A SIMULATION OF RACIAL TRANSITION IN NEIGHBORHOODS

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ABSTRACT

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This paper presents a model which simulates racial transition in neighborhoods. In the model there is one type of actor: households, which are differentiated by race and income. The decisions of when to move and where to move, in the face of changing racial patterns, is the primary operation of the model. The households make their decisions according to a utility function, by which they evaluate the various alternatives.

The action of the model is carried out by a simple dynamic model of the housing market. The market model consists of three parts. The first, MOVERS, generates lists of households seeking different housing and available sites. In the second part, CHOOSE, each mover makes one choice of the tract to which he would like to move. The third part, EXCHNG, rectifies the supply and demand in each tract, makes the transactions, and adjusts the price of housing in each tract for the next time period. Execution of these three sections of the market model constitutes completion of one time period.

A series of experiments are performed. These simulations test the dependence of the racial transition process on a variety of different household preference patterns and population distributions. Behavior in these experiments is typical of actual patterns of transition.

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CONTENTS

1. Introduction

The distribution of race in American cities has an important impact on many important social issues. Issues such as the provision of decent housing for minorities, metropolitan-wide government, and school busing are greatly affected, or even generated, by the racial patterns in cities. The subject of this paper is the process by which this pattern is created: the process of neighborhood racial transition. This process is examined through the development and operation of a simulation model.

There are several reasons for developing a formal model of racial transition. First and most importantly, it is needed to reconcile the stated preferences of both blacks and whites and the actual patterns found in American cities. Surveys and interviews indicate that most whites are indifferent or in favor of racially integrated housing; blacks are even more favorably disposed.[1] Yet empirical studies of housing patterns in American cities indicate that most are heavily segregated.[2] Moreover, this pattern is not changing significantly.[3] An obvious explanation is that people simply do not practice what they preach. However, another explanation is possible: the system by which an individual makes his housing choice, known as the housing market, produces an aggregate result dissimilar to the real preferences of the individual. A simulation model could analyze this explanation.

The model could also be used in other ways. The sensitivity of its output to its parameters indicates the stability of actual housing patterns in the face of corresponding changes in actual conditions. The model may also be used to evaluate the effects of a variety of practical and impractical policy alternatives: price supports in changing areas, the stability of forced integration after removal of the force, housing or income subsidies for poor blacks, etc. And finally, the model may be used to analyze some of the more controversial questions surrounding the process of racial transition. The most obvious of these are: what is the relation between property values and racial change, and what is the meaning of tipping and when does it occur.

The rest of this paper is divided into three main sections. The first of these outlines previous simulation models of racial transition. The second describes in detail the model developed in this study. And the last describes some experiments performed on the model.

2. Survey of Previous Models of Racial Transition

There have been five previous simulations of racial transition, each with its own approach. Despite their differences, they all have certain fundamental features in common. First, all are dynamic; each traces the spatial pattern of race through time. Secondly, whether explicitly stated or not, all the model are behavioral; each defines a decision rule by which the actors make their choices. Thirdly, they all define basic units of space and time, though the size of the units varies greatly. In the rest of this chapter, the main features of each of the models will be described.

The first simulation model of racial transition was developed by Morrill.[4] Space, in this case the city of Seattle, is divided into blocks; the temporal unit is one year. Racial transition is viewed as the expansion and diffusion of the Negro ghetto. The process is driven by significant increases in the Negro population caused by migration from outside the metropolitan area. Each year these migrants, plus 20% of the existing Negro population, move. The destinations of these movers is determined by a probablity field, which weights the likelihood of a move to a particular block by the distance of the move. Negro migrants are assumed to originate in the center of the Negro ghetto. Random numbers are generated and compared to the probability of each move as defined by the probability field; the random numbers determine the moves of the Negroes. If a block already has Negro residents then all moves generated by the random numbers are made. But if the block had been previously all white, the move is not made. Instead, a contact is registered. Once a block has been contacted a specified number of times, all future moves to that block are allowed to occur. The number of contacts required depends on the median property value of the block: the higher the value, the more contacts required. The procedure of generating movers and assigning them according to a probability distribution is repeated each time cycle.

By manipulating the number of contacts required, this simple model can replicate with good accuracy the general pattern of ghetto expansion, though for a few blocks major errors existed. Several comments concerning the decision rules of the model are in order. First, blacks and whites are viewed as fundamentally different types of actors. Second, blacks base their locational decision solely on the distance of the move; they do not consider any features of potential destinations other than location. Recognizing the importance of these other features, Morrill mechanisticly varies the required number of contacts by property values to capture some of the impact of block features on the probability of choice. Third, the rule for whites, though never explicitly stated, makes him simply a deserter; for each black which chooses a particular block, there is a white who immediately leaves the same block. In short, the model may be able to reproduce the pattern of race in cities by using these decision rules, but it offers little insight into the motives of the individuals which produce the pattern.

A similar model was later developed by Hansell and Clark.[5] Census tracts were the spacial unit and the time interval was two years. The actors were white and black households. As in Morrill's model, a probability field is computed, but the decisions rules which produce it are more complicated. The probability that a black family moves to a given tract depends on the distance of the move, the value of dwellings in the destination tract, and the number of black households already in the tract. The value of dwelling units is included by ranking the tract according to the median value of its units. For tracts ranking in the lowest two quartiles all blacks who eventually choose it are allowed to enter. In the next quartile, only 50% of the blacks choosing it are allowed to move in. In the highest quartile, no blacks are allowed to enter. The behavior of whites is also more complicated; they are no longer simple deserters, but will resist black incursion. This resistance depends on the strength of non-racial ethnicity in the tract. If 60% of the tract is foreign born or the children of foreign born, none of the black choosers are allowed in; if the tract is more than 30% foreign, only half of the blacks picking the tract get in; and if the tract is less than 30% foreign, no resistance is offered to black incursion. Once blacks successfully enter the tract all resistance ceases and whites desert as in Morrill's model.

The model was run using data from Milwaukee and was able to reproduce the pattern of race with about the same accuracy as Morrill's model. The model is noteworthy in two ways. First it increases the role of white households: they can offer resistance to black incursion.

However, their motives as movers remain fundamentally different than the motives of blacks. Secondly, though the method for including tract characteristics is contrived, it does seem preferable to Morrill's and should be regarded as a step in the right direction.

A different and more complicated approach has been taken by Freeman and Sunshine.[6] Space is a single neighborhood composed of 240 houses arranged in blocks. The time interval is not specified, but is clearly quite short, probably no longer than one month. The actors are households which are specified by race, income, "aquisitiveness" score, and prejudice score. Households act as buyers and sellers of housing.

Each time period a specified number of households leave the neighborhood and a number of blacks and whites consider moving into the neighborhood. The seller attempts to maximize his selling price, subject to his level of aquisitiveness. The higher the aquisitiveness, the higher the seller's initial asking price and the longer he is willing to wait to sell at a high price. The buyer will choose the lowest priced unit because all the units are identical. However, if the lowest price is greater than a specified fraction of the buyer's income, he chooses to go elsewhere and does not enter the neighborhood. The time cycle consists of letting each potential mover determine if he will enter the neighborhood.

Race enters the model in two ways: first, discrimination can block the entrance of blacks; and second, whites will be less willing to enter and more willing to leave as the percentage of blacks grows. For a black family to enter the neighborhood several hurdles must be passed: the black buyer must be shown the house, the white owner must be willing to sell to a black, and the black family, assuming that it does buy the house, must be willing to stay in the neighborhood in the face of white animosity. Each of these hurdles is defined as a probabilistic event; the probability that a black passes these hurdles depends on the prejudice score of the seller, the aquisitive-ness of the owner, the percentage of the neighborhood which is black, and the number of homes for sale in the neighborhood. Whites also react to race: for each white family there is a probability that it will move away. The probability is a function of the family's prejudice score and the percent black in the neighborhood. Moreover, there is also a probability that each of the potential white buyers decides not to move to the neighborhood because of the presence of blacks.

Operation of the model consists of letting buyers and sellers attempt to complete transactions, with the outcome of the hurdles decided by a stream of random numbers. By varying the many parameters of the model, its authors were able to reproduce a variety of neighborhood types: a neighborhood which blacks could not penetrate, one which became sharply divided into white and black sections, one which became eventually all black, and one in which the blacks and whites mixed freely.

Even the simplified description of the model presented here should indicate that the model is very complex. Because of its complexity some of the more important and fundamental features of the model may not be clear and deserve special note. Most importantly, this model is an attempt to model neighborhood transition as a market process. This achievement, however, is greatly diminished by the size of the market it models. The sample runs consisted of a mere 240 houses, nor because of its detailed nature and the amount of computations required, would it be practical to expand its scope greatly. The result is a tiny housing market model which is isolated from the rest of the city. The neighborhood stands alone; its surroundings have no impact. The basic demand for housing, both black and white, is constant; blacks and whites are not allowed to consider alternative neighborhoods. Despite the limitations of the market mechanism, its inculsion in the model is a major achievement. It is also important to note that the black and white households are really the same type of actors, both are households seeking to find new housing and to dispose of the old. They differ only in the manner in which race affects their choices. This model is able, unlike the previous two, to consider the effect of racial change on property values; this is clearly an advancement over previous efforts.

Another market model has been developed by Vandell.[7] Space consists of indivisible cells, each with a specified set of dwelling traits: market value and a level of housing service; and resident traits: income, job location, and color. The time unit is variable.

The black and white populations are initially in an equilibrium; each race lives in its own segregated neighborhoods (groupings of cells). The equilibrium is then upset by a disturbance, such as the migration of sizable numbers of blacks to the city. This migration produces a rise in the cost of housing in the black area. In the case of renters, this clearly decreases satisfaction with living in the ghetto. A number of blacks will decide to leave the ghetto and seek housing in the surrounding white areas. (Determinationn of which blacks move is made by ranking them by a function of their income and their present ratio of housing expenditures to income. Three restrictions are placed on the destinations of these movers. First, the blacks are not allowed to move to sites which are further than their present home from their place of work. Second, the new home must cost at least as much as the old. And third, the chosen home must maximize the exchange agent's profit.

This exchange agent is the first appearance of an actor which is not a housing consumer. He can obtain a profit because whites are willing to sell their homes below the true market value if the home is adjacent to black cells. The resident of a white cell examines the surrounding eight cells and he drops his selling price an amount depending on the number of black cells around him. (This procedure is referred to as the cellular autamata technique.) The agent is thus able to buy low and then sell the unit to blacks at the true market value. The process continues until the vacancy rate in the ghetto is higher than the rate in all the other neighborhoods, or until no black is both able and willing to move. Once this new equilibrium is established the clock may be advanced to the time of the next disturbance.

The important feature of this model, like the Freeman-Sunshine model, is the inclusion of a market process. Hence it is able to trace changes in the vacancy rate and market values in areas of racial transition. However, the model pre-determines that values in areas changing from white to black must drop. Moreover the size of the price drop and the strength of white resistance to change do not depend on the availability of alternative similar housing. A unique feature of the model is the exchange agent. The exchange agent is important for he determines the direction of ghetto expansion. Though real estate speculators of this type undoubtedly play a role in racial transition, the universality and centrality of this role is questionable.

Finally, there is the simple yet insightful model of Schelling.[8] He has developed a dynamic model of the type of segregation which arises from individual choice. The technique may be applied to any spatial segregation, but his ultimate concern is housing segregation by race. The effort was motivated by the observation that individual preferences and goals do not always correspond to the collective results.

Spatially, the model consists of an array of cells which can be occupied by one of two groups or allowed to stand vacant. Individuals move because they are dissatisfied with the (racial) composition of the surrounding cells. Each operates according to the following decision rule: examine a specified set of the surrounding cells, if the percentage of these cells occupied by the other group is greater than a specified number, then the individual decides to move. He will move to the nearest vacant cell which has acceptable surroundings. This rule is based on distaste for the other group; another rule was considered which was based on the desire to be with a specified number of one's own kind. (These rule are slightly different because some cells are vacant.) All individuals are continually considered for moving until all are happy with their location, when the system reaches equilibrium.

Numerous runs were made varying the size of the neighborhood, the relative sizes of the populations of the two groups, and the tolerance for the other group (or the need for one's own group). The results are striking; even for high tolerances, the equilibrium patterns are quite segregated. For example, if both groups require only two of the surrounding eight cells to be occupied by his own group, there is significant clustering of groups. If one group is very tolerant and less numerous, and the more numerous group demands majority status in its surroundings, the process eventually forces the minority group into well-defined ghetto areas.

This simple but abstract model makes its point well: the rules of micro behavior need not bear any resemblance to the macro pattern it produces. A group of individuals, each "looking out for himself," may produce an aggregate result which none would either expect or especially desire. The implications for the patterns of race in American cities are clear. However, because of its abstractness, interpretation of this model as a model of racial transition in cities is neither wise, nor intended by its author. But it indicates quite forcefully the potential value of models of racial transition.

3.1 An Overview of the Model

The model developed in this study has been formulated with the view that there are several characteristics which a model of racial transition should possess. First, the model must be explicitly behavioral, with reasonable decision rules. It was decided therefore, that blacks and whites would make their decisions in the same manner. Households of both races make their housing decisions according to a utility function. Differences in their decisions result from differences in the parameters of the utility function, but blacks and whites are fundamentally the same type of actors.

Secondly, the model should be simple. Racial transition in neighborhoods is clearly a complicated process. It should also be noted that this complexity has hindered analysis. The approach of this study has been to pull out of the complexity a few key features, or universals, and examine their relationships. Only two features of households, race and income, and two features of housing, a measure of housing service and the price of this service, have been included in the model. As a further simplification, the model deals with owner occupied units, though the model could be easily extended to include renters. Moreover, other potential actors, such as bankers and realtors do not appear in the model. Though there are serious dangers in limiting the complexity of the model, there are also important advantages. Most importantly, a simple model eases analysis of the relations between those features included in the model, relations which a more complicated and comprehensive model might miss. Moreover, as will be seen later, there are many interesting and important questions which can be examined with a simple model.

Thirdly, the nature of the subject necessitates a dynamic model. Important transitory phenomena do not appear in a model which outputs only final conditions or produces an equilibrium solution. For example, the behavior of market values during the period of transition (an important and controversial question) can only be examined with a dynamic model.

Fourthly, the model should be versatile. Hopefully, versatility is obtained in several ways. The spatial dimension is variable; it could be as small as a block or as large as a group of census tracts. Moreover, the size of the spatial units in a single simulation could vary greatly. For example, very high detail could be obtained in present areas of racial transition and their immediate surroundings, while in areas distant from change which will clearly remain white, the unit could be census tracts. Unlike most of the previous models, a rectangular grid arrangement of the spatial units is not required. The model is also versatile because it provides a systematic procedure for the creation of new decision rules. New decision rules can be added simply by appending terms to the utility function. (Most of the previous models, on the other hand, would require major alterations in their structure.)

A detailed description of the model follows.

3.2 The Actors and Their Decision Rules

Fundamentally, the model simulates the behavior of housing consumers and the housing market; the only actors in the model are housing consumers. These consumers are differentiated only by race, income, and tract of residence; no other features are known about the actors. Thus the population of the model consists of a three dimensional array:

POP_{RYT} = number of households by race (R), income (Y), and tract (T).

All individuals of the same race, income and tract are identical; or in other words, their decision rules are the same. Households make their decisions according to their evaluation of various alternatives. The evaluations are performed by a utility function which may be associated with each household. Unlike most of the other models, black and white actors are really the same: both simply evaluate housing alternatives. Of course their evaluations are different, but this is a result of preference differences and is expressed by different forms of their utility functions.

The utility function states that the utility of a given housing choice is a function of the income and race of the household and three characteristics of the housing unit: 1) its quantity of housing service, 2) the price of one unit of its housing service, and 3) the racial composition of the unit's tract. Symbolically:

$$U = U(Y, R, q, p, \%)$$

Y = income of household

R = race of household

q = quantity of housing service

p = price of one service unit

% = per cent black in the tract (T).

The utility function makes use of a well-known concept of housing analysis: the concept of the quantity of housing service.[9] It collapses all the numerous features of a housing unit, such as rooms, land, plumbing, condition, age, etc., into the one dimensional quantity of housing service, which crudely measures the quality of the unit. Other things being equal, a rich household will consume more service units than a poor household. It should also be noted that a small but well-kept dwelling may provide more service than a large but poorly maintained unit. If housing services were free, all households would try to consume more. But a price does exist and the more the household spends on housing, the less money it has for other goods; eventually greater consumption of housing reduces utility. In short, housing is viewed as a normal good of utility theory in classical microeconomics. Market value is simply the price of one service unit times the quantity of service the unit provides.

The conceptualization of market value as a quantity times a price is valuable because it permits clear analysis of changes in market value due solely to the actions of the market (changes in price) and formally excludes value changes resulting from changes in the condition of the unit (changes in its quantity of housing service). (In the model, the quantity of housing services provided by each unit does not change.) Thus when a price changes in the model, it should be clear that the unit does not change, but only the market's evaluation of the fixed bundle of services it provides.

The dependence of the utility function on the per cent black represents an important effect of the neighborhood on utility. It is a fundamental assumption of the model that households evaluate alternatives partially on the basis of the color of their neighbors. The percentage entered into the utility function may be the simple percentage of the tract's population which is black, or it may be a more complicated function. For example, it could also include a weighted average of the percentage in the surrounding tracts. Or since housing choices are long term decisions, the percentage may be an expected percentage, a projection of the per cent black at some time in the future.

As stated previously all housing units in a given tract are identical (they all supply the same number of service units). Hence the prevailing market price will be the same for all units in the same tract; since they are identical there is no reason to pay more for one than the other. Thus utility can be written as:

$$U = U(R,Y,T)$$

where for each tract T, there is a specified q(T), p(T), and %(T).

3.3 The Market Model

The action of the model is simply a simulation of the metropolitan housing market. The market model consists of three separate stages: 1) MOVERS: the decision to move, the decision to enter the housing market; 2) CHOOSE: the comparison of alternative housing choices and the selection of the move destination; 3) EXCHNG: the completion of the transaction, the purchase of the chosen unit.

MOVERS. MOVERS generates a list of households, by income, race, and location, who decide to move in the present time period. It also places the homes of the movers on the market. The movers are divided into two groups. First, there are those who move for reasons other than the changing racial composition of the area. These households, black and white, move for a variety of reasons which are not specified by the model: a new job, a change in income, a change in family size, etc. These diverse factors combine to produce an aggregate movement rate which is exogenous to the model and depends only on income. The number of movers of unspecified motivation, by race, income, and origin tract, is given by:

$${}_{u}m_{RYT} = r_Y \cdot POP_{RYT}$$

where ${}_{u}m_{RYT}$ = number of movers of unspecified motivation by race (R), income (Y), and origin tract (T),

r_Y = fraction of households which move each time period for reasons other than racial change by income (Y),

POP_{RYT} = number of households by race(R), income(Y), and tract (T).

Secondly, there are households which move as a reaction to the changing racial composition of the tract; they are responding to negative neighborhood externalities. Since utility depends on the racial composition, the effect of changing racial patterns can be measured through the utility function. The rate of racially motivated movement is a function of the fractional decrease in utility caused by changes in the racial composition of the tract. The fractional decrease in utility is given by:

$$\Delta_{RYT} = \frac{U^o_{RYT} - U_{RYT}}{U^o_{RYT}} \quad .$$

where Δ_{RYT} = fraction change in utility because of race by R,Y,T

U_{RYT} = present level of utility by R,Y,T

U^o_{RYT} = level of utility if independent of race.

The quantity Δ_{RYT} measures the loss of utility because of race and the rate of racially motivated movement depends on Δ_{RYT}:

$$\rho_{RYT} = f(\Delta_{RYT})$$

where ρ_{RYT} = fraction of households which move each time period because of race by R,Y,T

The specific form of $f(\Delta_{RYT})$ is unspecified, but it is clear that the larger the utility loss, the greater the rate of movement $(\frac{df}{d\Delta} > 0)$. It must also be pointed out that $f(\Delta_{RYT})$ is really a function of an ordinal scale, since classical utility functions produce ordinal values (ranked, but with no measure of the separation of the rankings). This usage is discussed in the appendix.

The number of racially motivated movers may be computed:

$$_{r}m_{RYT} = \rho_{RYT} \cdot POP_{RYT}$$

where $_{r}m_{RYT}$ = number of racially motivated movers by R,Y,T.

