Modeling and Simulation of the
Autonomous Underwater Vehicle, Autolycus

by

Sia Chuan, Tang

BSc., Mechanical Engineering
Nottingham University, UK (1990)

Submitted to the Department of Ocean Engineering
in partial fulfillment of the requirements for the degree of

Master of Science in Naval Architecture and Marine Engineering

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Author ........................................................................

Department of Ocean Engineering

January 26, 1999

Certified by ......................................................................

John J. Leonard

Assistant Professor of Ocean Engineering

Thesis Supervisor

Accepted by ..................................................................

Professor Arthur B. Baggeroer

Ford Professor of Engineering

Chairman, Departmental Committee on Graduate Students
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Abstract

This thesis has developed a model to simulate the motion responses of the AUV, Autolycus, in six degrees of freedom. The rigid body dynamics in this model is non-linear so as to retain the inherent non-linear behavior of an underwater vehicle. The hydrodynamic effects acting on the vehicle are modeled as components of the added mass, viscous damping, restoring and propulsion forces and moments. Simulations of the vehicle motions are obtained by solving the equations of motion using Matlab. Experimental data on Autolycus are used to enhance the estimation of the hydrodynamic coefficients, which are used to describe the hydrodynamic characteristics of the vehicle.

The model is able to reproduce the motions of Autolycus in the vertical and horizontal planes. In contrast with more complex models found in literature, the model presented in this thesis offers the benefits of being easy to understand, apply and further enhance or adapt to other vehicle designs.

Thesis Supervisor: John J. Leonard
Title: Assistant Professor of Ocean Engineering
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Many special people have given me the help, encouragement and advice so greatly needed for the completion of this thesis. Knowing and learning from each of these friends have been a real joy to me and I am thankful.

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I thank M. Nahon for sharing the model from [18], on which the Matlab scripts are based in part.

Robert Damus, Linda Kiley, Jennifer Tam and Ian Ingram have given me the insight and advice to Autolycus design and testing. I have a better understanding of the issues regarding the design and testing of an AUV.

I remember clearly that Dr Y. Liu and Meldon Wolfgang gave me a supplementary lecture on hydrodynamic forces on an underwater body. This lecture helped me to make a significant progress in Chapter Three of my thesis.

Thomas Fulton, Chris Cassidy, Sung-Joon Kim, Rick Rikoski and Rob Damus were most helpful during the two days of experiments with Autolycus. These are wonderful people to work with.

My sponsor, the Ministry of Defense, Singapore should be mentioned for offering me an opportunity to come back to school. It has been a refreshing learning experience.

Much thankfulness goes to my wife for laboring over the needs of the family so that I could be set free to attend to my work.
Dedication

Thou rulest the raging of the sea: when the waves thereof arise, thou stillest them.

Psalm 89:9 (KJV)

Now it came to pass on a certain day, that he went into a ship with his disciples; and he said to them, *Let us go over unto the other side of the lake.* And they launched forth.

But as they sailed, he fell asleep; and there came down a storm of wind on the lake; and were in jeopardy.

And they came to him, and awoke him, saying, Master, master, we perish. Then he arose, and rebuked the wind and the raging of the water: and they ceased, and there was a calm.

And he said unto them, *Where is your faith?* And they being afraid wondered, saying one to another, What manner of man is this! for he commandeth even the winds and water, and they obey him.

Luke 8:22-25 (KJV)

To Christ Jesus, my Lord and my God. John 20:28

To three beautiful ladies whom I love; Joyce, Hannah and Esther.
# Contents

1 Introduction ......................................................... 18
   1.1 Background .................................................... 18
   1.2 Overview of Autolycus Design ............................... 21
   1.3 Objective of Thesis ......................................... 21
   1.4 Novelty of This Work ........................................ 22
   1.5 Summary ....................................................... 25
   1.6 Document Roadmap ........................................... 25

2 A Review of Underwater Vehicle Simulation Models .............. 27
   2.1 Introduction ................................................... 27
   2.2 Experimental Based Model by Humphreys .................... 29
   2.3 Empirical Model by Nahon ................................... 32
   2.4 Approach to Modeling Autolycus ............................. 35
   2.5 Comparison to Fossen's Model ............................... 37
   2.6 Summary ....................................................... 38

3 Derivation of the Equations of Motion ........................... 39
   3.1 Introduction ................................................... 39
   3.2 Derivation of the Dynamics Equations ....................... 40
      3.2.1 Linear Momentum ....................................... 40
      3.2.2 Angular Momentum ..................................... 43
   3.3 External Forces and Moments ................................ 44
      3.3.1 Added Mass .............................................. 45
      3.3.2 Hydrodynamic Damping .................................. 49
      3.3.3 Restoring Forces ....................................... 50
      3.3.4 Environmental Forces ................................... 52
      3.3.5 Propulsion Forces and Moments ......................... 52
   3.4 Choosing the Body Fixed Origin ............................... 54
   3.5 Transformation to Earth Fixed Reference ..................... 55
   3.6 Assumptions Applied to Autolycus Simulation ............... 56
   3.7 Autolycus Motions Simulation Model ......................... 56
   3.8 Summary ....................................................... 59
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Estimation of Autolycus' Parameters.</td>
<td>60</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>60</td>
</tr>
<tr>
<td>4.2</td>
<td>Estimating KG and LCG</td>
<td>61</td>
</tr>
<tr>
<td>4.3</td>
<td>Estimating KB and LCB</td>
<td>62</td>
</tr>
<tr>
<td>4.4</td>
<td>Estimating the Mass Moment of Inertia.</td>
<td>63</td>
</tr>
<tr>
<td>4.5</td>
<td>Estimating the Added Mass Coefficients</td>
<td>71</td>
</tr>
<tr>
<td>4.5.1</td>
<td>Added Mass of a Cylinder</td>
<td>71</td>
</tr>
<tr>
<td>4.5.2</td>
<td>Added Mass of a Plate</td>
<td>72</td>
</tr>
<tr>
<td>4.5.3</td>
<td>Off Diagonal Added Mass Terms</td>
<td>72</td>
</tr>
<tr>
<td>4.5.4</td>
<td>Autolycus Added Mass Coefficients</td>
<td>73</td>
</tr>
<tr>
<td>4.6</td>
<td>Estimating the Hydrodynamic Damping Coefficients</td>
<td>77</td>
</tr>
<tr>
<td>4.7</td>
<td>Estimating the Propulsion Thruster Force</td>
<td>81</td>
</tr>
<tr>
<td>4.8</td>
<td>Comparing Autolycus Coefficients</td>
<td>82</td>
</tr>
<tr>
<td>4.9</td>
<td>Autolycus Motion Simulation Using Matlab</td>
<td>83</td>
</tr>
<tr>
<td>4.10</td>
<td>Summary</td>
<td>84</td>
</tr>
<tr>
<td>5</td>
<td>Comparing Autolycus Simulations with Experimental Data</td>
<td>85</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>85</td>
</tr>
<tr>
<td>5.2</td>
<td>Summary of the Simulation Process</td>
<td>88</td>
</tr>
<tr>
<td>5.3</td>
<td>Surge Maneuver</td>
<td>90</td>
</tr>
<tr>
<td>5.4</td>
<td>Heave Maneuver</td>
<td>94</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Zero Thrust, 20.9 grams weight added.</td>
<td>94</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Zero Thrust, 53.4 grams weight added.</td>
<td>97</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Full Thrust Heave Maneuver</td>
<td>100</td>
</tr>
<tr>
<td>5.5</td>
<td>Yaw Maneuver</td>
<td>102</td>
</tr>
<tr>
<td>5.5.1</td>
<td>Wide Turn</td>
<td>102</td>
</tr>
<tr>
<td>5.5.2</td>
<td>Narrow Turn</td>
<td>106</td>
</tr>
<tr>
<td>5.6</td>
<td>Summary</td>
<td>111</td>
</tr>
<tr>
<td>6</td>
<td>Conclusion</td>
<td>112</td>
</tr>
<tr>
<td>6.1</td>
<td>Summary</td>
<td>112</td>
</tr>
<tr>
<td>6.2</td>
<td>Future Work</td>
<td>113</td>
</tr>
<tr>
<td>6.3</td>
<td>Thruster Dynamics</td>
<td>113</td>
</tr>
<tr>
<td>6.4</td>
<td>Redesign of Autolycus</td>
<td>113</td>
</tr>
</tbody>
</table>

Appendix A: Matlab Scripts

A-1 autosc.m. ........................................................................ 115
A-2 autodync.m. ................................................................. 122
List of Figures

1 Autolycus, a small, low-cost student built AUV for ocean engineering education ........................................ 15
2 Positive Directions of Axes, Angles, Velocities, Forces and Moments ........................................ 16
3 Major Dimensions of Autolycus ........................................ 17
3-1 Body axis and earth fixed system ........................................ 40
3-2 Angular velocity of particle mass ........................................ 41
3-3 Gravity and buoyancy forces acting on the vehicle ........................................ 51
3-4 Thrusters forces and moment arms ........................................ 54
4-1 Mass moments of inertia ........................................ 63
4-2 Mass moments of inertia of hollow cylinder ........................................ 64
4-3 Mass moments of inertia of hemisphere ........................................ 65
4-4 Added mass of a cylinder ........................................ 72
4-5 Added mass of a plate ........................................ 72
4-6 Off diagonal added mass moment terms ........................................ 73
4-7 Added mass of circle with fins ........................................ 74
4-8 Damping due to rolling ........................................ 79
4-9 Damping due to pitching ........................................ 80
4-10 Autolycus resistance test data ........................................ 81
5-1 Overview of process to obtain hydrodynamic coefficients .................................. 87
5-2 Experimental data for surge maneuver ................................................................. 91
5-3 Initial simulation for surge maneuver ................................................................. 92
5-4 Final simulation for surge maneuver ................................................................. 93
5-5 Experimental data for heave maneuver (20.9g added) ....................................... 95
5-6 Simulation for heave maneuver (20.9g added) ................................................... 96
5-7 Experimental data for heave maneuver (53.4g added) ....................................... 98
5-8 Simulation for heave maneuver (53.4g added) ................................................... 99
5-9 Experimental data for heave maneuver (full thrust) ......................................... 100
5-10 Simulation for heave maneuver (full thrust) ..................................................... 101
5-11 Experimental data for yaw maneuver (wide turn) .......................................... 103
5-12 Simulation for yaw maneuver (wide turn) ....................................................... 104
5-13 Experimental data for yaw maneuver (narrow turn) ...................................... 107
5-14 Simulation for yaw maneuver (narrow turn) .................................................... 109
List of Tables

2-1 Comparison of terms used in simulation methods .................................. 36

4-1 Estimates of KG, LCG, Ixx, Iyy, Izz ......................................................... 67
4-2 Comparison of Autolycus data ............................................................... 83

5-1 Summary of coefficients obtained from simulation. .............................. 89
5-2 Visually estimated yaw rate and turning diameter for narrow turn .......... 108
Nomenclature

\[ A_o \] reference area; may be frontal area, wetted area or plane projected area
\[ B \] buoyancy force
\[ CB \] center of buoyancy
\[ CD \] coefficient of cross-flow drag
\[ Cf \] coefficient of frictional drag on wetted surface
\[ CG \] center of mass
\[ I_x \] mass moment of inertia of vehicle component about component x axis
\[ I_{xx} \] mass moment of inertia of vehicle about x axis
\[ I_{xy} \] product of inertia with respect to the x and y axes
\[ I_y \] mass moment of inertia of vehicle component about component y axis
\[ I_{yy} \] mass moment of inertia of vehicle about y axis
\[ I_{yz} \] product of inertia with respect to the y and z axes
\[ I_z \] mass moment of inertia of vehicle component about component z axis
\[ I_{zz} \] mass moment of inertia of vehicle about z axis
\[ I_{zx} \] product of inertia with respect to the z and x axes
\[ i, j, k, l \] indices
\[ K \] moment component about x axis (rolling moment)
\[ K_p \] roll added mass coefficient due to roll acceleration
\[ K_{\rho|\rho} \] roll damping coefficient due to roll
\[ K_A \] added mass in roll component
\[ K_D \] damping in roll component
\[ K_P \] propulsion in roll component
\[ K_R \] hydrostatic restoring moment in roll component
\[ L \] overall length of vehicle
\[ lh \] mean length of hull
\[ lph \] length of propulsion thruster housing
\[ m \] mass of vehicle, including water in free-flooding spaces
\[ m_i \] mass of a component of the vehicle
\[ M \] moment component about y axis (pitching moment)
\[ M_{\dot{\phi}} \] pitch added mass coefficient due to pitch acceleration
\[ M_{\dot{\psi}} \] pitch added mass coefficient due to heave acceleration
\[ M_{\phi|\phi} \] pitch damping coefficient due to pitch
\[ M_{\psi|\psi} \] pitch damping coefficient due to heave
\[ M_A \] added mass in pitch component
\[ M_D \] damping in pitch component
\[ M_P \] propulsion in pitch component
\[ M_R \] hydrostatic restoring moment in pitch component
\[ N \] moment component about z axis (yawing moment)
\[ N_{\dot{\chi}} \] yaw added mass coefficient due to yaw acceleration
\[ N_{\chi|\chi} \] yaw moment damping coefficient due to yaw
$N_\phi$ yaw moment damping coefficient due to sway

$N_\dot{\phi}$ yaw added mass coefficient due to sway acceleration

$N_A$ added mass in yaw component

$N_D$ damping in yaw component

$N_P$ propulsion in yaw component

$N_R$ hydrostatic restoring moment in yaw component

$p$ angular velocity component about $x$ axis relative to fluid (roll)

$\dot{p}$ angular acceleration component about $x$ axis relative to fluid

$q$ angular velocity component about $y$ axis relative to fluid (pitch)

$\dot{q}$ angular acceleration component about $y$ axis relative to fluid

$r$ angular velocity component about $z$ axis relative to fluid (yaw)

$\dot{r}$ angular acceleration component about $z$ axis relative to fluid

$T$ rotation matrix

$T_{fore}$ generalized propulsion thruster force

$U$ velocity of origin of vehicle axes relative to fluid

$u$ velocity component of $U$ in direction of the $x$ axis (surge velocity)

$\dot{u}$ acceleration in direction of $x$ axis

$v$ velocity component of $U$ in direction of the $y$ axis (sway velocity)

$\dot{v}$ acceleration in direction of $y$ axis

$\overline{V}$ volume of body

$w$ velocity component of $U$ in direction of the $z$ axis (heave velocity)

$\dot{w}$ acceleration in direction of $z$ axis

$W$ weight of vehicle, including water in free flooding spaces
\( x \) longituidinal body axis; also the coordinate of a point relative to the origin of the body axes
\( \dot{x} \) rate of change of displacement in direction of \( x \) axis
\( x_B \) the \( x \) coordinate of the CB
\( x_G \) the \( x \) coordinate of the CG
\( X \) force component along \( x \) axis (longitudinal or axial force)
\( X_a \) surge added mass coefficient due to surge acceleration
\( X_u \) surge damping coefficient due to surge
\( X_A \) added mass in axial component
\( X_D \) damping in axial component
\( X_P \) propulsion force in axial component
\( X_R \) hydrostatic restoring force in axial component
\( y \) lateral body axis; also the coordinate of a point relative to the origin of the body axes
\( \dot{y} \) rate of change of displacement in direction of \( y \) axis
\( y_B \) the \( y \) coordinate of the CB
\( y_G \) the \( y \) coordinate of the CG
\( Y \) force component along \( y \) axis (lateral force)
\( Y_s \) sway added mass coefficient due to yaw acceleration
\( Y_s \) sway added mass coefficient due to sway acceleration
\( Y_v \) sway damping coefficient due to sway
\( Y_A \) added mass in lateral component
\( Y_D \) sway damping force in lateral component
$Y_p$ sway propulsion force in lateral component

$Y_R$ sway hydrostatic restoring force in lateral component

$z$ normal body axis; also the coordinate of a point relative to the origin of the body axes

$\dot{z}$ rate of change of displacement in direction of $z$ axis

$z_B$ the $z$ coordinate of the CB

$z_G$ the $z$ coordinate of the CG

$zp$ the $z$ coordinate of the propulsion force

$Z$ force component along $z$ axis (normal force)

$Z_q$ heave added mass coefficient due to pitch acceleration

$Z_w$ heave added mass coefficient due to heave acceleration

$Z_{wz}$ heave damping coefficient due to heave

$Z_A$ added mass in normal component

$Z_D$ damping in normal component

$Z_P$ propulsion force in normal component

$Z_R$ hydrostatic restoring force in normal component

$\theta$ angle of pitch

$\dot{\theta}$ pitch rate

$\psi$ angle of yaw

$\dot{\psi}$ yaw rate

$\phi$ angle of roll

$\dot{\phi}$ roll rate

$\rho$ density of fluid

$\Omega$ rotational velocity
Figure 1: Autolycus, a small, low-cost student-built AUV for ocean engineering education.
Figure 2: Positive Directions of Axes, Angles, Velocities, Forces and Moments.
Figure 3: Major Dimensions of Autolycus.

Diagrams are not drawn to scale

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<td>63</td>
</tr>
</tbody>
</table>

All dimensions are in mm.
Chapter 1

Introduction

1.1 Background

The ocean holds immense resources that can enrich and benefit humanity. It supports the essential food chain and also holds high-value minerals that keep industries and economies functioning. But these resources are finite and the ecosystem is fragile. The ocean needs to be continually studied to gain an understanding of its dynamic behavior. This can lead to better management of the ocean’s resources, which is necessary for them to be sustained into the long term. The ocean also has strategic importance to the security of a nation. A nation needs to effectively and efficiently monitor its territorial waters and have a good knowledge of the environment in which it deploys its naval deterrence.

The study of the ocean typically involves underwater search and mapping, climate change assessment or marine habitat monitoring. These are critical problems that challenge the scientific and engineering community. The development of better ocean monitoring systems and more capable machines to survey the depths of the ocean will help the users of the ocean to find solutions that fulfil their respective needs. However, the surveying and monitoring tasks are inherently tedious, long in duration and the ocean depths are often hazardous. Traditionally, underwater vehicles have been developed to perform the surveying and monitoring functions with the objective of overcoming the time and risk problems.
Underwater vehicles can generally be classified into the following categories:

(a) manned vehicle,
(b) tethered manned vehicle,
(c) tethered unmanned vehicle, and
(d) untethered unmanned vehicle or Autonomous Underwater Vehicle (AUV).

This thesis concerns itself only with AUVs. AUVs offer the following advantages over the other categories of vehicles:

(a) there is no need for specialized operators, which manned vehicles require.
(b) there is no need to station an operator on the ocean surface, as would be required for a tethered vehicle. This can reduce the total labor cost for a long duration mission.
(c) an AUV can be programmed to operate in an area of interest without further human intervention. The control station can be at the shore, away from the harsh and unpredictable ocean.

It can be seen that deploying AUVs for oceanographic survey and monitoring can be cost-effective. The total operating cost can be significantly reduced since human intervention at a survey site can be eliminated. This is especially attractive since survey work is intensive and long in duration. Furthermore, AUVs do not suffer from fatigue, unlike humans, which enable them to work non-stop. However, energy is a serious concern in current AUV designs. An additional benefit of an autonomous method for oceanographic survey is that the AUV is deep beneath the ocean surface, consequently, the AUV is not subjected to any adverse weather conditions on the surface.

Some of the critical challenges facing current designs of AUVs are:

(a) energy efficiency and power management,
(b) communications,
(c) navigation,
(d) control.
The challenge to an AUV designer is to design a vehicle which has sufficient energy and sensing capability to fulfil an extended mission at an acceptable cost. Over the years, many different types of AUVs have been developed. Two such examples are Odyssey, built by the MIT Sea Grant Underwater Vehicle Laboratory [21] and the Autonomous Remotely Controlled Submersible (ARCS) built by International Submarine Engineering [17]. Unfortunately, the design and development of an AUV is complex and expensive. It involves developing the vehicle, subjecting it to tests, refining the design, and then subjecting it to further tests. The vehicle is indispensable to the researcher during the testing and refining process. Student access for learning becomes next to impossible. An alternative to intensive testing during the design and development phase is to create a simulation tool to predict the response of the vehicle when modified. A simulation tool can shorten the development time, which being available for educational purposes.

