14.461 Lectures 25-26
Liquidity Provision

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1 Lagos, Rochateau and Weill

Many financial assets, such as mortgage-backed securities or collateralized debt obligations, are traded in "over-the-counter" (OTC) markets, where investors use brokers/dealers, such as investment banks, to trade within each other. During financial disrubptions, dealers may provide liquidity by buying assets on their own account when the selling pressure is large and selling them when the selling pressure alleviates. During the current crisis, the dealers' liqudity provision has been proven inadequate and the Fed has pursued many interventions to increase the liquidity of the financial market, e.g. various lending facilities intended for the dealers, purchase of mortgage-backed securities, regulation of the OTC market.

This paper provides a model of liquidity provision in the OTC market, focusing on two frictions: search and bargaining. The model is based on Duffie, Garleanu, and Pederson (2007), with the key difference that here both dealers and investors can hold unrestricted inventories. The paper is very related to Weill (2007), where the dealers could hold unrestricted inventories, but investors were restricted to hold either 0 or 1 unit of the asset. This paper shows that the endogenous response of investors' asset holding may crucially affect the decision of liquidity provision of the dealers. In fact, Weill found that dealers always find optimal to provide liquidity but they may be constrained by limited capital. Here, instead, dealers may not find it optimal to provide enough liquidity in the market because of trading friction. Depending on the severity of the crisis and the strenght of the trading friction, it may be that they provide 0 liquidity! Clearly the policy implications may be very different when one analyze the trade-offs between recapitalizing banks or direct purchase of assets. This paper provides a rationale in favor of the second type of policy, clearly abstracting from credit-market frictions that may limit dealers' access to capital and speak in favor of the first type of policy.

1.1 Set Up

- time is continuous and run forever
- there is one asset in fixed supply A that is durable, perfectly divisible and yields a nontradable flow of services to its owner
- there is one perishable good, used as numeraire, produced and consumed by all agents
- there are two types of infinitely-lived agents who discount future at rate r: a unit mass of investors and a unit mass of dealers
- agents' utility function

$$\zeta(t)\,\varepsilon(t)\,u(a(t)) + c$$

  where a is investor's asset holdings, c net consumption of the perishable good, $\varepsilon(t)$ an idiosyncratic preference shock (to create trading motive), and $\zeta(t)$ an aggregate shock (to create the crisis)

- investors: $\varepsilon(t) \in \{\varepsilon_1, ..., \varepsilon_I\}$
  1. receive a new indiosyncratic preference shock at Poisson arrival rate δ
  2. new shock $\varepsilon(t) = \varepsilon_i$ with probability π_i, with
- dealers: $\varepsilon(t) = 0$ and short-selling constraint
- A financial crisis is determined by an aggregate shock at time 0 such that $\zeta(t) = \theta < 1$ for all $t \in [0, T)$ and $\zeta(t) = 1$ for all $t \geq T$, where T is the time when the recovery starts
- T is an exponentially distributed random variable with mean $1/\rho$

Market structure:

- dealers can continuously buy and sell the asset in the interdealer market at price $p(t)$
- investors can only trade periodically and through a dealer
- they contact a dealer at random with Poisson intensity α
- terms of trade in a dealer-investor match are determined with Nash bargaining
- negotiation over quantity and intermediation fee

1.2 Equilibrium

1.2.1 After the recovery

First, let us consider the equilibrium for all $t \geq T$, taking as given the realization of T and the dealers' inventories at that time, that is $A_d(T) \geq 0$ (later we endogenize it).

At each time a dealer:

1. may receive a call from an investor and trade on his behalf
2. chooses asset inventory $a_d \geq 0$

Because of continuous trading and no wealth effects the two problems can be solved separately! Hence, the dealer is choosing his inventories $a_d \geq 0$ optimally iff

$$\dot{p}(t) \leq rp(t) \text{ with equality if } a_d > 0.$$

This is the standard speculators' demand for assets. Notice that we are assuming that $p(t)$ is differentiable and we are excluding explosive solutions, that is, the no-bubble condition $\lim_{s\to\infty} e^{-rs}p(s) = 0$ is satisfied.