Finally, it is possible to compute the total number of movers by race (R), income (Y), and origin tract (T):

$$MOV_{RYT} = {_u}m_{RYT} + {_r}m_{RYT} \quad .$$

There is also a list of movers whose tract of origin is unspecified. Placed on this list are the net number of migrants to the metropolitan area and the net number of internal household formations. These households may be viewed as originating in a special tract, with designation T=0, which has an undefined spatial location. Thus,

$$MOV_{RY0} = GR_{RY}$$

where MOV_{RY0} = number of movers of unspecified origin (formally originating in tract T=0) by race (R) and income(Y).

GR_{RY} = net change in the total number of households in the entire region by R and Y.

The array GR_{RY} is exogenous to the model; therefore, the model does not determine the total metropolitan population by race and income, but only distributes it within the region. The following relation must hold:

$$\sum_{T} {_t}POP_{RYT} = \sum_{T} {_{t-1}}POP_{RYT} + GR_{RY}$$

where $\sum_{T} {_t}POP_{RYT}$ = total number of households by R,Y in the region at the present time.

$\sum_{T} {_{t-1}}POP_{RYT}$ = total number of households by R,Y in the last time period.

MOVERS also sets the number of houses on sale in each:

$$FS_T = \sum_{R,Y} MOV_{RYT} + VAC_T + NEW_T$$

where FS_T = total number for sale in tract T

MOV_{RYT} = number of movers by R,Y,T

VAC_T = number of vacant units in T; these are units unsold from the last time period

NEW_T = new units just constructed in T.

This equation merely expresses the three ways units may appear on the market in any time period: 1) the dwellings of the movers are placed on sale; 2) units unsold in the last time period remain on sale; 3) newly constructed units go on sale. The equation also points out that all the units in the tract are identical, whether old or newly constructed. The array NEW_T is exogenous to the model.

The arrays MOV_{RYT} and FS_T are the sole output of MOVERS; MOVERS has generated a list of movers and a list of dwellings to which the movers can go.

<u>CHOOSE</u>. In CHOOSE each mover is allowed to pick a destination of his move, though the actual moves will still not be completed. Three factors are postulated as affecting the choice of destination. First, the choice depends on the utility of the tract, This dependence will attract whites to white areas and blacks to black areas. It will also attract wealthy movers to tracts of high housing quality and poor movers to low quality housing. The strengths of these attractions depend on the form of the utility functions. Each race and income group will tend to move to the most useful housing.

Secondly, the likelihood that a tract is chosen depends on the number of units for sale in the tract. For example, consider two identical tracts with identical housing. One tract has two houses for sale and the other has only one. Also each house will have an equal likelihood of being chosen (since the units are completely indistinguishable). The likelihood that a tract is chosen is simply the sum of the likelihoods of choosing each of the houses for sale in the tract; hence the tract with two units is twice as likely to be chosen as the one with only one unit. In other words, the likelihood of choosing a tract depends on the number of units for sale because households actually choose houses and not tracts.

And finally, the likelihood of a move depends on the distance of the move.[10] This reflects informational factors: the further the available unit is from the mover, the less likely the mover will learn of it. Households will also generally prefer shorter moves to lessen the strain on established ties, such as church, clubs, and friendships.

By combining these three factors the likelihood of the move from one tract to another for each racial and income group can be computed:

$$L_{RYTT'} = H(U_{RYT'}) \cdot FS_{T'} \cdot D(d_{TT'})$$

where $L_{RYTT'}$ = likelihood of a move from tract T to tract T' for household of race (R) and income (Y)

$U_{RYT'}$ = utility of tract T' to household of R,Y.

$FS_{T'}$ = number of units for sale in T'

$d_{TT'}$ = distance from center (center of gravity) of tract T to center of tract T'.

The functional form of $H(U_{RYT'})$ is critical for it relates the likelihood of a choice with the utility of the choice. (Once again the model requires a function of an ordinal variable; see the Appendix.) Whatever, its precise form, it is clear that as $U_{RYT'}$ increases, so should $H(U_{RYT'})$: the greater the utility of a move, the more likely the move, other things being equal. The opposite is true for $D(d_{TT'})$: as $d_{TT'}$ increases, $D(d_{TT'})$ decreases.

By this point each mover, by race, income, and origin, has examined all the tracts and has computed the relative likelihood of choosing each tract. The probability that the mover chooses each tract is simply the normalized likelihood of choosing the tract:

$$P_{RYTT'} = \frac{L_{RYTT'}}{\sum_{T'} L_{RYTT'}}$$

where $P_{RYTT'}$ = probability that a mover of race (R), income (Y), and origin (T) chooses to move to tract T'.

The sum of these probabilities over T', for each R,Y,T class, is one; this is equivalent to forcing each mover to make a choice of one and only one destination tract. (Those movers choosing to leave the metropolitan area are accounted for in the net growth array, GR_{RY}.) Movers make their choices proportionally to $P_{RYTT'}$:

$$CHO_{RYTT'} = MOV_{RYT} \cdot P_{RYTT'}$$

where $CHO_{RYTT'}$ = number of choosers of race (R), income (Y), choosing to move from T to T'.

The total number choosing each tract, by race and income, is simply the sum of $CHO_{RYTT'}$ over T, the origin tracts:

$$CHO_{RYT'} = \sum_{T} CHO_{RYTT'}$$

where $CHO_{RYT'}$ = number of choosers of tract T' by R,Y.

(It should be pointed out that the movers with unspecified origins, MOV_{RY0}, are treated in precisely the same manner. The only special feature for these movers is that $D(d_{TT'})$ for T=0 is always unity, which formally states that their likelihoods do not depend on the distance of the move.) The array $CHO_{RYT'}$ is the sole output of the second section of the market.

EXCHNG. The final section is EXCHNG and it actually comletes the transactions. It also adjusts the prices of housing. By the beginning of EXCHNG movers have been generated, units have been placed on the market, and movers have chosen their destinations. However, the actual transactions, the changing of ownership, have not taken place. The total number of choosers in each tract is compared with the number of units available in the tract:

$$N_{T'} = \sum_{R,Y} CHO_{RYT'} - FS_{T'}$$

Two cases are possible: 1) $N_{T'} \leq 0$ and $N_{T'} > 0$. In case 1 there are more units available that there are choosers. Hence all the choosers may be assigned to their chosen destinations:

$$BUY_{RYT'} = CHO_{RYT'}$$

where $BUY_{RYT'}$ = number of choosers who are successful (able to purchase their chosen unit), by race (R), income (Y), and tract (T').

Unless $\sum_{R,Y} CHO_{RYT'} = FS_{T'}$, some of the units will be left unsold. These units are designated as "vacant."

$$VAC_{T'} = -N_{T'} = FS_{T'} - \sum_{R,Y} CHO_{RYT'} \quad \text{for } N_{T'} \leq 0$$

Designation of the unit as vacant does not necessarily mean that the unit is actually unoccupied. Vacant units in the model are units which are not occupied by a household which intends to stay in the unit. It may in fact still be occupied by the household which decided to move away from it, since he may be required to sell his old unit before he is able to move into his new one. On the other hand, the seller may have the financial resources (downpayment) to move before his old dwelling is sold; in this case the unit would be unoccupied. The model does not attempt to distinguish between these two situations. Conversely, a unit which is empty for a time between the departure of the old resident and the arrival of the new does not appear on the vacant list, provided it does have a definite buyer (it was chosen by household from $BUY_{RYT'}$ list). Vacant units are units which do not have buyers; vacant units are units whose owner does not wish to live there any longer. In this sense, the model simulates housing transactions (the buying and selling of units by owner-occupants) and not actual moves. No doubt moves will follow the transactions, but there will be some time lags.

This rather peculiar definition of vacancies not only simplifies the model of the market, but it also is most appropriate for this analysis: the great symptom of racial transition in not unoccupied units, but a high percentage of homes displaying the "for sale" sign on their lawn. In some ways a unit with a resident who intends to move as soon as possible has the same effect on the neighborhood as an unoccupied unit; in either case the future status of the unit is unknown to the neighbors. To repeat, vacant units are units **remaining** unsold after completion of the present time period.

In case 2 there are more choosers than units available and no units will be classified as vacant. Instead a number of choosers, equal to $N_{T'}$, will be unsuccessful: they do not obtain the unit which they had chosen in this round. Choosers are proportionally rejected: if 10% of all choosers must be rejected, then 10% of each racial and income class is rejected. (This assumes that sellers do not discriminate against choosers; sellers are not concerned with the race of the chooser.) Formally,

$$BUY_{RYT'} = CHO_{RYT'} - \frac{N_{T'}}{\sum_{R,Y} CHO_{RYT'}} \cdot CHO_{RYT'}$$

where $BUY_{RYT'}$ = number of choosers who are successful in tract T' by race (R) and income (Y).

$CHO_{RYT'}$ = number of choosers of tract T' by R,Y.

$N_{T'}$ = total number of rejected chooser of T'.

It should be noted that the following necessary conditions hold

$$\sum_{R,Y} BUY_{RYT'} = \sum_{R,Y}\left(CHO_{RYT'} - \frac{N_{T'}}{\sum_{R,Y} CHO_{RYT'}} \cdot CHO_{RYT'}\right)$$

$$\text{hence} \quad \sum_{R,Y} BUY_{RYT'} = \sum_{R,Y} CHO_{RYT'} - N_{T'}$$

$$\text{and} \quad \sum_{R,Y} BUY_{RYT'} = FS_{T'} \quad .$$

Those who have failed in their choice are placed on the unspecified movers list for the next time period (MOV_{RY0}) and will continue to search for housing in the next time period. Placing those rejected on this list implies that they will broaden their search and will examine the entire market with full intensity: $D(d_{0T'}) = 1$ for all T'.

Finally, the transactions can take place; those leaving turn their units over to those entering:

$$_{t+1}POP_{RYT'} = {}_{t}POP_{RYT'} + BUY_{RYT'} - MOV_{RYT'}$$

where $_{t+1}POP_{RYT'}$ = population in tract T' by race (R) and income (Y) in the next time period.

$_{t}POP_{RYT'}$ = population in T' by R,Y in present time period.

It should be noted that these procedures account for all movers. Conservation of households is maintained through the market process; nobody is lost or disappears in the search for housing. Of course population does change, but only through the net growth array, GR_{RY}:

$$\sum_{T} {}_{t+1}POP_{RYT'} = \sum_{T} {}_{t}POP_{RYT'} + GR_{RY} \quad .$$

One final task remains before the market process is completed: the adjustment of prices. In each tract there is an established price of a unit of housing service; it is at this price that sellers offer their units and it is this price that buyers enter into their utility function. This price will now change in response to new conditions in the tract: if units remain unsold after transactions the price will fall; if choosers were rejected the price will rise. This price adjustment is critical for it affects the choices which will be made in the next time period. A lower price will increase the tract's utility to all movers and if the racial compostition had not changed, the number of choosers will be increased. On the other hand, an increased price will tend to yield fewer choosers in the next period.

Let

$$N_{T'} = \sum_{R,Y} CHO_{RYT'} - FS_{T'}$$

and

$$\delta_{T'} = f(N_{T'}/FS_{T'})$$

where $CHO_{RYT'}$ = number of choosers of T' by R,Y

$FS_{T'}$ = number of units offered in T' at beginning of the market.

then

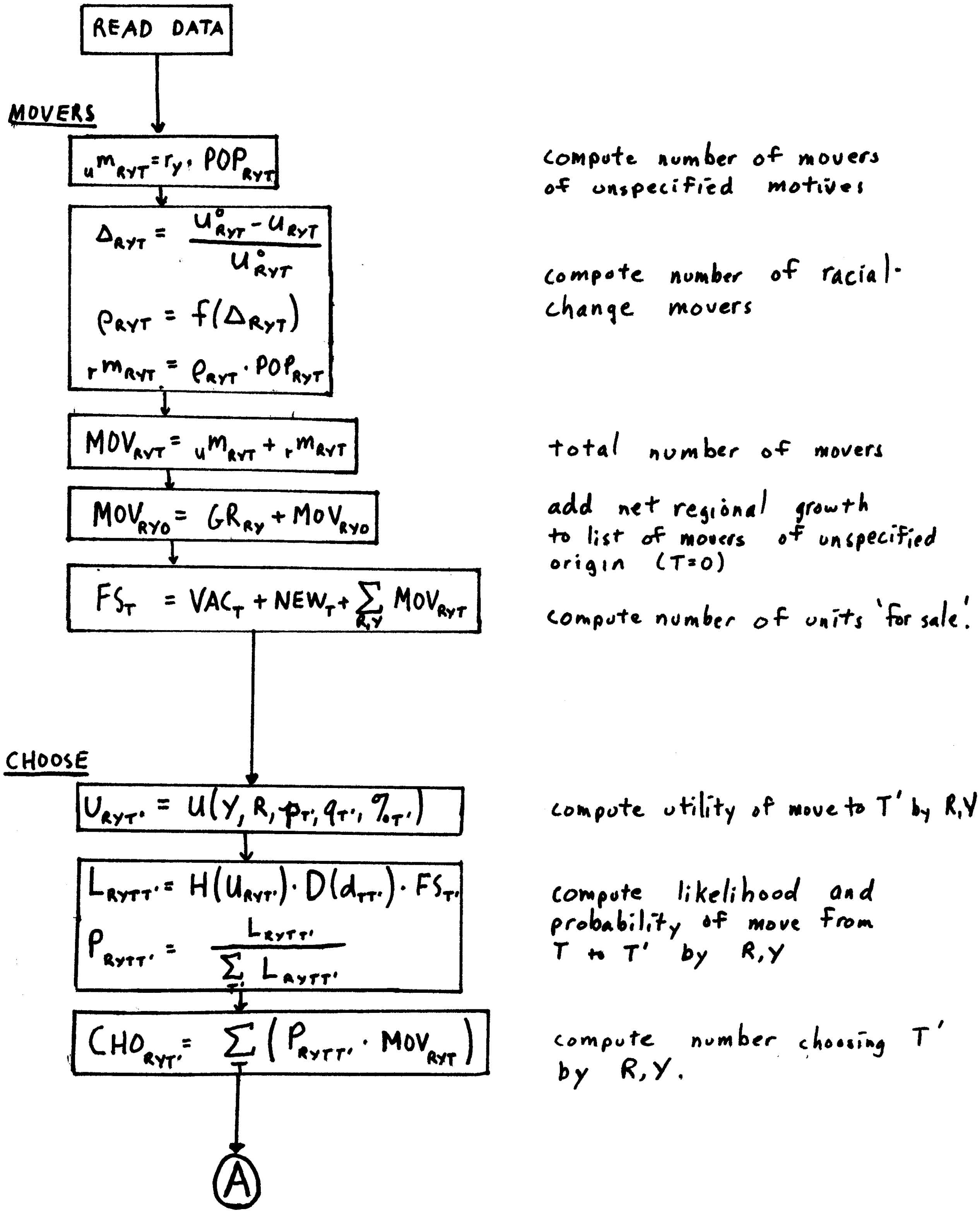
$$_{t+1}p_{T'} = {}_{t}p_{T'} \cdot (1 + \delta_{T'})$$

where $_{t+1}p_{T'}$ = price of a unit of housing service in tract T' in the next time period.

$_{t}p_{T'}$ = price of a unit of housing service in tract T' in the present period.

Thus the market price of housing is established in each tract. These relations state that the change in price is a function of the fractional excess or shortage of offered units. When $N_{T'} < 0$, $\delta_{T'} < 0$; when $N_{T'} > 0$, $\delta_{T'} > 0$. The price adjustment will tend to limit the concentration of vacancies in any one area; the lower prices in high vacancy areas will attract more choosers in the next time period.

After the prices have been adjusted, the simulation of the present time period is completeed. The clock may be advanced to the next time period and the market process can begin again. A flow chart of the process is presented on the next page.


READ DATA
MOVERS
$_u m_{RYT} = r_Y \cdot POP_{RYT}$
compute number of movers of unspecified motives
$\Delta_{RYT} = \frac{u^\circ_{RYT} - u_{RYT}}{u^\circ_{RYT}}$
$\rho_{RYT} = f(\Delta_{RYT})$
$_r m_{RYT} = \rho_{RYT} \cdot POP_{RYT}$
compute number of racial-change movers
$MOV_{RYT} = {_u m_{RYT}} + {_r m_{RYT}}$
total number of movers
$MOV_{RYO} = GR_{RY} + MOV_{RYO}$
add net regional growth to list of movers of unspecified origin (T=0)
$FS_T = VAC_T + NEW_T + \sum_{R,Y} MOV_{RYT}$
compute number of units 'for sale'.
CHOOSE
$U_{RYT'} = U(Y, R, p_{T'}, q_{T'}, \%_{oT'})$
compute utility of move to T' by R,Y
$L_{RYTT'} = H(U_{RYT'}) \cdot D(d_{TT'}) \cdot FS_{T'}$
$P_{RYTT'} = \frac{L_{RYTT'}}{\sum_{T'} L_{RYTT'}}$
compute likelihood and probability of move from T to T' by R,Y
$CHO_{RYT'} = \sum_T (P_{RYTT'} \cdot MOV_{RYT})$
compute number choosing T' by R,Y.
A

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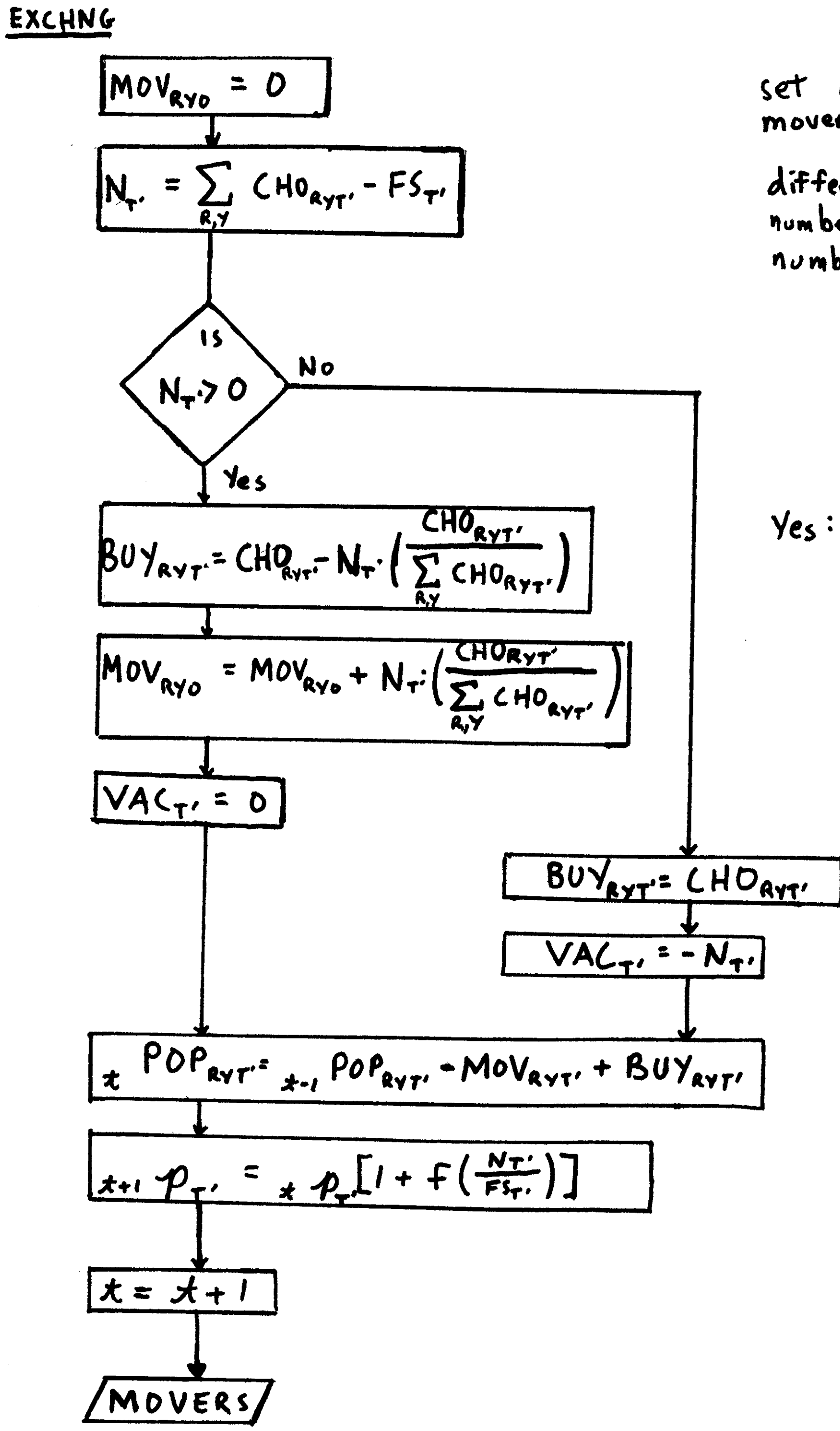


set origin unspecified movers to zero

difference between total number of choosers and number 'for sale'

Yes: too many choosers choosers proportionally rejected; others buy

those rejected are placed on origin unspecified movers list

vacancies set to zero

No: All choosers buy

Excess units set vacant

Transactions are made

prices are adjusted

update time

recycle to <u>MOVERS</u>

3.4 Limitations of the Model

Before proceeding further it is necessary to point out some of the limitations of the model which result from its fundamental simplicity. Most importantly, actual housing decisions are far more complicated than decisions in the model. Obviously there are characteristics other than income and race which affect the decisions of when to move and where to move; without doubt the age and education of the household head, the household size, non-racial ethnicity, and many other factors affect decisions. Moreover, the model does not clearly distinguish between willingness to move and ability to move. Each time period a number of racially motivated movers are generated; all of these households try to complete a transaction. The decision to move is not affected by the price of their present unit. In fact, many households in racially changing areas do not decide to move because they do not believe that can sell their home at its "true" value. As a result households which are willing to move are unable to move because they cannot get enough for the old home. The model may therefore over-estimate rates of racial change.