At MIT, the problem of accessing an AUV for Marine Robotics education was overcome through the Autolycus program. Autolycus, as shown in Figure 1, is a small underwater vehicle that was designed and built by the undergraduates of courses 13.017 and 13.018 "Design of Ocean Systems". In order to refine or redevelop Autolycus, it is essential to be able to predict the changes in the response of the vehicle resulting from changes made to the vehicle’s payload, rearrangement of the internal components which shifts the center of bodies, or vehicle external modifications. This need lead to this simulation tool for Autolycus. With a physical vehicle (Autolycus) and a simulation model, research on underwater vehicles can be cost and time effective. The problem of student inaccessibility is also solved.

An AUV design and simulation may include a number of elements, such as payload, vehicle sizing, power, and/or mission planning. These elements can then be used to create a suitable mathematical model of the AUV. One possible mathematical model represents the vehicle’s dynamics and its interaction with the fluid in which it operates. The focus of this thesis is to create such a model and to show that it is a close representation of Autolycus.
1.2 Overview of Autolycus Design

Figure 1 shows a view of Autolycus. Figures 2 defines the positive directions of axes, angles, velocities, forces and moments. Figure 3 shows the major dimensions of the vehicle.

Autolycus can be decomposed into the following constituent components:

(a) a hemispherical nose cap,
(b) an axis-symmetric cylindrical mid-body (hull),
(c) a hemispherical tail cap,
(d) two horizontal thrusters, port and starboard (stbd) side,
(e) two horizontal struts supporting the thrusters – these struts are not designed as lifting surfaces, and
(f) two vertical thrusters embedded in the body, forward and aft.

Within the watertight hull are electronic cards to control the vehicle and to process the signals of the sensors. The batteries that power the thrusters and the computers are also stored in the hull. At the base of the hull is a lead plate acting as ballast to balance the buoyancy force such that a neutral buoyancy condition is achieved. The buoyancy force is equal to the weight of the water displaced by the vehicle. A neutral buoyancy condition is achieved when the vehicle weight, including entrained water, is equal to the weight of water displaced by the vehicle.

1.3 Objectives of Thesis

The objective of this thesis is to model and simulate the response motions of Autolycus. The model will capture the rigid body dynamics, hydrostatics and hydrodynamic effects of the underwater vehicle.

Important issues concerning modeling and simulation of AUV’s are model complexity, ease of implementation and accuracy of prediction. A complex model, such as suggested by Humphreys [12], is difficult to implement but the result is highly
accurate. The main reason is that the coefficients used to describe the vehicle characteristics are obtained from experiments carried out on the particular vehicle. Since experimental facilities are costly, an experimentally based model is unattractive. The simplified model suggested by Nahon [18], overcomes the problem of costly experiments by using a simplified hydrodynamic model of the vehicle. Nahon [18] uses only body lift and drag forces to model the hydrodynamic characteristics of the vehicle. The lift and drag force coefficients of the vehicle, based on its geometric features, can be found in references such as [10] and [11]. However, an oversimplified hydrodynamic model, as seen in [18], is not likely to produce good results for a wide range of simulation cases.

This thesis suggests an approach to simplifying and computing the hydrodynamics of an AUV. Autolycus is used as a case for study because research work using Autolycus is ongoing [16]. Modeling and simulation of Autolycus thus offer value to future research efforts. It will overcome the difficulty of accessing an underwater vehicle and thus enable students of marine robotics to further their understanding. The simulation is written in Matlab, since Matlab is easy to learn and is widely used at MIT.

The result of this thesis can thus be used in three ways. One, it can serve as an effective educational tool for learning the hydrodynamic behavior of an underwater vehicle. Two, it can be used to design Autolycus-like vehicles in a shorter time frame and at a lower cost than with an experimental method. Three, it can benefit future research work that needs to incorporate the hydrodynamic behavior of an Autolycus-like vehicle.

1.4 Novelty of This Work

The design and development of an AUV is both complex and costly. The number of variations, such as a vehicle's geometry and controllers, involved in a developmental system is expected to be large and in this respect, the designer cannot solely rely on prototyping or experimental work. The cost and time required to test each variation is
unacceptable. Hence, a good solution would be to develop computational modeling tools to help the designer, particularly during the initial phases of feasibility studies and prototyping.

Leonard [16] has identified the lack of a single course in MIT, as well as other universities, that provides exposure to the unique issues of modeling and simulation of ocean vehicles, sensors, systems and environments. The plan is to develop a comprehensive graduate course that provides an introduction to the field of marine robotics for new students. The teaching of this envisaged course can be enhanced with a simulation tool that captures the dominant effects on an underwater vehicle.

There is no doubt that the above requirements can be met with underwater vehicle simulation tools currently available. There are also journals that report such simulation work. Some examples of these are [6], [7], [8], [12], [18] and [19]. However, the full programming code is not available from such references. Also, Fidler [6], Gertier [8] and Humphreys [12] used very complex hydrodynamic models. These references did not give an elaborate account of what terms should be used to represent the hydrodynamic model and how these terms can be derived, which makes understanding them more difficult. Also, the relationship between vehicle geometry and the response of the vehicle is not obvious.

This thesis is an attempt to make available a design and simulation tool for an underwater vehicle. It combines basic knowledge of submarine design and hydrodynamic effects on an underwater vehicle.

Autolycus can be thought of as being composed of many components as described in Section 1.2. All these components have their individual masses, dimensions and centers of gravity referenced with respect to the vehicle fixed origin. With these data, the position of the vehicle's center of gravity (CG) and its mass moments of inertia with respect to the vehicle origin can be computed. The hydrodynamic damping and added mass coefficients can be estimated from the vehicle geometry. Having represented the
vehicle in terms of the hydrodynamic coefficients, the dynamic response of the vehicle can then be found by solving the 6 D-O-F equations.

The dynamic characteristics of the vehicle such as CG and moments of inertia are computed using a spreadsheet program as shown in Table 4-1. It lists all the components of the vehicle and the respective masses, dimensions and centers of gravity. The hydrodynamic coefficients are estimated in the early section of the Matlab script enclosed as Appendix A. The program subsequently computes the vehicle response motions.

The result of this thesis is thus a low cost design and simulation iteration tool for an underwater vehicle. The dynamic characteristics and hydrodynamic coefficients of the vehicle are first estimated. A simulation of its response is then performed. The designer can alter the parameters in the simulation so that the desired vehicle response is obtained. The designer then reverts to the spreadsheet to shift the masses of the vehicle components to obtain a match with the changes made in the simulation. The cycle proceeds until an optimized design is obtained. Chapter Five shows the process of estimating the vehicle parameters and the hydrodynamic coefficients that can effectively model the vehicle’s behavior based on experimental data of the vehicle. Further simulations of other types of maneuvers may be performed using the updated model to predict the responses of the vehicle. Furthermore, if a specific response of the real vehicle is desired, then the simulation model can be used to reproduce that motion by changing the coefficients and parameters. These new changes are then converted to the required changes on the real vehicle, such as the addition of a control fin or the shifting of the vehicle’s center of mass.

1.5 Summary
This chapter has briefly reviewed the advantages in using AUVs as ocean surveillance instruments. It explains the benefits of having a physical vehicle for research purposes and that the research can be more effective when supplemented by a motion response
simulation tool. The availability of these two assets can enhance the teaching and learning of underwater robotics.

1.6 Document Roadmap

Chapter Two will provide a summary of previous research on underwater vehicle modeling and simulation. Each of these models offers an important lesson for modeling Autolycus. The approach taken to modeling the motions of Autolycus will be outlined.

Chapter Three will step through the derivation of the 6 D-O-F equations for a rigid body. It will proceed to identify the external forces that are applicable to an underwater vehicle. The simple geometric features of Autolycus help to reduce the hydrodynamic complexity. Further assumptions are applied to simplify the equations used for the dynamic simulation of Autolycus. The equations for surge, sway, heave, roll, pitch and yaw motions are presented.

Chapter Four will show in detail the methods for determining the static characteristics and hydrodynamic coefficients of Autolycus with reference to the body fixed axes. The ability to closely estimate these parameters is essential to successfully modeling the dynamic response of Autolycus. Static characteristics are principally the vehicle center of gravity, center of buoyancy and mass moment of inertia about the vehicle axes. Hydrodynamic coefficients are the added mass and hydrodynamic damping. Other contributors of external forces are hydrostatic restoring forces and propulsion thrust. The static and dynamic parameters are substituted into the motion equations to obtain the resultant dynamic response of Autolycus. The simulation is performed using Matlab, based in part on a program kindly provided by Nahon used in [18].

Chapter Five will show how the proposed model is verified and modified using experimental data of Autolycus. Open-loop simulations are compared to the open-loop
maneuvers of Autolycus to better estimate the hydrodynamic coefficients and vehicle parameters that describe the real responses of Autolycus. The successful model can then be extended in future research on Autolycus and for educational purposes.

Finally, Chapter Six will conclude this thesis by summarizing the results of the thesis and suggesting potential areas for further research.
Chapter 2

A Review of Underwater Vehicle Simulation Models

2.1 Introduction

Much work has been done on modeling and simulating the motion of an underwater vehicle. In 1967 the foundation of submarine motion simulation was laid with the publication of Standard Equations of Motion for Submarine Simulation by Gertler [8]. This was the standard upon which a subsequent revision was made in 1979 by Feldman [5]. The six degree of freedom equations of motion for submarine simulation are general enough to simulate the trajectories and responses of the submarine resulting from normal maneuvers as well as for extreme maneuvers such as those associated with emergency recoveries [8]. Understandably, these equations provide a highly accurate and robust model of the submarine responses. The key feature of these equations is the hydrodynamic coefficients. Hydrodynamic coefficients are related to the fluid forces and moments acting on the underwater body as a result of its motion in the fluid. A successful prediction of the submarine responses depends on the ability to determine these coefficients accurately. For a submarine, the numerical values of the hydrodynamic coefficients are determined solely from experiments to eliminate doubts. Theoretical methods of obtaining these coefficients can offer a reasonable success only for simple forms without appendages, such as bodies of revolution.
Critical and complex simulations for a vehicle such as for a submarine require substantial computing and experimental resources. Due to its complexity, the designer has to have insights regarding how changes to a particular hydrodynamic coefficient, for example increasing the fin area, could affect the submarine responses. Only then can the designer use the simulation tool effectively.

The motion simulation of an AUV such as Autolycus is in many aspects similar to a submarine. Major differences between AUV’s and submarine’s with respect to motion are:

(a) an AUV’s translational and angular motions are small,
(b) the rate of translational and angular changes are small,
(c) an AUV has simpler forms, and
(d) an AUV simulation is generally less demanding so as to reduce the total cost of its development.

These differences offer an opportunity to use a simplified version of the submarine model for simulating the responses of an AUV. Furthermore, since the result of this thesis will be used as a teaching and learning tool, simplification is beneficial for imparting the basic concepts of simulating an underwater vehicle without undue complexity.

This chapter will proceed to review two simulation models that represent distinct approaches to simulating the motions of an underwater vehicle. The simulation model developed by Humphreys [12] adopted a set of linearized equations of motion but maintained a rigorous approach to obtaining the hydrodynamic coefficients. In contrast, the model proposed by Nahon [18] retained the non-linear equations of motion but avoided the rigorous hydrodynamic coefficients by calculating the hydrodynamic forces directly from known relations which govern the flow around simple shapes. This thesis is in essence a combination of these two approaches. It adopts the non-linear equation of motions but applies strip theory to estimate the hydrodynamic coefficients of Autolycus.
The latter is possible because Autolyus has predominantly a simple circular cylinder form. To ensure that the necessary hydrodynamic effects are included, a term by term comparison is made with the model proposed by Humphreys [12]. The comparison is tabulated in Table 2-1. The approach to simulating Autolyus is outlined in Section 2.4.

This review of notable work is useful because it addresses two main issues in the simulation of an AUV:

(a) different models can be developed for simulating the motions of an underwater vehicle, and

(b) it shows that there are several possible ways to obtain the hydrodynamic coefficients for the vehicle.

A judgement on the part of the designer is required to choose an appropriate simulation model for the vehicle in question because each class of AUV design has its own particular differences that may require slightly different treatment. The most important aspect is that the result of the simulation must be comparable to the experimental data of the vehicle concerned. Hydrodynamic coefficients of the vehicle can be obtained by testing, predictive or theoretical methods. Experimental data will certainly give the best accuracy for the simulation but lower accuracy may have to be accepted if testing resources are lacking.

2.2 Experimental Based Model by Humphreys

In 1976, Humphreys [12] derived the linear, small-perturbation equations of motion for a self-propelled underwater vehicle. The equations include the inertial, hydrodynamic and gravity-buoyancy forces and moments. Assumptions are made to linearize the equations and to decouple the longitudinal motions from the lateral motions. Laplace transform
techniques are applied to obtain the solutions to the equations. The expressions for the transfer functions are given in non-dimensionalized hydrodynamic coefficients.

Humphreys argued that in the context of small perturbations and small angles, the products and squares of the changes in velocities are negligible in comparison with the changes themselves. He thus showed that the classical Euler equation for a rigid body with respect to a set of axes fixed at the body center-of-gravity could be linearized and reduced to:

\[
\begin{align*}
\Sigma X &= m\ddot{x}, \\
\Sigma Y &= m(\dot{y} + rU_o), \\
\Sigma Z &= m(\dot{z} - qU_o), \\
\Sigma K &= pI_{xx} - rI_{xz}, \\
\Sigma M &= qI_{yy}, \text{ and} \\
\Sigma N &= rI_{zz} - \dot{p}I_{xz}.
\end{align*}
\] (2.1)

Using the small perturbation assumption, the relationship between the angular velocities and the rate of change of the angles \(\dot{\phi}, \dot{\theta}, \dot{\psi}\) can be reduced to:

\[
\begin{align*}
p &= \dot{\phi}, \\
q &= \dot{\theta}, \text{ and} \\
r &= \dot{\psi}.
\end{align*}
\] (2.2)

Further assumptions made in the course of deriving the above equations are:

(a) the vehicle is assumed to be a rigid body,
(b) the mass, and mass distribution of the vehicle are assumed to be constant, and
(c) the \(xz\) plane is assumed to be a plane of symmetry.

In a strict mathematical sense, Equations (2.1) are applicable only to small disturbances but in Humphreys' experience a quick and accurate result can be obtained even when the equations are applied to larger disturbances.
The left-hand sides of Equations (2.1) represent the hydrodynamic effects acting on the vehicle. The hydrodynamic effects are known to be functions of the relative velocity, acceleration and position as well as control deflections, $\delta$. These can be written as

$$ F = f(\dot{u}, \dot{v}, \dot{w}, \dot{\theta}, \dot{\phi}, \dot{\psi}, p, \theta, \phi, r, \psi, \delta). \quad (2.3) $$

Taking a Taylor expansion about the equilibrium condition and ignoring the second order and higher terms, the expressions for the hydrodynamic forces and moments, denoted with subscript $h$, are:

$$ X_h = X_o + X_u \dot{u} + X_v \dot{v} + X_w \dot{w} + X_q \dot{q} + X_\theta \dot{\theta} + X_\phi \dot{\phi} + X_\psi \dot{\psi}, $$

$$ Y_h = Y_o + Y_u \dot{u} + Y_v \dot{v} + Y_p \dot{p} + Y_\phi \dot{\phi} + Y_\theta \dot{\theta} + Y_r \dot{r} + Y_\psi \dot{\psi}, $$

$$ Z_h = Z_o + Z_u \dot{u} + Z_v \dot{v} + Z_w \dot{w} + Z_q \dot{q} + Z_\theta \dot{\theta} + Z_\phi \dot{\phi} + Z_\psi \dot{\psi}, $$

$$ K_h = K_o + K_u \dot{u} + K_v \dot{v} + K_p \dot{p} + K_\phi \dot{\phi} + K_\theta \dot{\theta} + K_r \dot{r} + K_\psi \dot{\psi}, $$

$$ M_h = M_o + M_u \dot{u} + M_v \dot{v} + M_p \dot{p} + M_\phi \dot{\phi} + M_\theta \dot{\theta} + M_r \dot{r} + M_\psi \dot{\psi}, $$

$$ N_h = N_o + N_u \dot{u} + N_v \dot{v} + N_p \dot{p} + N_\phi \dot{\phi} + N_\theta \dot{\theta} + N_r \dot{r} + N_\psi \dot{\psi}. $$

Equations (2.4) have included the added mass terms $X_u, Y_v$, etc, and the gravity and buoyancy disturbance terms $X_\phi, Y_\theta$, etc. Terms such as $X_u, Y_v$, etc are associated with hydrodynamic damping, lift and drag forces and moments.

Substituting Equations (2.2) into Equations (2.1) and then equating to Equations (2.4) yields the complete equations that Humphreys [12] uses to simulate the motion response of an AUV.

It appears that Humphreys obtained the hydrodynamic coefficients from experimental data, which probably explains his success with his method. Certain terms used by Humphreys are different from those used in this thesis. For example, the hydrostatic restoring forces are expressed in the form of $X_o$ and $Y_o$ where
\[ X_\phi = -(W - B) \cos \theta, \text{ and} \\
Y_\phi = (W - B) \cos \theta. \]  \hfill (2.5)

In summary, Humphreys reduced the rigid body dynamic equations to a simple linearized form by applying the theory of small perturbations. These simplified equations are combined with an elaborate set of hydrodynamic coefficients that describe fully the hydrodynamic characteristics of the vehicle. In the case of a vehicle still in the design and development phase, such elaborate coefficients will probably be unavailable. Thus the student will not benefit from the simulation model presented by Humphreys. Also, the lack of experimental testing facilities will contribute to the difficulty in using Humphreys model.

2.3 Empirical Model by Nahon

In 1993, Nahon [19] investigated the use of empirical methods to estimate the hydrodynamic coefficients of streamlined underwater vehicles. Nahon applied an empirical method known as the “Stability and Control Data Compendium”, otherwise commonly referred to as Datcom. The Datcom is a database of lift and drag coefficients of aircraft components such as fuselages and fins, expressed in non-dimensionalized form. AUVs can benefit from this database simply because their geometric forms very often match those of aircraft fuselages. The required hydrodynamic coefficients of the AUV are predicted based on the equivalent aerodynamic coefficient in the Datcom. For example, \( Y \), of an AUV with ellipsoidal hull of some \( l/d \) ratio can be predicted using the experimental data performed on an ellipsoidal blimp with similar \( l/d \) ratio. To apply this experimental data to the AUV, compressibility effects are neglected and the density of air is replaced with that of water. However, Datcom does not account for added mass, which is a dominant influence on an AUV. It is thus necessary to determine the added mass of the AUV from other sources such as Newman [20]. Hoerner [10] and [11] also have a rich databases on lift and drag coefficients of bodies.
A predictive method such as proposed by Nahon is an attractive alternative to the test-based method discussed in Section 2.2. Nahon found that the Datcom could be used to provide good estimates for the hydrodynamic coefficients of an AUV.

As an aside note, Fidler [6] has documented an alternative methodology and has compiled data for estimating the lift and drag coefficients of an underwater vehicle as a result of vortex shedding from the vehicle.

In 1996, Nahon [18] proposed a simplified hydrodynamic model of an AUV. The model is based on directly computing the hydrodynamic forces and moments acting on each constituent component of the vehicle. These constituent forces and moments are then summed to produce the total lift and drag forces on the vehicle. This method works best for a vehicle that is streamlined and made up of simple symmetrical shapes. Fortunately, most AUVs, including Autolycus, fall into such a category.