Next, let us consider the bargaining of dealers and investors. Consider a bilateral meeting at $t \geq T$ of an investor with assets a and a dealer with assets a_d. Call $V_i(a,t)$ the continuation utility at time t of an investor with shock ε_i and assets a, and $W(a_d,t)$ the value function of a dealer with assets a_d at time t. They bargain over the post-trade asset holding of the investor a' and the dealer fee ϕ. The bargaining problem when the bargaining power of the dealer is η can be written as

$$\max_{a',\phi} \left[V_i(a',t) - V_i(a,t) - p(t)(a'-a) - \phi\right]^{1-\eta} \phi^{\eta}.$$

Notice that the net surplus of the dealer is linear in the intermediation fee because there are no wealth effects and he has continuous access to the market! Hence, the intermediation fee is a fraction η of the surplus

$$\phi = \eta \left[V_i \left(a', t \right) - V_i \left(a, t \right) - p \left(t \right) \left(a' - a \right) \right],$$

while the investor will keep a fraction $1 - \eta$ of the surplus of the match. Hence, trading through dealers with bargaining power $1 - \eta$ is payoff equivalent to having direct access to the market with intensity $\kappa = \alpha \left(1 - \eta \right)$.

Also, from the bargaining problem, the investor new asset holding must solve

$$\max_{a'} \left\{ V_i \left(a', t \right) - V_i \left(a, t \right) - p \left(t \right) \left(a' - a \right) \right\}$$

and hence

$$\max_{a'} \left\{ V_i \left(a', t \right) - p \left(t \right) a' \right\}. \tag{1}$$

The investor's value function is

$$V_i \left(a, t \right) = E_i \left[\left(\int_t^\tau e^{-r(s-t)} \varepsilon_{k(s)} ds \right) u \left(a' \right) + e^{-r(\tau - t)} \left\{ V_{k(\tau)} \left(a, \tau \right) + \max_{a'} \left[V_{k(\tau)} \left(a', \tau \right) - V_{k(\tau)} \left(a, \tau \right) - p \left(\tau \right) \left(\right. \right. \right. \right.$$

and hence

$$V_i \left(a, t \right) = E_i \left[\left(\int_t^\tau e^{-r(s-t)} \varepsilon_{k(s)} ds \right) u \left(a \right) + e^{-r(\tau - t)} \left\{ p \left(\tau \right) a + \max_{a'} \left[V_{k(\tau)} \left(a', \tau \right) - p \left(\tau \right) a' \right] \right\} \right],$$

where $k \left(s \right)$ denotes the type at time s. Plugging this expression into problem (1), we obtain that an investor who contacts a dealer chooses a' to maximize

$$\max_{a'} E_i \left[\left(\int_t^\tau e^{-r(s-t)} \varepsilon_{k(s)} ds \right) u \left(a' \right) - \left(p \left(t \right) - e^{-r(\tau - t)} p \left(\tau \right) a' \right) \right].$$

We can rewrite this problem as

$$\max_{a'} \left[\bar{\varepsilon}_i u \left(a' \right) - \xi \left(t \right) a' \right]$$

where $\bar{\varepsilon}_i u \left(a' \right)$ is the flow utility until next "effective contact", $\xi \left(t \right)$ the flow cost of buying now and reselling at the next "effective contact", where the next "effective contact" happens at rate κ. This implies that the asset demand $a_i \left(t \right)$ must satisfy

$$\bar{\varepsilon}_i u' \left(a_i \left(t \right) \right) = \xi \left(t \right). \tag{2}$$

where

$$\bar{\varepsilon}_i = (r+\kappa) E_t \left[\int_t^{\tau} e^{-r(s-t)} \varepsilon_{k(s)} ds \right] = \frac{r+\kappa}{r+\kappa+\delta} \varepsilon_i + \frac{\delta}{r+\kappa+\delta} \sum_j \pi_j \varepsilon_j$$

and

$$\xi(t) = (r+\kappa) \left(p(t) - E_t \left[e^{-r(\tau-t)} p(\tau) \right] \right).$$

Differentiating this expression, we get

$$\frac{\dot{\xi}(t)}{(r+\kappa)} - \xi(t) = \dot{p}(t) - rp(t).$$

Notice that the dealers' foc can be rewritten as

$$\begin{aligned} a_d &\geq 0 \text{ if } \frac{\dot{\xi}(t)}{(r+\kappa)} = \xi(t) \\ a_d &= 0 \text{ if } \frac{\dot{\xi}(t)}{(r+\kappa)} < \xi(t). \end{aligned} \tag{3}$$

Notice that the dealer makes an extra return because of his continuous access to the asset market! In fact the marginal cost of buying an extra asset for the investor is equal to $\xi(t)$, while the extra cost for a dealer is $\xi(t) - \dot{\xi}(t) / (r+\kappa)$, when the market is active.