In like manner, housing is more complicated than the one dimensional quality of housing service: the unit's age, its history of maintenance, its school district, its accessibility, and many of its detailed features will affect the unit's attractiveness to the different types of movers.

Another set of simplifications appear in the market process.

Important actors, such as lenders, real estate investors, and realtors, do not play an active role in the model market. Moreover, the model does not consider the role of a home as a major capital asset. The decision rule for movers only considers the annual payment owners of housing must make (quantity of service units times the price of a service unit) and does consider the effect the downpayment has on the kind of unit which is purchased. The real market also exerts forces on the housing stock. First, the location and nature of new construction, though subject to various constraints, does respond to market changes. In the model both the location, type, and quantity of new construction is exogenous. Secondly, housing for many households, particularly poor households, is provided through the conversion process: units are allowed to deteriorate until their market value is low enough for poorer households to afford. In terms of the model, conversion of a unit consists of a change in the number of service units the dwelling provides (a change in q). However, changes in q are not allowed in the model.

These limitations are significant. Clearly, the ability of the model, when applied to real data, to duplicate actual racial patterns or to predict future changes is questionable. But this is not to say that the model is useless. As noted in the Introduction, a simple model is still of value. In a topic as important as racial transition in neighborhoods, even this limited value is significant.

4.1 Operation of the Model

Operation of the model requires the specification of a variety of arrays, parameters, and functions. The following arrays describe the initial characteristics of the model region:

POP_{RYT} = number of households in each tract by race and income.

GR_{RY} = net regional growth by race and income.

r_Y = fraction of households moving each time period for reasons other than race by income.

Y = income of each income class.

q_T = number of housing service units of dwellings in tract T.

p_T = price of unit of housing service in tract T.

x_T = x-coordinate of center of gravity of tract T.

y_T = y-coordinate of center of gravity of tract T.

VAC_T = initial vacancies in tract T.

NEW_T = number of new dwellings constructed in T each time period.

Besides the arrays, there are several functions which must be set:

$\%(T)$ = perceived per cent black for tract T. (This may be the actual per cent black in T, or it may be a more complicated function, including for example, a projection of what the percentage will be or the per cent in adjacent tracts.

$U(R,Y,T)$ = utility of choice of tract T for household of race (R) and income (Y).

$f(\Delta_{RYT})$ = rate of racially motivated movement as function of Δ_{RYT}, the fractional loss of utility because of race.

$D(d_{TT'})$ = dependence of likelihood of move on distance.

$H(U(R,Y,T))$ = dependence of likelihood of move on utilty.

$\delta(N_T/FS_T)$ = fraction response of price to market condition in tract T.

Besides, the utility function makes use of the parameters α_Y; the use of the parameters α_Y will be discussed later. These arrays and functions are the only data required by the model.

The next section will describe a series of experiments performed with the model.

4.2 Experiments Performed with the Model

A series of simulation runs were made using data generated specifically for the model; no data derived from a real city was used. A model city was created which consists of 16 tracts. The tracts were arranged in a 4x4 grid; this was done only for display purposes and is not required by the model. (The model will accept any arrangement of tracts of any size.) The population is divided into two races (black and white) and three income classes (low, middle, and high). The time period in all the simulations was 6 months and all runs covered 20 periods or 10 years. Unless otherwise specifically noted, the following values were used:

POP_{RYT}

tract	white low	white mid	white high	black low	black mid	black high
1	0	300	1500	0	0	0
2	0	1550	1500	0	0	0
3	0	1500	1500	0	0	0
4	200	1500	1000	0	0	0
5	0	0	2000	0	0	0
6	0	1000	1000	0	500	1000
7	1000	1000	1000	500	500	0
8	2000	1000	0	1000	200	0
9	0	3000	0	0	0	0
10	500	1000	500	200	800	100
11	3000	1000	0	50	0	0
12	0	0	0	2000	2000	0
13	200	1500	300	0	0	0
14	0	0	2000	0	0	0
15	100	1500	180	100	2000	300
16	0	0	0	4000	1000	0

GR_{RY}

	low	mid	high
white	100	100	100
black	300	500	300

	low	mid	high
Y	8000	15000	25000
r_Y	.15	.08	.08

tract	q_T	p_T	location (x,y)	initial VAC_T
1	5000	1.00	(1,1)	0
2	5000	1.00	(2,1)	0
3	5000	1.00	(3,1)	0
4	3750	1.00	(4,1)	0
5	5000	1.00	(1,2)	0
6	3750	1.00	(2,2)	0
7	3500	1.00	(3,2)	0
8	3000	1.00	(4,2)	0
9	3750	1.00	(1,3)	0
10	4000	1.00	(2,3)	0
11	3000	1.00	(3,3)	0
12	2800	1.00	(4,3)	0
13	3500	1.00	(1,4)	0
14	5000	1.00	(2,4)	0
15	3750	1.00	(3,4)	0
16	2800	1.00	(4,4)	0

The values of q_T and p_T were were set with respect to each other and the income levels of the households. The price of a unit of housing service was initially set at $1.00. Census data indicates that households spend roughly 25% of their income on housing (less at higher incomes and more at lower). Thus if incomes range from $8000 to $25,000, housing expenditures would tend to range from $2750 to $5000. Since the price of a unit of service is $1.00, this means that bundles of housing services from 2750 to 5000 units are appropriate.

The utility function used in all the experiments is a variation of the standard Cobb-Douglass function of microeconomics.[11]

$$U(R,Y,T) = (Y - q_T p_T)^{\alpha_Y} \cdot q_T^{1-\alpha_Y} \cdot f_R(\%)$$

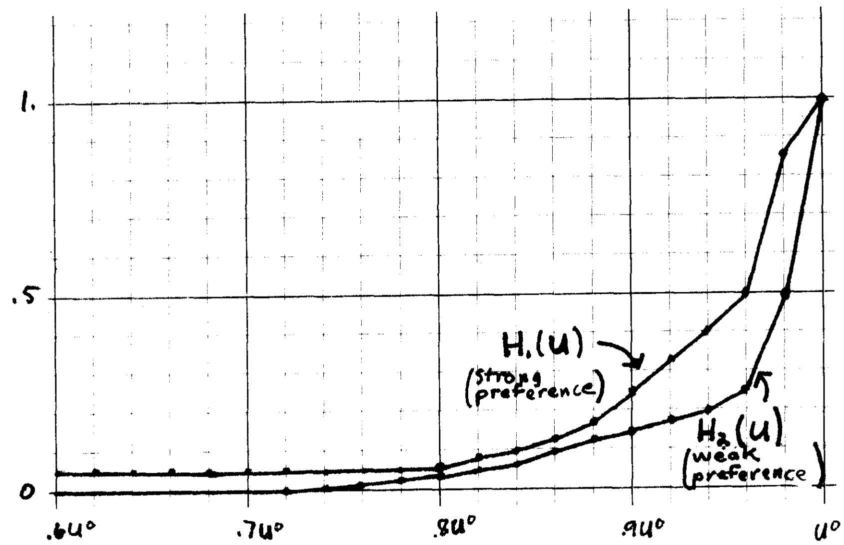
The first factor represents the utility derived from non-housing expenditures. The housing expenditures are $q_T p_T$; therefore it has $Y - q_T p_T$ left over for all other expenditures. The second term $(q_T p_T^{1-\alpha_Y})$ represents the utility derived from the consumption of housing. The last term $(f_R(\%))$ measures the effect of the racial composition of the tract on its utility. This is the only term of the utility function which is different for whites and blacks. The racial effect is not described as an analytic function but as a table function; its form will be described in each experiment.

The Cobb-Douglass form has one major advantage for this model. Through the parameter α_Y it is possible to specify the percentage of its income that a household must spend on housing in order to maximize its utility. In all experiments:

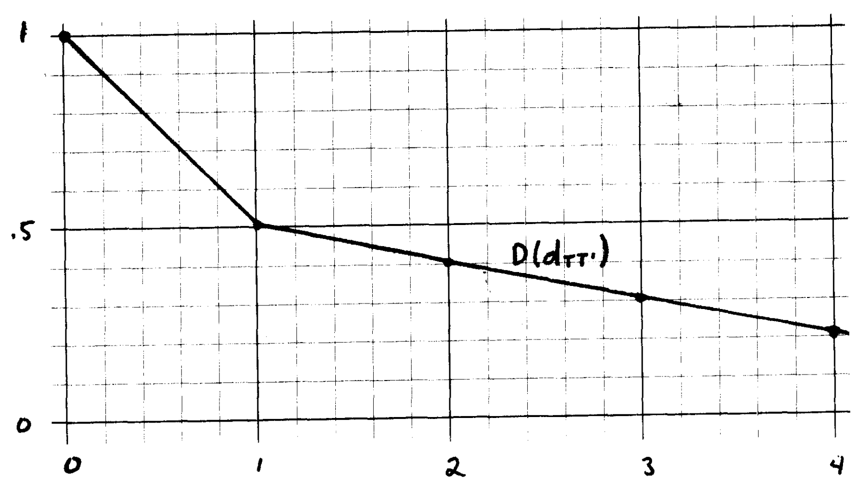
	low	mid	high
α_Y	.65	.75	.80 .

Households maximize when they spend $1 - \alpha_Y$ of their income on housing. The parameter α_Y controls which type of housing each income group is directed to. The major drawback of the Cobb-Douglass form is that it is rather insensitive to different combinations of income and housing. Disregarding the racial term, the utility of all the housing bundles for a given household will vary by only 2%-3%. This is not a serious problem because the insensitivity can be compensated by making H(U(R,Y,T)) very sensitive to U(R,Y,T). Thus it is possible to make the small differences in utility have a large effect on the likelihoods of moves.

Two different forms of $H(U(R,Y,T))$ were used in the experiment. Both are very sensitive to changes in utility, but the first one is more sensitive than the second; households using the first function are more particular in choosing a house to meet their needs as determined by their race and income.

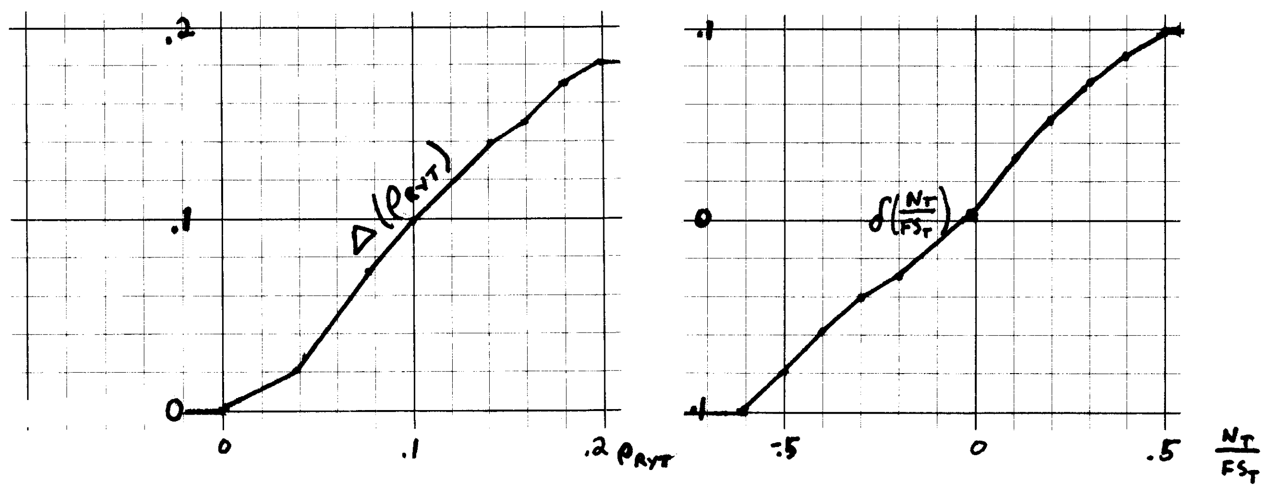


The dependency of likelihood on distance ($D(d_{TT'})$) was not varied. The form of $D(d_{TT'})$ was:



The distance axis is in the same units as the grid coordinates; thus tracts at (3,1) and (1,1) are two units apart. The actual size of these distance units is unspecified, but is of the order of one mile.

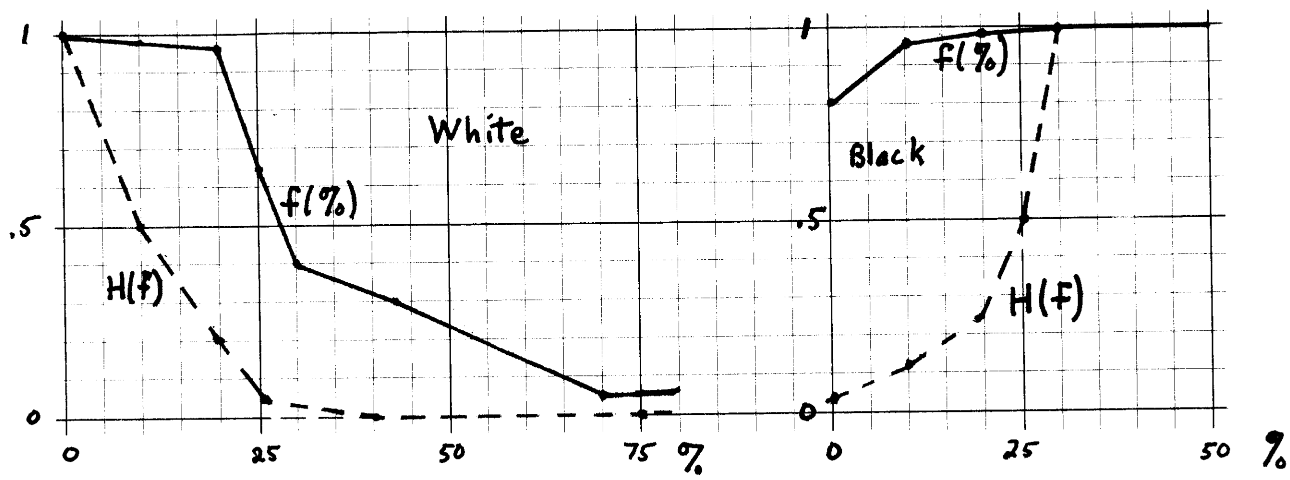
Shown below are the final two functions which must be specified: the rate of movement because of race as a function of the fractional utility loss $\Delta(\rho_{RYT})$, and the response of prices to the fractional excess or shortage of units, $\delta(N_T/FS_T)$.



Unless otherwise noted, the decision to move (computation of ρ_{RYT}) uses the present per cent black in the tract. The decision to enter a tract, as determined by H(U) is based on a percentage which is an average of the per cent black in the tract and the mean per cent black in those tracts less than 1.1 units away. Thus the potential buyer considers the four adjacent tracts when computing his likelihoods.

EXPERIMENT #1: Base run.

Besides the specifications described in the preceeding section, the following forms of $f_R(\%)$ were used:



The solid curve in each graph is the function entered into the utilty function for each race. The broken curve is H(f(%)) and displays the effect of race on behavior much more clearly. H(f(%)) measures the change in the likelihood of a move because of race, holding all other factors constant. For example, consider a mover faced with two alternatives of identical housing (same q_T and p_T) the same distance away, but in tracts of different racial composition. The relative likelihood of each move is simple the value of H(f(%)) in each tract. In the curves above, a white is not very likely to move to a tract which is more than 20% black. Blacks on the other hand, are more willing to accept inter-racial housing, but they are unlikely to enter a tract which is almost exclusively white. This experiment uses the strong form of H(U); likelihoods depend strongly on utility.

The results of this experiment are striking and typical of racial transition. In the ten years of the simulation, the black population has nearly tripled and the black area has extended into many formerly all-white areas. The solidly black area was concentrated into two tracts (12 and 16), but after ten years nine tractswere almost exclusively black. In fact, by the end of the run only one tract (1) had not begun the process of racial transition. In black areas, and those in the process of becoming black, there are significant numbers of units remaining unsold (vacant). This results from the rapid flight of whites which brings more units to the market than the growing black population is able to absorb. Thus blacks can obtain housing at a reduced price, while whites pay more for segregated housing.

It is important to note the effect of racial transition on particular tracts. Tract 1 is the only tract which blacks do not enter. Initially, the high rate of white flight from tracts near the initial concentration of blacks forces up prices in the outlying white areas (1,2,3,4,5, and 9). However, the continued heavy construction in these tracts eases supply and prices drop in tract 1 and the others. But after 15 time periods ($7\frac{1}{2}$ years) racial transition has begun in all the tracts except 1. Thus a new white flight begins from these outlying tracts and is aimed soley at tract 1. In response, prices rise in tract 1, despite the continued construction. Tract 1 is the last haven for whites.

Tracts 2,3,4,5,9, and 13 display, by time period 20, different phases of the racial transition process. Typical features can be seen in each:

#1

PRICE - "P"
MEAN INCOME - "Y"
% BLACK - "%"
VACANCY RATE - "V"

T= 0

1	%=0.0	V=0.0	P=1.00	Y=23333.	Q= 5000.
2	%=0.0	V=0.0	P=1.00	Y=19918.	Q= 5000.
3	%=0.0	V=0.0	P=1.00	Y=20000.	Q= 5000.
4	%=0.0	V=0.0	P=1.00	Y=18185.	Q= 3750.
5	%=0.0	V=0.0	P=1.00	Y=25000.	Q= 5000.
6	%=0.43	V=0.0	P=1.00	Y=20714.	Q= 3750.
7	%=0.25	V=0.0	P=1.00	Y=14875.	Q= 3500.
8	%=0.29	V=0.0	P=1.00	Y=10000.	Q= 3000.
9	%=0.0	V=0.0	P=1.00	Y=15000.	Q= 3750.
10	%=0.35	V=0.0	P=1.00	Y=15355.	Q= 4000.
11	%=0.01	V=0.0	P=1.00	Y= 9728.	Q= 3000.
12	%=1.00	V=0.0	P=1.00	Y=11500.	Q= 2800.
13	%=0.0	V=0.0	P=1.00	Y=15800.	Q= 3500.
14	%=0.0	V=0.0	P=1.00	Y=25000.	Q= 5000.
15	%=0.57	V=0.0	P=1.00	Y=15813.	Q= 3750.
16	%=1.00	V=0.0	P=1.00	Y= 9400.	Q= 2800.