The rigid body equation in Nahon’s model is not linearized to retain the vehicle’s fundamental non-linear behavior [18]. The body-fixed frame has its origin at the vehicle mass center. The model accounts for forces and moments arising due to the following effects:

(a) hydrostatic restoring forces due to weight and buoyancy: \( W, B \)
(b) propulsive thrust: \( T \)
(c) control: \( F_c, M_c \)
(d) hydrodynamic: \( F_h, M_h \)
The complete equations of motion used by Nahon to simulate an AUV are:

\[-(W - B) \sin \theta + T_x + F_{cx} + F_{hx} = m(\dot{u} + qw - rv),\]
\[(W - B) \sin \phi \cos \theta + T_y + F_{cy} + F_{hy} = m(\dot{v} + ru - pw),\]
\[(W - B) \cos \phi \cos \theta + T_z + F_{cz} + F_{hz} = m(\dot{w} + pv - qu),\]

\[z_B \sin \phi \cos \theta + S_x - T_y z_p + M_{cx} + M_{hx} = I_{xx} \dot{\theta} - (I_{yy} - I_{zz})qr - I_{xz} (\dot{r} + pq),\]
\[x_B \cos \phi \cos \theta + z_B \sin \theta + S_y + T_x z_p - T_z x_p + M_{cy} + M_{hy} = I_{yy} \dot{\theta} - (I_{zz} - I_{xx})pr - I_{yz} (\dot{r} - p),\]
\[-x_B \sin \phi \cos \theta + S_z + T_y x_p + M_{cz} + M_{hz} = I_{zz} \dot{\theta} - (I_{xx} - I_{yy})pq - I_{xz} (\dot{p} - qr).\]

\[x_b\] and \[y_b\] are the positions of the center of buoyancy, subscripts \(x,y,z\) denote the direction of the body-fixed axes, \(S\) is the thruster reaction torque and \(x_p\) and \(z_p\) are the distances from the origin to the line of thrust.

This approach is straightforward. On the right-hand side are the rigid body dynamics equations while on the left-hand side are the external forces and moments. Definitions of external forces and moments are given in Section 3.3. The hydrodynamic forces and moments are derived from well-established empirical relations for lift and drag forces on a body immersed in a fluid.

The simulation result was compared with the experimental motions of ARCS [17]. It appears that Nahon [18] has not included the added mass term in the simulation. The added mass coefficient is an important hydrodynamic characteristic of the underwater body, except in the case of a steady translation where the viscous drag dominates.

Several comments can be made on the model suggested by Nahon:

(a) the non-linear rigid body dynamic equation should be retained to capture the inherent non-linear behavior of an underwater vehicle.
(b) the body fixed origin should not be placed at the body center of gravity as done by Nahon. Instead, it should be placed at the vehicle geometric center so that some of the added mass terms can be made zero as shown in Chapters Three and Four.

(c) the hydrodynamic forces and moments must include the added mass terms.

2.4 Approach to Modeling Autolycus

The lesson that can be learned from Humphreys’ model is that hydrodynamic effects must be modeled as accurately as possible while Nahon’s model says that the non-linear rigid body dynamics must be retained.

The approach taken for the simulation of Autolycus will be to retain the non-linear rigid body dynamics and to build up a hydrodynamic model for the vehicle using theoretical estimation. For Autolycus, the hydrodynamic model will account for the following effects:

(a) added mass,
(b) viscous damping, otherwise known as drag,
(c) hydrostatics, and
(d) propulsive thrust.

Autolycus does not have control surfaces. Therefore, lift and drag forces due to fins need not be accounted. The thruster reaction torque is assumed small compared to the inertia of the vehicle and is therefore neglected.

The simple form of Autolycus is an advantage because well-established theoretical methods can be applied to estimate its hydrodynamic coefficients, as shown in Chapters Three and Four. However, in practice, modifications must be made to the model to match the behavior of the real vehicle as discussed in Chapter Five.
Table 2-1 shows a tabulated comparison of the terms used in the models proposed by Humphreys and Nahon. Alongside these are those terms used in the Autolycus simulation.

Table 2-1: Comparison of terms used in simulation models.

<table>
<thead>
<tr>
<th>Simulation Terms</th>
<th>Humphreys Model</th>
<th>Nahon Model</th>
<th>Autolycus Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid body dynamics</td>
<td>small perturbations theory, origin at center of gravity</td>
<td>origin at center of gravity</td>
<td>origin at center of vehicle</td>
</tr>
<tr>
<td>- linearized</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- non-linear</td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Hydrodynamic coefficients</td>
<td>experimental only</td>
<td>empirical method</td>
<td>theoretical method (but to be changed based on experimental data)</td>
</tr>
<tr>
<td>- added mass</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>- viscous damping or body drag</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>- body lift</td>
<td>✓</td>
<td>✓</td>
<td>not applicable</td>
</tr>
<tr>
<td>- control surfaces</td>
<td>✓</td>
<td>✓</td>
<td>not applicable</td>
</tr>
<tr>
<td>Hydrostatic</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Propulsive thrust</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Thruster torque</td>
<td>✓</td>
<td></td>
<td>assume small</td>
</tr>
</tbody>
</table>

The process of simulating the motions of Autolycus can be broken down into four sub-tasks:

(a) derive the governing equations to describe the dynamics of the vehicle,

(i) estimate the position of the center of gravity,

(ii) estimate the mass moments of inertia, and

(iii) estimate vehicle mass and buoyancy.
(b) calculate the hydrodynamic coefficients,
   (i) use strip theory to estimate added mass terms, and
   (ii) use strip theory to estimate the viscous damping terms.

(c) solve the equations of motion to obtain the vehicle response for a known set of control inputs, and

(d) compare with the real data of the vehicle and iterate the process until a desired accuracy is achieved.

2.5 Comparison to Fossen’s Model

The model developed in this thesis is primarily the adaptation of both Humphreys’ and Nahon’s models. It has a non-linear rigid body dynamics and a model to describe the hydrodynamic forces and moments acting on the vehicle. This structure is similar to that of Fossen’s model [7]. Fossen describes the hydrodynamic model as a sum of the physical forces and moments acting on the vehicle, instead of relying solely on experimental results to define the hydrodynamic model as done by Humphreys. These physical forces and moments are classified as radiation-induced, environmental and propulsion. The breakdown of these categories of forces and moments is shown in Section 3.3. This approach for the hydrodynamic model is more rigorous than the approach taken by Nahon, where only body lift and drag forces are considered. This thesis has adopted the hydrodynamic model suggested by Fossen [7]. The methods of estimating each of these hydrodynamic forces and moments are then in accordance to [22] and [23], as shown in Chapter Four.

The hydrodynamic effects on an underwater body is an inherently complex issue. Fossen’s work in [7] has enabled a better grasp of the subject by treating each category of
the hydrodynamic effects separately and representing them with physical forces and moments acting on the body.

2.6 Summary

This chapter has reviewed two simulation models that are relevant to the simulation of Autolycus. Two important lessons can be learned; one, that the dynamic model of a rigid body should be non-linear to retain the inherent non-linear behavior of the AUV and two, the hydrodynamic model should be extensive to include all the hydrodynamic forces and moments acting on the vehicle.

A review of the models used by Humphreys [12] and Nahon [18] shows that the Autolycus model has accounted for the external forces and moments necessary to enable preliminary simulations. The forces and moments coefficients can be refined when experimental comparisons are available. The approach to modeling the responses of Autolycus is laid out in Section 2.4.

Chapter Three will derive the equations that represent the dynamics of a rigid body. The external forces will also be clearly defined. Equating the dynamics of the rigid body to the external forces will provide the equations used for simulating the response motions of Autolycus.
Chapter 3

Derivation of the Equations of Motion

3.1 Introduction

This chapter will derive the complete three-dimensional, six degree-of-freedom (D-O-F) rigid body governing equations. This chapter will briefly work through the derivation of the dynamic equations of the general form \( F = ma \), where \( F \) is the external force acting at the center of mass of the body and \( ma \) are the body mass and acceleration respectively.

The external forces consist of radiation-induced forces, environmental forces and propulsive forces. Radiation-induced forces include added mass forces, hydrodynamic damping forces and restoring forces. All these forces will be treated in detail in this chapter.

Several assumptions, which will be elaborated, are applied to simplify the computation to obtain the vehicle’s resultant motions with respect to the body fixed axes. It is then desirable to transform the vehicle motions to obtain the translational position of the vehicle in terms of an inertial reference frame. Typically for an underwater vehicle, earth is chosen as the inertial reference frame.
3.2 Derivation of the Dynamics Equations

Although the derivation of the dynamic equations may be readily obtainable from several sources or reference texts, it is summarized in this section for the reason of making this document complete in itself. The material in this section is obtained from Humphreys [12], Nahon [18] and Triantafyllou [22].

3.2.1 Linear Momentum

The translational motions are in accordance with Newton’s law, which can be written as

$$\ddot{\mathbf{F}} = \frac{d}{dt}(m\dot{\mathbf{V}}),$$

(3.1)

for a rigid body with axes (x,y,z) rigidly attached on the body and whose origin O moves with velocity $V_0$ relative to the earth fixed system $(X_0,Y_0,Z_0)$ as shown in Figure 3-1.

![Figure 3-1: Body axis and earth fixed system.](image)

The velocity of a particle that is rigidly attached to the body is

$$\ddot{\mathbf{V}}_i = \ddot{\mathbf{V}}_0 + \ddot{\omega} \times \ddot{r}_i,$$

(3.2)

as shown in Figure 3-2. $\ddot{\omega}$ denotes the angular velocity of the rigid body.
Summing the forces of all particles of the body, the total force is

\[
\vec{F} = \sum_{i=1}^{N} \frac{d}{dt}[m_i(\vec{V}_0 + \vec{\omega} \times \vec{r}_i)],
\]

\[
= m \frac{d\vec{V}_0}{dt} + \frac{d}{dt} \{\vec{\omega} \times [\sum_{i=1}^{N} m_i \vec{r}_i]\}. \tag{3.3}
\]

\(m = \sum_{i=1}^{N} m_i\) and \(i = 1, 2, 3, \ldots N\).

If the center of gravity of a body, \(G\), is placed at a distance \(\vec{r}_G\) from the origin, its mass moment can be expressed as

\[m \vec{r}_G = \sum_{i=1}^{N} m_i \vec{r}_i. \tag{3.4}\]

Therefore, Equation (3.3) can be rewritten as

\[
\sum_{i=1}^{N} \vec{F}_i = m \frac{d\vec{V}_0}{dt} + m \frac{d}{dt} (\vec{\omega} \times \vec{r}_G). \tag{3.5}
\]

A vector \(\vec{f}\) projected on a rotating axes system \((x, y, z)\) has components \(f_x, f_y\) and \(f_z\). These components change because \((x, y, z)\) is rotating in addition to the angular change of the vector, \(\frac{df}{dt}\). This can be expressed as
where \( r \) denotes the relative derivative with respect to \((x,y,z)\).

Using the expression in (3.6), Equation (3.5) becomes

\[
\ddot{\vec{F}} = \sum_{i=1}^{N} \vec{F}_i = m \left\{ \frac{d\vec{V}_o}{dt} + \vec{\omega} \times \vec{V}_o + \frac{d\vec{\omega}}{dt} \times \vec{r}_G + \vec{\omega} \times (\vec{\omega} \times \vec{r}_G) \right\},
\]

(3.7)

\[
= m \left\{ \frac{d\vec{V}_o}{dt} + \vec{\omega} \times \vec{V}_o + \frac{d\vec{\omega}}{dt} \times \vec{r}_G + (\vec{\omega} \cdot \vec{r}_G) \vec{\omega} - (\vec{\omega} \cdot \vec{\omega}) \vec{r}_G \right\}.
\]

(3.8)

Substituting \( \vec{V}_o = (u,v,w), \vec{r}_G = (x_G,y_G,z_G), \vec{\omega} = (p,q,r) \) and \( \vec{F} = (X,Y,Z) \) into Equation (3.8), the force equations in the three axes can be obtained.

The axial force equation is

\[
\Sigma X = m \left\{ \frac{du}{dt} + qw - rv + \frac{dq}{dt} z_G - \frac{dr}{dt} y_G + (px_G + qy_G + rz_G) p - (p^2 + q^2 + r^2) x_G \right\},
\]

(3.9)

\[
= m \left\{ \dot{u} - vr + wq - x_G (q^2 + r^2) + y_G (pq - \dot{r}) + z_G (pr + \dot{q}) \right\}.
\]

(3.10a)

The Lateral force equation is

\[
\Sigma Y = m \left\{ \dot{v} - wp + ur - y_G (r^2 + p^2) + z_G (qr - \dot{p}) + x_G (qp + \dot{r}) \right\}.
\]

(3.10b)

The Normal force equation is

\[
\Sigma Z = m \left\{ \dot{w} - uq + vp - z_G (p^2 + q^2) + x_G (rp - \dot{q}) + y_G (rq + \dot{p}) \right\}.
\]

(3.10c)
3.2.2 Angular Momentum

Take the moments of all the forces with respect to the origin gives the expression
\[
\sum_{i=1}^{N} (\vec{\Theta}_i + \vec{r}_i \times \vec{F}_i) = \sum_{i=1}^{N} \vec{r}_i \times \frac{d}{dt} (m_i \vec{V}_i). \tag{3.11}
\]

Using the same derivation as that for linear momentum gives
\[
\vec{\Theta} = \sum_{i=1}^{N} (\vec{\Theta}_i + \vec{r}_i \times \vec{F}_i) = \sum_{i=1}^{N} \vec{r}_i \times \left[ m_i \frac{d\vec{V}_i}{dt} + m_i \frac{d}{dt} (\vec{\omega} \times \vec{r}_i) \right], \tag{3.12}
\]
\[
= \left( \sum_{i=1}^{N} m_i \vec{r}_i \right) \times \left( \frac{d\vec{V}_0}{dt} \right) + \sum_{i=1}^{N} (m_i \vec{r}_i) \times \left( \frac{d\vec{\omega}}{dt} \times \vec{r}_i \right) + \sum_{i=1}^{N} (m \vec{r}_i) \times (\vec{\omega} \times (\vec{\omega} \times \vec{r}_i)). \tag{3.13}
\]

The matrix of inertia can be written as
\[
\vec{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \tag{3.14}
\]

Expanding Equation (3.13) and comparing it term by term with \( \vec{I} \frac{d\vec{\omega}}{dt} \), and writing \( \vec{\Theta} = (K, M, N) \), the moment equations can be obtained.

The rolling moment equation is
\[
\sum K = I_{xx} \dot{\theta} + (I_{zz} - I_{yy}) \dot{r} - (\dot{p} + pq)J_{xz} + (r^2 - q^2)J_{yz} + (pr - \dot{q})J_{xy} + m[y_G (\dot{w} - uq + vp) - z_G (\dot{v} - wp + ur)]. \tag{3.15a}
\]

The pitching moment equation is
\[
\sum M = I_{yy} \dot{\phi} + (I_{xx} - I_{zz}) \dot{p} - (\dot{p} + qr)J_{yx} + (p^2 - r^2)J_{yx} + (q - \dot{r})J_{xy} + m[z_G (\dot{u} - vr + wq) - x_G (\dot{v} - uq + vp)]. \tag{3.15b}
\]
The yawing moment equation is
\[ \Sigma N = I_{zz} \ddot{r} + \left( I_{yy} - I_{yx} \right) pq - \left( q + rp \right) I_{yx} + \left( q^2 - p^2 \right) I_{yx} + \left( rq - p \right) I_{yx} + m \left( x_G (\dot{v} - wp + ur) - y_G (\dot{u} - vr + wq) \right). \] (3.15c)

The relationship between the angular velocity and the rate of change of the angles (roll rate \( \dot{\phi} \), pitch rate \( \dot{\theta} \), yaw rate \( \dot{\psi} \)) are given by
\[ \begin{align*}
    p &= \dot{\phi} - \dot{\psi} \sin \theta, \\
    q &= \dot{\theta} \cos \phi + \dot{\psi} \cos \theta \sin \phi, \text{ and} \\
    r &= \dot{\psi} \cos \theta \cos \phi - \dot{\theta} \sin \phi.
\end{align*} \] (3.16)

\( p, q, r \) are the angular velocities measured about the vehicle body axes and \( \dot{\phi}, \dot{\theta}, \dot{\psi} \) are the angular velocities measured about the earth fixed axes system.

### 3.3 External Forces and Moments

The left-hand side of Equations (3.10) and Equations (3.15) represent the sum of all the external forces and moments acting on the vehicle. Fossen [7] classified these forces as:

- **a)** radiation-induced forces and moments which include:
  - i) added mass due to the inertia of the surrounding fluid producing the added mass force and moment terms,
  - ii) hydrodynamic damping, and
  - iii) restoring forces due to Archimedes (weight and buoyancy).

- **b)** environmental forces which include ocean currents, waves and wind, and

- **c)** propulsion forces which include thruster/propeller forces and moment and forces generated by control surfaces. Thruster reaction torque should also be taken into account.
There is another hydrodynamic lift phenomena that was documented by Hoerner [11] page 19-4 and Fidler [6]. Flow separation occurs at the tail end of a streamlined body placed inclined to the fluid flow. The fluid rolls off the body forming a vortex trail, known as vortex shedding. A vortex can be viewed as a suction device and it has the effect of generating a lift force at the point of separation on the body. While this may be applicable to other AUVs, it was thought that such a lifting force could not exist on Autolycus. It was observed that Autolycus has many irregularities on its external surface such as grooves, threaded tie rods and cables which disturb the fluid flow around the vehicle. Autolycus is thus not a streamlined body. Therefore, the vortex shedding phenomena which creates a lift force does not take place on the vehicle body and hence such a lifting force will not be simulated for Autolycus.

3.3.1 Added Mass

Added mass is understood as the pressure-induced forces and moments due to a forced motion of the body that is proportional to the acceleration of the body. For a completely submerged vehicle, the added mass coefficients are assumed constant and thus independent of the wave circular frequency [7]. Further treatment on the subject of added mass can be found in Newman [20]. The added mass force and moment equations can be expressed in the form of

\[ F_j = -\dot{U}_i m_{ji} - \varepsilon_{jkl} U_i \Omega_k m_{li}, \]  
\[ M_j = -\dot{U}_i m_{j+3,i} - \varepsilon_{jkl} U_i \Omega_k m_{l+3,i} - \varepsilon_{jkl} U_j U_k m_{li}. \]  

\( j,k,l \) take on values 1,2,3, and the index \( i \) is used to denote the six velocity components. \( U_i \) are the three components of translational velocity (surge, heave and sway) and \( \Omega_1 = U_4, \Omega_2 = U_5, \Omega_3 = U_6 \) are the corresponding rotational velocity components (roll, yaw and pitch). The alternating tensor \( \varepsilon_{jkl} \) is equal to +1 if the indices are in cyclic order (123, 231, 312), equal to −1 if the indices are acyclic (132, 213, 321), and equal to zero if any pair of the indices are equal.
Expanding Equations (3.17) and (3.18) and using notations consistent with those used by the Society of Naval Architects and Marine Engineers (SNAME), the added mass forces and moments are expressed as shown below.