Let us define the aggregate demand of the dealers as $A_d(t)$. During a small interval of lenght dt, the flow supply of assets is $\alpha (A - A_d(t))$ and the flow demand is $\alpha \sum_i \pi_i a_i(t)$ where $a_i(t)$ solves

$$\bar{\varepsilon}_i u'(a_i(t)) = \xi_t.$$

The net demand of dealers is equal to $\dot{A}_d(t)$ and hence market clearing requires

$$\alpha [A - A_d(t)] = \alpha \sum_i \pi_i a_i(t) + \dot{A}_d(t). \tag{4}$$

An equilibrium after the recovery is given by a flow cost of holding assets until next effective contact $\xi(t)$, asset demands $a_i(t)$ for all i, and aggregate inventories $A_d(t)$ such that the optimality conditions of dealers and investors are satisfied (2) and (3) and market clearing (4) holds for all t. The equilibrium can be reduced to the following system of differential equations:

$$\begin{aligned} &A_d(t) \left[(r+\kappa) \xi(t) - \dot{\xi}(t) \right] = 0, \\ &\alpha [A - A_d(t)] = \alpha \sum_i \pi_i u'^{-1} (\xi(t) / \bar{\varepsilon}_i) + \dot{A}_d(t). \end{aligned}$$

In SS, $\dot{\xi}(t) = \dot{A}_d(t) = 0$ and hence from the first condition $A_d = 0$. Moreover, ξ is going to be pinned down by

$$A = \sum_i \pi_i u'^{-1}\left(\frac{\xi}{\bar{\varepsilon}_i}\right).$$

There are two possibilities:

1. $A_d(T) = 0$, that is, dealers do not provide liquidity during the crisis, in which cases A_d stays at zero and the prices is constant

2. $A_d(T) > 0$, that is, dealers do provide liquidity during the crisis, in which case A_d slowly decreases to 0 and prices increase slowly.

As the economy converge to its steady state, where $\dot{p}(t) = 0$ and hence $A_d(t) = 0$, dealers unwind their inventories. However, because search frictions and non-linear utility, it takes time for dealers to unwind their inventories so A_d stays positive for a while. As A_d is positive it must be that dealers make capital gains, and hence also $p(t)$ must increase slowly over time.

One can show that there exists a unique saddle-path. Let us call $\psi(A_d)$ such path in the space (A_d, ξ). This implies that $\xi(T) = \psi(A_d(T))$, where ψ is a decreasing function.

1.3 Equilibrium During the Crisis

The analysis is very similar. Denote by a subscript C the variables during the crisis.

First, the dealers' first-order condition for the choice of $a_d(t)$ is now given by

$$\begin{aligned} a_d(t) &\geq 0 \text{ if } \dot{p}^C(t) + \rho\left[p(t|t) - p^C(t)\right] = p^C(t)\, r \\ a_d(t) &= 0 \text{ otherwise} \end{aligned}$$

where $p^C(t)$ is the price at time t during the crisis and $p(t|t)$ is the price during recovery at the time of the recovery (conditional on $T = t$).