T= 4

1	%=0.0	V=0.0	P=1.33	Y=21717.
2	%=0.00	V=0.0	P=1.37	Y=19919.
3	%=0.0	V=0.0	P=1.29	Y=20126.
4	%=0.00	V=0.0	P=1.46	Y=17090.
5	%=0.00	V=0.0	P=1.37	Y=22484.
6	%=0.80	V=0.21	P=0.72	Y=18502.
7	%=0.76	V=0.25	P=0.72	Y=14262.
8	%=0.80	V=0.26	P=0.73	Y=11369.
9	%=0.0	V=0.0	P=1.46	Y=15331.
10	%=0.79	V=0.22	P=0.72	Y=15210.
11	%=0.43	V=0.27	P=0.72	Y=11296.
12	%=1.00	V=0.15	P=0.74	Y=11893.
13	%=0.0	V=0.0	P=1.46	Y=15033.
14	%=0.01	V=0.0	P=1.27	Y=22998.
15	%=0.86	V=0.17	P=0.72	Y=15318.
16	%=1.00	V=0.18	P=0.74	Y=10291.

T= 8

1	%=0.0	V=0.0	P=1.35	Y=20793.
2	%=0.00	V=0.09	P=1.19	Y=20165.
3	%=0.00	V=0.0	P=1.32	Y=19899.
4	%=0.00	V=0.12	P=1.40	Y=17510.
5	%=0.00	V=0.10	P=1.19	Y=21747.
6	%=0.95	V=0.19	P=0.65	Y=16692.
7	%=0.94	V=0.19	P=0.65	Y=13993.
8	%=0.96	V=0.20	P=0.65	Y=12723.
9	%=0.00	V=0.05	P=1.37	Y=16573.
10	%=0.95	V=0.19	P=0.65	Y=14712.
11	%=0.89	V=0.22	P=0.65	Y=13101.
12	%=1.00	V=0.16	P=0.65	Y=12676.
13	%=0.0	V=0.08	P=1.50	Y=15880.
14	%=0.03	V=0.26	P=0.91	Y=22946.
15	%=0.96	V=0.16	P=0.65	Y=14779.
16	%=1.00	V=0.18	P=0.65	Y=11740.

T=12

1	%=0.0	V=0.09	P=1.00	Y=20747.
2	%=0.01	V=0.11	P=0.97	Y=19860.
3	%=0.01	V=0.11	P=0.95	Y=19968.
4	%=0.02	V=0.14	P=1.01	Y=17648.
5	%=0.01	V=0.11	P=0.97	Y=20778.
6	%=0.99	V=0.12	P=0.65	Y=15768.
7	%=0.98	V=0.11	P=0.65	Y=14070.
8	%=0.99	V=0.12	P=0.65	Y=13417.
9	%=0.01	V=0.11	P=1.06	Y=17164.
10	%=0.98	V=0.11	P=0.65	Y=14535.
11	%=0.98	V=0.10	P=0.65	Y=13689.
12	%=1.00	V=0.10	P=0.65	Y=13266.
13	%=0.0	V=0.10	P=1.11	Y=16680.
14	%=0.61	V=0.20	P=0.65	Y=17537.
15	%=0.99	V=0.10	P=0.65	Y=14533.
16	%=1.00	V=0.12	P=0.65	Y=12794.

#1

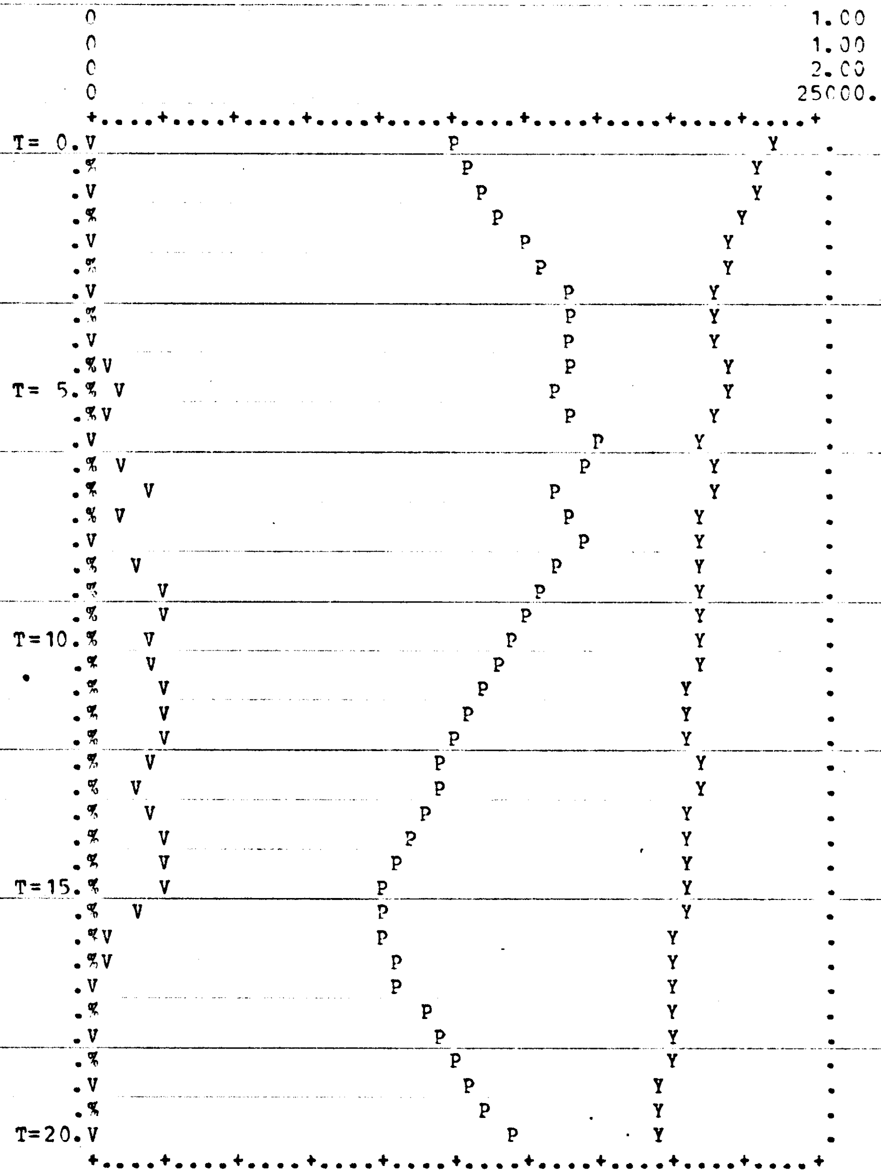
T=16

1 %=0.00	2 %=0.09	3 %=0.13	4 %=0.35
V=0.03	V=0.0	V=0.11	V=0.26
P=0.78	P=0.84	P=0.72	P=0.72
Y=20204.	Y=19196.	Y=19102.	Y=16140.
5 %=0.10	6 %=0.99	7 %=0.99	8 %=1.00
V=0.0	V=0.14	V=0.13	V=0.14
P=0.82	P=0.65	P=0.65	P=0.65
Y=19673.	Y=15495.	Y=14305.	Y=13899.
9 %=0.16	10 %=0.99	11 %=0.99	12 %=1.00
V=0.18	V=0.13	V=0.12	V=0.13
P=0.76	P=0.65	P=0.65	P=0.65
Y=16306.	Y=14616.	Y=14076.	Y=13721.
13 %=0.10	14 %=0.89	15 %=0.99	16 %=1.00
V=0.13	V=0.19	V=0.13	V=0.14
P=0.80	P=0.65	P=0.65	P=0.65
Y=16190.	Y=15005.	Y=14554.	Y=13448.

T=20

1 %=0.00	2 %=0.24	3 %=0.55	4 %=0.81
V=0.0	V=0.25	V=0.39	V=0.40
P=1.14	P=0.65	P=0.65	P=0.65
Y=19665.	Y=18226.	Y=16630.	Y=14599.
5 %=0.32	6 %=1.00	7 %=1.00	8 %=1.00
V=0.33	V=0.22	V=0.21	V=0.22
P=0.65	P=0.65	P=0.65	P=0.65
Y=18240.	Y=15501.	Y=14604.	Y=14333.
9 %=0.66	10 %=0.99	11 %=1.00	12 %=1.00
V=0.42	V=0.21	V=0.21	V=0.22
P=0.65	P=0.65	P=0.65	P=0.65
Y=14922.	Y=14827.	Y=14436.	Y=14162.
13 %=0.41	14 %=0.96	15 %=0.99	16 %=1.00
V=0.37	V=0.26	V=0.22	V=0.23
P=0.65	P=0.65	P=0.65	P=0.65
Y=15493.	Y=14716.	Y=14765.	Y=14006.

TIME PATH OF CHARACTERISTICS OF TRACT NO. 1   Q(1)= 5000.   RUN NO. 1

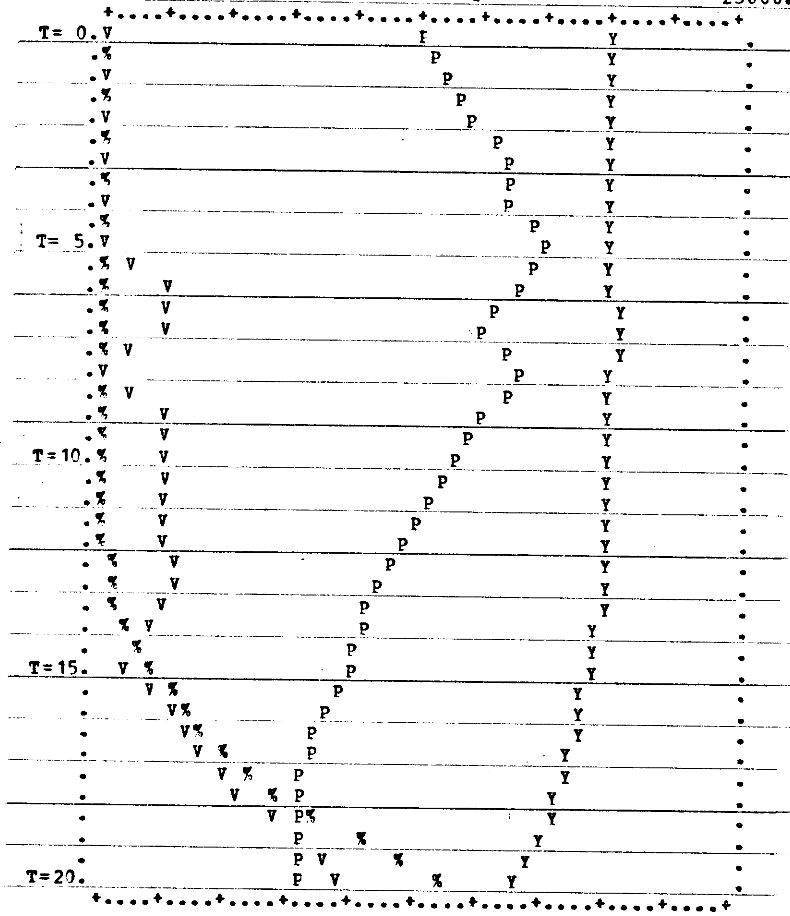


TIME	% BLACK	VACANCIES	PRICE	INCOME
0	0.0	0.0	1.00	23333.
1	0.0	0.0	1.08	22829.
2	0.0	0.0	1.18	22179.
3	0.0	0.0	1.30	21748.
4	0.0	0.0	1.33	21717.
5	0.0	0.04	1.27	22143.
6	0.0	0.0	1.40	21233.
7	0.0	0.08	1.29	21512.
8	0.0	0.0	1.35	20793.
9	0.0	0.10	1.24	20972.
10	0.0	0.08	1.17	20796.
11	0.0	0.09	1.08	20733.
12	0.0	0.09	1.00	20747.
13	0.0	0.07	0.94	20754.
14	0.0	0.10	0.87	20645.
15	0.0	0.11	0.80	20545.
16	0.00	0.03	0.78	20204.
17	0.00	0.0	0.86	19971.
18	0.00	0.0	0.94	19818.
19	0.00	0.0	1.04	19688.
20	0.00	0.0	1.14	19665.

1

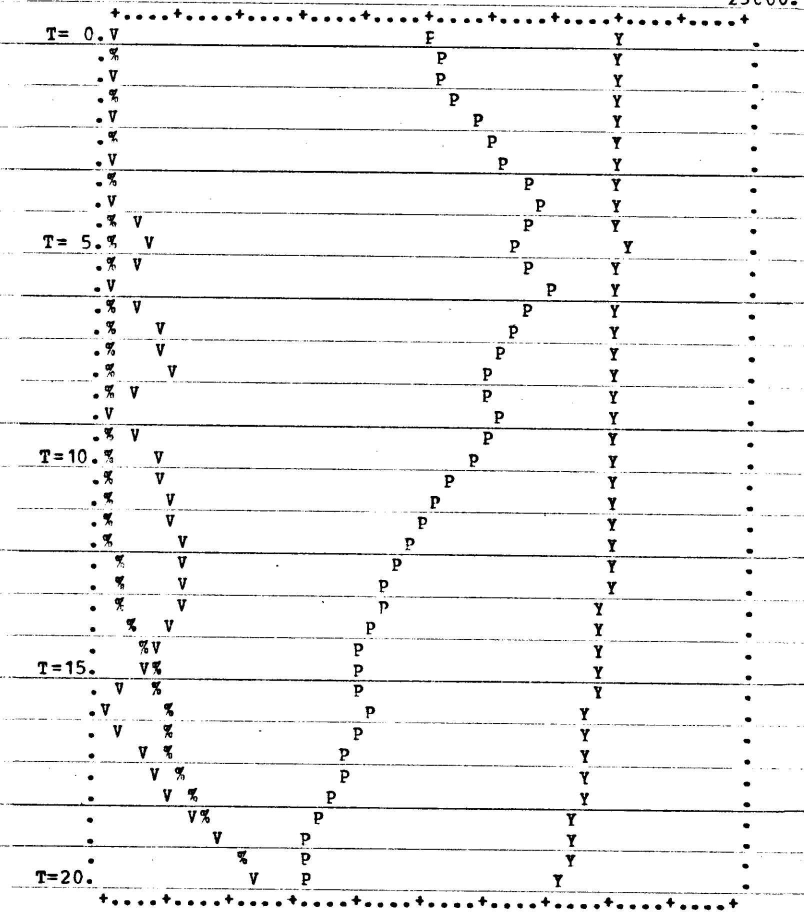
TIME PATH OF CHARACTERISTICS CF TRACT NO. 3   Q(3) = 5000

C   % BLACK - "%"   1.00
0   VACANCY RATE - "V"   1.CO
0   PRICE - "P"   2.00
0   MEAN INCOME - "Y"   25000.



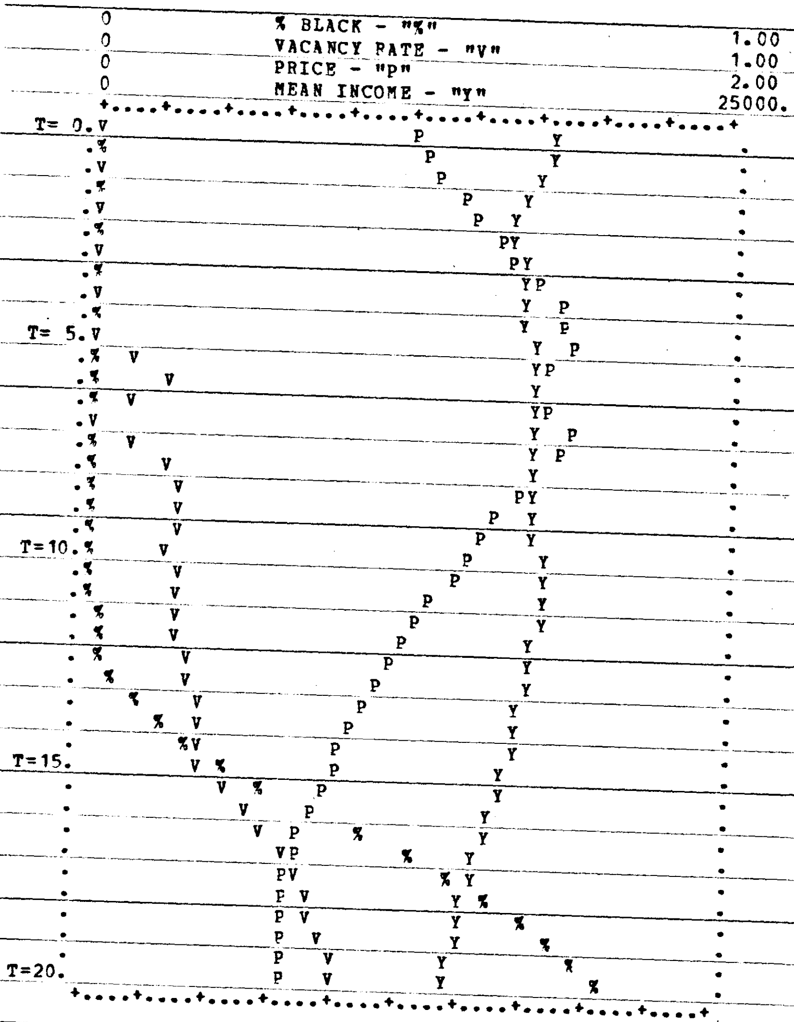
TIME PATH OF CHARACTERISTICS OF TRACT NO. 2   Q(2) = 5000

0   1.00
0   1.00
0   2.00
0   25000.

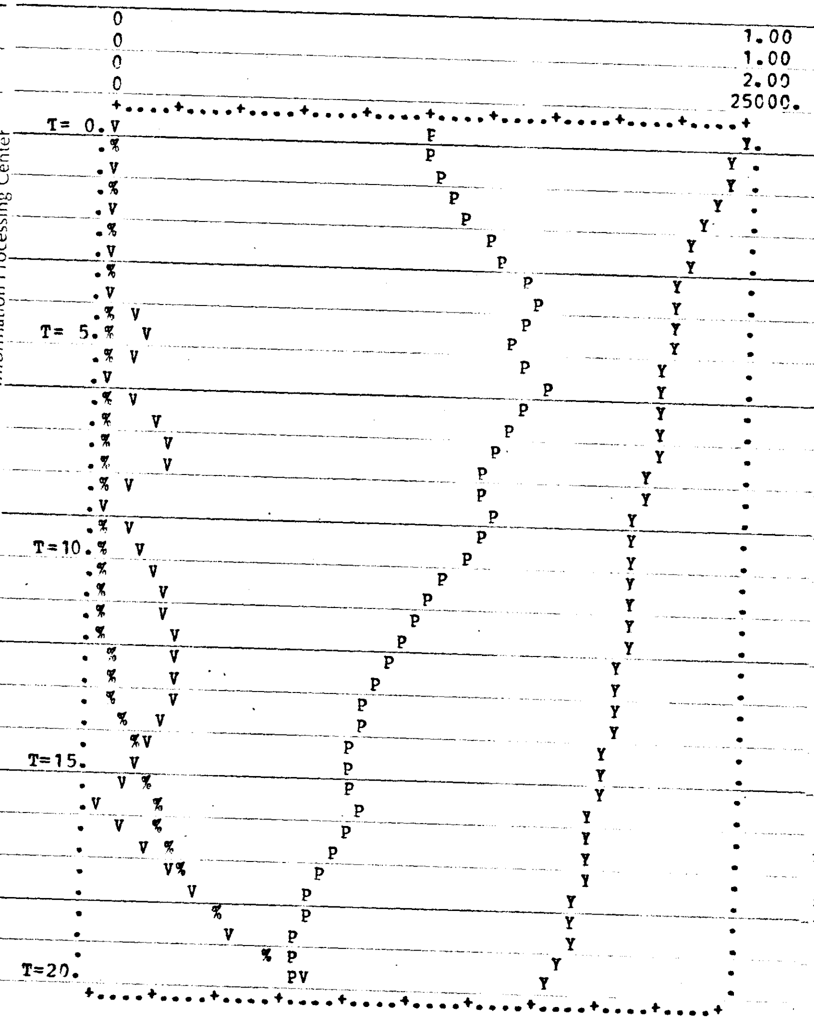


#1

TIME PATH OF CHARACTERISTICS OF TRACT NO. 4   Q(4)= 3750.