**Surge added mass force**

\[
X_A = X_u \ddot{u} + X_w (\dot{w} + uq) + X_q \dot{q} + Z_u wq + Z_q q^2 \\
+ X_v \dot{v} + X_p \dot{p} + X_r \dot{r} - Y_v vr - Y_p rp - Y_r r^2 \\
- X_v ur + Y_w wr \\
+ Y_w vq + Z_p pq - (Y_q - Z_i) qr.
\]

\[(3.19a)\]

**Sway added mass force**

\[
Y_A = X_v \ddot{v} + Y_w \dot{w} + Y_q \dot{q} \\
+ Y_v \dot{v} + Y_p \dot{p} + Y_r \dot{r} + X_v vr - Y_w wq + X_r r^2 + (X_p - Z_r) rp - Z_p p^2 \\
- X_v (u_p - w_r) + X_u ur - Z_u wp \\
- Z_q pq + X_q qr.
\]

\[(3.19b)\]

**Heave added mass force**

\[
Z_A = X_u (\ddot{u} - wq) + Z_w \dot{w} + Z_q \dot{q} - X_u uq - X_q q^2 \\
+ Y_u \dot{v} + Z_p \dot{p} + Z_r \dot{r} + Y_v vp + Y_r rp + Y_p p^2 \\
+ X_v up + Y_w wp \\
- X_v vq - (X_p - Y_q) pq - X_i qr.
\]

\[(3.19c)\]

**Roll added mass moment**

\[
K_A = X_p \ddot{p} + Z_p \dot{w} + K_q \dot{q} - X_v wu + X_u uq - Y_w w^2 - (Y_q - Z_i) wq + M_i q^2 \\
+ Y_p \dot{v} + K_p \dot{p} + K_r \dot{r} + Y_v v^2 - (Y_q - Z_i) vr + Z_p wp - M_p r^2 - K_p rp \\
+ X_v uv - (Y_v - Z_u) vw - (Y_r + Z_q) wr - Y_p wp - X_i ur \\
+ (Y_v + Z_q) vq + K_i pq - (M_i - N_i) qr.
\]

\[(3.19d)\]

**Pitch added mass moment**

\[
M_A = X_q (\ddot{u} + wq) + Z_q (\dot{w} - uq) + M_q \dot{q} - X_v (u^2 - w^2) - (Z_u - X_u) wu \\
+ Y_q \dot{v} + K_q \dot{p} + M_p \dot{r} + Y_v v r - Y_p w p - K_i (p^2 - r^2) + (K_p - N_i) r p \\
- Y_q uv + X_i vw - (X_r + Z_p) (u p - w r) + (X_p - Z_i) (w p + u r) \\
- M_i pq + K_q qr.
\]

\[(3.19e)\]
**Yaw added mass moment**

\[
N_A = X_u \ddot{u} + Z_r \dot{w} + M_r \dot{q} + X_\dot{u} u + Y_\dot{w} w - (X_p - Y_q) uq - Z_p wq - K_q q^2 \\
+ Y_r \dot{v} + K_r \dot{p} + N_r \dot{r} - X_r v^2 - X_r vr - (X_p - Y_q) vp + M_r rp + K_q p^2 \\
- (X_u - Y_v) uv - X_\dot{w} v w + (X_q + Y_p) up + Y_r ur + Z_q wp \\
- (X_q + Y_p) vq - (K_p - M_q) pq - K_r qr.
\] (3.19f)

Equations (3.19) are arranged in four lines. The first line consists of longitudinal components and lateral components are on the second line. The third line consists of mixed terms (also known as cross-coupling terms) involving \( u \) or \( w \). The fourth line consists of mixed terms that are second order terms, which are usually neglected [7]. However, the latter are retained for the Autolycus simulation. The \((Z_w - X_\dot{u})wu\) and \((X_u - Y_v)uv\) terms are the Munk moments.

The 6 x 6 added mass matrix is defined as

\[
M_A = \begin{bmatrix}
X_u & X_v & X_\dot{w} & X_\dot{p} & X_\dot{q} & X_\dot{r} \\
Y_u & Y_v & Y_\dot{w} & Y_\dot{p} & Y_\dot{q} & Y_\dot{r} \\
Z_u & Z_v & Z_\dot{w} & Z_\dot{p} & Z_\dot{q} & Z_\dot{r} \\
K_u & K_v & K_\dot{w} & K_\dot{p} & K_\dot{q} & K_\dot{r} \\
M_\dot{u} & M_\dot{v} & M_\dot{w} & M_\dot{p} & M_\dot{q} & M_\dot{r} \\
N_\dot{u} & N_\dot{v} & N_\dot{w} & N_\dot{p} & N_\dot{q} & N_\dot{r}
\end{bmatrix}
\] (3.20)

By inspection of Autolycus and placing the center of origin at the geometric center of the vehicle, the non-zero added mass elements are identified and the above matrix can be simplified. These non-zero elements are then substituted into Equations (3.19) to obtain the added mass forces and moments applicable to Autolycus.
The non-zero added mass matrix for Autolycus is

\[
M_A = -\begin{bmatrix}
X_u & 0 & 0 & 0 & 0 & 0 \\
0 & Y_v & 0 & 0 & 0 & Y_r \\
0 & 0 & Z_v & 0 & Z_q & 0 \\
0 & 0 & 0 & K_p & 0 & 0 \\
0 & 0 & M_w & 0 & M_q & 0 \\
0 & N_v & 0 & 0 & 0 & N_r \\
\end{bmatrix}.
\]  \hspace{1cm} (3.21)

By symmetry, \( N_v = Y_r \) and \( M_w = Z_q \).

Substituting Equation (3.21) into Equations (3.19), we obtain the added mass forces and moments for Autolycus as follows:

**Surge added mass force**

\[
X_A = X_u \dot{u} + Z_v q + Z_q q^2 \\
- Y_v \dot{v} r - Y_r r^2. \hspace{1cm} (3.22a)
\]

**Sway added mass force**

\[
Y_A = Y_v \dot{v} + Y_r \dot{r} \\
+ X_u ur - Z_v wp \\
- Z_q pq. \hspace{1cm} (3.22b)
\]

**Heave added mass force**

\[
Z_A = Z_v \dot{w} + Z_q \dot{q} - X_u uq \\
+ Y_v \dot{v} p + Y_r r p. \hspace{1cm} (3.22c)
\]

**Roll added mass moment**

\[
K_A = K_p \dot{p} \\
- (Y_v - Z_w) \dot{w} - (Y_r + Z_q) \dot{r} \\
+ (Y_v + Z_w) q - (M_i - N_i) q r. \hspace{1cm} (3.22d)
\]

**Pitch added mass moment**

\[
M_A = Z_q (\dot{q} - uq) + M_i \dot{q} - (Z_w - X_u) wu \\
- Y_r \dot{v} p + (K_p - N_p) r p. \hspace{1cm} (3.22e)
\]
Yaw added mass moment

\[ N_A = Y_v \dot{\psi} + N_v \dot{\psi} \]
\[ - (X_a - Y_a) uv + Y_r ur + Z_w wp \]
\[ - (K_p - M_q) pq. \]  

(3.22f)

The added mass coefficients may be estimated using various techniques such as slender body theory or strip theory. The real geometry of Autolycus is complex because of the external irregularities such as the exposed cables, propellers and the grooves in which the connectors are mounted. For modeling purpose, Autolycus is assumed to have simple shapes such as hemispheres, cylinders and plates. Strip theory is used to compute the added mass coefficients since the technique is suitable for the simple shapes that are assumed for Autolycus. For a more detailed explanation of estimating added mass coefficients, the reader may refer to Newman [20].

The computation of Autolycus' added mass coefficients will be shown in Chapter Four.

3.3.2 Hydrodynamic Damping

The damping of an underwater vehicle moving in 6 DOF at high speed is non-linear and coupled. An approximation for simplification is to assume that the vehicle is performing a non-coupled motion. Quadratic damping terms dominate in the case of an underwater vehicle that is operating in an unbounded fluid. Two effects that are most prominent are:

a) the quadratic or non-linear skin friction due to turbulent boundary layer theory, and

b) the viscous damping due to vortex shedding.

The Reynolds number for Autolycus, whose length and speed are 1.4m and 0.3m/s respectively, is about $4.25 \times 10^5$. This places the operating point of the vehicle in the transition regime. However, the disturbed flow over the groove and cables at the nose of the vehicle is expected to cause the operating point to shift into the turbulent regime.
The hydrodynamic damping force can be modeled as:

\[ D(u) = -\frac{1}{2} \rho C_o A_o |u| u \]  

(3.23)

where \( C_o \) and \( A_o \) are the applicable drag or frictional coefficients and the reference area respectively.

For convenience, the damping forces in the three vehicle axes and the damping moments about the axes are written as \( X_D, Y_D, Z_D, K_D, M_D, N_D \), where

\[
\begin{align*}
X_D &= X_{u|u|} |u| |u|, \\
Y_D &= Y_{v|v|} |v| |v|, \\
Z_D &= Z_{w|w|} |w| |w|, \\
K_D &= K_{p|p|} |p| |p|, \\
M_D &= M_{q|q|} q|q| + M_{w|w|} |w| |w|, \text{ and} \\
N_D &= N_{r|r|} r|r| + N_{v|v|} |v| |v|. 
\end{align*}
\]

(3.24)

The computation for the hydrodynamic damping coefficients (forces and moments) will be shown in Chapter Four. The damping coefficients are related to the resistive forces and they have negative values.

3.3.3 Restoring Forces

The gravitational and buoyant forces on the vehicle cause the restoring forces. The gravitational force acts through the center of gravity \((x_G, y_G, z_G)\) of the vehicle while the buoyant force acts through the center of buoyancy \((x_B, y_B, z_B)\). The buoyant force also creates a moment since it is displaced from the center of gravity by the distance \(x_B, y_B, z_B\).

In an underwater vehicle design, it is desirable to locate the center of gravity in-line with the center of buoyancy. This way, the static heel (roll angle) and trim (pitch angle) are both level (zero angles). To set the static heel to zero, \(y_G = y_B = 0\) must be satisfied and to set trim to zero, \(x_G = x_B\) must be satisfied.
In a submerged vehicle, hydrostatic stability, both transverse (heel) and longitudinal (pitch), requires that the center of gravity CG should be below the center of buoyancy CB. The size of the distance CG-CB is commonly known as BG. BG determines the restoring moment when the vehicle experiences an angle of heel or an angle of pitch [2].

In Figure 3-3, the $x_B$ and $z_B$ are shown in a negative sense and the figure is shown for a stable configuration.

Figure 3-3: Gravity and buoyancy forces acting on the vehicle.

There is no necessity to show the derivation of the restoring forces equations in this report. These equations can be readily found in many of the references such as [2], [8], [9] and [12].

---

1 B is used to denote the buoyancy force. But it is also commonly used to denote the center of buoyancy. After this section, B will be used to denote force while the position of buoyancy will be denoted by $x_B$, $y_B$ and $z_B$. 
The restoring forces and moments, denoted by subscript $R$, are shown below.

$$
X_R = -(W - B) \sin \theta, \\
Y_R = (W - B) \cos \theta \sin \phi, \\
Z_R = (W - B) \cos \theta \cos \phi, \\
K_R = (y_G W - y_B B) \cos \theta \cos \phi - (z_G W - z_B B) \cos \theta \sin \phi, \\
M_R = -(z_G W - z_B B) \sin \theta - (x_G W - x_B B) \cos \theta \cos \phi, \text{ and} \\
N_R = (x_G W - x_B B) \cos \theta \sin \phi + (y_G W - y_B B) \sin \theta. 
$$

\begin{equation}
3.25
\end{equation}

### 3.3.4 Environmental Forces

As described in the earlier section, environmental forces are ocean currents, surface waves and wind. For the Autolycus simulation, it is assumed that the vehicle is submerged, which allows surface wave and wind to be neglected. It is further assumed that there are no underwater currents acting on the vehicle. A reason for the latter assumption is that creating an underwater current in a laboratory experiment is not practical and it is usually not performed. Even if a simulation with underwater current is performed, there will be no experimental data to compare it with. Hence, it is reasonable to neglect current. A closed-loop control system will often be able to correct the effects of currents by applying the appropriate thrust to compensate for the vehicle drift.

### 3.3.5 Propulsion Forces and Moments

$T_{port}$, $T_{stbd}$, $T_{fore}$ and $T_{aft}$ are the forces of the respective thrusters. The thruster force can either be obtained from the manufacturer or through experiments. The moment is simply the thrust force multiplied by the distance from the line of thrust to the vehicle reference origin. There is also a need to account for the thruster torque reaction. For convenience, the propulsion forces in the three vehicle axes and the propulsion moments about the axes are written as $X_P$, $Y_P$, $Z_P$, $K_P$, $M_P$, $N_P$, where
In Autolycus’ case, the port and starboard thrusters’ rotations are arranged so as to cancel out the reaction torque. Likewise, the reaction torques of the forward and aft thrusters cancel out. If a thruster is operated singly, there should rightfully be a reaction torque on the vehicle. However, knowing that the thrusters are of relatively low power, the reaction torque is expected to be insignificant with respect to the inertia of the vehicle. The reaction torque is thus assumed to be small and it is not simulated.

Autolycus was previously tested in a water tank where the total axial resistance was measured by a load cell. A separate test was done to measure the maximum speed achieved by Autolycus [3]. Correlating this maximum achievable speed to the resistance data will give the maximum thrust force each of the thruster, assuming that all the thrusters are uniform. The resistance and speed data will be presented in Chapter Four.

Autolycus does not have control surfaces. The struts that hold the port and starboard thrusters are not designed as lifting foils and they do not generate lifting forces. Similarly, the hull is not expected to experience hydrodynamic lift force as explained in Section 3.3.
In Figure 3-4, the thrusters' forces shown are positive and acting on the vehicle. The moment $M$ is also shown in the positive direction.

### 3.4 Choosing the Body Fixed Origin

It is convenient to choose the body axes such that they coincide with the principal axes of inertia or the longitudinal, lateral and normal symmetry axes of the vehicle. That is, the body-fixed origin is placed at the geometry center of the vehicle. Strictly speaking, Autolycus has only two planes of geometric symmetry — top/bottom and port/starboard. Only the circular cylinder hull satisfies the bow/stern symmetry.

By fixing the body axes at these planes of symmetry, the inertia tensor is reduced to only the diagonal elements. Equations (3.15) are thus simplified as
Furthermore, some of the added mass and hydrodynamic coefficients can be made zero due to symmetry [22], [23]. This will be elaborated in Chapter Four.

3.5 Transformation to Earth Fixed Reference

All the motion equations and forces derived thus far are with respect to the vehicle body-axes. It is desirable to know the translational position for the vehicle with respect to the inertial reference frame. Typically for an underwater vehicle, earth is chosen as the inertial reference frame.

The vehicle velocity vector \([u, v, w]^{T}\) is premultiplied by the rotation matrix \(T\), where

\[
T = \begin{bmatrix}
    \cos \psi \cos \theta & -\sin \psi \cos \phi + \cos \psi \sin \theta \sin \phi & \sin \psi \sin \phi + \cos \psi \cos \phi \sin \theta \\
    \sin \psi \cos \theta & \cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi & -\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi \\
    -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi 
\end{bmatrix}.
\]

This gives the acceleration in the three axes with respect to the earth frame reference. \(T_{1,1}\) and the like refer to the elements in the matrix \(T\).

\[
\dot{x} = T_{1,1}u + T_{1,2}v + T_{1,3}w, \\
\dot{y} = T_{2,1}u + T_{2,2}v + T_{2,3}w, \text{ and} \\
\dot{z} = T_{3,1}u + T_{3,2}v + T_{3,3}w. 
\]

The resulting velocity vector is then integrated to produce the translational position of the vehicle, expressed as components in the inertial frame [18].

The calculation of the transformation matrix and the integration to obtain the translational position of the vehicle are performed using Matlab as part of the simulation.
3.6 Assumptions Applied to Autolycus Simulation

Thus far, the dynamic equations have been derived and all the applicable external forces and moments have been discussed. Assumptions are now applied to simplify the equations used for simulation. These assumptions are:

a) the vehicle is immersed in an unbounded body of fluid;
b) the vehicle is away from the free surface, walls and the bottom;
c) there are no underwater currents;
d) the vehicle is rigid;
e) the mass and mass distribution of the vehicle do not change during its operation;
f) the vehicle is initially at zero trim and heel angles;
g) the vehicle undergoes small angles and slow changes due to small disturbances;
h) the thruster force is a step function;
i) the thruster force is equal in both forward and reverse direction.

3.7 Autolycus Motions Simulation Model

The modeling of Autolycus is completed. Now the assumptions listed above are applied to simplify the equations of motion.

From Equation (3.10), the simplified force equations are:

Axial force
\[ \sum X_{\text{ext}} = m \left[ i - vr + wq - x_G \left( q^2 + r^2 \right) + z_G \left( pr + \dot{q} \right) \right] \] \quad (3.30a)

Lateral force
\[ \sum Y_{\text{ext}} = m \left[ \dot{v} - wp + ur + z_G \left( qr - p \right) + x_G \left( qp + r \right) \right] \] \quad (3.30b)

Normal force
\[ \sum Z_{\text{ext}} = m \left[ \dot{v} uq + vp - z_G \left( p^2 + q^2 \right) + x_G \left( rp - \dot{q} \right) \right] \] \quad (3.30c)
From Equation (3.15), the simplified moment equations are:

**Rolling moment**
\[\sum K_{ext} = l_{zz} \dot{p} + (l_{xx} - l_{yy}) \dot{q} r - m [z_G (\dot{v} - w p + u r)].\] (3.31a)

**Pitching moment**
\[\sum M_{ext} = l_{xy} \dot{q} + (l_{xx} - l_{yy}) \dot{r} p + m [z_G (\dot{u} - v r + w q) - x_G (\dot{w} - u q + v p)].\] (3.31b)

**Yawing moment**
\[\sum N_{ext} = l_{xz} \dot{r} + (l_{yy} - l_{xx}) \dot{p} q + m [x_G (\dot{v} - w p + u r)].\] (3.31c)

The external surge force \(\Sigma X_{ext}\) is sum of the added mass forces, viscous damping forces, restoring forces and propulsion forces. This is written as
\[\sum X_{ext} = X_A + X_D + X_R + X_P.\] (3.32)

The other forces and moments acting on the vehicle have similar expressions as the surge force.

To obtain the vehicle’s motions in the six degrees of freedom, the external forces and moments are incorporated into the force and moment equations, (3.28) and (3.29). The resultant body motions can then be transformed to give the translational positions of the vehicle with respect to the earth fixed reference frame as discussed in Section 3.5.

Hence, the complete six D-O-F equations arranged in the vertical and horizontal planes used to model the motions of Autolycus are:

**Surge equation**
\[m [\ddot{u} - v r + w q - x_G (q^2 + r^2) + z_G (\dot{p} r + \dot{q})] = X_{\Sigma u} + Z_w w q + Z_q q^2 - Y_v v r - Y_r r^2 + X_{\Sigma u} + X_{\Sigma d} |d| - (W - B) \sin \theta + T_{port} + T_{subd}.\] (3.33)
Vertical Plane

Heave equation

\[
m\left[\ddot{w} - uq + vp - z_G (\dot{p}^2 + q^2) + x_G (\dot{\phi} - \dot{q})\right] = \]
\[
Z\dddot{w} + Z\ddot{q} - X_u uq + Y_v vp + Y_r rp + Z_{\dot{w}|w|} w|w| + (W - B) \cos \theta \cos \phi + T_{\text{fore}} + T_{\text{aft}}. \] (3.34)

Pitch equation

\[
I_{yy}\ddot{\phi} + (I_{x\phi} - I_{zh}) rp + m[z_G (\dot{u} - vr + wz) - x_G (\dot{w} - uq + vp)] = \]
\[
Zq (\ddot{w} - uq) + Mq \ddot{q} - (Zw - X_u) wu - Y_v vp + (K_p - N_\theta) rp + M_{\dot{q}|q|} q|q| \]
\[
+ M_{\dot{w}|w|} w|w| - (z_G W - z_B B) \sin \theta - x_G (W - B) \cos \theta \cos \phi + (T_{\text{port}} + T_{\text{stbd}}) z_p \]
\[
+ T_{\text{aft}} xp_{\text{aft}} - T_{\text{fore}} xp_{\text{fore}}. \] (3.35)

Horizontal Plane

Sway equation

\[
m\left[\ddot{v} - wp + ur + z_G (qr - \dot{r}) + x_G (qp + \dot{r})\right] = \]
\[
Y_v \dddot{v} + Y_r \dddot{r} + X_u ur - Z_w wp - Z_q pq + Y_{\dot{v}|v|} v|v| + (W - B) \cos \theta \sin \phi + 0. \] (3.36)

Roll equation

\[
I_{xx}\ddot{r} + (I_{x\phi} - I_{zh}) qr - m[z_G (\dot{v} - wp + ur)] = \]
\[
K_p \ddot{p} - (Y_r - Z_w) vw - (Y_r + Z_q) wr + (Y_r + Z_q) vw - (M_q - N_r) qr + K_{\dot{p}|p|} p|p| \]
\[
- (z_G W - z_B B) \cos \theta \sin \phi + 0. \] (3.37)

Yaw equation

\[
I_{zz}\ddot{r} + (I_{y\phi} - I_{zh}) pq + m[z_G (\dot{v} - wp + ur)] = \]
\[
Y_r \dddot{v} + N_r \dddot{r} - (X_q - Y_r) uv + Y_r ur + Z_w wp - (K_p - M_q) pq + N_{\dot{r}|r|} r|r| \]
\[
+ N_{\dot{v}|v|} v|v| + x_G (W - B) \cos \theta \sin \phi + (T_{\text{port}} - T_{\text{stbd}}) yp. \] (3.38)
3.8 Summary

This chapter has worked through the vehicle dynamic equations with respect to the axes fixed to the vehicle. The relevant external forces and moments acting on the vehicle are introduced. Assumptions are then applied to simplify the equations used for simulating the resultant body motions of the vehicle. From the resultant velocity of the vehicle, the translational position of the vehicle with reference to the earth fixed frame can be obtained.