Following the same steps of before, we can show that the investor will choose a' to solve

$$\max_{a'} E_i\left[\left(\int_t^\tau e^{-r(s-t)}\varepsilon_{k(s)}\left(\theta + (1-\theta) I_{\{s\geq T\}}\right) ds\right) u(a') - \left(p(t) - e^{-r(\tau-t)}p(\tau)\right) a'\right],$$

where

$$p(\tau) = I_{\{s<T\}}p^C(\tau) + I_{\{s\geq T\}}p(\tau|T).$$

Then $a_i^C(t)$ has to satisfy the following optimality condition

$$\bar{\varepsilon}_i^C u'\left(a_i^C(t)\right) = \xi^C(t),$$

where

$$\bar{\varepsilon}_i^C = \frac{r+\kappa+\rho}{r+\kappa+\rho+\delta}\varepsilon_i^C + \frac{\delta}{r+\kappa+\rho+\delta}\sum_j \pi_j \varepsilon_j^C$$

where

$$\varepsilon_i^C = \frac{r+\kappa}{r+\kappa+\rho}\theta\varepsilon_i + \frac{\rho}{r+\kappa+\rho}\varepsilon_i$$

and

$$\xi^C(t) = (r+\kappa)\left(p^C(t) - \int_t^\infty \kappa e^{-(r+\kappa)(\tau_k - t)}\left[e^{-\rho(\tau_k - t)}p^C(\tau) + \int_0^{\tau_k} e^{-\rho(\tau_\rho - t)}p(\tau_k|\tau_\rho)\,d\tau_\rho\right]d\tau_k\right).$$

Differentiating this last expression you get

$$\frac{\dot{\xi}^C(t) + \rho\left[\psi\left(A_d^C(t)\right) - \xi^C(t)\right]}{r+\kappa} - \xi^C(t) = \dot{p}^C(t) + \rho\left[p(t|t) - p^C(t)\right] - rp^C(t).$$

Hence, you can rewrite the dealer optimality condition as

$$\frac{\dot{\xi}^C(t) + \rho\left[\psi\left(A_d^C(t)\right) - \xi^C(t)\right]}{r+\kappa} - \xi^C(t) \le 0 \text{ with equality if } A_d(t) > 0. \tag{5}$$

The market clearing condition is similar to the one above

$$\alpha\left[A - A_d^C(t)\right] = \alpha\sum_i \pi_i u'^{-1}\left(\xi^C(t)/\bar{\varepsilon}_i^C\right) + \dot{A}_d^C(t). \tag{6}$$

The equilibrium during the crisis is given by a path for $\xi^C(t)$, $A_d^C(t)$, and $\left\{a_i^C(t)\right\}_i$ such that (5) and (6) are satisfied. You can represent the equilibrium in a phase diagram and show that there exists a unique saddle-path as we did in class.

Notice that, under some conditions such that $A_d^{C*} > 0$, when a crisis hit at $t = 0$ when $A_d(0) = 0$, ξ^C (and hence p^C) jumps down at $t = 0$, then dealers start cumulating inventories so that A_d increase and ξ^C (and hence p^C) starts decreasing further. Why? Because as dealers cumulate inventories, they will need more time to unfold their inventories during the recovery, which means they need to expect capital gains for longer, and hence price will need to be lower when the recovery starts. This pushes prices down even during the crisis! When T happens, and the recover starts $\xi(t)$ jumps up to the saddle path of the recovery, prices start increasing towards the steady state and inventories to decrease towards 0.

What is interesting in this model is that during the crisis prices keep decreasing, even when dealers cumulate inventories, without any additional negative shock after time 0. This is just because of the trading friction and the expectiation that during the recovery, dealers won't be able to unfold their inventories immediately, but it will take time. This implies that as the inventories increase, prices keep decreasing to be consistent with the expectation of higher appreciation in the future. This implies that even if the dealers provide some liquidity, there is a bound to the amount of liquidity they are willing to provide (given by the steady state value of the dynamic system during the crisis) coming from the trading frictions of the economy.

The paper shows that it may be that $A_d(T) = 0$ when $a \to \infty$ or $r + \alpha(1-\eta) \to 0$. If trading frictions are very small, we converge to a case where also investors have continuous access to a Walrasian market and hence the dealer cannot gain anything by being intermediaries. If trading frictions are very large, on the other hand, investors trade so infrequently that they are going to value their assets putting more weight on their avarage type rather than on their current one. Hence, they won't trade much and the demand for liquidity during the crisis will be low giving no incentive to the dealers to cumulate inventories. Also dealers tend to provide more liquidity when θ is lower (crisis deeper) and ρ lower (crisis more short-lived) and they can make their gains faster!

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