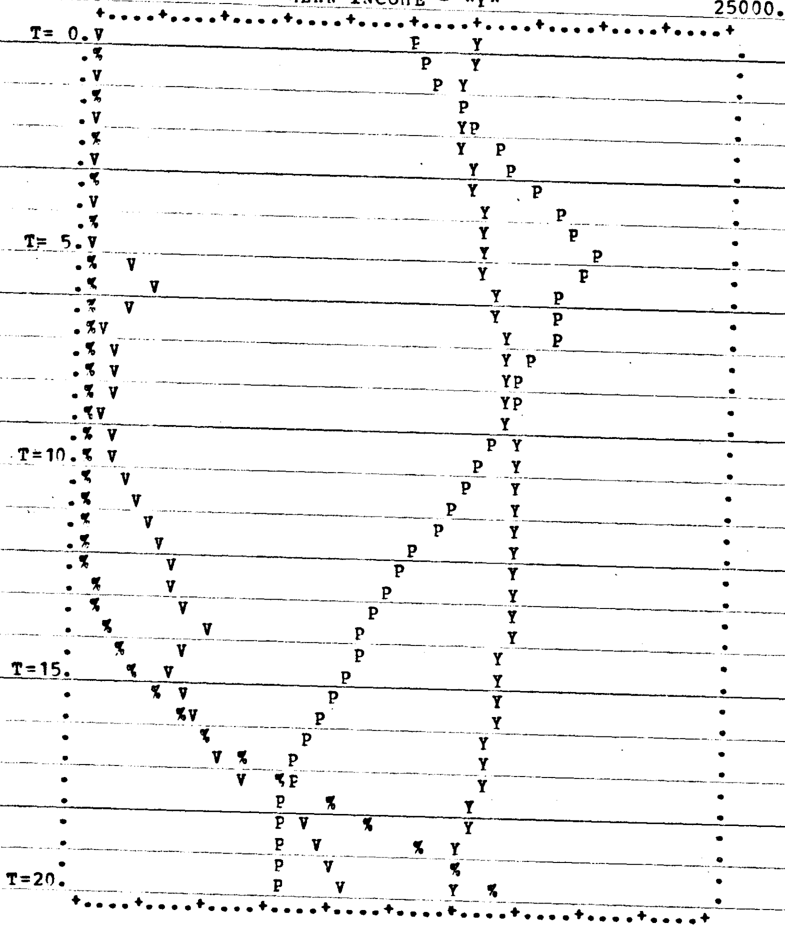
TIME PATH OF CHARACTERISTICS OF TRACT NO. 5   Q(5)= 5000.



#1

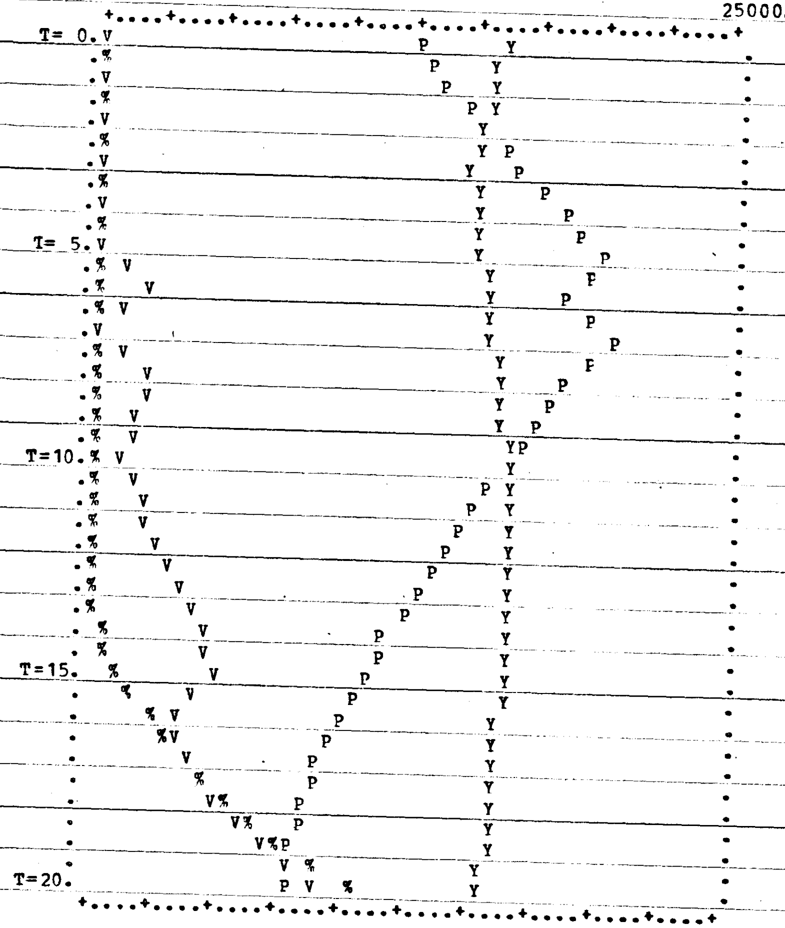
TIME PATH OF CHARACTERISTICS OF TRACT NO. 9 Q(9)= 3750.

0	% BLACK - "%"	1.00
0	VACANCY RATE - "V"	1.00
0	PRICE - "P"	2.00
0	MEAN INCOME - "Y"	25000.



TIME PATH OF CHARACTERISTICS OF TRACT NO.13 Q(13)= 3500.

0	1.00
0	1.00
0	2.00
0	25000.

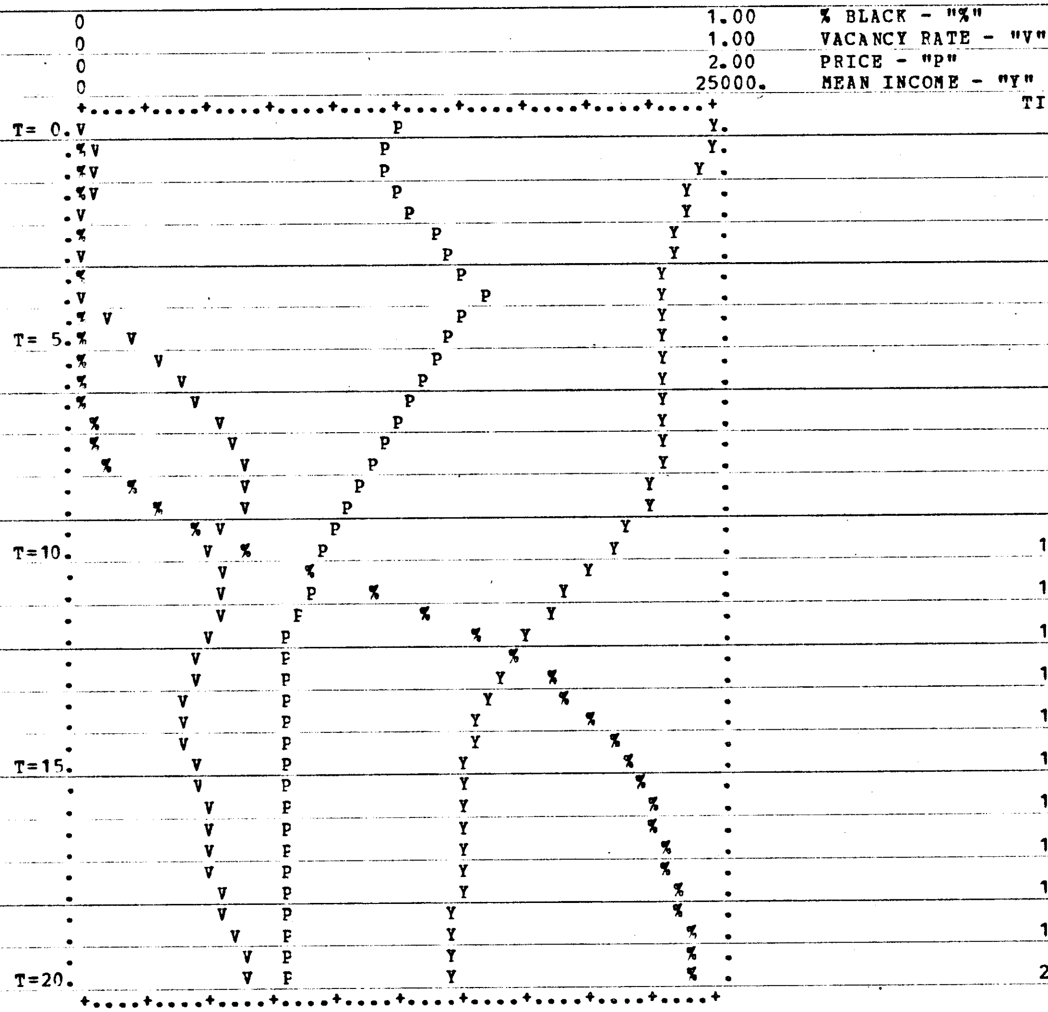


a rise in vacancies as neighboring tracts begin to change, declining prices, and an increasingly steep rate of change from white to black.

The entire transition processs occurs in tract 14. At the beginning of the simulation the tract is all-white, with high quality units and high income residents. At first very few units going on the market are not sold (low vacancies) and as whites are fleeing the first neighborhoods to change, prices rise. A trace appearance of blacks after one year has little immediate effect. However, after 2½ years two neighboring tracts (10 and 15) have built up substantial black populations and this affects the willingness of whites to move to tract 14. After 3½ years 22% of the units cannot be sold and prices are dropping. At this point the number of black entrants begins to sharply increase; the tract has now clearly entered the transition process. At the fourth year of the simulation the tract is 3% black; during the next three years this figure rises to 81%. In this same period the mean income of the tract's residents falls from $23,000 to $15,000. Thus housing in the tract is not only filtering over (from white to black), but also filtering down (from upper to middle income groups). It should be noted that prices remain quite low during the entire transition process.

It is important to note that prices do not rise after transition is completed. The tract's price does conform to the prices in other black tracts; prices are low in all black tracts. By the final year tract 14 is part of the ghetto.

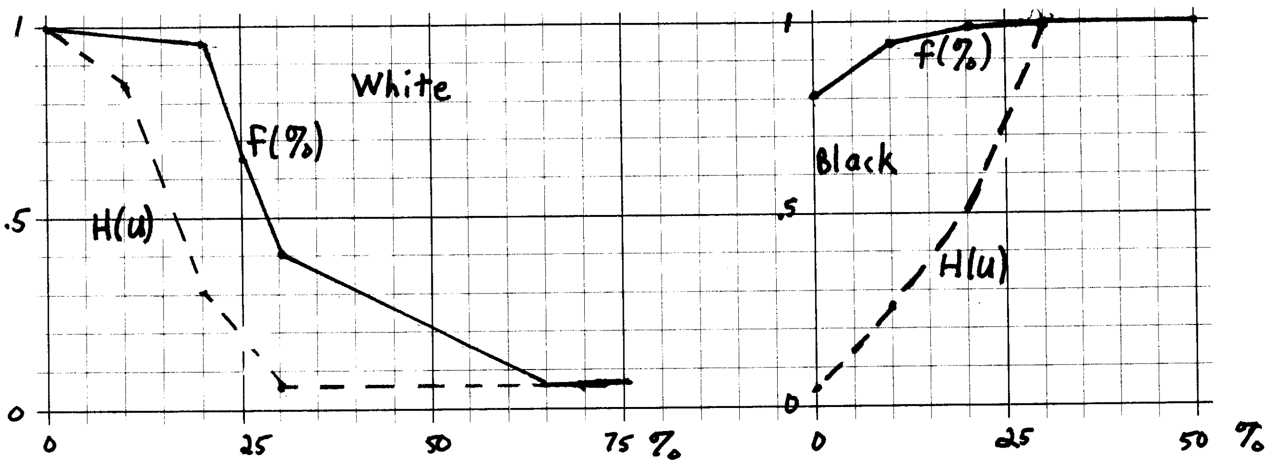
TIME PATH OF CHARACTERISTICS OF TRACT NO.14 Q(14)= 5000. RUN NO. 1



TIME	% BLACK	VACANCIES	PRICE	INCOME
0	0.0	0.0	1.00	25000.
1	0.00	0.02	0.95	24668.
2	0.01	0.0	1.05	23810.
3	0.01	0.0	1.15	23385.
4	0.01	0.0	1.27	22998.
5	0.01	0.08	1.17	23007.
6	0.01	0.15	1.07	23014.
7	0.01	0.22	0.99	23000.
8	0.03	0.26	0.91	22946.
9	0.12	0.26	0.84	22437.
10	0.26	0.20	0.77	20967.
11	0.45	0.22	0.71	19107.
12	0.61	0.20	0.65	17537.
13	0.73	0.17	0.65	16396.
14	0.81	0.16	0.65	15685.
15	0.86	0.17	0.65	15239.
16	0.89	0.19	0.65	15005.
17	0.92	0.20	0.65	14826.
18	0.94	0.22	0.65	14757.
19	0.95	0.24	0.65	14727.
20	0.96	0.26	0.65	14716.

EXPERIMENT #2 A Lessening of Preferences

This experiment is identical to the previous one, except that households are slightly less particular: the dependence of the likelihood of a move does not depend as heavily on the utility of the move. Households are more likely to pick a less than perfect match of housing service units and income, and more willing to live in racially mixed tracts.



As might be expected the greater tolerances slowed the rates of racial transition. For example, after 10 years tract 4 was 81% black in experiment 1, while in this experiment it had only reached 26%. The rapid transition in tract 14 was delayed four years in this simulation. The number of vacant units was generally lower. Interestingly, the slightly greater tolerances made it possible for blacks to enter all the tracts; even tract 1 was 2% black by time period 20.

The response of whites to black incursions was slowed in this run: whites did not flee as rapidly. Also, blacks continued to enter the region at the same rate. These two facts combined to produce an

#2

% BLACK - "%"
VACANCY RATE - "V"
PRICE - "P"
MEAN INCOME - "Y"

T= 0

1 %=0.0 V=0.0 P=1.00 Y=23333. Q= 5000.	2 %=0.0 V=0.0 P=1.00 Y=19918. Q= 5000.	3 %=0.0 V=0.0 P=1.00 Y=20000. Q= 5000.	4 %=0.0 V=0.0 P=1.00 Y=18185. Q= 3750.
5 %=0.0 V=0.0 P=1.00 Y=25000. Q= 5000.	6 %=0.43 V=0.0 P=1.00 Y=20714. Q= 3750.	7 %=0.25 V=0.0 P=1.00 Y=14875. Q= 3500.	8 %=0.29 V=0.0 P=1.00 Y=10000. Q= 3000.
9 %=0.0 V=0.0 P=1.00 Y=15000. Q= 3750.	10 %=0.35 V=0.0 P=1.00 Y=15355. Q= 4000.	11 %=0.01 V=0.0 P=1.00 Y= 9728. Q= 3000.	12 %=1.00 V=0.0 P=1.00 Y=11500. Q= 2800.
13 %=0.0 V=0.0 P=1.00 Y=15800. Q= 3500.	14 %=0.0 V=0.0 P=1.00 Y=25000. Q= 5000.	15 %=0.57 V=0.0 P=1.00 Y=15813. Q= 3750.	16 %=1.00 V=0.0 P=1.00 Y= 9400. Q= 2800.

T= 8

1 %=0.00 V=0.0 P=2.09 Y=19951.	2 %=0.00 V=0.0 P=2.09 Y=19038.	3 %=0.00 V=0.0 P=2.08 Y=19091.	4 %=0.00 V=0.0 P=2.14 Y=15967.
5 %=0.00 V=0.0 P=2.08 Y=20505.	6 %=0.95 V=0.19 P=0.65 Y=16586.	7 %=0.94 V=0.20 P=0.65 Y=13889.	8 %=0.96 V=0.20 P=0.65 Y=12904.
9 %=0.00 V=0.0 P=2.14 Y=15073.	10 %=0.95 V=0.19 P=0.65 Y=14544.	11 %=0.87 V=0.22 P=0.65 Y=13096.	12 %=1.00 V=0.15 P=0.65 Y=12834.
13 %=0.00 V=0.0 P=2.14 Y=14475.	14 %=0.01 V=0.01 P=1.56 Y=21606.	15 %=0.96 V=0.17 P=0.65 Y=14678.	16 %=1.00 V=0.18 P=0.65 Y=11947.

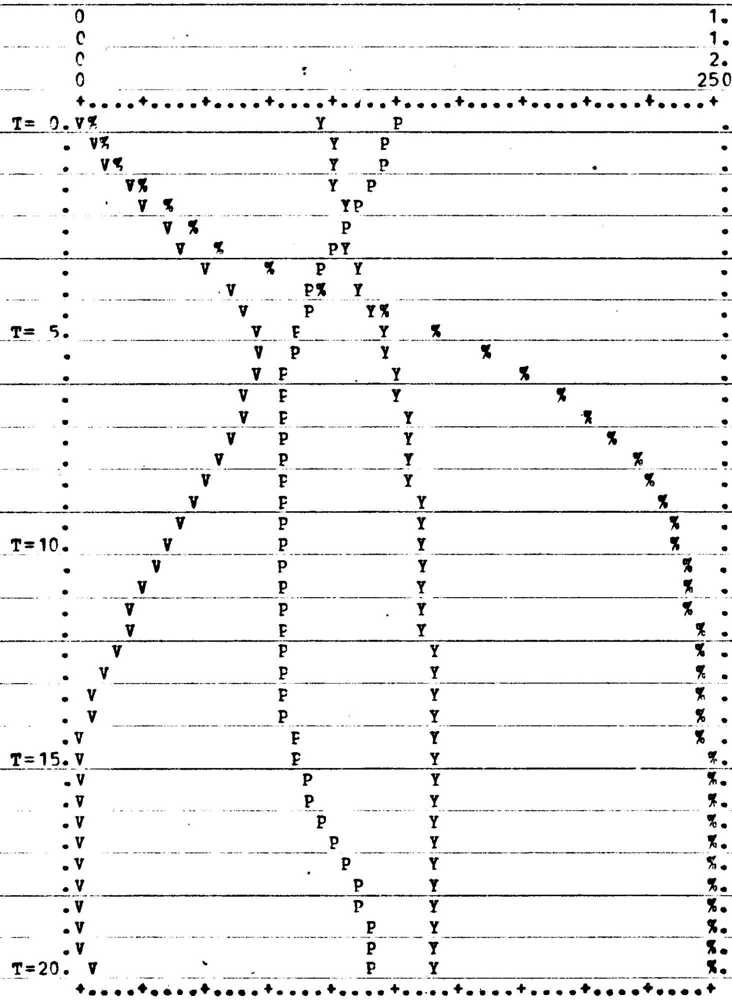
T=12

1 %=0.00 V=0.12 P=1.67 Y=19979.	2 %=0.00 V=0.11 P=1.67 Y=19390.	3 %=0.00 V=0.11 P=1.66 Y=19377.	4 %=0.00 V=0.12 P=1.88 Y=17338.
5 %=0.00 V=0.11 P=1.66 Y=20336.	6 %=0.99 V=0.09 P=0.65 Y=15645.	7 %=0.98 V=0.08 P=0.65 Y=14019.	8 %=0.99 V=0.09 P=0.65 Y=13567.
9 %=0.00 V=0.10 P=1.95 Y=16685.	10 %=0.98 V=0.08 P=0.65 Y=14438.	11 %=0.97 V=0.07 P=0.65 Y=13716.	12 %=1.00 V=0.07 P=0.65 Y=13407.
13 %=0.00 V=0.09 P=1.94 Y=16005.	14 %=0.01 V=0.12 P=1.17 Y=21117.	15 %=0.99 V=0.07 P=0.65 Y=14476.	16 %=1.00 V=0.08 P=0.65 Y=12965.

T=20

1 %=0.02 V=0.0 P=1.06 Y=19588.	2 %=0.15 V=0.16 P=0.86 Y=18735.	3 %=0.18 V=0.15 P=0.85 Y=18672.	4 %=0.26 V=0.28 P=0.97 Y=16746.
5 %=0.15 V=0.15 P=0.85 Y=19059.	6 %=1.00 V=0.02 P=0.79 Y=14809.	7 %=1.00 V=0.01. P=0.86 Y=14076.	8 %=1.00 V=0.02 P=0.82 Y=13878.
9 %=0.10 V=0.21 P=1.00 Y=16868.	10 %=1.00 V=0.01 P=0.82 Y=14240.	11 %=1.00 V=0.01 P=0.90 Y=13667.	12 %=1.00 V=0.02 P=0.86 Y=13599.
13 %=0.05 V=0.14 P=1.00 Y=16359.	14 %=0.74 V=0.06 P=0.80 Y=16076.	15 %=1.00 V=0.01 P=0.84 Y=14214.	16 %=1.00 V=0.02 P=0.80 Y=13553.