The next chapter will compute all the relevant parameters for the Autolycus motion simulation. These are the vehicle’s center of gravity position, mass moments of inertia, propulsion thruster force, restoring forces and the hydrodynamic coefficients.
Chapter 4

Estimation of Autolycus' Parameters

4.1 Introduction

This chapter will show the process of estimating Autolycus' parameters that are used for simulation in the Matlab program. These include:

(a) the vertical center of gravity with reference to the keel (base), KG,
(b) the longitudinal center of gravity with reference to the bow (nose) of the vehicle, LCG,
(c) the vertical center of buoyancy with reference to the keel, KB,
(d) the longitudinal center of buoyancy with reference to the nose, LCB,
(e) the mass moment of inertia of the vehicle in water about the three principal axes with reference to the vehicle origin which is set at the geometric center of the vehicle,
(f) the hydrodynamic derivatives which include the added mass terms and the hydrodynamic damping terms, and
(g) the thruster force.

The ability to estimate these parameters well is crucial to the success of simulating Autolycus' motions. The spreadsheet in Table 4-1 is adaptable to progressive changes to the vehicle design while the Matlab program is set up to compute the hydrodynamic coefficients from several key dimensions of the vehicle. These dimensions can be changed as part of the process of redesigning the vehicle.
The positions of the vehicle's LCG and KG are necessary to determine the distance of the center of gravity with respect to the vehicle. The method of estimating the LCB and KG can be found in Burcher [2] and Gillmer [9].

Typically, an AUV's hull is a streamlined body with simple or common shapes as appendages. The advantage in designing an AUV with simple forms is that the characteristics of these forms have been studied and documented such as in [6], [10], [11] and [20]. The AUV designer can also gain from studies relating to submarines since these two types of vehicles closely resemble each other in many aspects. However, unlike a submarine, the positions of the center of gravity and buoyancy in an AUV remain constant since it remains submerged throughout. Also, there are no consumables in an AUV.

The added mass coefficients are based on a circular cylinder and a flat plate, which are the predominant shapes of Autolycus. The method can be found in [20] and [23]. The hydrodynamic coefficients relating to the viscous and frictional drag over a body can be found in any standard engineering references such as [1] and [4]. These references are useful for working out the hydrodynamic damping coefficients.

4.2 Estimating KG and LCG

Just as in a typical ship design, it is essential to maintain a record of all the components of the vehicle, documenting each individual component's mass and its position of center of gravity with respect to an origin in the vehicle. It is also important to note subsequent changes.

For Autolycus, a complete listing of its components was documented. The masses and the positions of the center of masses with respect to the vehicle origin were tabulated using a spreadsheet as shown in Table 4-1.
The vertical position of the vehicle center of mass with respect to the base (keel), KG, is given as:

\[ KG = \frac{\Sigma (m_i \times v_{cg_i})}{\Sigma m_i} \] (4.1)

The longitudinal position of the vehicle center of mass with respect to the bow, LCG, is given as:

\[ LCG = \frac{\Sigma (m_i \times l_{cg_i})}{\Sigma m_i} \] (4.2)

\( m_i \) is the mass of the individual component with its vertical and longitudinal center of mass taken with respect to the keel and nose denoted by \( v_{cg_i} \) and \( l_{cg_i} \), respectively.

For the present Autolycus configuration, KG is estimated to be 55.2 mm and LCG is estimated to be 738.2 mm. The overall length of Autolycus is 1487 mm. The longitudinal origin measured from the bow, \( x_o \), is 743.5 mm. Hence, \( x_G = 5.3 \) mm (+ve being fwd of the origin). The overall diameter of Autolycus is 127 mm, which makes \( z_o = 63.5 \) mm and \( z_G = 8.3 \) mm.

### 4.3 Estimating KB and LCB

The hull of Autolycus is a circular cylinder. Thus, the position of its center of buoyancy is one half of the hull diameter. The position of the center of buoyancy referenced with respect to the base of the vehicle, KB, is 63.5 mm. The position of the center of buoyancy referenced with respect to the origin, \( z_b \), is 0. This gives us the distance between the center of buoyancy and the center of gravity, BG, which is equal to 8.3 mm (BG = \( z_b - z_G \)). BG is an indicator of the vehicle's stability in the vertical plane.

To achieve static intact stability of the vehicle in water, LCB and LCG must coincide on the horizontal and transverse planes so that the trim and heel angles are zero. It is often easier to achieve a balance by adding small weights to the hull (while the vehicle is fully immersed in water) to shift the LCG to coincide with the LCB (\( x_b = x_G \)). An even
heel condition is achieved when the vehicle weight is equally distributed to the port and starboard sides. Then the lateral locations of the center of gravity and the center of buoyancy coincide and they are both at the lateral center of origin. In mathematical form, this condition is expressed as \( y_G = y_B = 0 \). It should be noted that the weight of the water entrained in the vehicle’s voids must also be accounted for as part of the total vehicle weight.

### 4.4 Estimating the Mass Moments of Inertia

As defined in Section 3.4, the vehicle origin is set at the geometric center of the vehicle. The mass moments of inertia of the vehicle are estimated with reference to the center of origin. Standard expressions of the mass moments of inertia for simple geometric shapes can be found in typical engineering handbooks such as [4]. This section of the report will reproduce only the expressions for the shapes relevant to Autolycus.

The mass moments of inertia about the three axes as shown in Figure 4-1 are given as

\[
I_x = \int r_x^2 \, dm = \int (y^2 + z^2) \, dm ,
\]

\[
I_y = \int r_y^2 \, dm = \int (x^2 + z^2) \, dm ,
\]

\[
I_z = \int r_z^2 \, dm = \int (x^2 + y^2) \, dm .
\]

![Figure 4-1: Mass moments of inertia.](image-url)
The hull, the port and starboard thruster housings of Autolycus are modeled as hollow cylinders as shown in Figure 4-2. The masses and mass moments of inertia are given as

\[ m_s = \rho \pi \left( r_o^2 - r_i^2 \right) h, \]
\[ I_x = \frac{1}{2} m_s \left( r_o^2 + r_i^2 \right), \text{ and} \]
\[ I_y = I_z = \frac{1}{4} m_s \left( r_o^2 + r_i^2 + \frac{h^2}{3} \right). \]  \hspace{1cm} (4.4)

The water bodies within the thruster housings and the batteries are modeled as solid cylinders, whose masses and mass moments of inertia are given as

\[ m_s = \rho \pi r_o^2 h, \]
\[ I_x = \frac{1}{2} m_s r_o^2, \text{ and} \]
\[ I_y = I_z = \frac{1}{12} m_s \left( 3r_o^2 + h^2 \right). \]  \hspace{1cm} (4.5)

The nose and tail caps are modeled as hollow hemispheres, whose masses and mass moments of inertia are given as

\[ m_s = \frac{2}{3} \rho \pi \left( r_o^3 - r_i^3 \right), \]
\[ I_y = I_z = \frac{83}{320} m_s \left( r_o^2 - r_i^2 \right), \text{ and} \]
\[ I_x = \frac{2}{5} m_s \left( r_o^2 - r_i^2 \right). \]  \hspace{1cm} (4.6)
The water bodies within the nose and tail caps are modeled as solid hemispheres as shown in Figure 4-3. The masses and mass moments of inertia are given as

\[ m_s = \frac{2}{3} \rho \pi r^3, \]

\[ I_{xy} = I_{yz} = \frac{83}{320} m_s r^2, \text{ and} \]

\[ I_{zx} = \frac{2}{5} m_s r^2. \] (4.7)

The moments of inertia for each individual component that makes up Autolycus can thus be calculated referenced with respect to the component’s axes. Each of these elemental moments of inertia is then referenced with respect to the vehicle’s axes. The equations to relate the moments of inertial of the individual components to the vehicle’s axes are

\[ I_{xx} = I_{xx} + m_s \left( y_s^2 + z_s^2 \right), \]

\[ I_{yy} = I_{yy} + m_s \left( z_s^2 + x_s^2 \right), \text{ and} \]

\[ I_{zz} = I_{zz} + m_s \left( x_s^2 + y_s^2 \right). \] (4.8)

\( \bar{x}, \bar{y} \) and \( \bar{z} \) are axes parallel to the vehicle axes, \( x, y \) and \( z \). \( I_{xx}, I_{yy} \) and \( I_{zz} \) are the mass moments of inertia referenced with respect to vehicle origin. \( I_{\bar{x}}, I_{\bar{y}} \) and \( I_{\bar{z}} \) are the mass moments of inertia referenced with respect to the component axes. \( m_s \) is the component mass and \( x_s, y_s \) and \( z_s \) are distances from the components’ centers of mass to the vehicle origin along the respective axes.
The $I_x, I_y$ and $I_z$ of irregularly shaped components are difficult to estimate. For components with relatively small masses, $I_x, I_y$ and $I_z$ may be neglected but the $\int r^2 dm$ term must be accounted for. This is an acceptable estimation since the former terms are small with respect to the latter terms. Examples of these situations are the computer card and the sensor card.

The mass moments of inertia of Autolycus about its origin are computed as shown in Table 4-1. It is found that $I_{xx} = 0.05 \text{ kg/m}^2$, $I_{yy} = 3.98 \text{ kg/m}^2$ and $I_{zz} = 3.99 \text{ kg/m}^2$. The tabulation is checked against the weight and the LCG of Autolycus when the vehicle is in the dry condition. The LCG (dry vehicle) is found by balancing Autolycus on a sharp edge while its weight is measured with an electronic scale. The accuracy of the estimated values for the LCG and the weight (dry vehicle) will give a higher confidence that the values obtained for a submerged vehicle is reasonably accurate. In dry condition, the measured weight and LCG of Autolycus are 15.6 kg and 773 mm respectively while the estimated weight and LCG using Table 4-1 are 16.0 kg and 772.7 mm respectively. It is thus assumed that the values estimated for a submerged vehicle will be reasonably accurate.
Table 4-1: Estimates of KG, LCG, Ixx, Iyy, Izz.

<table>
<thead>
<tr>
<th>Item</th>
<th>Components of Autolycus</th>
<th>mass(kg)</th>
<th>vcg (mm)</th>
<th>leg (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nose Hemisphere Shell</td>
<td>0.08</td>
<td>63.5</td>
<td>32.5</td>
</tr>
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<td>2</td>
<td>Nose Shell Water Enclosed, excl. ballast</td>
<td>0.36</td>
<td>63.5</td>
<td>40.9</td>
</tr>
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<td>3</td>
<td>Speed Sensor</td>
<td>0.1</td>
<td>63.5</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>Fwd Ballast Weight</td>
<td>0.11</td>
<td>0</td>
<td>63.5</td>
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<tr>
<td>5a</td>
<td>Terminating Block +</td>
<td>0.85</td>
<td>63.5</td>
<td>84.5</td>
</tr>
<tr>
<td>5b</td>
<td>Housing Lid +</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Block for housing altimeter</td>
<td>1.5</td>
<td>63.5</td>
<td>164</td>
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<tr>
<td>7</td>
<td>Altimeter</td>
<td>0.59</td>
<td>62</td>
<td>143.5</td>
</tr>
<tr>
<td>8</td>
<td>Housing for depth sensor</td>
<td>0.18</td>
<td>63.5</td>
<td>285.5</td>
</tr>
<tr>
<td>9</td>
<td>Water Enclosed in Housing</td>
<td>1.24</td>
<td>63.5</td>
<td>285.5</td>
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<td>Fwd Vertical Thruster Motor +</td>
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<td></td>
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<td>12c</td>
<td>Fwd Pressure Housing End Cap +</td>
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<td></td>
<td></td>
</tr>
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<td>Horizontal Thruster Block +</td>
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<tr>
<td>20e</td>
<td>Port Horizontal Thruster Strut +</td>
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<td>20f</td>
<td>Stbd. Horizontal Thruster Strut +</td>
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<td>20g</td>
<td>Port Thruster Duct +</td>
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</table>

**SUM** 18.04

Total vehicle length, \( L = \frac{1487}{2} = 743.5 \) mm

VCG of vehicle = (sum m*vcg)/total m = 55.2 mm

LCG of vehicle = (sum m*lcg)/total m = 738.2 mm

for even trim, LCB = LCG

BG = D/2 - VCG = 8.3 mm \( zB=0 \)

xG = Xo - LCG = 5.3 mm (-ve aft of origin)

Origin \( Xo = L / 2 = 743.5 \) mm

**Note** +' these items have been assembled and are not readily removable.

But this is not a problem for the purpose of the above calculation
Table 4-1: Estimates of KG, LCG, Ixx, Iyy, Izz, continued.

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<th>Item</th>
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<th>m*lcg (kg mm)</th>
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996.232  13316.279

Notes:

*Italic Numbers are Computed*

Worksheet to estimate the LCG, VCG and moments of inertia for Autolycus

Weights, position, masses taken respect to the "keel" and bow of the vehicle

Find LCG and then shift counter weights such that LCG and LCB coincides

Density of fresh water assumed to be 1000 kg/m^3

68
Table 4-1: Estimates of KG, LCG, Ixx, Iyy, Izz, continued.

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Table 4-1: Estimates of KG, LCG, Ixx, Iyy, Izz, continued.

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49334.06714 3978063.066 3986861.446 kg*mm^2
0.049334067 3.978063066 3.986861446 kg*m^2

mass moment of inertia about the vehicle’s reference axes
4.5 Estimating the Added Mass Coefficients

This section will work out the added mass elements of Equation (3.20). Strip theory is used where necessary. References [20] and [23] are used for this section.

For simplicity, the hemispherical nose and tail are neglected. The length overall of Autolycus is about 1487 mm. So as not to over estimate the hydrodynamic coefficient of the hull, the mean length of the hull with and without the hemispherical ends will be used. This length will be denoted as \( l_h \) where \( l_h = 1423.5 \) mm.

The added mass of Autolycus is the summation of the added mass of all the components of the vehicle. Autolycus can be modeled as consisting of three circular cylinders and two flat plates. In the following sections, the added mass of a cylinder and a flat plate are described.

4.5.1 Added Mass of a Cylinder

Figure 4.4 shows a cylinder with the appropriate dimensions and axes. The non-zero added masses of a cylinder in the respective axes are

\[
\begin{align*}
m_{11} &= 5 \text{ to } 10\% \text{ of mass (slender body) } [7], \\
m_{22} &= m_{33} = \int_{-L/2}^{L/2} \rho \pi r^2 \, dx = \rho \pi r^2 L = \rho \bar{V}, \\
m_{55} &= m_{66} = \int_{-L/2}^{L/2} x^3 \rho \pi r^2 \, dx = \frac{L^3}{12} \rho \pi r^2 = \frac{L^2}{12} \rho \bar{V}, \text{ and } \\
m_{44} &= 0.
\end{align*}
\]

All other elements are zero. \( \bar{V} \) is the volume of the cylinder and \( \rho \) is the density of the fluid.
4.5.2 Added Mass of a Plate

The added mass of a thin plate in the transverse direction as shown in Figure 4.5 is

\[ m_{33} = \frac{1}{4} \pi \rho \text{ chord}^2 \text{ span}, \]  

(4.10)

where the span and chord are labeled in accordance to lifting foil nomenclature.

4.5.3 Off Diagonal Added Mass Terms

For a mass with an offset from the origin as shown in Figure 4.6, the off diagonal added mass moment terms can be approximated as

\[
\begin{align*}
    m_{35} &= m_{33} L, \\
    m_{26} &= -m_{35}, \text{ and} \\
    m_{55} &= m_{66} = -L^2 m_{22}. 
\end{align*}
\]  

(4.11)
4.5.4 Autolycus Added Mass Coefficients

It is now appropriate to work out each of the added mass coefficients applicable to Autolycus. The reader should see Figure 3 for the relevant dimensions used in this section.

The surge added mass coefficient due to surge acceleration is

\[ X_u \equiv m_{11} = -0.1m, \quad (4.12) \]

where \( m \) is mass of the vehicle in water [7]. For streamlined bodies, \( X_u \approx 5 \) to 10\% of \( m \).

The sway added mass coefficient due to sway acceleration is

\[ Y_v \equiv m_{22} = -\left( \rho \, \overline{V}_h + 2\rho \, \overline{V}_{ph} \right), \quad (4.13) \]

\( \overline{V}_h \) and \( \overline{V}_{ph} \) are the volume of the vehicle main hull and the volume of the water enclosed within the propeller housing respectively.

The heave added mass coefficient due to heave acceleration is

\[ Z_\omega \equiv m_{33} = -\left[ \rho \, \overline{V}_h + 2(\rho \, \overline{V}_{ph}) + 2\left( \frac{1}{4} \pi \rho \, chord^2 \, span \right) \right]. \quad (4.14) \]
The roll added mass coefficient due to roll acceleration is

\[ K_p \equiv m_{44} = - \left( 0 + 2 \left( \rho \bar{V}_{ph} \right) y_p^2 + K_{struts} \right) \tag{4.15} \]

where \( y_p \) is the distance from the center of the propeller housing to the origin.

The added mass of a circle with fin as shown in Figure 4-7 is

\[ K_{struts} = \text{chord} \ \rho \ a^4 \left( \pi^{-1} \cos \epsilon \alpha \left( 2 \alpha^2 - \alpha \sin 4 \alpha + \frac{1}{2} \sin^2 2 \alpha \right) - \frac{\pi}{2} \right), \tag{4.16} \]

where \( \sin \alpha = \frac{2ab}{a^2 + b^2} \).

\( K_{struts} \approx 0.000653 \text{ kgm}^2 \) for Autolycus in its present configuration.

\[ \text{Figure 4-7: Added mass of a circle with fins [20].} \]

The pitch added mass coefficient due to pitch acceleration is

\[ M_q = - \left[ \left( \frac{lh^2}{12} \rho \bar{V}_h \right) + 2 \left( \rho \bar{V}_{ph} \right) r_1^2 + 2 \left( \frac{1}{4} \pi \rho \text{ chord}^2 \text{ span} \right) r_1^2 \right]. \tag{4.17} \]

The yaw added mass coefficient due to yaw acceleration is

\[ N_r = - \left[ \left( \frac{lh^2}{12} \rho \bar{V}_h \right) + 2 \left( \rho \bar{V}_{ph} \right) r_1^2 + y_p^2 \right]. \tag{4.18} \]

Due to the port and starboard symmetry, the surge added mass due to sway acceleration and the sway added mass due to surge acceleration are

\[ X_s = Y_u \equiv m_{12} = 0. \tag{4.19} \]
Due to the top and bottom symmetry, the surge added mass due to heave acceleration and the heave added mass due to surge acceleration are

\[ X_\dot{u} = Z_\dot{u} \equiv m_{13} = 0. \quad (4.20) \]

Since the struts are not control surfaces, the surge added mass due to roll acceleration and the roll added mass due to surge acceleration are

\[ X_\dot{\phi} = K_\phi \equiv m_{14} = 0. \quad (4.21) \]

Due to the top and bottom symmetry, the surge added mass due to pitch acceleration and the pitch added mass due to surge acceleration are

\[ X_\dot{\theta} = M_\phi \equiv m_{15} = 0. \quad (4.22) \]

Due to the port and starboard symmetry, the surge added mass due to yaw acceleration and the yaw added mass due to surge acceleration are

\[ X_\dot{\psi} = N_\phi \equiv m_{16} = 0. \quad (4.23) \]

Since there is no hydrodynamic lifting force on the hull, the sway added mass due to heave acceleration and the heave added mass due to sway acceleration are

\[ Y_\dot{u} = Z_\dot{v} \equiv m_{23} = 0. \quad (4.24) \]

Due to the top and bottom symmetry, the sway added mass due to roll acceleration and the roll added mass due to sway acceleration are

\[ Y_\dot{\phi} = K_\psi \equiv m_{24} = 0. \quad (4.25) \]

Due to the body symmetry, the sway added mass due to pitch acceleration and the pitch added mass due to sway acceleration are

\[ Y_\dot{\theta} = M_\psi \equiv m_{25} = 0. \quad (4.26) \]
and the sway added mass due to yaw acceleration and the yaw added mass due to sway acceleration are

\[ Y_s = N_s = m_{2b} \approx 2 (\rho \bar{V}_{ph}) \left( \frac{r_i^2}{y_p^2} \right). \]  

(4.27)

Note that \( Y_s \) and \( N_s \) are both positive because the hydrodynamic effect of the thruster housings at the rear dominate.

