w)^{k-1}$ .

**Theorem 1** *There are three possible firm equilibria:*

1. given  $(\langle q_n \rangle_{n=1}^{\infty}, \tilde{w})$  with  $q_1 = 1$ , the unique firm equilibrium is monopolistic, that is,

$$\Pi = \mu(y - \tilde{w}) \quad \text{and} \quad F(w) = \begin{cases} 1 & \text{if } w \geq \tilde{w} \\ 0 & \text{if } w < \tilde{w} \end{cases} ;$$

2. given  $(\langle q_n \rangle_{n=1}^{\infty}, \tilde{w})$  with  $q_1 = 0$ , the unique firm equilibrium is competitive, that is

$$\Pi = 0 \quad \text{and} \quad F(w) = \begin{cases} 1 & \text{if } w \geq y \\ 0 & \text{if } w < y \end{cases} ;$$

3. given  $(\langle q_n \rangle_{n=1}^{\infty}, \tilde{w})$  with  $q_1 = (0, 1)$  and  $\tilde{w} < y$ , the unique firm equilibrium features dispersed wage, with  $F$  continuous with compact support on  $[\tilde{w}, \bar{w}]$  with  $\bar{w} \in (\tilde{w}, y)$  and

$$\Pi = \mu q_1 (y - \tilde{w}) = \mu (y - \bar{w}) \sum_{k=1}^{\infty} k q_k > 0$$

define  $\Pi$  and  $\bar{w}$ . If  $q_1 \in (0, 1)$  and  $\tilde{w} = y$ , the unique equilibrium is where firms charge  $y$  and  $\Pi = 0$ .

Sketch of the Proof. Claim (1) comes straight from the expression for expected profits. If  $q_1 = 1$  then firms will choose the lowest possible wage, otherwise there would be a profitable deviation. Now let us establish claim (2). Suppose that  $q_1 = 0$ . Then from Lemma 1 we know that any equilibrium is either concentrated at  $y$  or continuous and strictly increasing on the support. First you can see that the monopolistic equilibrium is an equilibrium, given that if any firm reduces the wage will loose all the workers. Moreover, suppose there is another firm equilibrium with  $F$  continuous and with compact support. Let  $w^* = \inf_{F(w) > 0} w$ . As  $w \rightarrow w^*$ ,  $F(w) \rightarrow 0$  and since  $q_1 = 0$

$$\Pi(w) \rightarrow (y - w^*) \sum_{k=1}^{\infty} q_k k \mu F(w)^{k-1} = 0.$$

But  $\Pi(w) = \Pi$  for all  $w$  in the support of  $F$  and hence  $\Pi = 0$ . However, at any  $w$  with  $F(w) \in (0, 1)$

$$\Pi(w) = (y - w) \sum_{k=1}^{\infty} q_k k \mu F(w)^{k-1} > 0$$

which gives a contradiction.

Finally let us prove claim (3). Suppose  $q_1 \in (0, 1)$ . It follows that if  $\tilde{w} < y$ , there is no firm equilibrium where all firms charge  $y$ , otherwise there would be incentive to deviate, a firm would infinitesimally decrease the wage, keep some workers and make higher profits. Hence  $F$  must be continuous with compact support. Hence, for all  $w$  in the support of  $F$

$$\Pi = \Pi(w) = (y - w) \sum_{k=1}^{\infty} q_k k \mu F(w)^{k-1}.$$

This implies

$$\frac{\Pi}{(y - w) \mu} = \sum_{k=1}^{\infty} q_k k F(w)^{k-1}.$$

The RHS is a  $C^\infty$  monotone increasing function of  $F(w)$  and hence has an increasing inverse  $\Phi$  so that

$$F(w) = \Phi(\Pi / ((y - w) \mu)).$$

You can check that  $\inf_{F(w)>0} w = \tilde{w}$ . Hence  $\Pi(\tilde{w}) = (y - \tilde{w})\mu q_1$  and the equal profit condition implies

$$\Pi(\bar{w}) = (y - \bar{w}) \sum_{k=1}^{\infty} q_k k \mu = \Pi \text{ where } \bar{w} = \sup_{F(w)>0} w.$$

### 1.3 Market Equilibrium

Imagine that the cost of sampling  $n$  wage is  $cn$ . Then the a worker chooses  $n$  to maximize his expected utility

$$\max_n \int_0^{\infty} nwF(w)^{n-1} dF(w) - cn.$$

The objective function is concave and hence there is a unique maximum when  $n$  is a real number (or two if is an integer). The worker will choose the highest wage if it is higher than the reservation wage  $\tilde{w}$ .

**Definition 2** *The triple  $(F(\cdot), \Pi, \langle q_n \rangle_{n=1}^{\infty})$  is a market equilibrium iff for fixed  $\tilde{w}$  and  $c$  (a)  $(F(\cdot), \Pi)$  is a firm equilibrium given  $(\langle q_n \rangle_{n=1}^{\infty}, \tilde{w})$ , and (b)  $\langle q_n \rangle_{n=1}^{\infty}$  is generated from the expected cost minimizing strategies of the workers given  $F(\cdot)$ .*

As we have shown before, there are three types of firm equilibria.

**Theorem 2** *If  $c > 0$  and if  $(F(\cdot), \Pi, \langle q_n \rangle_{n=1}^{\infty})$  is a market equilibrium then it is either a monopoly equilibrium or a dispersed wage equilibrium. Moreover, a monopoly equilibrium always exists.*

Sketch of the proof. Suppose all firms charge  $y$ , then all workers would search only once. However, then firms would decrease wages. If instead all firm charge  $\tilde{w}$  then all firms apply only once and there is no profitable deviation.

**Theorem 3** *Suppose all workers face the same search cost  $c$ . There are one, two or three market equilibria, one with monopoly and zero, one, or two with dispersed wages. There exists a  $c^*$  such that there are two dispersed wage equilibria if  $c < c^*$  and there are none otherwise.*

Proof.

Step 1. In any equilibrium  $q_1 + q_2 = 1$  and  $0 < q_1 \leq 1$ . All workers will see the same number of wages or will be indifferent between  $n$  and  $n + 1$ . If they search all more than once, all firms will charge  $y$  but then they would search once. Then  $q_1 > 0$  and  $q_1 + q_2 = 1$ .

Step 2. For any fixed  $q \in (0, 1)$  the unique associated firm equilibrium  $(F^q(\cdot), \Pi^q)$  has a strictly increasing distribution.

Step 3. Define  $V(q)$  as the difference between the utility from sampling two wages instead than 1, that is

$$V(q) = \int_0^\infty 2wF^q(w) dF^q(w) - \int_0^\infty wdF^q(w),$$

where  $F$  depends on  $q$ . This function has a unique maximum  $c^*$ . It is then easy to see graphically that the claim must be true.

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