TIME PATH OF CHARACTERISTICS OF TRACT NO. 11 Q(11) = 3000. RUN NO. 2



TIME	% BLACK	VACANCIES	PRICE	INCOME
0	0.01	0.0	1.00	9728.
1	0.06	0.05	0.94	9982.
2	0.13	0.11	0.87	10274.
3	0.23	0.15	0.80	10659.
4	0.38	0.24	0.73	11249.
5	0.56	0.28	0.67	11884.
6	0.70	0.28	0.65	12414.
7	0.81	0.26	0.65	12805.
8	0.87	0.22	0.65	13096.
9	0.92	0.18	0.65	13308.
10	0.95	0.14	0.65	13473.
11	0.96	0.11	0.65	13609.
12	0.97	0.07	0.65	13716.
13	0.98	0.04	0.65	13805.
14	0.99	0.01	0.65	13878.
15	0.99	0.0	0.68	13950.
16	0.99	0.0	0.73	14008.
17	0.99	0.0	0.80	14063.
18	0.99	0.0	0.87	14078.
19	1.00	0.0	0.92	14012.
20	1.00	0.01	0.90	13867.

important price effect. The decreased white flight drastically reduced the number of units becoming available for blacks. This tighter market for blacks forced prices to remain near their initial values in black areas. In transition areas, such as tract 11, prices fall as the changeover gathers momentum, but once the tract approaches 100% black, prices begin to rise.

EXPERIMENT #3 More Demanding Blacks

This simulation is identical to the first, except that blacks are now less willing to accept minority status. The likelihood that a black will choose a tract does not become large until the expected per cent black (as stated earlier, this considers the per cent in adjacent tracts) is 40%-50%. This change did not have a major effect on the outcomes. Rates of change were slowed because transtion has a harder time getting started, but the final outcomes were essentially the same. (See figures on the next page.)

EXPERIMENT #4 Reduced Population Growth and New Construction

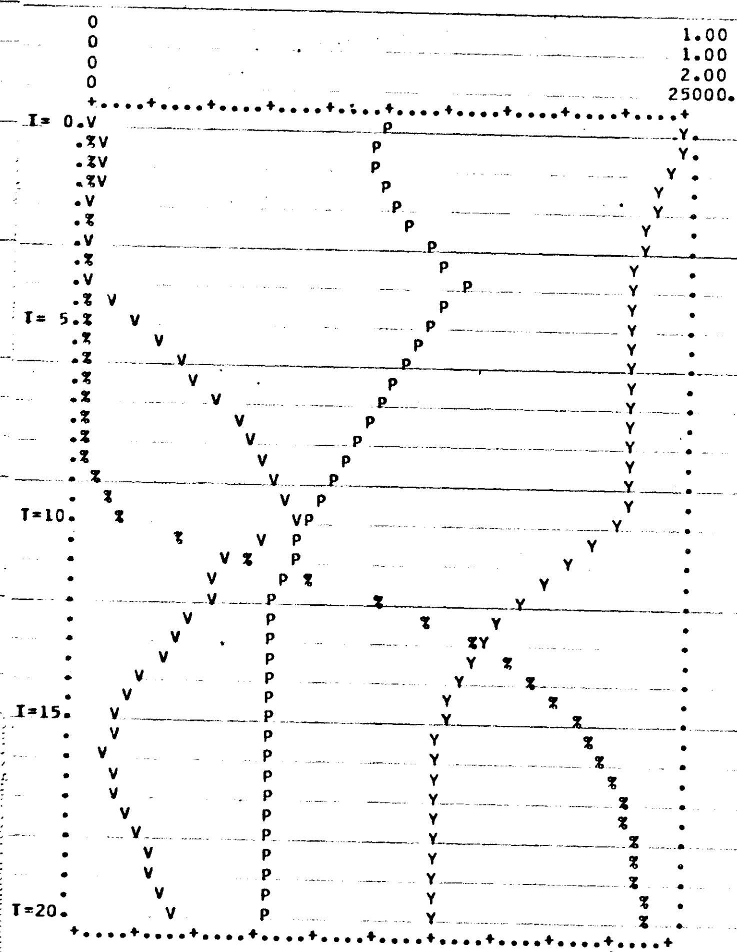
This experiment uses the same preferences as Experiment #1, but the rates of population growth and new construction have been reduced. The following values were used:

GR_{RY}		low	mid	high
	white	50	175	125
	black	150	300	100

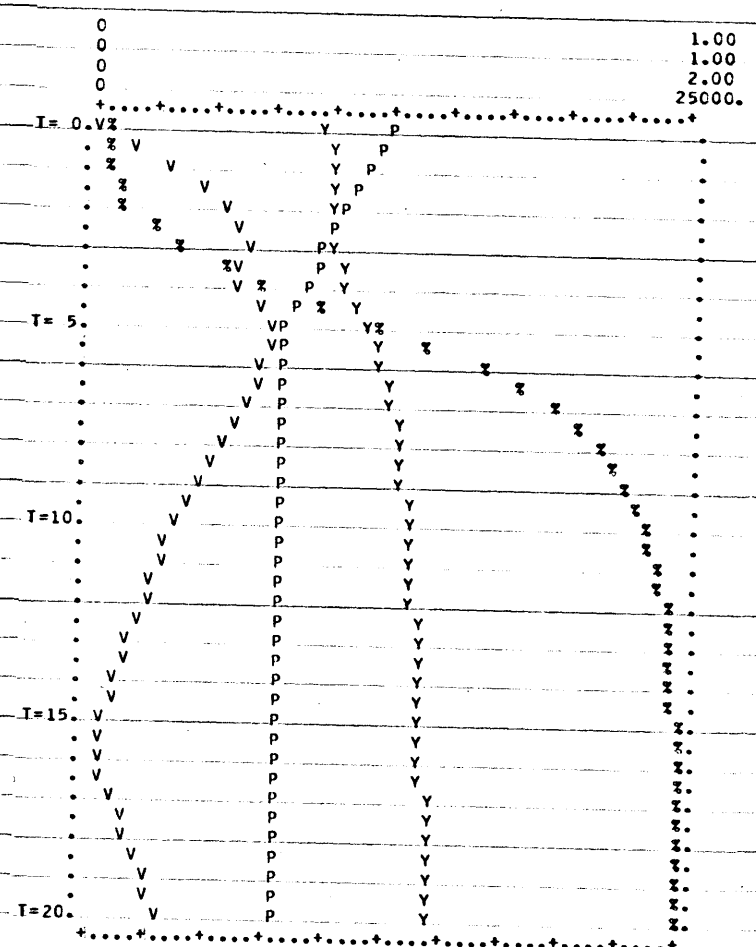
NEW_T	tract	1	2	3	4	5	9	all others
		200	200	100	100	100	200	0

3

TIME PATH OF CHARACTERISTICS OF TRACT NO.14 Q(14)= 5000.



TIME PATH OF CHARACTERISTICS OF TRACT NO.11 Q(11)= 3000.



Not surprisingly, the results are quite different than the base case results. The significant reduction in black population growth produces seven tracts which do not undergo transition (there was only one such tract in Experiment #1). The vacancy rates tend to be lower. This indicates a reduced disturbance in the housing market; fewer units going on sale will tend to leave fewer units unsold after the market process is completed in a given time period.

EXPERIMENT #5 Reduced Population Growth and Increased Tolerances

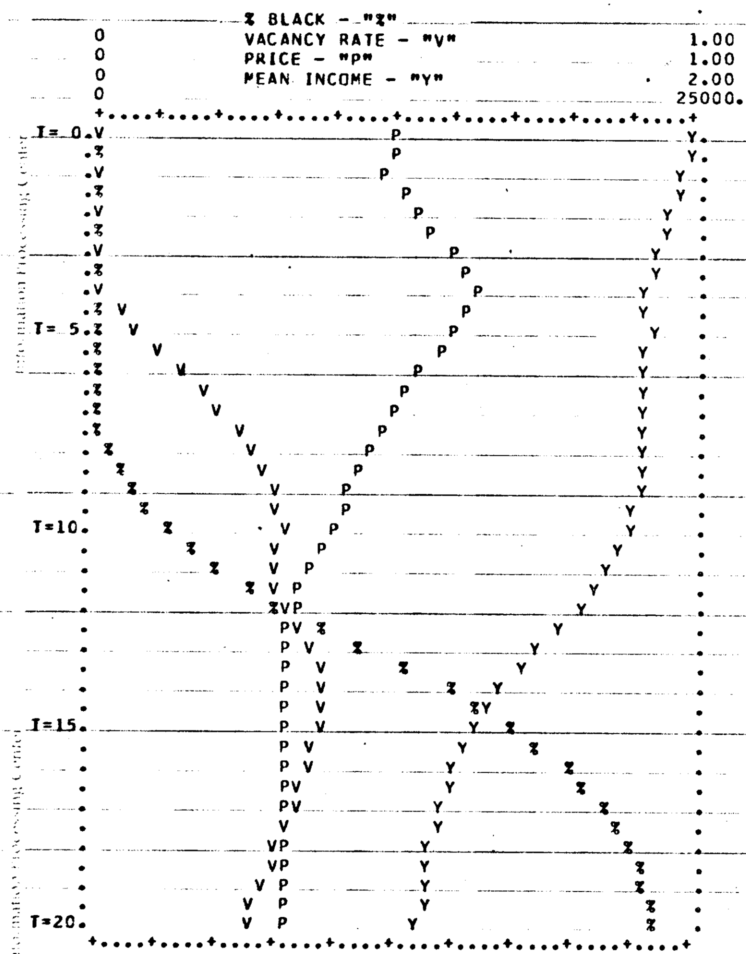
Experiment #5 replicates Expermiment #2; only the population growth has been reduced. GR_{RY} and NEW_{T} are the same as in the previous experiment. Like Experiment #4, fewer tracts underwent transition. However, unlike any of the previous experiments, including #4 which had the same population and construction changes, tract 14 did not begin to change from white to black. After 10 years tract 14 remained 99% white. (See graph on the next page.) At least in one tract, a slight change in attitudes made a significant difference. Surprisingly, an increase in tolerance stopped significant black incursion into tract 14 (at least the incursion was delayed many years to a point beyond the simulation period). It appears that some threshold phenomenon is operating. (These results and their relation to tipping will be discussed later.)

EXPERIMENT #6 No Interaction Amongst Tracts

The importance of the racial composition in neighboring tracts in the determination of eventual patterns was tested in this experi-

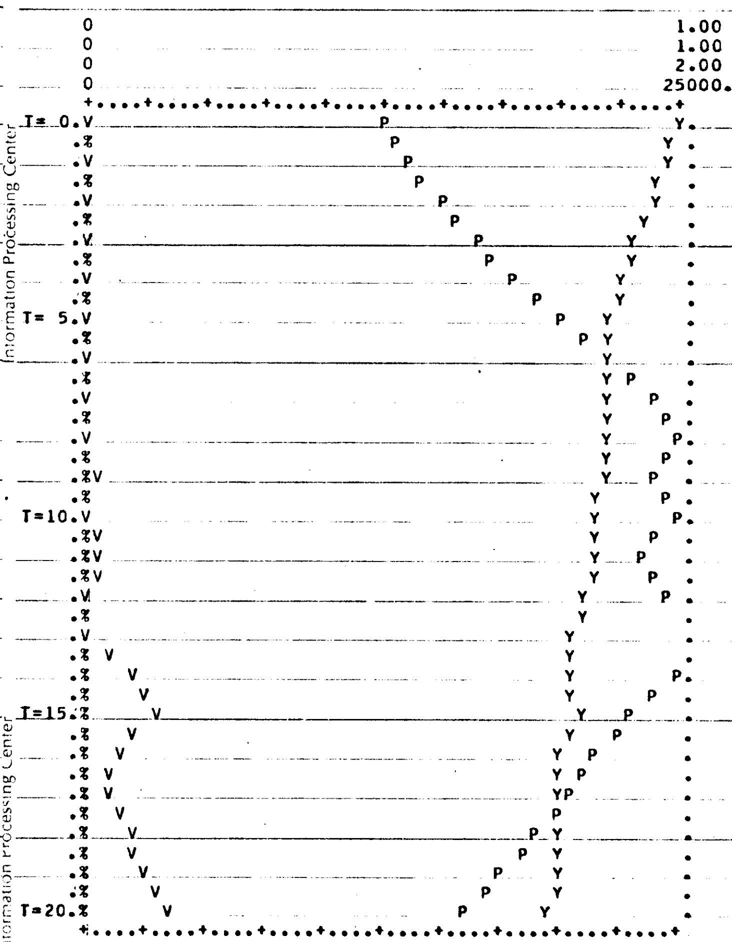
#4

TIME PATH OF CHARACTERISTICS OF TRACT NO.14 Q(14)= 5000.



#5

TIME PATH OF CHARACTERISTICS OF TRACT NO.14 Q(14)= 5000.



#4
#5

T= 0

	1	2	3	4
%	0.0	0.0	0.0	0.0
V	0.0	0.0	0.0	0.0
P	1.00	1.00	1.00	1.00
Y	23333.	19918.	20000.	18185.
Q	5000.	5000.	5000.	3750.

	5	6	7	8
%	0.0	0.43	0.25	0.29
V	0.0	0.0	0.0	0.0
P	1.00	1.00	1.00	1.00
Y	25000.	20714.	14875.	10000.
Q	5000.	3750.	3500.	3000.

	9	10	11	12
%	0.0	0.35	0.01	1.00
V	0.0	0.0	0.0	0.0
P	1.00	1.00	1.00	1.00
Y	15000.	15355.	9728.	11500.
Q	3750.	4000.	3000.	2800.

	13	14	15	16
%	0.0	0.0	0.57	1.00
V	0.0	0.0	0.0	0.0
P	1.00	1.00	1.00	1.00
Y	15800.	25000.	15813.	9400.
Q	3500.	5000.	3750.	2800.

% BLACK - "%"
VACANCY RATE - "V"
PRICE - "P"
MEAN INCOME - "Y"

#4

T=20

	1	2	3	4
%	0.0	0.00	0.00	0.00
V	0.05	0.04	0.08	0.06
P	1.16	1.11	1.11	1.16
Y	20997.	20086.	19965.	17887.

	5	6	7	8
%	0.00	1.00	1.00	1.00
V	0.08	0.21	0.19	0.21
P	1.11	0.65	0.65	0.65
Y	20824.	14390.	13428.	13194.

	9	10	11	12
%	0.00	0.99	1.00	1.00
V	C.05	0.19	0.18	0.19
P	1.22	0.65	0.65	0.65
Y	17877.	13713.	13263.	13185.

	13	14	15	16
%	0.0	0.95	1.00	1.00
V	0.06	0.25	0.19	0.21
P	1.18	0.65	0.65	0.65
Y	17341.	13693.	13691.	12991.

#5

T=20

	1	2	3	4
%	0.00	0.00	0.00	0.C0
V	0.10	0.09	0.09	0.12
P	1.73	1.73	1.71	2.14
Y	20224.	19921.	19805.	18296.

	5	6	7	8
%	0.00	1.00	1.00	1.00
V	0.09	0.17	0.15	0.16
P	1.73	0.65	0.65	0.65
Y	20290.	14315.	13389.	13223.

	9	10	11	12
%	0.00	0.99	0.99	1.C0
V	0.11	0.16	0.14	0.14
P	2.18	0.65	0.65	0.65
Y	18137.	13645.	13280.	13212.

	13	14	15	16
%	0.0	0.01	1.00	1.00
V	0.10	0.13	0.15	0.16
P	2.32	1.29	0.65	0.65
Y	17294.	19743.	13669.	13035.

ment. Households no longer consider the composition in adjacent tracts; choosers act only on the per cent black of the tract in question. Each tract is isolated from changes in neighboring tracts. Otherwise, this experiment is identical to the preceeding one.

This change had a considerable impact on the resulting patterns, especially in a few tracts. For example, tract 11 remains predominately white, whereas in the previous experiment tract 11 was almost all black in five years. Clearly, the transition in tract 11 is driven by its proximity to major concentrations of black population. Unlike the previous experiments, prices tended to be higher in black tracts than in white tracts. The reason is obvious: whites no longer panic when transition takes place in a neighboring tract; they ignore changes in the next tract. Likewise, blacks are less willing to enter white tracts which are contiguous to black tracts. As a result, forces for transition in the border tracts are greatly reduced and the ghetto is confined. This confinement raises prices in black tracts since the black population is still growing at a substantial rate. At the same time, white tracts, in which all the new construction is concentrated, experience an oversupply of units and prices are low.

EXPERIMENT #7 Color Blind Households

This experiment tests the behavior of the model if blacks and whites did not consider race in their housing decisions. This was done by making $f_R(\%)$ identically one for both races. All other features are the same as Experiment #5. As might be expected, blacks soon enter

#6

% BLACK - "%"
VACANCY RATE - "V"
PRICE - "P"
MEAN INCOME - "Y"

T= 0

1 %=0.0 V=0.0 P=1.00 Y=23333. Q= 5000.	2 %=0.0 V=0.0 P=1.00 Y=19918. Q= 5000.	3 %=0.0 V=0.0 P=1.00 Y=20000. Q= 5000.	4 %=0.0 V=0.0 P=1.00 Y=18185. Q= 3750.
5 %=0.0 V=0.0 P=1.00 Y=25000. Q= 5000.	6 %=0.43 V=0.0 P=1.00 Y=20714. Q= 3750.	7 %=0.25 V=0.0 P=1.00 Y=14875. Q= 3500.	8 %=0.29 V=0.0 P=1.00 Y=10000. Q= 3000.
9 %=0.0 V=0.0 P=1.00 Y=15000. Q= 3750.	10 %=0.35 V=0.0 P=1.00 Y=15355. Q= 4000.	11 %=0.01 V=0.0 P=1.00 Y= 9728. Q= 3000.	12 %=1.00 V=0.0 P=1.00 Y=11500. Q= 2800.
13 %=0.0 V=0.0 P=1.00 Y=15800. Q= 3500.	14 %=0.0 V=0.0 P=1.00 Y=25000. Q= 5000.	15 %=0.57 V=0.0 P=1.00 Y=15813. Q= 3750.	16 %=1.00 V=0.0 P=1.00 Y= 9400. Q= 2800.

T=10

1 %=0.01 V=0.04 P=0.79 Y=18908.	2 %=0.01 V=0.0 P=0.85 Y=18766.	3 %=0.01 V=0.05 P=0.82 Y=19197.	4 %=0.02 V=0.08 P=0.86 Y=16991.
5 %=0.01 V=0.00 P=0.83 Y=18861.	6 %=0.95 V=0.20 P=0.92 Y=14407.	7 %=0.03 V=0.0 P=0.93 Y=15646.	8 %=0.05 V=0.05 P=0.76 Y=13693.
9 %=0.02 V=0.07 P=0.89 Y=17223.	10 %=0.91 V=0.0 P=0.95 Y=12237.	11 %=0.02 V=0.05 P=0.88 Y=14635.	12 %=1.00 V=0.0 P=1.13 Y=12762.
13 %=0.02 V=0.07 P=0.83 Y=16290.	14 %=0.02 V=0.04 P=0.78 Y=18478.	15 %=0.98 V=0.20 P=0.93 Y=14285.	16 %=1.00 V=0.0 P=1.13 Y=12717.

T=20

1 %=0.05 V=0.11 P=0.65 Y=17091.	2 %=0.04 V=0.08 P=0.65 Y=17077.	3 %=0.05 V=0.09 P=0.65 Y=17053.	4 %=0.05 V=0.14 P=0.65 Y=16949.
5 %=0.05 V=0.09 P=0.65 Y=17079.	6 %=0.99 V=0.16 P=0.98 Y=14740.	7 %=0.04 V=0.07 P=0.65 Y=16954.	8 %=0.05 V=0.12 P=0.65 Y=16822.
9 %=0.05 V=0.11 P=0.65 Y=17007.	10 %=0.99 V=0.14 P=0.97 Y=15018.	11 %=0.05 V=0.10 P=0.65 Y=16806.	12 %=1.00 V=0.0 P=1.30 Y=12856.
13 %=0.06 V=0.17 P=0.65 Y=16963.	14 %=0.06 V=0.14 P=0.65 Y=16988.	15 %=1.00 V=0.17 P=1.00 Y=14639.	16 %=1.00 V=0.0 P=1.29 Y=12887.