Due to the port and starboard symmetry, the heave added mass due to roll acceleration and the roll added mass due to heave acceleration are

\[ Z_p = K_w = m_{34} = 0, \]  

(4.28)

and the heave added mass due to pitch acceleration and the pitch added mass due to heave acceleration are

\[ Z_q = M_w = m_{35} = -2 \left( \rho \bar{V}_{ph} + \left( \frac{1}{4} \pi \rho \text{chord}^2 \text{span} \right) \right) r_i. \]  

(4.29)

Due to the port and starboard symmetry, the heave added mass due to yaw acceleration and the yaw added mass due to heave acceleration are

\[ Z_r = N_w = m_{36} = 0. \]  

(4.30)

Since there are no control surfaces, the roll added mass due to pitch acceleration and the pitch added mass due to roll acceleration are

\[ K_q = M_p = m_{45} = 0. \]  

(4.31)

Due to the top and bottom symmetry, the roll added mass due to yaw acceleration and the yaw added mass due to roll acceleration are

\[ K_s = N_p = m_{46} = 0. \]  

(4.32)

There is no hydrodynamic lift force on the thruster housing. The pitch added mass due to yaw acceleration and the yaw added mass due to pitch acceleration are
\[ M_{\dot{x}} = N_{\dot{q}} = m_{s6} = 0. \] (4.33)

Substituting the dimensions of Autolycus into Equations (4.12) to (4.33), the added mass coefficients can be obtained as listed below.

\[
\begin{align*}
X_{\dot{u}} &= -1.8040 \text{ kg}, \\
Y_{\dot{v}} &= -18.1047 \text{ kg}, \\
Y_{\dot{r}} &= 0.0457 \text{ kgm}, \\
Z_{\dot{w}} &= -18.6658 \text{ kg}, \\
Z_{\dot{q}} &= 0.3800 \text{ kgm}, \\
K_{\dot{p}} &= -0.0035 \text{ kgm}^2, \\
M_{\dot{q}} &= -3.2730 \text{ kgm}^2, \\
M_{\dot{w}} &= 0.3800 \text{ kgm}, \\
N_{\dot{r}} &= -3.0739 \text{ kgm}^2, \text{ and} \\
N_{\dot{v}} &= 0.0457 \text{ kgm}. 
\end{align*}
\] (4.34)

### 4.6 Estimating the Hydrodynamic Damping Coefficients

This section will estimate the damping coefficients as shown in Equation (3.24). The hydrodynamic damping force can be modeled as

\[ F_D (u) = -\frac{1}{2} \rho C_o A_o |u| u, \] (4.35)

where \( C_o \) and \( A_o \) are the reference drag coefficient and area respectively. When calculating the form drag, \( C_D \) will replace \( C_o \) and \( A_c \) is the frontal projected area. When calculating frictional drag, \( C_f \) will replace \( C_o \) and \( A_o \) is the wetted surface area.

The skin frictional force acting on the wetted surface of the vehicle and the viscous force acting on the frontal area of the vehicle body, including the propeller housings, cause the damping coefficient in the \( x \) direction. The effects of the struts can be neglected. The damping coefficient in the \( x \) direction is given as
\[ X_{u|w|} = -\frac{1}{2}\rho \left[ (C_D A_{frontal})_{hull} + (C_f A_{wetted})_{hull} + 2(C_D A_{axial})_{housing} \right]. \]

\[ = -\frac{1}{2}\rho \pi \left[ (C_D r^2)_{hull} + (C_f 2 r l)_{hull} + 2(C_D r^2)_{housing} \right]. \tag{4.36} \]

\(C_D,\text{axial}\) is the base drag coefficient over the housing which has the shape of a ring. This data can be found in Hoerner [10], page 7-13.

The damping coefficients in the \(y\) and \(z\) axes are due to viscous effects on the side projected areas of the hull and the propeller housings. Frictional effects can be neglected. The effect of the struts can be neglected for the coefficient in the \(y\) axis. The damping coefficient in the \(y\) direction is given as

\[ Y_{e|y|} = -\frac{1}{2}\rho \left[ (C_D 2 r l)_{hull} + 2(C_D 2 r l)_{housing} \right], \tag{4.37} \]

the damping coefficient in the \(z\) direction is given as

\[ Z_{e|z|} = -\frac{1}{2}\rho \left[ (C_D 2 r l)_{hull} + 2(C_D 2 r l)_{housing} + 2(C_D \text{span chord})_{struts} \right]. \tag{4.38} \]

Only the struts and the propeller housings contribute to the viscous damping moment in roll as shown in Figure 4-8. The hull has no effect. Using strip theory, the damping force due to rolling can be estimated as

\[ K_{\phi|\phi|} = -2\left[ \frac{1}{2}\rho \ \frac{w|w|}{\pi} \left[ \left( \int_{\alpha+b}^{a+b+c} C_D l \ dy y \right)_{housing} + \left( \int_{a}^{a+b} C_D \text{chord dy} \right)_{struts} \right] \right]. \tag{4.39} \]

Assuming small angular motions, \(w \approx \rho y\). Hence, damping coefficient due to roll motion is

\[ K_{\phi|\phi|} = -2\left[ \frac{1}{2}\rho \left[ \left( \int_{\alpha+b}^{a+b+c} C_D l \ y^2 \dy \right)_{housing} + \left( \int_{a}^{a+b} C_D \text{chord} y^2 \dy \right)_{struts} \right] \right], \tag{4.40} \]

\[ = -\frac{1}{3}\rho \left[ \left( C_D l \ (a + b + c)^3 - (a + b)^3 \right) c \right]_{housing} + \left( C_D \text{chord} ((a + b)^3 - a^3) \right)_{struts}. \]
Apply the same technique used in the above to the pitching motion. The damping force due to pitching, as shown in Figure 4-9, is given as

\[
M_{\phi|q} = -\frac{1}{2} \rho w |w| \left[ \left( \int_{-h/2}^{h/2} C_D 2 r \, dx \right)_{null} + \right] \left[ \int_{r_1}^{r_1 + l_{ph}} C_D 2 r \, dx \right]_{hous|g} + \left[ \int_{r_2 + chord}^{r_2} C_D \, span \, dx \right]_{trut}.
\]

Assuming small angular motions, \( w = q x \). Hence, the damping coefficient due to pitch motion is

\[
M_{\phi|q} = -\rho \left[ \frac{1}{12} C_D r l_{h}^4 \right]_{null} + \left( \frac{2}{3} C_D r l_{ph} ((r_1 + l_{ph})^3 - r_1^3) \right)_{hous|g} + \left[ \frac{1}{3} C_D \, span \, chord \, ((r_2 + chord)^3 - r_2^3) \right]_{trut}.
\]
The vertical drag forces on the struts and the propeller housings contribute to the pitch damping moment due to heave motion, whose coefficient is given as

\[ M_{\theta|\theta} = -\frac{1}{2} \rho \left[ 2(C_D \ 2 \ r \ l)_{\text{hosing}} + 2(C_D \ \text{span chord})_{\text{struts}} \right] \xi. \]  

(4.43)

The horizontal drag forces on the hull and the propeller housings contribute to the yaw damping moment due to yaw, whose coefficient is given as

\[ N_{\phi|\phi} = -\rho \left[ \left( \frac{1}{12} \ C_D \ r \ l h^4 \right)_{\text{hull}} + \right. \]

\[ \left. \frac{2}{3} \left( C_D \ r \ l \ p h \ ((\xi + l \ p h)^3 - \xi^3) \right)_{\text{hosing}} \right]. \]  

(4.44)

The horizontal drag forces on the propeller housings contribute to the yaw damping moment due to the sway motion, whose coefficient is given as

\[ N_{\psi|\psi} = \frac{1}{2} \rho \left[ 2(C_D \ 2 \ r \ l)_{\text{hosing}} \right] \xi. \]  

(4.45)

\( N_{\psi|\psi} \) is positive.

Substituting the dimensions of Autolycus into Equations (4.40) to (4.45), the viscous damping coefficients can be obtained as listed below.
4.7 Estimating the Propulsion Thruster Force

In the case of Autolycus, the propulsion thruster force is estimated from experimental data obtained by Damus et al [3]. A resistance verse speed test was performed and the resistance force was measured with load cells. Another experiment performed by Kreamer et al [14] using a spring gauge yielded similar results. The results obtained from [3] are plotted in Figure 4-10.

![Figure 4-10: Autolycus resistance test data [3].](image)

It will be initially assumed that the port and starboard thrusters used on Autolycus are the same, neglecting the possible differences in frictional resistance in each of the motors. It is further assumed that the forward and reverse thrusts are the same, neglecting the reduced thrust efficiency for reverse fluid flow referenced with respect to the propeller.
In a separate test, the maximum speed of Autolycus was found to be about 0.3 m/s. Using the data in Figure 4-10, each of the port and starboard thrusters is estimated to produce about 1.75 N of thrust force. Therefore,

\[ T_{\text{port}} = T_{\text{starboard}} \approx 1.75 \text{ N}. \]  

No experiments were performed to measure the thrust forces of the forward and aft thrusters. For the initial simulations, it will be assumed that these thrusters produce the same forces as the port and starboard thrusters,

\[ T_{\text{fore}} = T_{\text{aft}} \approx 1.75 \text{ N}. \]  

It is noted the forward and aft propellers are smaller in size than the port and starboard propellers. In Chapter Five, all the thruster forces will be verified using the experimental data.

4.8 Comparing Autolycus Coefficients

The best way to check that the coefficients estimated in this chapter are reasonable is to compare them to coefficients published for vehicles similar to Autolycus. The data for a similar vehicle (Edge Tech Model 272 Side Scan Sonar Towfish) was obtained from [22].

Listed in Table 4-2 are the key parameters of Autolycus and the towfish. Using strip theory and assuming that Autolycus is a streamlined body, its estimated coefficients are comparable to that of a typical streamlined underwater vehicle.

In Chapter Five, the hydrodynamic coefficients for Autolycus will be verified by comparing the simulation results with the experimental data. It will be found that Autolycus is not a streamlined body at all and strictly speaking using strip theory to estimate the hydrodynamic coefficients is not appropriate. However, the theoretical estimates provide an essential start-up for the initial simulations. They also act as a basis for reference during the subsequent simulations in which the hydrodynamic coefficients are systematically changed to obtain a better match between the simulation results and the

82
experimental data. In fact, working out Autolycus' coefficients using a theoretical method has provided an insight into understanding the effects of these hydrodynamic coefficients on the vehicle.

Table 4-2: Comparison of Autolycus data.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Autolycus</th>
<th>Towfish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall length, L (m)</td>
<td>1.49</td>
<td>1.44</td>
</tr>
<tr>
<td>Max. diameter, D (m)</td>
<td>0.127</td>
<td>0.110</td>
</tr>
<tr>
<td>L/D ratio</td>
<td>11.7</td>
<td>13.1</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>18.04</td>
<td>25</td>
</tr>
<tr>
<td>Moment of inertia (kg m²)</td>
<td>3.99</td>
<td>3.76</td>
</tr>
<tr>
<td>$X_{u</td>
<td>w</td>
<td>}$ (kg/m)</td>
</tr>
<tr>
<td>$Z_{u}$ (kg)</td>
<td>18.66</td>
<td>14.98</td>
</tr>
</tbody>
</table>

4.9 Autolycus Motion Simulation Using Matlab

The motions of Autolycus, described by Equations (3.33) to (3.38) are now fully defined. The non-linear dynamic equations worked out in Chapter Three and the hydrodynamic coefficients worked out in Chapter Four are incorporated into a Matlab program previously set up by Leonard [15], in part based on Nahon's work in [18].

The Matlab program can calculate the angular acceleration, linear acceleration, speed and displacement of Autolycus for any of the following maneuvers:

(a) open loop forward (surge),
(b) open loop downward (heave),
(c) open loop forward and down,
(d) closed loop depth control, and
(e) closed loop pitch control.
Only the open-loop maneuvers are required to be used for verifying and modifying the hydrodynamic coefficients and vehicle parameters of the model. These are presented in Chapter Five.

4.10 Summary

This chapter has described a method that may be used for estimating the weight distribution, mass moments of inertia and the hydrodynamic coefficients of Autolycus. The calculations are set up to be able to accommodate design changes made to the vehicle and the resulting changes in its hydrodynamic behavior as part of the design iteration cycle. As long as the changes do not severely change the Autolycus-like features, the model suggested in Chapters Three and Four will be valid.

Chapter Five will present the results of the simulation for several open-loop maneuvers. These simulation results are compared with the experimental data. From these comparisons, better estimates of the hydrodynamic coefficients may be obtained to enable the model to reproduce the behavior of the real vehicle.
5.1 Introduction

This chapter compares the motion simulations of Autolycus with the experimental results. The purpose is to estimate the hydrodynamic coefficients that are used by the simulation model to reproduce the real behavior of Autolycus. Three types of open-loop maneuvers were performed with Autolycus for comparison with the simulation results. These were:

(a) forward only motion (surge), with full thrust applied by the port and starboard thrusters. The vehicle was adjusted to be neutrally buoyant.

(b) downward only motion (heave), and
   (i) at neutrally buoyant and with zero thrust. A 10.45-gram weight was added to each end of Autolycus to obtain a slow descent. Total weight added was 20.90g, making \( B = W - 0.205 \) N.
   (ii) at neutrally buoyant and with zero thrust. A 26.71-gram weight was added to each end of Autolycus to obtain a faster descent. Total weight added was 53.41g, making \( B = W - 0.524 \) N.
(iii) at neutrally buoyant. The forward and aft thrusters applied full downward thrust.

(c) turning motion (yaw).

(i) only one of either port or starboard thrusters was used to make a wide turning circle.

(ii) both the port and starboard thrusters were used, but in the opposite direction to make a narrow turning circle.

The equations of motion shown in Section 3.7 were solved for all the above maneuvers using Matlab. The hydrodynamic coefficients and the vehicle parameters estimated in Chapter 4 were used as a baseline condition for the simulations. These simulations were compared with the experimental data to obtain insights into the effects caused by each of these coefficients and parameters. Further simulations were performed with updated coefficients to make the simulation results match with the experimental data. The model was deemed to be successful when the simulations match the real behavior of Autolycus.

The chart below gives an overview of the simulation process to obtain better estimates of the hydrodynamic coefficients and vehicle parameters \((x_G, z_G, z_p)\) for Autolycus.
Set up equations to model Autolycus

Estimate the vehicle parameters and hydrodynamic coefficients using appropriate techniques

Simulate forward only maneuver. Compare with experimental data and modify coefficients in simulation to match real vehicle behavior, update all coefficients and parameters

Simulate downward only maneuvers. Compare with experimental data and modify coefficients in simulation to match real vehicle behavior, update all coefficients and parameters

Simulate turning (yaw) maneuvers. Compare with experimental data and modify coefficients in simulation to match real vehicle behavior, update all coefficients and parameters

Do all the simulations of the maneuvers match experimental data?

If no

If yes, then end

Figure 5-1: Overview of process to obtain hydrodynamic coefficients.
Although the initially estimated coefficients using theoretical methods do not produce results that are comparable to the real vehicle behavior, they provide a set of useful baseline assumptions for the initial simulations. From the initial results, we can obtain insights into how each coefficient affects the simulation by comparing the output with the experimental data. Varying each coefficient in turn during subsequent simulations can eventually lead to better insights into the dynamics of the model, defined by the six D-O-F equations. With these insights, a set of coefficients can be chosen to fit the model so that the real behavior of the vehicle can be reproduced.

5.2 Summary of the Simulation Process

Approximately four cycles of the process outlined in Figure 5-1 were performed to arrive at a set of hydrodynamic coefficients and vehicle parameters that could match the real behavior of Autolycus for all the maneuvers listed in Section 5.1. The number of simulations was extensive and it will be cumbersome to present each and every case. As an illustration, Table 5-1 shows the process of estimating and updating these coefficients and parameters during the simulation of each maneuver. The experimental data was used to guide towards better estimates. Also, only the essential simulation plots are presented in the subsequent sections for each of the maneuvers.

No experiments were conducted to verify the roll added mass and roll damping coefficients, $K_p$ and $K_{pp}$, respectively. In all the simulations, roll motion was observed to be insignificant. Hence, roll added mass and damping coefficients were not evaluated further.
### Table 5-1: Summary of coefficients obtained from simulation.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Baseline</th>
<th>Forward only</th>
<th>Zero thrust descend, 20.90g added</th>
<th>Zero thrust descend, 53.41g added</th>
<th>Full thrust descend</th>
<th>Wide turning</th>
<th>Narrow turning</th>
<th>Finally, for all cases, use</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_G$</td>
<td>0.0053mm</td>
<td>0.0053</td>
<td>0.0053</td>
<td>0.0053</td>
<td>0.0053</td>
<td>0.0053</td>
<td>0.0053</td>
<td>0.0053</td>
</tr>
<tr>
<td>$z_G$</td>
<td>0.0083mm</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0083</td>
<td>0.0083</td>
</tr>
<tr>
<td>$s_p$</td>
<td>0mm</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.004</td>
<td>0.0083</td>
<td>0.0083</td>
<td>-0.002</td>
</tr>
<tr>
<td>$T_{port}$</td>
<td>1.75N</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>-1.5</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
</tr>
<tr>
<td>$T_{thd}$</td>
<td>1.75N</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.75</td>
<td>1.6</td>
<td>1.75</td>
</tr>
<tr>
<td>$T_{fem}$</td>
<td>1.75N</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$T_{ah}$</td>
<td>1.75N</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$X_\dot{u}$</td>
<td>-10% m kg</td>
<td>-40% m</td>
<td>-40% m</td>
<td>-40% m</td>
<td>-40% m</td>
<td>-40% m</td>
<td>-40% m</td>
<td>-40% m</td>
</tr>
<tr>
<td>$X_{uu}$</td>
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<td>-35</td>
<td>-35</td>
<td>-35</td>
<td>-35</td>
<td>-35</td>
<td>-35</td>
<td>-35</td>
</tr>
<tr>
<td>$Y_v$</td>
<td>0.0457 kg/m</td>
<td>0.0457</td>
<td>0.0457</td>
<td>0.0457</td>
<td>0.0457</td>
<td>0.0457</td>
<td>0.0457</td>
<td>1</td>
</tr>
<tr>
<td>$Y_{v}$</td>
<td>0.0457 kg/m</td>
<td>0.0457</td>
<td>0.0457</td>
<td>0.0457</td>
<td>0.0457</td>
<td>0.0457</td>
<td>0.0457</td>
<td>1</td>
</tr>
<tr>
<td>$Y_{vv}$</td>
<td>-75.4592 kg/m</td>
<td>-75.4592</td>
<td>-75.4592</td>
<td>-75.4592</td>
<td>-75.4592</td>
<td>-250</td>
<td>-250</td>
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<td>$Z_{\dot{w}}$</td>
<td>-18.6658 kg</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
<td>-30</td>
</tr>
<tr>
<td>$Z_{ww}$</td>
<td>-81.1292 kg/m</td>
<td>-270</td>
<td>-270</td>
<td>-270</td>
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<td>-270</td>
<td>-270</td>
<td>-270</td>
</tr>
<tr>
<td>$Z_{\dot{w}}$</td>
<td>-0.38 kg/m</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>$M_{\dot{w}}$</td>
<td>-3.2130 kgm²</td>
<td>-3.2130</td>
<td>-3.2130</td>
<td>-3.2130</td>
<td>-6</td>
<td>-6</td>
<td>-6</td>
<td>-4</td>
</tr>
<tr>
<td>$M_{ww}$</td>
<td>-17.9998 kgm²</td>
<td>-40</td>
<td>-40</td>
<td>-40</td>
<td>-40</td>
<td>-40</td>
<td>-40</td>
<td>-70</td>
</tr>
<tr>
<td>$N_{rr}$</td>
<td>-17.8704 kg</td>
<td>-17.8704</td>
<td>-17.8704</td>
<td>-17.8704</td>
<td>-17.8704</td>
<td>-50</td>
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<td>-55</td>
</tr>
<tr>
<td>$N_{\dot{r}}$</td>
<td>-3.0739 kgm²</td>
<td>-3.0739</td>
<td>-3.0739</td>
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<td>-10</td>
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<td>-10</td>
</tr>
<tr>
<td>$N_{vv}$</td>
<td>0.8026 kg</td>
<td>0.8026</td>
<td>0.8026</td>
<td>0.8026</td>
<td>0.8026</td>
<td>2</td>
<td>8</td>
<td>14</td>
</tr>
</tbody>
</table>

where $m$ is the total mass of the vehicle, which includes the entrained water in the vehicle.

1. use -1.5 for narrow turn maneuver
2. use 1.6 for narrow turn maneuver
5.3 Surge Maneuver

The experimental data for a surge only motion is shown in Figure 5-2. This test was done in the smaller of the MIT Alumni swimming pools. The port side motor/thruster was set to produce 90% of its available thrust while the starboard side motor/thruster was set to produce 100%. This was found to be necessary because the vehicle tended to yaw toward starboard if both motors/thrusters were to be set at 100%. The different thrust output is mainly due to the different frictional resistance of the motor. The maximum speed recorded by the on-board speed sensor was about 0.3 to 0.35 m/s. The time taken for the vehicle to reach maximum speed was about 8 seconds. The vehicle appeared to be pitching at ±20° and a period in the region of 15 seconds. The vehicle was able to hold a constant altitude throughout its 25-second maneuver.

A surge simulation was performed using the theoretically estimated hydrodynamic coefficients, i.e. the baseline condition. The simulation output is shown in Figure 5-3. In the simulation, both the port and starboard thrusters were assumed to provide equal thrust of 1.75 N each. The initial simulation produced a highly unstable pitch motion. The forward speed (surge) was also too high, at about 1 m/s compared to the expected 0.3 m/s. This suggested that the surge damping coefficient $X_{uu}$ used for the initial simulation was too low. When $X_{uu} = -35$ kg/m and $X_{u} = -10\%$ m kg were applied, it was found that the maximum speed was in the region of 0.3 m/s but the time taken to reach maximum speed was about 6 seconds compared to the expected 8 seconds. This indicated that the order of magnitude for $X_{uu}$ was appropriate, but that $X_{u}$ was too low. Figure 5-4 shows the surge motion simulation result when $X_{uu} = -40$ kg/m and $X_{u} = -40\%$ m kg were used.

Figure 5-4 shows the final simulation for the surge maneuver. The maximum surge obtained was about 0.3 m/s. The time taken to settle at maximum speed was about 7 second. The pitch angle was in the region of ±20° with pitch period of about 27 seconds. The altitude was fairly constant. The simulation compared reasonably well with the experimental data shown in Fig 5-2.
Figure 5-2: Experimental data for surge maneuver.
Figure 5-3: Initial simulation of surge maneuver.
Figure 5-4: Final simulation of surge maneuver.
5.4 Heave Maneuver

Three downward maneuvers were performed in the deep end of the larger MIT Alumni swimming pool. Each of these maneuvers will be presented individually.

5.4.1 Zero Thrust, 20.9 grams weight added

The vehicle was initially neutrally buoyant. A 10.45-gram weight was added to each end of the vehicle (total of 20.9 g) and the thrusters applied no thrust. Since the vehicle weight was more than the buoyancy force (B=W-0.205 N), the vehicle descended slowly. The experimental data for this condition is shown in Figure 5-5. The average descent rate was about 0.027 m/s and pitch was in the region of ± 2°. It was not possible to estimate the time taken to reach its steady state descent rate for this case.

Figure 5-6 shows the simulation for this case using the final set of coefficients and parameters in Table 5-1. The steady state descent rate was about 0.028 m/s and the pitch was about -0.6°. The simulation model was not able to reproduce the pitching characteristics of the vehicle. The vehicle experiences a forward glide due to the negative pitch angle, as seen in Figure 5-6(c). Other than the pitch characteristics, the simulation result compared reasonably well with the experimental data as shown in Figure 5-5.
Figure 5-5: Experimental data for heave maneuver (20.9g added).
Figure 5-6: Simulation for heave maneuver (20.9g added).
5.4.2 Zero Thrust, 53.4 grams weight added

The vehicle was initially neutrally buoyant. A 26.71-gram weight was added to each end of the vehicle (total of 53.42 g) and no thrust was applied. The vehicle weight was more than the buoyancy force \( B = W - 0.524 \, \text{N} \). The experimental data for this condition is shown in Figure 5-7. The vehicle took about 65 seconds to bottom, as can be seen in Figure 5-7(a). The average descent rate was about 0.04 m/s. The pitch varied within the region of \( 0^0 \) to \(-5^0\). It was not possible to estimate the time taken to reach its steady state descent rate for this case. There were two descent rates seen in this experiment. The vehicle initially descended at \( \approx 0.04 \, \text{m/s} \) for about 15 seconds, followed by a faster descent rate of \( \approx 0.07 \, \text{m/s} \). The faster descent rate was probably due to the loss of buoyancy when the external water pressure compressed the vehicle hull. It was observed during the experiment that when the vehicle was set near the water surface it was neutrally buoyant. However, when it was submerged further (about 1.5 m), the vehicle tended to sink. Fundamentally, this is due to a breakdown of the rigid body assumption. As the vehicle was subjected to higher ambient pressure, its volume was reduced, leading to a decreased buoyancy force.

Figure 5-8 shows the simulation results, again using the final set of coefficients and parameters in Table 5-1. The steady state descent rate is about 0.046 m/s and the pitch was about \(-1^0\). The simulation model was not able to reproduce the pitching characteristics of the vehicle. The vehicle experiences a forward glide due to the negative pitch angle, as seen in Figure 5-8(c). As in the previous case, the simulation result was acceptable.
Figure 5-7: Experimental data for heave maneuver (53.4g added).
(a): Heave velocity (m/s) versus time (s).

(b): Pitch angle (deg) versus time (s).

(c): z-inertial (m) versus x-inertial (m).

Figure 5-8: Simulation for heave maneuver (53.4g added).
5.4.3 Full Thrust Heave Maneuver

The vehicle was neutrally buoyant (W=B). The forward and aft vertical thrusters applied full thrust on the vehicle for its descent. The experimental data for this condition is shown in Figure 5-9. The average descend rate was about 0.075 m/s. The pitch varied within the region of $0^\circ$ to $-6^\circ$.

Figure 5-10 shows the simulation results. The steady state descent rate was about 0.073 m/s. The pitch was about $-1.5^\circ$ to $-2^\circ$. The simulation model was not able to reproduce the pitching characteristics of the vehicle. In this case the forward glide was negligible. The simulation results for this case was reasonably close to the experimental data.

![Graph](image1)

(a): Altitude (m) versus time (s).

![Graph](image2)

(b): Pitch angle (deg) versus time (s).

Figure 5-9: Experimental data for heave maneuver (full thrust).
Figure 5-10: Simulation for heave maneuver (full thrust).
5.5 Yaw Maneuver

Two turning maneuvers were performed. Each of these maneuvers will be presented individually.

5.5.1 Wide Turn

Only one of either the port or starboard thrusters was providing maximum thrust. The other thruster was idle. This experiment was performed in the larger MIT Alumni swimming pool. The experimental data is shown in Figure 5-11. In this case, the starboard thruster applied maximum thrust. The time taken for the vehicle to yaw increments of 90\(^\circ\) and the turning diameter of the vehicle were visually estimated. The average yaw rate was found to be about 5.3 deg/s. The turning diameter was approximately 4.5 m. Figure 5-11 shows that the vehicle was able to hold a consistent mean depth. The pitch angle predominantly ranged between 0\(^\circ\) to –6\(^\circ\). The surge velocity was about 0.2 to 0.25 m/s.

Figure 5-12 shows the simulation results for the wide turn maneuver. Thrust applied by the starboard thruster was 1.75 N and that of the port thruster was 0. The predicted steady state yaw rate was about 5.6 deg/s. The turning diameter was about 4 m. The surge velocity was about 0.19 m/s. The simulation was not able to model the pitch characteristics of the vehicle. The simulation results matched the experimental data fairly well.
Figure 5-11: Experimental data for yaw maneuver (wide turn).
(a): Yaw rate (deg/s) versus time (s).

(b): Surge velocity (m/s) versus time (s).

(c): Pitch angle (deg) versus time (s).
(d): Sway velocity (m/s) versus time (s).

(e): y-inertial (m) versus x-inertial (m).

(f): z-inertial (m) versus x-inertial (m).

Figure 5-12: Simulation for yaw maneuver (wide turn).
5.5.2 Narrow Turn

In this case, the port and starboard thrusters applied a force couple. This experiment was performed in the testing tank located in MIT OE Department, Building 1-225. The experimental data is shown in Figure 5-13. The starboard thruster was set to apply 90% thrust in the positive direction while the port thruster was set to apply 100% thrust in the negative direction. The time taken for the vehicle to yaw increments of 90° and the turning diameter of the vehicle were visually estimated. The average yaw rate ranged from 6.4 deg/s to 7.7 deg/s. The turning diameter was about 0.5 m. The vehicle was able to hold a consistent mean depth. The pitch angle ranged between -2° to +6°. The surge velocity was negligible. The data for the visually estimated yaw rate and turning diameter is shown in Table 5-2 and the yaw rate is plotted in Figure 5-13(d).

Figure 5-14 shows the simulation results for the narrow turn maneuver. The thrust applied by the starboard thruster was 1.6 N and that of the port thruster was -1.5 N. The starboard thruster was set at 1.6 N because the starboard thrust setting on Autolycus was 90%. The port thruster was derated for reduced efficiency in the reverse flow. The predicted steady state yaw rate was about 6.1 deg/s. The turning diameter was about 0.63 m for the first circle and then 0.44 m for the subsequent circles. The surge velocity was negligible. The simulation was not able to model the pitch characteristic of the vehicle. The simulation results were generally acceptable.
(a): Altitude (m) versus time (s).

(b): Pitch angle (deg) versus time (s).

(c): Surge velocity (m/s) versus time (s).
(d): Visually estimated yaw rate.

Figure 5-13: Experimental data for yaw maneuver (narrow turn).

Table 5-2: Visually estimated yaw rate and turning diameter for narrow turn.

<table>
<thead>
<tr>
<th>time (s)</th>
<th>yaw angle (deg)</th>
<th>turning mark (inch)</th>
<th>yaw rate (deg/s)</th>
<th>turning diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>119</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14.1</td>
<td>90</td>
<td>6.38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26.6</td>
<td>180</td>
<td>98</td>
<td>6.76</td>
<td>0.53</td>
</tr>
<tr>
<td>38.2</td>
<td>270</td>
<td>7.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>49.1</td>
<td>360</td>
<td>119</td>
<td>7.33</td>
<td>0.53</td>
</tr>
<tr>
<td>60</td>
<td>450</td>
<td>7.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>71.6</td>
<td>540</td>
<td>102</td>
<td>7.54</td>
<td>0.43</td>
</tr>
<tr>
<td>82.7</td>
<td>630</td>
<td>7.61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>93.6</td>
<td>720</td>
<td>122</td>
<td>7.69</td>
<td>0.50</td>
</tr>
</tbody>
</table>
(a): Yaw rate (deg/s) versus time (s).

(b): Surge velocity (m/s) versus time (s).

(c): Pitch angle (deg) versus time (s).
Figure 5-14: Simulation for yaw maneuver (narrow turn).
5.6 Summary

This chapter has presented both the experimental data and the simulation results of three types of open-loop maneuvers. The simulation model was able to reproduce the general characteristics of the real behavior for all the maneuvers described in this chapter, with the exception of the pitch angle and pitch period characteristics. The equations of motion used for the simulation model are highly coupled and it will require additional effort to isolate the terms that determine the pitch angle and period responses. It may also be that the model developed in Chapter 3 was not able to describe the subtle characteristics of the real vehicle. Extensive experience in AUV modeling and simulation work is prerequisite to accurately model and reproduce the real behavior of the vehicle.

In conclusion, the model that is proposed in this thesis has been fairly successful in simulating the motions of Autolycus in the vertical and horizontal planes using a common set of hydrodynamic coefficients and vehicle parameters. A mathematical model to describe the 6 D-O-F motion of Autolycus in its present configuration is now available to facilitate further research and educational purposes.
Chapter 6

Conclusion

6.1 Summary

In this thesis, an approach to modeling and simulation of the autonomous underwater vehicle, Autolycus has been presented. The model, expressed as equations of motion in the six degrees of freedom, was used to predict the motion response of Autolycus. The model was developed as shown in Chapter Three. The vehicle parameters and the hydrodynamic coefficients were estimated using theoretical methods as shown in Chapter Four.

Simulations of the vehicle motions were obtained by solving the six equations of motion using Matlab. The initial simulations were performed using the vehicle parameters and hydrodynamic coefficients estimated from theory.

Experiments were conducted on Autolycus to obtain the vehicle’s surge, heave, pitch and yaw motion characteristics. The simulation results were compared with the experimental data for the corresponding motion. From the initial simulations, it was shown that Autolycus was not a streamlined body and applying strip theory methods to estimate the hydrodynamic coefficients was not strictly applicable. However, the theoretical estimates provided an essential start-up for the simulations. The also provided a basis for reference in the course of further simulations when the hydrodynamic
coefficients systematically varied. The objective was to deduce a set of vehicle parameters and hydrodynamic coefficients that would produce simulation results that match the behavior of the real vehicle for all types of maneuvers. When this is achieved, the model can be used to simulate other types of vehicle maneuvers without resorting to further experiments.

6.2 Future Work

The simulation model developed in this thesis can be extended in future research in a number of ways. Two issues in particular are:

(a) modeling of the thruster dynamics, and
(b) redesign of Autolycus.

6.3 Thruster Dynamics

In the current model, the thruster force is assumed to be a step function in which the thrust is constant and applied at the instant of time $0^t$. This model has neglected the dynamics of the thruster. The Autolycus model can be enhanced with a closed-loop controller, which needs a dynamic model for the thruster. In reality, thruster dynamics is a highly complex subject. A simpler thruster model could be developed, which accounts for the slight delay in thrust production from the instant the propeller starts to spin. This delay is the time taken for 2 or 3 vortices to form and shed at the propeller in order to produce the thrust force [22]. Furthermore, the propeller takes time to ramp up to its maximum thrust. A closed-loop controller requires this thrust delay feature to model the behavior of the real vehicle.

6.4 Redesign of Autolycus

The experimental plots of Autolycus’ motions show that the vehicle’s motions in the vertical and horizontal planes are stable. For example, the pitch angles are less than $15^0$
and the pitch does not grow in time. Thus, the design of the vehicle is fundamentally sound. The position of the center of mass, \( x_G, y_G \) and \( z_G \) are well placed to provide the required recovery forces. However, the vehicle should be made more streamlined if Autolycus were to be further developed into a full fledged research vehicle. This would reduce its resistance \( (X_{\text{in}}) \), in turn improving its power consumption of the vehicle and extending its mission time.
Appendix A-1

autosc.m

% autosc.m - Autolycus simulation, created by wrk/jleonard fall 97
% adapted from arcsim by Meyer Nahon, UVic (see AUV96 paper)
% last mod: jleonard 2/13/98
% re-worked by S.C. Tang; Fall 1998

% x0 = [Vxi; Vyi; Vzi; phidot; thetadot; psidot; xi; yi; xi; phi; theta; psi]  global co-ords

global m Ixx Iyy Izz
global xplore xpaft yp zp zb B W rho
global rm rml
global rh lh Ah Cdh Vh ma_h
global span chord Ast Cdst ma_st
global rph lph Aph Cdph Vph ma_ph
global Xuu Xud
global Yvd Yrd Yvv
global Zwd Zww Zqd
global Kpp Kpd
global Mqd Mqq Mwd Mww
global Nrr Nvd Nrd Nvv

global Thrust cur_step max_steps  % for saving thrust values in autodync()
global Command  % save commanded values

global controller_type

global OPEN_LOOP_FORWARD OPEN_LOOP_DOWN DEPTH_CONTROL
DEPTH_PITCH_CONTROL

global Tforce  % for thruster force

radtodeg = 180/pi;
degtorad = pi/180;

if (exist('run_number')==0)
run_number = 1;
else
    run_number = run_number+1;
end
dstring = date;
run_name = ['Sim#' num2str(run_number) ' ' dstring];
doprint=1;

% types of controllers
OPEN_LOOP_FORWARD = 0;
OPEN_LOOP_DOWN = 1;
OPEN_LOOP = 2; % forward and down (all thrusters on full)
DEPTH_CONTROL = 3;
DEPTH_PITCH_CONTROL=4;

controller_type =OPEN_LOOP_DOWN; % specify type of simulation maneuver

%%%%%% set values of AUV hydro coefficients %%%%%%%
m = 18.04; % vehicle mass, including the entrained water in kg.
W = m*9.81; % weight (N) = mg
%B = W; % neutral buoyant condition
%B=W-0.205; % 10.45grams added to each end of vehicle, total 0.205 N
B=W-0.524; % 26.71grams added to each end of vehicle, total 0.524 N
rho = 1000; % assume density of fresh water [kg/m^3]
Ixx = 0.04933; % mass moment of inertia, in kg-m^2
Iyy = 3.97806; % see spreadsheet for Ixx, Iyy, Izz estimates
Izz = 3.98686;

xpfore = .327; % distance in x-axis from origin to forward thruster (m)
xpaf = .510; % distance from origin to aft thruster (m)
yp = .2; % distance in y-axis from origin to port and starboard thrusters
zp = -0.002; % position at which port/stbd thrusts are applied w.r.t. origin
zb = 0; % vertical center of buoyancy for a circular cylinder main hull

% sections below input vehicle geometry for computing hydrodynamic coefficients
% the main vehicle hull
rh = 0.0635; % outer radius of the hull in meter
lh = 1.4235; % mean length of the hull. Account for one half of hemisphere
% origin is at one half of total vehicle length
Ah = 2*rh*lh; % projected area of hull in heave and sway
Cdh = 0.82; % cross flow: drag coefficient of cylinder with l/d ratio of 10
Cfh = 0.004; % axial flow, wetted area: skin friction drag on cylinder with l/d = 10
Cbh = 0.2; % axial flow: base drag coefficient (see Hoerner - Drag, pg3-12)
Vh = pi*rh^2*lh; % volume displaced by the hull
ma_h = rho*Vh; % added mass of circular cylinder in heave and sway

% the struts data. struts are considered as thin plates
span = 0.090; % distance from hull to propeller housing
chord = 0.063; % span and chord seen from bow of vehicle, % nomenclature similar to lifting fin Ast = span*chord; % projected area of each strut in heave Cdst = 1.0; % drag coefficient of plate in heave ma_st = 0.25*pi*rho*chord^2*span; % added mass of each strut in heave, % other directions are zero

% Note: the propeller guards were removed in late Fall 98. % So, instead of modeling propeller guards (housing), the motor casing is modeled % port/stbd propeller motor casing
rph = 0.011; % outer radius of motor casing
lph = 0.095; % length of port/stbd thruster motor
Aph = 2*rph*lph; % projected area of each housing in heave and sway
Cdph = 0.64; % cross flow: drag coefficient of cylinder
Cbph = 0.2; % axial flow: base drag over cylinder
% axial flow: skin friction drag about zero pg3-12
Vph = pi*rph^2*lph; % volume displaced by each motor housing
ma_ph = rho*Vph; % added mass of each housing in heave and sway

r1 = 0.600; % distance from origin to center of mass of housing and struts
r2 = 0.570; % distance from origin to forward edge of strut w.r.t vehicle origin