#7

```
T= 0
  1 %=0.0       2 %=0.0       3 %=0.0       4 %=0.0
    V=0.0         V=0.0         V=0.0         V=0.0
    P=1.00        P=1.00        P=1.00        P=1.00
    Y=23333.      Y=19918.      Y=20000.      Y=18185.
    Q= 5000.      Q= 5000.      Q= 5000.      Q= 3750.

  5 %=0.0       6 %=0.43      7 %=0.25      8 %=0.29
    V=0.0         V=0.0         V=0.0         V=0.0
    P=1.00        P=1.00        P=1.00        P=1.00
    Y=25000.      Y=20714.      Y=14875.      Y=10000.
    Q= 5000.      Q= 3750.      Q= 3500.      Q= 3000.

  9 %=0.0      10 %=0.35     11 %=0.01     12 %=1.00
    V=0.0         V=0.0         V=0.0         V=0.0
    P=1.00        P=1.00        P=1.00        P=1.00
    Y=15000.      Y=15355.      Y= 9728.      Y=11500.
    Q= 3750.      Q= 4000.      Q= 3000.      Q= 2800.

 13 %=0.0      14 %=0.0      15 %=0.57     16 %=1.00
    V=0.0         V=0.0         V=0.0         V=0.0
    P=1.00        P=1.00        P=1.00        P=1.00
    Y=15800.      Y=25000.      Y=15813.      Y= 9400.
    Q= 3500.      Q= 5000.      Q= 3750.      Q= 2800.
```

```
T=10
  1 %=0.27      2 %=0.24      3 %=0.22      4 %=0.27
    V=0.00        V=0.00        V=0.00        V=0.00
    P=0.86        P=0.88        P=0.88        P=0.98
    Y=18150.      Y=18043.      Y=18110.      Y=16311.

  5 %=0.23      6 %=0.39      7 %=0.33      8 %=0.40
    V=0.00        V=0.0         V=0.02        V=0.04
    P=0.87        P=1.04        P=1.01        P=1.01
    Y=19565.      Y=18098.      Y=15464.      Y=12290.

  9 %=0.27     10 %=0.35     11 %=0.33     12 %=0.66
    V=0.00        V=0.0         V=0.04        V=0.04
    P=0.99        P=1.00        P=1.02        P=1.02
    Y=15670.      Y=16642.      Y=12387.      Y=12218.

 13 %=0.24     14 %=0.21     15 %=0.48     16 %=0.64
    V=0.0         V=0.00        V=0.0         V=0.0
    P=1.03        P=0.87        P=1.00        P=1.06
    Y=15067.      Y=20258.      Y=15606.      Y=11294.
```

```
T=20
  1 %=0.35      2 %=0.34      3 %=0.34      4 %=0.36
    V=0.02        V=0.00        V=0.00        V=0.02
    P=0.72        P=0.83        P=0.83        P=0.85
    Y=16669.      Y=16699.      Y=16712.      Y=15927.

  5 %=0.34      6 %=0.38      7 %=0.38      8 %=0.43
    V=0.01        V=0.00        V=0.00        V=0.01
    P=0.81        P=0.97        P=0.98        P=0.88
    Y=17186.      Y=17178.      Y=15775.      Y=13759.

  9 %=0.36     10 %=0.37     11 %=0.41     12 %=0.53
    V=0.01        V=0.0         V=0.01        V=0.01
    P=0.91        P=0.94        P=0.94        P=0.84
    Y=15850.      Y=16616.      Y=13658.      Y=13330.

 13 %=0.33     14 %=0.32     15 %=0.45     16 %=0.52
    V=0.02        V=0.01        V=0.00        V=0.02
    P=0.76        P=0.78        P=0.92        P=0.76
    Y=15553.      Y=17681.      Y=15591.      Y=13263.
```

all tracts and whites enter black areas. Whether or not housholds ever really behave in this manner, such a pattern should result given this preference pattern.

EXPERIMENT #8 Stability of Integrated Housing

The experiment also examines an unlikely situation. Controlling for income, blacks and whites are distributed uniformly: 10% of upper income residents in each tract are black; 25% of all middle income residents; and 50% of all lower income residents in each tract are black. Preferences, were the same as in Experiment #2. There were no total population changes and no new construction.

The initial pattern of integration quickly begins to break down. After five years, four tracts are clearly moving to the formation of a ghetto. In nine other tracts there is a steady decrease in the per cent black. However, in one tract (15), the eventual outcome was still uncertain after all twenty time periods. After 10 years (time period 20) the tract was 24% black; initially it was 23% black. At first the per cent dropped, since the tract was as attractive to whites as other tracts of medium quality. However, during the first eight years the black population is busy concentrating itself in the neighboring tracts of low quality (11,12, and 16). This adjacent concentration made tract 15 more appealing to blacks. As a result the per cent black rose from 9% to 24% in the last two years. Although it is dangerous to predict the final outcome, it appears that tract 15 has entered upon an inevitable path of transition.

8

T= 0

1	%=0.12	V=0.00	P=1.00	Y=22253.	Q= 5000.
2	%=0.16	V=0.00	P=1.00	Y=18977.	Q= 5000.
3	%=0.17	V=0.00	P=1.00	Y=17850.	Q= 5000.
4	%=0.18	V=0.00	P=1.00	Y=15631.	Q= 3750.
5	%=0.10	V=0.00	P=1.00	Y=23980.	Q= 5000.
6	%=0.21	V=0.00	P=1.00	Y=15040.	Q= 3750.
7	%=0.21	V=0.00	P=1.00	Y=15625.	Q= 3500.
8	%=0.29	V=0.00	P=1.00	Y=10500.	Q= 3000.
9	%=0.20	V=0.00	P=1.00	Y=15401.	Q= 3750.
10	%=0.22	V=0.00	P=1.00	Y=15098.	Q= 4000.
11	%=0.29	V=0.00	P=1.00	Y=10059.	Q= 3000.
12	%=0.30	V=0.00	P=1.00	Y= 9522.	Q= 2800.
13	%=0.21	V=0.00	P=1.00	Y=15079.	Q= 3500.
14	%=0.14	V=0.00	P=1.00	Y=20686.	Q= 5000.
15	%=0.23	V=0.00	P=1.00	Y=13901.	Q= 3750.
16	%=0.29	V=0.00	P=1.00	Y=10059.	Q= 2800.

T=10

1	%=0.02	V=0.00	P=1.37	Y=18573.
2	%=0.04	V=0.00	P=1.37	Y=17265.
3	%=0.05	V=0.00	P=1.37	Y=16260.
4	%=0.08	V=0.00	P=1.34	Y=13736.
5	%=0.03	V=0.01	P=1.35	Y=18865.
6	%=0.07	V=0.00	P=1.73	Y=14790.
7	%=0.07	V=0.00	P=1.61	Y=14374.
8	%=0.92	V=0.10	P=0.65	Y=12963.
9	%=0.06	V=0.00	P=1.67	Y=14304.
10	%=0.07	V=0.00	P=1.60	Y=14890.
11	%=0.66	V=0.09	P=0.87	Y=12412.
12	%=0.95	V=0.08	P=0.65	Y=12580.
13	%=0.12	V=0.00	P=1.50	Y=13640.
14	%=0.05	V=0.00	P=1.35	Y=16746.
15	%=0.09	V=0.00	P=1.39	Y=13739.
16	%=0.95	V=0.10	P=0.65	Y=12598.

T=20

1	%=0.03	V=0.01	P=1.13	Y=17650.
2	%=0.03	V=0.01	P=1.14	Y=17095.
3	%=0.04	V=0.01	P=1.12	Y=16531.
4	%=0.12	V=0.11	P=1.01	Y=13428.
5	%=0.03	V=0.01	P=1.15	Y=17567.
6	%=0.05	V=0.05	P=1.30	Y=14787.
7	%=0.08	V=0.09	P=1.12	Y=13991.
8	%=0.99	V=0.00	P=1.08	Y=13444.
9	%=0.04	V=0.07	P=1.27	Y=14464.
10	%=0.06	V=0.06	P=1.15	Y=14518.
11	%=0.99	V=0.02	P=1.06	Y=13305.
12	%=1.00	V=0.00	P=1.13	Y=13169.
13	%=0.06	V=0.08	P=1.23	Y=13805.
14	%=0.04	V=0.02	P=1.09	Y=16414.
15	%=0.24	V=0.05	P=0.99	Y=13669.
16	%=1.00	V=0.00	P=1.09	Y=13197.

EXPERIMENTS #9 and #10 Different Initial Population

The only difference between these experiments and the first two is that the initial population distributions have been changed. In the first time period blacks account for less than 10% of the total population, though the growth remains at the high level described in Experiment #1. (The figure on the next page describes the initial pattern of race.)

Generally, the behavior is quite similar to the behavior in the first two experiments. One unique feature appears however: the vacancy rate in black tracts oscillates, with a period of about seven years. The explanation of this phenomenon is complicated and depends greatly on the transition process. Initially, the black population is concentrated in tract 16, with a few blacks in tract 12. In the first few time periods, the black population is directed almost exclusively at tract 12, which changes from 17% to 53% black in one year. A small number makes an initial incursion into tract 15 at the same time. The white flight from tracts 12 and 15 makes more units available than is necessary to meet the needs of the steadily growing black population. Hence vacancies, or unsold units, begin to rise in black areas, as in tract 16 at time period 2. However, the opportunities in tract 12 and 15 are soon filled as space in black areas is in high demand; vacancies fall to zero in tract 16 in the next year. However, by the third year of the simulation two more tracts have experienced significant incursion (8 and 11). Once again the the swift white reaction brings many units to market and vacancies

RUN NO. 9 + 10

T= 0

1 %=0.0	2 %=0.0	3 %=0.0	4 %=0.0
V=0.0	V=0.0	V=0.0	V=0.0
P=1.00	P=1.00	P=1.00	P=1.00
Y=19714.	Y=19125.	Y=16857.	Y=15750.
Q= 5000.	Q= 5000.	Q= 5000.	Q= 3750.
5 %=0.0	6 %=0.0	7 %=0.0	8 %=0.0
V=0.0	V=0.0	V=0.0	V=0.0
P=1.00	P=1.00	P=1.00	P=1.00
Y=21666.	Y=11500.	Y=14428.	Y=14199.
Q= 5000.	Q= 3750.	Q= 3500.	Q= 3000.
9 %=0.0	10 %=0.0	11 %=0.0	12 %=0.17
V=0.0	V=0.0	V=0.0	V=0.0
P=1.00	P=1.00	P=1.00	P=1.00
Y=15750.	Y=14000.	Y= 8778.	Y=10333.
Q= 3750.	Q= 4000.	Q= 3000.	Q= 2800.
13 %=0.0	14 %=0.0	15 %=0.0	16 %=1.00
V=0.0	V=0.0	V=0.0	V=0.0
P=1.00	P=1.00	P=1.00	P=1.00
Y=16777.	Y=20499.	Y=14428.	Y= 9750.
Q= 3500.	Q= 5000.	Q= 3750.	Q= 2800.

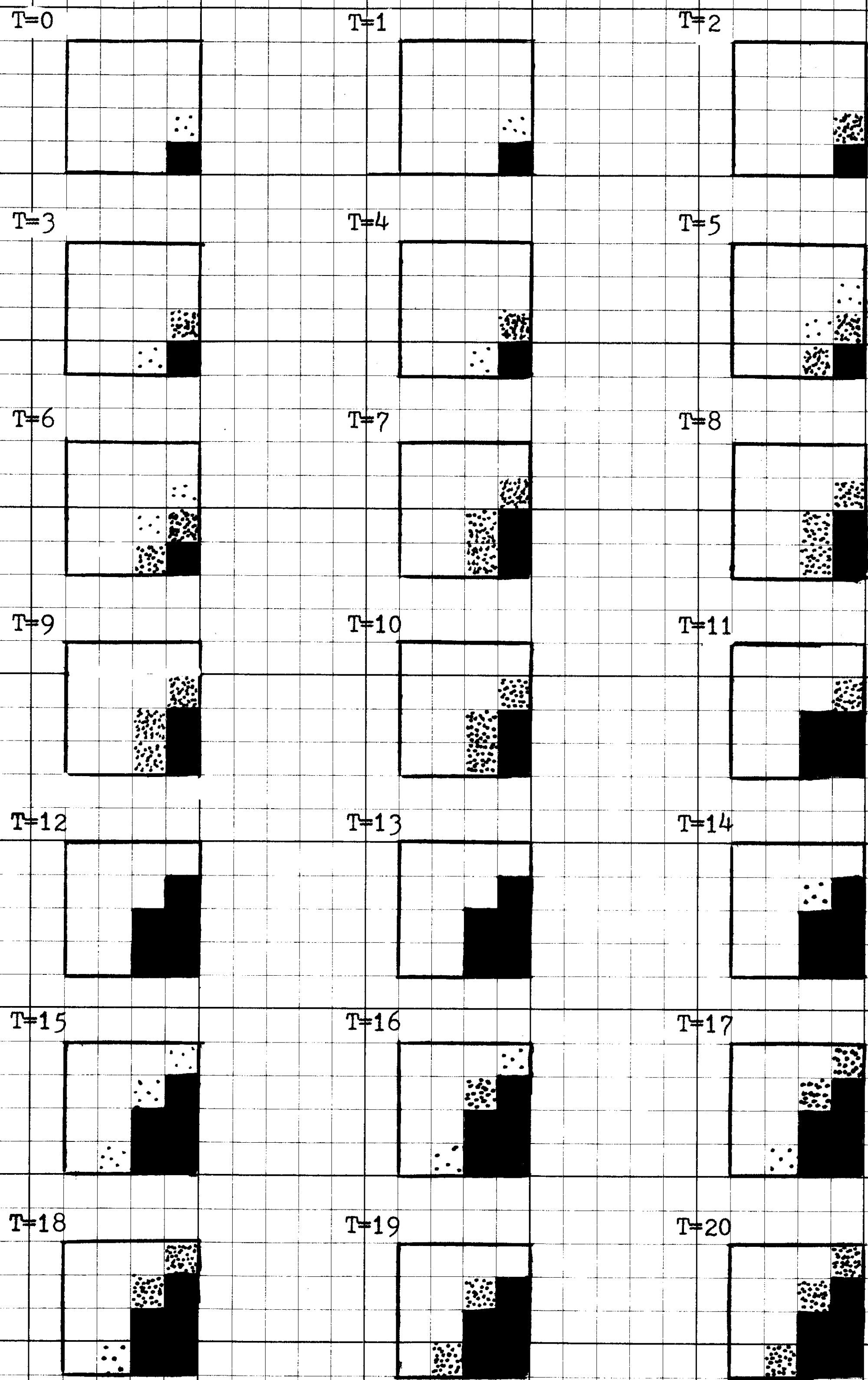
begin appearing (tract 16, time period 8). By time period 13 the blacks have just filled the tracts from which whites have fled. At this point a ghetto is clearly defined; it consists of tracts 8,11,12,15, and 16. There are no evenly mixed tracts; no tract is more than 6% and less than 94% black.

Having filled up the ghetto, the next year (time period 15) blacks make significant incursion into three more tracts (4,7, and 14), all becoming more than 10% black. Transition has now begun in ernest in these three tracts. For a third time, whites leaving transition areas loosen the housing market for blacks. As blacks pour into these transition tracts vacancies return to black areas.

The diagram on the next page depicts the expansion of the ghetto. The ghetto expands in discreet jumps, as groups of tracts go through transition at the same time. Following each expansion is a period of consolidation, when the transition tracts meet the needs of the growing population. Discontinuities in the rate of ghetto expansion did not appear in earlier runs because tracts entered transition individually and not in groups. In this experiment the transitions seemed to be in phase with each other. Such discontinuities are not likely to have a broad effect on markets in real cities. Such jumps in the size of the ghetto are probably of the size of city blocks; such a small unit will not likely affect the market. Moreover, the small size means that there will be many small bursts of expansion and therefore a greater chance for the bursts to be out of phase.

EXPANSION OF THE GHETTO

EXPERIMENT 10



less than 10% black

greater than 10% and less than 50%

greater than 50% and less than 90%

greater than 90% black

4.3 Some Conclusions

These experiments should be viewed only as preliminary investigations. Many more simulations need to be made to test the sensitivity of the model to all of its parameters and functions. However, some tentative conclusions are indicated by the results of the first few experiments.

First, the model is able to portray the time dynamics of racial transition. The concensus of students of transition is that the per cent black in a tract as a function of time is an S-curve: rising slowly at first, but accelerating, and eventually slowing down as the percenatge approaches 100%. In the experiments transition always took this form.

Secondly, the behavior of prices is sensitive to changes in preferences. As the first two experiments indicate, slight changes in preferences can significantly affect the price of housing in both black and white areas; either blacks or whites can pay more. However, in all the experiments, prices were lower in tracts while they were undergoing transition. It seems that the contradictions of the empirical studies may be an actual reflection of reality:[12] slight differences in conditions and attitudes can greatly affect market values; market values will most likely move differently in differnt areas.

Thirdly, most of the experiments display a phenomenon which could be defined as tipping. The tipping point may be defined as as the per cent black at which the neighborhood's conversion to 100% black becomes clearly inevitable. In experiments #4 and #5 a threshold effect appears to be present: with only a slight change in preferences the eventual racial composition of tract 14 is completely altered. In one experiment the tract crossed a threshold which carried it to an all-black population; in the other this threshold was not crossed. The threshold may be viewed as the tipping point: in one experiment tract 14 tipped; in the other it did not tip. Furthermore, the experiments indicate that the tipping point is quite low; no tract reached 10% black without beginning the process of racial transition. (The only exception is the experiment where households had no racial preferences.) Though its value is low, the precise value of the tipping point will depend heavily on the particular circumstances, such as the rates of population growth and the preferences of each racial group.

And fourthly, in all the experiments, except the color-blind experiment, there was a definite tendency for tracts to move one way or the other. There were no stable patterns of integration, though it did take tract 15 in experiment #8 the full 10 years to determine its eventual color. This affirms the hypothesis of Schelling; the system has imbedded within it forces favoring segregation. However, further investigation with widely ranging sets of preferences are needed before definitive conclusions may be drawn.

Despite its limitations, the experiments indicate that the model does capture some of the important systemic features of racial transition. Certainly, such a simple model cannot predict what will happen in a particular neighborhood, but it can provide important insights into how racial transition takes place.

APPENDIX - The role of the Utility Function

In two separate instances, the model requires functions of the utility computation: first, the rate of racially motivated movement depends on utility; and second, the likelihood of moving to a tract depends on the utility of the tract. The difficulty is that utility, with its vague units, is normally considered to be an ordinal scale: utilities may be ranked, but it is impossible to attach meaning to the numerical differences between levels of utility. In classical microeconomics this is not a serious problem, because ordinal utilities appear in optimization analysis: maximize utility, subject to various constraints. In this role ordinal utilities are sufficient.

However, the actors in this model do not maximize utility and many movers, in fact, make sub-optimal choices: they choose a tract which does not maximize their utility. The model maps the computed utilities of the various alternatives into probabilities. As such utilities play only an intermediate role. Utility is defined as a function of the race and income of the household and three features of the housing in the tract: the quantity of housing servicce available, the price of one unit of housing service, and the racial composition of the tract. Formally, $U = f(R,Y,T)$. The likelihood of a move is defined as a function of the utility: $L = H(U)$. Combining these expressions:

$$L = H(U) = H(f(R,Y,T)) = G(R,Y,T)$$

where $G = H(f)$

Thus the model defines the likelihood of a move as a function of the race and income of the mover and of the features of the tract. The utility function could have been by-passed; it could have computed likelihoods from a function which is represented here as G(R,Y,T). As was seen in Chapter 4, peculiarities in the utility function may easily be offset by an appropriate form of H(U). Use of the utility function does entail one restriction: alternatives of equal utility must be equally likely. If the concept of ordinal utilities is accepted, then this restriction must also be accepted.

Though the utility formulation could have been avoided, it was included for two reasons. First, it greatly emphasizes the behavioral approach of the model, for the utility function is a concise statement of the decision rule. And second, the formal structure it provides for stating decision rules eases the inclusion of new rules or the alteration of the old ones. Changes in the rules are simply changes in the utility function.