% Xu = -0.5*rho*pi*(Cfh*2*rh*lh + Cbh*rh^2 + 2*Cbph*rph^2) % account for friction and axial drag
Xu = -40;
% Xud = -0.1*m % X u-dot added mass in surge (5% to 10% of m for streamlined bodies)
Xud = -0.4*m; % adjusted

% Yvd = -(ma_h + 2*ma_ph) % Y v-dot added mass in sway [kg]
Yvd = -29;
% Yrd = 2*((r1^2+yp^2)^0.5)*ma_ph % added mass in sway due to yaw
Yrd = 1;
% Yvv = -0.5*rho*(Cdh*Ah + 2*Cdph*Aph) % viscous damping coefficient in sway
Yvv = -200;

% Zwd = -(ma_h + 2*ma_ph + 2*ma_st) % added mass in heave % adjusted
Zwd = -30;
% Zww = -0.5*rho*(Cdh*Ah + 2*Cdph*Aph + 2*Cdst*Ast) % viscous damping coefficient in heave
Zww = -250;
% Zqd = -2*r1*(ma_st + ma_ph) % added mass in heave due to pitch
Zqd = -0.5;
% Kpd = -(2*yp^2*ma_ph + 0.00065268) % added moment in roll
Kpd = -1;
### Roll Damping

\[
K_{pp} = -\frac{1}{3} \rho (C_{dph} l_{ph}^2 2 r_{ph} ((r_{h} + s_{pan} + 2 r_{ph})^3 - (r_{h} + s_{pan})^3) + ... \\
C_{dst} c_{hord} s_{pan} ((r_{h} + s_{pan})^3 - r_{h}^3)) \]  
\]  
% roll damping

\[
K_{pp} = -0.07; \]  
% roll damping, adjusted

### Added Moment in Pitch

\[
M_{qd} = -\frac{1}{12} m_{a} h l_{h}^2 - 2 m_{a} p_{h} r_{1}^2 - 2 m_{a} s_{t} r_{1}^2 \]  
% added moment in pitch

\[
M_{qd} = -4; \]  
% adjusted

\[
M_{wd} = Z_{qd} \]  
% added moment in pitch due to heave

### Damping Moment in Pitch

\[
M_{qq} = -\rho \frac{1}{12} C_{dh} l_{h}^4 - 2/3 \rho r_{ph} C_{dph} l_{ph} ((r_{1} + l_{ph})^3 - r_{1}^3) - ... \\
1/3 \rho s_{pan} C_{dst} c_{hord} ((r_{2} + c_{hord})^3 - r_{2}^3) \]  
% damping moment in pitch

\[
M_{qq} = -70; \]  
% adjusted

### Pitch Damping Due to Heave Motion

\[
M_{ww} = -0.5 \rho (2 C_{dph} A_{ph} + 2 C_{dst} A_{st}) r_{1} \]  
% Pitch damping due to heave motion

\[
M_{ww} = -20; \]  
% adjusted

### Damping Moment in Yaw

\[
N_{rr} = -\rho \frac{1}{12} C_{dh} l_{h}^4 - 2/3 \rho r_{ph} C_{dph} l_{ph} ((r_{1} + l_{ph})^3 - r_{1}^3) \]  
% damping moment in yaw

\[
N_{rr} = -55; \]  
% adjusted

### Added Moment in Yaw

\[
N_{rd} = -\left( \frac{1}{12} l_{h}^2 m_{a} h + 2 m_{a} p_{h} (r_{1}^2 + y_{p}^2) \right) \]  
% added moment in yaw

\[
N_{rd} = -10; \]  
% adjusted

\[
N_{vd} = Y_{rd} \]  
% added moment in yaw due to sway.

% Nvd is positive

### Yaw Damping Due to Sway Motion

\[
N_{vv} = 0.5 \rho (2 C_{dph} A_{ph}) r_{1} \]  
% yaw damping due to sway motion.

% Nvv is positive.

\[
N_{vv} = 14; \]  
% adjusted

### Initial Condition

\[
x_0 = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0]; \]  
% initial condition

\[
T = zeros(6); \]  
TT = zeros(6);

\[
\text{phi} = x_0(7); \]  
the = x_0(8);
\psi = x_0(9);

\[
c_{phi} = \cos(\text{phi}); \]  
c_{the} = \cos(\text{the});
\c_{psi} = \cos(\psi);
\text{s}_{phi} = \sin(\phi);
\text{s}_{the} = \sin(\text{the});
\s_{psi} = \sin(\psi);

\[
T(1,1) = \c_{psi} \c_{the}; \]  
T(1,2) = \c_{psi} \s_{the} \s_{phi} - \s_{psi} \c_{phi};
\text{T(1,3) = s}_{psi} \s_{phi} + \c_{psi} \s_{the} \c_{phi};
\text{T(2,1) = s}_{phi} \c_{the};
\text{T(2,2) = s}_{phi} \c_{the} \c_{phi} \c_{psi} + \c_{psi} \c_{the};
\text{T(2,3) = -c}_{phi} \c_{phi} + \s_{psi} \s_{the} \c_{phi};

118
T(3,1) = -sthe ;
T(3,2) = cthe*sphi ;
T(3,3) = cthe*cphi ;

TT(1:3,1:3) = T(1:3,1:3)' ;
TT(4,4) = 1 ;
T(4,6) = -sthe ;
T(5,5) = cphi ;
TT(5,6) = sphi*cthe ;
TT(6,5) = -sphi ;
TT(6,6) = cphi*cthe ;
X0 = TT*x0(1:6) ;

X0(7:12) = [0 0 0 x0(10) x0(11) x0(12)];
X0(13:16) = [0 0 0 0];

%%%%% specify initial qdot,udot etc %%%%
udot = 0;
vdot = 0;
wdot = 0;
pdot = 0;
qdot = 0;
rdot = 0;

dt = 0.01;
tspan = [0:dt:120];

cur_step = 1;
max_steps = 10000;
Thrust = zeros(max_steps,5);

Command = zeros(max_steps,5);

[t,X] = ode45('autodynsc',tspan,X0);
delta_t = diff(t);

%%%%% plot the results of the simulation %%%%

figure(1) % plots of variables in the vehicle frame
subplot(321), plot(t,X(:,1)); grid; title('surge-velocity u (m/s)')
subplot(321), plot(t,X(:,1)); grid; title('sway-velocity v (m/s)')
subplot(323), plot(t,X(:,2)); grid; title('heave-velocity w (m/s)')
subplot(325), plot(t,X(:,3)); grid; title('roll-velocity p (deg/s)')
subplot(324), plot(t,radsdeg*X(:,5)); grid; title('pitch-velocity q (deg/s)')
subplot(326), plot(t,radsdeg*X(:,6)); grid; title('yaw-velocity r (deg/s)');
if (doprint) 
    print([dstring(4:6) dstring(1:2) '_' num2str(run_number) '_f' num2str(1)]);
end
figure(2) % plots of variables in the inertial frame
subplot(321), plot(t,X(:,7)); grid; title('Inertial frame x (m)')
subplot(323), plot(t,X(:,8)); grid; title('Inertial frame y (m)')
subplot(325), plot(t,X(:,9)); grid; title('Inertial frame z (m)')
set(gca, 'Ydir', 'reverse');
subplot(322), plot(t,radtodeg*X(:,10)); grid; title('Roll phi (deg)')
subplot(324), plot(t,radtodeg*X(:,11)); grid; title('Pitch theta (deg)')
subplot(326), plot(t,radtodeg*X(:,12)); grid; title('Yaw psi (deg)');
% print('plot2')
zoom on
if (doprint)
    print([dstring(4:6) dstring(1:2) '_ num2str(run_number) _f' num2str(2)]);
end

figure(3) % plots of the vehicle trajectory in the x-y and x-z planes
clf
subplot(211), plot(X(1,7),X(1,8), 'o');
hold on
subplot(211), plot(X(:,7),X(:,8));
title('Birds eye view (y-inertial vs. x-inertial)')
axis('equal');
grid on
zoom on
subplot(212), plot(X(1,7),X(1,9), 'o');
hold on
subplot(212), plot(X(:,7),X(:,9));
axis('equal');
grid on;
title('Side view (z-inertial vs. x-inertial)')
set(gca, 'Ydir', 'reverse');
zoom on
if (doprint)
    print([dstring(4:6) dstring(1:2) '_ num2str(run_number) _f' num2str(3)]);
end

figure(4) % plots of commanded thruster values
clf
thtime = Thrust(1:(cur_step-1),1);
thsbd = Thrust(1:(cur_step-1),2);	haft = Thrust(1:(cur_step-1),3);
thport = Thrust(1:(cur_step-1),4);
thfore = Thrust(1:(cur_step-1),5);

subplot(411);
hold off
ii = find(thstbd==0);
if (length(ii)>0) stem(thtime(ii), thstbd(ii)); end;
hold on

120
ii = find(thstbd>0);
if (length(ii)>0) stem(thtime(ii), thstbd(ii), 'g'); end;
ii = find(thstbd<0);
if (length(ii)>0) stem(thtime(ii), thstbd(ii), 'r'); end;
title('starboard thrust');
zoom on

subplot(412);
hold off
ii = find(thaft==0);
if (length(ii)>0) stem(thtime(ii), thaft(ii)); end;
hold on
ii = find(thaft>0);
if (length(ii)>0) stem(thtime(ii), thaft(ii), 'g'); end;
ii = find(thaft<0);
if (length(ii)>0) stem(thtime(ii), thaft(ii), 'r'); end;
title('aft thrust');
zoom on

subplot(413);
hold off
ii = find(thport==0);
if (length(ii)>0) stem(thtime(ii), thport(ii)); end;
hold on
ii = find(thport>0);
if (length(ii)>0) stem(thtime(ii), thport(ii), 'g'); end;
ii = find(thport<0);
if (length(ii)>0) stem(thtime(ii), thport(ii), 'r'); end;
title('port thrust');
zoom on

subplot(414);
hold off
ii = find(thfore==0);
if (length(ii)>0) stem(thtime(ii), thfore(ii)); end;
hold on
ii = find(thfore>0);
if (length(ii)>0) stem(thtime(ii), thfore(ii), 'g'); end;
ii = find(thfore<0);
if (length(ii)>0) stem(thtime(ii), thfore(ii), 'r'); end;
title('fore thrust');
zoom on

if (doprint)
    print([dstring(4:6) dstring(1:2) '_' num2str(run_number) '_' f' num2str(4)]);
end
Appendix A-2

autodynsc.m

function qd = autodynsc(t,Y)

% written by wrk/jleonard fall 97/spring 98
% status: 2/12/98 - adding closed loop control
% re-worked by SC Tang; Fall 1998

% sol = [u v w p q r x y z phi theta psi Tstbd Taft Tport Tfore]
% = [y1 y2 y3 y4 y5 y6 y7 y8 y9 y10 y11 y12 y13 y14 y15 y16]

% Vehicle parameters and hydrodynamic coefficients are global
% variables that are defined in auto.m
global m Ixx Iyy Izz
global xpfore xpaft yp zp zb B W rho
global rm rm1
global rh lh Ah Cdh Vh ma_h
global span chord Ast Cdst ma_st
global rph lph Aphi Cdpi Vph ma_ph
global xG yG zG
global Xuu Xud
global Yvd Yrd Yvv
global Zwd Zww Zqd
global Kpp Kpd
global Mqd Mqq Mwd Mww
global Nrr Nvd Nrd Nvv
global Thrust cur_step max_steps % for saving thrust values
global Command
global controller_type
global OPEN_LOOP_FORWARD OPEN_LOOP_DOWN DEPTH_CONTROL
DEPTH_PITCH_CONTROL
global OPEN_LOOP
global Tforce
radtodeg = 180/pi;
degtorad = pi/180;

u = Y(1);
v = Y(2);
w = Y(3);
p = Y(4);
q = Y(5);
r = Y(6);

x = Y(7);
y = Y(8);
z = Y(9);
phi = Y(10);
theta = Y(11);
psi = Y(12);

xG = 0.0053; % center of mass is fwd of vehicle reference origin in x-axis (m)
yG = 0; % assume no heeling (m)
zG = 0.0083; % center of mass is below the origin in +ve z-axis (m)

Tforce = 1.75; % thruster force in N, usage is optional

commanded_depth = 10.0;
commanded_speed = 0.0;
commanded_heading = 0.0;
commanded_pitch = 0.0;
depth_tolerance = 0.1;
speed_tolerance = 0.1;
heading_tolerance = 0.1;
pitch_tolerance = 2.0*degtorad;

switch controller_type
    case OPENLOOPFORWARD,
        if (t==0)
            disp('open loop forward selected')
        end
    Tstbd = 1.6; % thruster force
    Taft = 0;
    Tport = -1.5;
    Tfore = 0;

    case OPENLOOP_DOWN,
        if (t==0)
            disp('open loop down selected')
        end
    Tstbd = 0;
    Taft = 0.6;
    Tport = 0;
    Tfore = 0.75;
case OPEN_LOOP,
    if (t==0)
        disp('open loop forward and down selected')
    end
    Tstbd = Tforce;
    Taft = Tforce;
    Tport = Tforce;
    Tfore = Tforce;
end

case DEPTH_CONTROL,
    if (t==0)
        disp('depth control selected -- linear controller')
    end
    Tport = commanded_speed;
    Tstbd = commanded_speed;

    kdp = 0.1; % proportional depth gain
    depth_error = commanded_depth-z;
    depth_cmd = depth_error * kdp;

    if (depth_cmd > 1) depth_cmd=1; end;
    if (depth_cmd < -1) depth_cmd=-1; end;

    kpp = 1; % proportional pitch gain
    kpd = 100; % derivative pitch gain
    pitch_error = commanded_pitch-z;

    % q is pitch rate
    pitch_cmd = pitch_error * kpp + kpd*q;

    % Do
    Taft = depth_cmd + pitch_cmd;
    Tfore = depth_cmd - pitch_cmd;

    if (Taft > Tforce) Taft=Tforce; end;
    if (Taft < -Tforce) Taft=-Tforce; end;
    if (Tfore > Tforce) Tfore=Tforce; end;
    if (Tfore < -Tforce) Tfore=-Tforce; end;
end

case DEPTHPITCH_CONTROL,
    if (t==0)
        disp('depth pitch control selected')
    end
end

Tport = commanded_speed;
Tstbd = commanded_speed;
if (z<(commanded_depth-depth_tolerance))
    Taft =Tforce;
elseif (z>(commanded_depth+depth_tolerance))
    Taft = -Tforce;
else
    Taft = 0;
end;
if (theta<(commanded_pitch-pitch_tolerance))
    Tfore = Tforce;
elseif (theta>(commanded_pitch+pitch_tolerance))
    Tfore = -Tforce;
else
    Tfore = 0;
end;
otherwise,
    disp('Unknown controller type in autodyn');
    Taft = 0;
    Tfore = 0;
    Tport = 0;
    Tstbd = 0;
end

%%% specify initial qdot,udot etc %%%
udot =0;
vdot =0;
wdot =0;
pdot =0;
qdot =0;
rdot =0;

% Surge Equation
udot = (1/(m-Xud))*( Zwd*w*q + Zqd*q^2 -Yvd*v*r - Yrd*r^2 + ...
    Xuu*u*abs(u) + Tport + Tstbd - (W-B)*sin(theta) + ...
    m*v*r - m*w*q + m*xG*(q^2 + r^2) - m*zG*(p*r + qdot) ) ;

% Motions in Vertical Plane
% Heave Equation
wdot = (1/(m-Zwd))*( Zqd*qdot - Xud*u*q + Yvd*v*p + Yrd*r*p + ...
    Zww*w*abs(w) + Taft + Tfore + (W-B)*cos(theta)*cos(phi) + ...
    m*u*q - m*v*p + m*xG*(p^2+q^2) - m*zG*(r*p-qdot) ) ;

% Pitch Equation
qdot = (1/(Iyy-Mqd))*( Zqd*(wdot-u*q) - (Zwd-Xud)*w*u - Yrd*v*p + ...
    (Kpd-Nrd)*r*p + Mqq*q*abs(q) + Mww*w*abs(w) + ...
    (Tport+Tstbd)*zp - Tfore*xpfore + Taft*xpaf - ...
    (zG*W-zb*B)*sin(theta) - xG*(W-B)*cos(theta)*cos(phi) - ...
    (Ixx-Izz)*r*p - m*( zG*(udot-v*r+w*q) - xG*(wdot-u*q+v*p)) ) ;

vertical_plane_only=0;
if (vertical_plane_only==1)
    vdot = 0 ;
pdot = 0 ;
rdot = 0 ;
else
125
% Motions in Horizontal Plane
% Sway Equation
vdot = \((1/(m-Yvd))*(Yrd*r*rdot + Xud*u*r - Zwd*w*p - Zqd*p*q + ...\)
Yvv*v*abs(v) + (W-B)*cos(theta)*sin(phi) + ...
\(m*p*w - m*u*r - m*zG*(q*p-rdot) - m*xG*(q*p+rdot)\)

% Roll Equation, Thruster reaction torque is ignored
pdot = \((1/(Ixx-Kpd))*(-(Yvd-Zwd)*v*w - (Yrd+Zqd)*w*r + ...
(Yrd+Zqd)*v*q + (Mqd-Nrd)*p*q - (zG*W-b*B)*cos(theta)*sin(phi) - ...
(Izz - Iyy)*q*r + m*zG*(vdot-w*p+u*r)\)

% Yaw Equation, Thruster reaction torque is ignored
rdot = \((1/(Izz-Nrd))*(Yrd*vdot - (Xud-Yvd)*u*v + Yrd*u*r + Zqd*w*p - ...
(Kpd-Mqd)*p*q + Nrr*r*abs(r) + Nvv*v*abs(v) - Tstd*yp + Tport*yp + ...
xG*(W-B)*cos(theta)*sin(phi) - (Iyy - Ixx)*p*q - m*xG*(vdot-w*p+u*r)\)

%% Calculate the rate of change of the Euler angles phi, theta, psi
phidot = \(p + (\sin(phi)*\tan(theta))*q + (\cos(phi)*\tan(theta))*r;\)
thetadot = \((\cos(phi)*q) - (\sin(phi))*r;\)
psidot = \((\sin(phi)*\sec(theta))*q + (\cos(phi)*\sec(theta))*r;\)

%% Calculate transformation matrix T
%% and use it to calculate xdot, ydot, and zdot

cphi = \cos(phi);\nctheta = \cos(theta);\ncpsi = \cos(psi);\nsphi = \sin(phi);\nstheta = \sin(theta);\nspsi = \sin(psi);\n
T = zeros(6,6);\nT(1,1) = cpsi*ctheta;\nT(1,2) = cpsi*stheta*sphi - spsi*cphi;\nT(1,3) = spsi*sphi + cpsi*stheta*cphi;\nT(2,1) = cpsi*ctheta;\nT(2,2) = spsi*sphi + cpsi*cphi;\nT(2,3) = -cpsi*sphi + cpsi*stheta*cphi;\nT(3,1) = -stheta;\nT(3,2) = ctheta*sphi;\nT(3,3) = ctheta*cphi;

xdot = T(1,1)*u + T(1,2)*v + T(1,3)*w;\nydot = T(2,1)*u + T(2,2)*v + T(2,3)*w;\nzdot = T(3,1)*u + T(3,2)*v + T(3,3)*w;
% save thruster values for plotting
Thrust(cur_step,1) = t;
Thrust(cur_step,2) = Tstbd;
Thrust(cur_step,3) = Taft;
Thrust(cur_step,4) = Tport;
Thrust(cur_step,5) = Tfore;

% save commanded values for plotting
Command(cur_step,1) = t;
Command(cur_step,2) = commanded_depth;
Command(cur_step,3) = commanded_speed;
Command(cur_step,4) = commanded_heading;
Command(cur_step,5) = commanded_pitch;

if (cur_step<max_steps)
    cur_step = cur_step + 1;
else
    disp(['max_steps value of ' num2str(max_steps) ' exceeded']);
end

qd = [udot; vdot; wdot; pdot; qdot; rdot; xdot; ydot; zdot; phidot;
     thetadot; psidot; Tstbd; Taft; Tport; Tfore];
Bibliography