APPENDIX - PROGRAM LISTING

```
      COMMON NJ(16),NEW(16),RATE(3),PER(16),U(16),Q(16),FOR(16),AL(3),
     1DX(16),DY(16),RL(16),NY,NT,RAT
      INTEGER WO(3,16),BO(3,16),WOM(3,16),BOM(3,16),WOC(3,16),BOC(3,16),
     1GWO(3),GBO(3),NLW(3),NLB(3),POP(16)
      DIMENSION Y(3),P(16),YBAR(16),WHITE(15),BLACK(15),DIST(15),U
     1TIL(15),PRICE(15),VAC(16),DIJ(16,16)
      REAL MOBIL(15)
      DIMENSION D(21,16,4),N(16),DMAX(4)
         INTEGER A(51),SYM(5)
      DATA WOM/48*0/,BOM/48*0/,NLW/3*0/,NLB/3*0/
      DATA P/16*1./
    1 FORMAT(3I4)
    2 FORMAT(16I5)
    3 FORMAT(8F10.3)
    5 FORMAT(15F5.2)
    6 FORMAT(15F5.3)
    7 FORMAT(5A1)
      NY=3
      NT=16
      READ(5,1)NRUN,NIT
      READ(5,1) ((WO(L,J),L=1,NY),J=1,NT)
      READ(5,1) ((BO(L,J),L=1,NY),J=1,NT)
      READ(5,2) (NEW(J),J=1,NT)
      READ(5,2) (GWO(L),L=1,NY)
      READ(5,2) (GBO(L),L=1,NY)
      READ(5,3) (RATE(L),L=1,NY),RAT
      READ(5,3) (Y(L),L=1,NY)
      READ(5,3) (Q(J),J=1,NT)
      READ(5,3) (AL(L),L=1,NY)
      READ(5,3) (DX(J),J=1,NT)
      READ(5,3) (DY(J),J=1,NT)
      READ(5,2) (NJ(J),J=1,NT)
      READ(5,5) (WHITE(I),I=1,15)
      READ(5,5) (BLACK(I),I=1,15)
      READ(5,5) (MOBIL(I),I=1,15)
      READ(5,5) (DIST  (I),I=1,15)
      READ(5,6) (UTIL  (I),I=1,15)
      READ(5,5) (PRICE(I),I=1,15)
      READ(5,2) (N(J),J=1,NT)
      READ(5,7) (SYM(K),K=1,5)
      READ(5,3) (DMAX(K),K=1,4)
      WRITE(6,3) (AL(L),L=1,NY)
      WRITE(6,3) (DX(J),J=1,NT)
      WRITE(6,3) (DY(J),J=1,NT)
      DO 22 J=1,NT
      DO 22 I=1,NT
   22 DIJ(I,J)=SQRT((DX(I)-DX(J))**2+(DY(I)-DY(J))**2)
   31 CONTINUE
      WRITE(6,5) (WHITE(I),I=1,15)
      WRITE(6,5) (BLACK(I),I=1,15)
      WRITE(6,5) (MOBIL(I),I=1,15)
      WRITE(6,5) (DIST  (I),I=1,15)
      WRITE(6,5) (UTIL  (I),I=1,15)
      WRITE(6,5) (PRICE(I),I=1,15)
```

```
      CALL STATS(WO,BO,YBAR,POP,Y,VAC,0)
      DO 21 J=1,NT
      D(1,J,1)=PER(J)
      D(1,J,2)=VAC(J)
      D(1,J,3)=P(J)
      D(1,J,4)=YBAR(J)
   21 WRITE(6,3)PER(J),P(J),VAC(J),YBAR(J)
      DO 99 IT=1,NIT
      CALL EXPECT(DIJ)
      CALL MOVERS(BO,BOM,NLB,GBO,2,BLACK,MOBIL)
      CALL MOVERS(WO,WOM,NLW,GWO,1,WHITE,MOBIL)
      CALL CHOOSE(NLW,WOM,WOC,1,WHITE,UTIL,DIST,Y,P,DIJ)
      CALL CHOOSE(NLB,BOM,BOC,2,BLACK,UTIL,DIST,Y,P,DIJ)
      CALL EXCHNG(WO,WOC,WOM,NLW,BO,BOC,BOM,NLB,P,PRICE)
      CALL STATS(WO,BO,YBAR,POP,Y,VAC,1)
      DO 90 J=1,NT
      D(IT+1,J,1)=PER(J)
      D(IT+1,J,2)=VAC(J)
      D(IT+1,J,3)=P(J)
      D(IT+1,J,4)=YBAR(J)
   90 CONTINUE
   99 CONTINUE
      NIT=NIT+1
      CALL TABLE(D,NIT,Q,NRUN,NT)
      CALL DISPLA(D,N,DMAX,A,SYM,NIT,NT,Q,NRUN)
      NIT=NIT-1
      READ(5,1)NRUN
      IF(NRUN .EQ.0)GOTO 999
      WRITE(6,32)
   32 FORMAT(1H1)
      READ(5,1) ((WO(L,J),L=1,NY),J=1,NT)
      READ(5,1) ((BO(L,J),L=1,NY),J=1,NT)
      DO 33 J=1,NT
      NJ(J)=0
      P(J)=1.
      DO 33 L=1,NY
      WOM(L,J)=0
      BOM(L,J)=0
   33 CONTINUE
      DO 34 L=1,NY
      NLW(L)=0
   34 NLB(L)=0
      READ(5,2) (NEW(J),J=1,NT)
      READ(5,2) (GWO(L),L=1,NY)
      READ(5,2) (GBO(L),L=1,NY)
      READ(5,5) (WHITE(I),I=1,15)
      READ(5,5) (BLACK(I),I=1,15)
      READ(5,5) (UTIL(I),I=1,15)
      GOTO 31
  999 STOP
      END
```

```
      SUBROUTINE MOVERS(Y,YM,NL,GR,IR,RACE,MOBIL)
      COMMON NJ(16),NEW(16),RATE(3),PER(16),U(16),Q(16),FOR(16),AL(3),
     1DX(16),DY(16),RL(16),NY,NT,RAT
      DIMENSION RACE(15)
      REAL MOBIL(15)
      INTEGER Y(NY,NT),YM(NY,NT),NL(NY),GR(NY)
      NSUM=0
      DO 15 L=1,NY
      DO 10 J=1,NT
      N=INT(RATE(L)*Y(L,J))
      NSUM=NSUM+N
   10 YM(L,J)=N
   15 NL(L)=NL(L)+GR(L)
      DO 25 J=1,NT
      DEL=1.-FUNC(RACE,PER(J))
      DO 20 L=1,NY
      R=FUNC(MOBIL,DEL)
      N=INT(R*Y(L,J))
      YM(L,J)=YM(L,J)+N
   20 NSUM=NSUM+N
   25 CONTINUE
      DO 40 J=1,NT
      DO 30 L=1,NY
      IF(YM(L,J).GT.Y(L,J))YM(L,J)=Y(L,J)
   30 NJ(J)=NJ(J)+YM(L,J)
      IF(IR.EQ.1)NJ(J)=NJ(J)+NEW(J)
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CHOOSE(NL,NM,NC,IR,RACE,UTIL,DIST,Y,P,DIJ)
      COMMON NJ(16),NEW(16),RATE(3),PER(16),U(16),Q(16),FOR(16),AL(3),
     1DX(16),DY(16),RL(16),NY,NT,RAT
      DIMENSION Y(NY),P(NT),RACE(15),UTIL(15),DIST(15),DIJ(NT,NT)
      INTEGER NL(NY),NC(NY,NT),NM(NY,NT)
      DO 10 L=1,NY
      DO 10 J=1,NT
   10 NC(L,J)=0
      DO 70 L=1,NY
      QM=(1.-AL(L))*Y(L)
      UM=UTLTY(Y(L),QM,1.,AL(L),1.)
      DO 20 I=1,NT
      F=FUNC(RACE,FOR(I))
   20 U(I)=UTLTY(Y(L),Q(I),P(I),AL(L),F)/UM
      DO 40 J=1,NT
      S=.1
      DO 30 I=1,NT
      RL(I)=FUNC(UTIL,U(I))*NJ(I)*FUNC(DIST,DIJ(I,J))
      IF(RL(I).LT.0.)RL(I)=0.
      IF(RL(I).LT.0.)WRITE(6,11)RL(I),NJ(I)
   30 S=S+RL(I)
      DO 40 I=1,NT
   40 NC(L,I)=NC(L,I)+INT(RL(I)/S*NM(L,J)+.5)
      S=.1
      DO 50 I=1,NT
      RL(I)=FUNC(UTIL,U(I))*NJ(I)
      IF(RL(I).LT.0.)RL(I)=0.
      IF(RL(I).LT.0.)WRITE(6,12)RL(I),NJ(I)
   50 S=S+RL(I)
      DO 60 I=1,NT
      IF(NC(L,I).LT.0)WRITE(6,13)NC(L,I)
   60 NC(L,I)=NC(L,I)+INT(RL(I)/S*NL(L)+.5)
   70 CONTINUE
      DO 80 L=1,NY
   80 NL(L)=0
   11 FORMAT('   11',F7.2,I6)
   12 FORMAT('   12',F7.2,I6)
   13 FORMAT(' 13',I8)
      RETURN
      END
```

```
      SUBROUTINE EXCHNG(W,WC,WM,NW,B,BC,BM,NB,P,PRICE)
      COMMON NJ(16),NEW(16),RATE(3),PER(16),U(16),Q(16),FOR(16),AL(3),
     1DX(16),DY(16),RL(16),NY,NT,RAT
      DIMENSION P(NT),PRICE(15)
      INTEGER W(NY,NT),WC(NY,NT),WM(NY,NT),NW(NY),B(NY,NT),BC(NY,NT),
     1BM(NY,NT),NB(NY)
      PMIN=.65
      DO 110 J=1,NT
      V=NJ(J)
      NM=0
      NC=0
      DO 5 L=1,NY
      NM=NM+WM(L,J)+BM(L,J)
    5 NC=NC+WC(L,J)+BC(L,J)
      N=NC-NJ(J)
      NJ(J)=0
      IF(N)30,35,10
   10 DO 20 L=1,NY
      Z=NC
      IF(Z.EQ.0.)Z=1.
      X=WC(L,J)/Z
      NF=INT(X*N+.5)
      NW(L)=NW(L)+NF
      WC(L,J)=WC(L,J)-NF
      W(L,J)=W(L,J)+WC(L,J)-WM(L,J)
      X=BC(L,J)/Z
      NF=INT(X*N+.5)
      NB(L)=NB(L)+NF
      BC(L,J)=BC(L,J)-NF
   20 B(L,J)=B(L,J)+BC(L,J)-BM(L,J)
      GOTO 100
   30 NJ(J)=-N
   35 DO 40 L=1,NY
      W(L,J)=W(L,J)+WC(L,J)-WM(L,J)
   40 B(L,J)=B(L,J)+BC(L,J)-BM(L,J)
  100 V=N/(V+.01)
      P(J)=P(J)+P(J)*FUNC(PRICE,V)
      IF(P(J).LT.PMIN)P(J)=PMIN
  110 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EXPECT(DIJ)
      COMMON NJ(16),NEW(16),RATE(3),PER(16),U(16),Q(16),FOR(16),AL(3),
     1DX(16),DY(16),RL(16),NY,NT,RAT
      DIMENSION DIJ(NT,NT)
      DO 10 J=1,NT
      C=PER(J)-FOR(J)
      S=0.
      DO 5 I=1,5
      X=I*C+PER(J)
      IF(X.GT.1.)X=1.
      IF(X.LT.0.)X=0.
    5 S=S+(6-I)/30.*X
   10 FOR(J)=S+.5*PER(J)
      DO 30 J=1,NT
      NS=0
      S=0.
      DO 20 I=1,NT
      IF(DIJ(I,J).GT.1.2)GOTO 20
      NS=NS+1
      S=S+PER(I)
   20 CONTINUE
      IF(FOR(J).LT.S/NS)FOR(J)=.1*(9.*FOR(J)+S/NS)
      IF(FOR(J).GT.1.)FOR(J)=1.
      IF(FOR(J).LT.0.)FOR(J)=0.
   30 CONTINUE
      RETURN
      END
```

```
      FUNCTION FUNC(F,X)
      DIMENSION F(15)
      IF(X.LT.F(1))GOTO 100
      DO 10 I=1,12
      IF(X.LT.F(1)+I*F(2)) GOTO 20
   10 CONTINUE
      I=12
   20 FUNC=F(I+3)+(X-F(1)-I*F(2))*(F(I+3)-F(I+2))/F(2)
      RETURN
  100 FUNC=F(3)+(X-F(1))*(F(4)-F(3))/F(2)
      RETURN
      END
```

```
      FUNCTION UTLTY(Y,Q,P,AL,F)
      X=Y-Q*P
      IF(X.LT.0.)X=0.
      UTLTY=X**AL*Q**(1.-AL)*F
      RETURN
      END
```

```
      SUBROUTINE STATS(WO,BO,YBAR,POP,Y,VAC,ICODE)
      COMMON NJ(16),NEW(16),RATE(3),PER(16),U(16),Q(16),FOR(16),AL(3),
     1DX(16),DY(16),RL(16),NY,NT,RAT
      INTEGER BO(NY,NT),WO(NY,NT),POP(NT)
      DIMENSION YBAR(NT),Y(NY),VAC(NT)
      DO 20 J=1,NT
      IS=0
      T=0.
      Z=.1
      DO 10 L=1,NY
      IF(BO(L,J).LT.0)BO(L,J)=0
      IF(WO(L,J).LT.0)WO(L,J)=0
      IS=IS+BO(L,J)
      X=BO(L,J)+WO(L,J)
      T=T+Y(L)*X
   10 Z=Z+X
      IF(ICODE.EQ.1)FOR(J)=PER(J)
      PER(J)=IS/Z
      IF(ICODE.EQ.0)FOR(J)=PER(J)
      YBAR(J)=T/Z
      POP(J)=Z
      Z=POP(J)+NJ(J)
   20 VAC(J)=NJ(J)/Z
      RETURN
      END
```

```
      SUBROUTINE DISPLA(D,N,DMAX,A,SYM,NIT,NT,Q,NRUN)
      DIMENSION D(NIT,NT,4),N(NT),DMAX(4) ,Q(NT)
      INTEGER SYM(5),A(51)
    1 FORMAT('1TIME PATH OF CHARACTERISTICS OF TRACT NO.',I2,
     15X,'Q(',I2,')=',F7.0,5X,'RUN NO.',I3//)
    2 FORMAT(5X,'.',51A1,'.',23X,I3,5X,F4.2,7X,F4.2,5X,F4.2,3X,F7.0)
    3 FORMAT( 6X,10('+....'),'+')
    4 FORMAT(1H+,'T=',I2)
    5 FORMAT(6X,'0',49X,F4.2,5X,'% BLACK - "%"'/
     1       6X,'0',49X,F4.2,5X,'VACANCY RATE - "V"'/
     2       6X,'0',49X,F4.2,5X,'PRICE - "P"'/
     3       6X,'0',47X,F7.0,4X,'MEAN INCOME - "Y"')
    6 FORMAT(1H+,80X,'TIME    % BLACK    VACANCIES   PRICE    INCOME')
      DO 60 J=1,NT
      IF(N(J).EQ.0)GOTO 70
      WRITE(6,1)N(J),N(J),Q(N(J)),NRUN
      WRITE(6,5) (DMAX(K),K=1,4)
      WRITE(6,3)
      WRITE(6,6)
      DO 50 IT=1,NIT
      ITR=IT-1
      TR=ITR
      DO 10 I=1,51
   10 A(I)=SYM(5)
      DO 20 K=1,4
      M=INT(D(IT,N(J),K)/DMAX(K)*50.+.5)  +1
      IF(M.GT.51)GOTO 20
      A(M)=SYM(K)
   20 CONTINUE
      WRITE(6,2) (A(I),I=1,51),ITR,(D(IT,N(J),K),K=1,4)
      IF(TR/5..EQ.INT(TR/5.))WRITE(6,4)ITR
      IF(IT.EQ.NIT)GOTO 45
      DO 30 I=1,51
   30 A(I)=SYM(5)
      DO 40 KK=1,4
      K=5-KK
      DD=(D(IT,N(J),K)+D(IT+1,N(J),K))/2.
      M=INT(DD/DMAX(K)*50.+.5)+1
      IF(M.GT.51)GOTO 40
      A(M)=SYM(K)
   40 CONTINUE
      WRITE(6,2) (A(I),I=1,51)
   45 CONTINUE
   50 CONTINUE
      WRITE(6,3)
   60 CONTINUE
   70 RETURN
      END
```

```
      SUBROUTINE TABLE(D,NIT,Q,NRUN,NT)
      DIMENSION D(NIT,NT,4),Q(NT)
    1 FORMAT('-T=',I2)
    2 FORMAT('+', 9X,4('%=',F4.2,8X)/
     1            10X,4('V=',F4.2,8X)/
     2            10X,4('P=',F4.2,8X)/
     3            10X,4('Y=',F6.0,6X))
    4 FORMAT(7X,4(I2,12X))
    5 FORMAT('1RUN NO.',I2)
    6 FORMAT(10X,4('Q=',F6.0,6X))
    7 FORMAT('-')
      WRITE(6,5)NRUN
      DO 20 IT=1,NIT
      ITR=IT-1
      WRITE(6,1)ITR
      DO 10 L=1,4
      L1=4*(L-1)+1
      L2=L1+3
      WRITE(6,4) (J,J=L1,L2)
      WRITE(6,2) ((D(IT,J,K),J=L1,L2),K=1,4)
      IF(IT.EQ.1)WRITE(6,6) (Q(J),J=L1,L2)
      WRITE(6,7)
   10 CONTINUE
   20 WRITE(6,7)
      RETURN
      END
```

NOTES

[1]A short sampling of the many studies of attitudes about race: Chester L. Hunt, "Negro-White Perceptions of Inter-racial Housing," Journal of Social Issues, XV(October, 1959), 24-29; Arnold Rose, "Inconsistencies in Attitudes toward Negro Housing," VII (Spring, 1961), 286-292; Paul B. Sheatsley, "White Attitudes Toward the Negro," Daedalus, XCV(Winter, 1966), 217-238.

[2]Karl and Alma Taeuber, Negroes in Cities: Residential Segregation and Neighborhood Change(Chicago, 1965). See also: Morton Grodzins, "Metropolitan Segregation", Scientific American, CXCVII(October, 1957), 33-41.

[3]Taueber and Tauber.

[4]Richard Morrill, "The Negro Ghetto: Problems and Alternatives," Geographic Review, LV(July, 1965), 350. A slightly different version has also been published: "A Geographic Perspective on the Black Ghetto," in Geography of the Ghetto, ed. Harold Rose, (DeKalb IL, 1972).

[5]Ch. R. Hansell and W. A. V. Clark, "The Expansion of the Negro Ghetto in Milwaukee, Tijdschrift Voor Econ. En Soc. Geografie, (Sept./Oct., 1970).

[6]Linton Freeman and Morris Sunshine, Patterns of Residential Segregation, (Cambridge MA, 1970).

[7]Kerry Vandell, "A Simulation Model of the Ghetto Expansion Process," Massachusetts Institute of Technology, unpublished.

[8]Thomas C. Schelling, "Dynamic Models of Segregation," Journal of Mathematical Sociology, I(1971) 143-186.

[9]A concise discussion of the quantity of housing service, known as "q", can be found in Edgar O. Olsen, "A Competitive Theory of the Housing Market," American Economic Review ,(September, 1969). A more exhaustive treatment may be found in Richard F. Muth, Cities and Housing, (Chicago, 1969).

[10]This is an adaptation of the approach used by David Birch and others in The New Haven Laboratory, a Testbed for Planning, 1973, V-35.

[11]A utility function of very similar form is used by J. Fereira and D. Carlton, "Simulation and Analysis of Three Hybrid Housing Allowance Payment Formulas," Report to Abt Associates and HUD, January, 1975.

[12] Scores of articles have been written on market values and race and there are many contradictory findings. Perhaps the most famous study is Luigi Laurenti, Property Values and Race: Studies in Seven Cities, (Berkeley, 1960). Some other views: Anthony Downs, "An Economic Analysis of Property Values and Race (Laurenti)," Land Economics, XXXVI (May, 1960), 181-188; Martin J. Bailey, "Effects of Race and Other Dempgraphic Factors on the Values of Single Family Homes," Land Economics, XLII (May, 1966), 215-220 E. F. Shietinger, "Race and Residential Market Values in Chicago," Land Economics, XXX(Nov., 1954), 301-308; Alfred N. Page, "Race and Property Values," The Appraisal Journal, XXXVI(July, 1968), 334-342; and Charles L. Osenbaugh, "Integrated Housing and Value," The Appraisal Journal, XXXV (July, 1967), 17-20.

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